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Microscopic study of total reaction cross-sections for proton on deuteron, ^3He and ^4He at energy range 20 to 50 MeV

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Abstract

Working within the framework of Coulomb modified Galuber model and using harmonic oscillator shell model target density, the proton total reaction cross-sections on the deuteron, ^3He and ^4He have been analyzed in the beam energy range 20 to 50 MeV. A very satisfactory agreement with experimental data for *p*-deuteron and to some extent for p-^3He have also been achieved. For p-^4He, the calculated reaction cross-section becomes negative at lower energies which is absolutely unphysical.

Keywords: Coulomb modified Glauber model; Microscopic analysis; Harmonic oscillator shell model density
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Introduction

In the study of nucleon scattering from nuclei, the total reaction cross-section σ_R is one of the most basic nuclear observables which provides us some important information such as the size of nucleus, nuclear structure and density distribution of nucleons in nucleus (Arafah, 2010). Apart from its application in several fields namely chemistry, medicine, astrophysics etc., with the advent of radioactive ion-beam technique, the interest in the measurement of σ_R is revitalized to study the structure of nuclei even far from the valley of stability (Ozawa, 2004).

Since several years, extensive analyses of proton reaction cross-section have been performed in a wide range of energies and mass numbers. In most of these analyses generally, the effort is devoted to the determination of either general parameterization of σ_R (Carlson *et al.*,1994; Wellisch & Axen, 1996) or global nucleon optical potential (Auce *et al.*, 2005). In recent years, Glauber multiple scattering model (Glauber, 1959) has also been applied to study the energy and mass dependence of proton reaction cross-sections. It is found that the Glauber model, though based on high energy approximation, works well down to energy about 20 MeV/nucleon provided that the model is suitably modified to account for the deviation of projectile trajectory due to Coulomb field (Charagi & Gupta, 1990; Madani, 2002). Recently, Coulomb modified Glauber model (Charagi & Gupta, 1990) has been applied by Alvi (2007) to derive an analytical expression for proton reaction cross-section. Using an approximate form for the Helm model form factor for the distribution of nucleons in nuclei, the proposed expression (Alvi, 2007) reproduces nicely the proton σ_R data at energies spanning 20-860 MeV. But strictly speaking, almost in all the application of Glauber model to study the proton σ_R , the optical limit approximation which is valid for sufficiently large mass nuclei, has been used.

Despite the enormous work, perhaps microscopic study of proton reaction cross-section on light nuclei seems not to have been addressed yet. Therefore, in the present work, using a single particle Gaussian model target density, Coulomb modified Glauber model is being applied to obtain a simple expression for the proton total reaction cross-section on light nuclei.

In this paper, we use the expression mentioned above to analyze proton reaction cross sections on deuteron (*d*), ^3He and ^4He in the energy range about 20 to 50 MeV and examine the predictive ability of this microscopic method. We will see shortly, without any free parameter, our expression satisfactorily reproduces the *p*-*d* and -^3He but fails to account p-^4He total reaction cross-section data.

Mathematical formulation

In the Glauber multiple scattering theory (Glauber, 1959) for hadron-nucleus scattering, the expression for total reaction cross-section σ_R for the collision of a nucleon on a target described by the ground state wave function Ψ_O and of mass number A may be written as:

$$\sigma_R = 2\pi \int_0^\infty bdb \left[1 - \left|S_{el}(b)\right|^2\right], \qquad (1)$$

where

$$S_{el}(b) = \left\langle \Psi_O \left| \prod_{j=1}^{A} \left[1 - \Gamma(\boldsymbol{b} - \boldsymbol{s}_j)\right] \right| \Psi_O \right\rangle . \qquad (2)$$

In the above Eq. (2), $\boldsymbol{b}$ is the impact parameter, $\boldsymbol{s}_j$ is the projection of the *j*-th target nucleon coordinate on the impact parameter plane and $\Gamma(\boldsymbol{b})$ the nucleon-nucleon (*NN*) profile function which is the two dimensional Fourier transform of *NN* scattering amplitude *f(q)*:

$$\Gamma(b) = \frac{1}{2\pi i k} \int e^{-i\boldsymbol{q}.\boldsymbol{b}}\, f(q)\, d^2\boldsymbol{q}, \qquad (3)$$

where k is the momentum of the projectile nucleon. It is worth to mention that the following parameterization of *f(q)* has been shown in several works (Charagi & Gupta, 1990; Madani, 2002) that it is very successful in at least explaining the total reaction cross section experiments (which is sensitive to nuclear surface region) down to energy about 20 MeV/nucleon provided the Glauber model is modified to take into account the projectile trajectory deviation due to Coulomb field. We, therefore, also take $f(q)$ in the usual Gaussian form:

$$f(q)=\frac{i\,k\,\sigma\,(1-i\gamma)}{4\pi}\,e^{-\beta^2 q^2/2}\ , \qquad (4)$$

where σ is the *NN* total cross-section, γ is the ratio of the real to the imaginary parts of the forward scattering amplitude and β^2 is the slope parameter.

In the literature, there are several methods (Harrington & Verma, 1978; Ahmad *et al*., 1997) for calculating $S_{el}(b)$ in terms of the target ground state density $\rho(r)$ and the pair correlation function $C(\boldsymbol{r}_1,\boldsymbol{r}_2)$. For example in the effective profile function expansion approach as developed in (Ahmad *et al*., 1997), the quantity $S_{el}(b)$ is expanded as:

$$S_{el}(b)=S_0(b)+S_2(b)+----------, \qquad (5)$$

where

$$S_0(b)=[1-\Gamma_0(b)]^A \qquad (6)$$

$$S_2(b)=\frac{A(A-1)}{2}\left[1-\Gamma_0(b)\right]^{A-2} I(b) \qquad (7)$$

with

$$\Gamma_0(b)=\int\rho(\boldsymbol{r})\,\Gamma(\boldsymbol{b}-\boldsymbol{s})\,d\boldsymbol{r}=\frac{1}{ik}\int_0^\infty qdqJ_0(qb)\ f(q)F(q)$$

$$I(b)=\int C(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\Gamma(\boldsymbol{b}-\boldsymbol{s}_1)\,\Gamma(\boldsymbol{b}-\boldsymbol{s}_2)\,d\boldsymbol{r}_1\,d\boldsymbol{r}_2$$

After neglecting the three-body and the higher order correlation terms, the $S_{el}(b)$ is written as:

$$S_{el}(b)=S_0(b)+\frac{A(A-1)}{2}\left[1-\Gamma_0(b)\right]^{A-2} I(b)\,, \qquad (8)$$

The first term in Eq. (8) corresponds to the so-called optical limit approximation in which the excitations of target nucleus during successive collisions with the projectile nucleon are neglected. In other words the target nucleus always remains in the ground state. For sufficiently large A, the $S_0(b)$ can approximately be written as:

$$S_0(b)\approx e^{-A\int\rho(\boldsymbol{r})\Gamma(\boldsymbol{b}-\boldsymbol{r})d\boldsymbol{r}}\ . \qquad (9)$$

The above expression (9) is called the optical limit approximation which is obtained in the phase expansion approach (Glauber, 1959).

For heavy nuclei the proton-nucleus reaction cross-sections are fairly well reproduced by evaluating $S_{el}(b)$ using Eq. (6) and or Eq. (9). As mentioned earlier, in this work we are concerned for *p*-nucleus system for light nuclei, the double and other higher order scattering terms in the evaluation of $S_{el}(b)$ are important and can not be neglected. The evaluation of higher order terms is not only difficult for realistic description of nuclei but is also beset with poor knowledge of higher order correlations in nuclei. However assuming the target nuclei may adequately be described by independent particle model Gaussian density, a closed expression for $S_{el}(b)$ with full scattering terms can be obtained. In order to calculate it, we define model elastic *S*-matrix as:

$$S_{el}^{(m)}(b)=\int\left|\Psi_0^{(m)}(\boldsymbol{r}_1,......\boldsymbol{r}_2)\right|^2\prod_{i=1}^{A}\left[1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_i)\right]d\boldsymbol{r}_1.....d\boldsymbol{r}_A\,, \qquad (10)$$

where $\Psi_0^{(m)}(\boldsymbol{r}_1,......\boldsymbol{r}_2)$ is the ground state model wave function of the target nucleus. Now if the target nucleus is described by the harmonic oscillator shell model, the independent particle model density is given by:

$$\left|\Psi_0^{(m)}(\boldsymbol{r}_1,.....\boldsymbol{r}_2)\right|^2=\rho^{(m)}(\boldsymbol{r}_1)\rho^{(m)}(\boldsymbol{r}_2).........\rho^{(m)}(\boldsymbol{r}_A)\,, \qquad (11)$$

where the model one-body density has the form:

$$\rho^{(m)}(\boldsymbol{r})=\left(\frac{\alpha_m^2}{\pi}\right)^{3/2} e^{-\alpha_m^2 r^2}\,.$$

For simplicity, treating the target nucleons as identical, the model elastic *S*-matrix now takes the form:

$$S_{el}^{(m)}(b)=\left[1-\overline{\Gamma}(\boldsymbol{b})\right]^A, \qquad (12)$$

where

$$\overline{\Gamma}(\boldsymbol{b})=\int\rho^{(m)}(\boldsymbol{r})\,\Gamma(\boldsymbol{b}-\boldsymbol{s})\,d\boldsymbol{r}\,. \qquad (13)$$

In the Glauber multiple scattering theory, the model elastic scattering amplitude is written as:

$$F_{el}^{(m)}(q)=\frac{i\,k_{cm}}{2\pi}\int d^2\boldsymbol{b}\ e^{i\boldsymbol{q}.\boldsymbol{b}}\left[1-S_{el}^{(m)}(b)\ \right], \qquad (14)$$

where k_{cm} is the c.m. momentum of nucleon-nucleus system. Now using the relation:

$$F_{el}(q)=\Theta(q)\,F_{el}^{(m)}(q)\,,$$

where $F_{el}(q)$ is the *N*-nucleus elastic scattering amplitude and $\Theta(q)$ is the usual c.m. correction factor, the expression for $S_{el}(b)$ may be written as:

$$S_{el}(b)=1-\frac{1}{2\pi i\,k_{cm}}\int d^2\boldsymbol{q}\ e^{-i\boldsymbol{q}.\boldsymbol{b}}\,F_{el}(q)\,. \qquad (15)$$

An analytical expression for $F_{el}(q)$ was derived long ago by Czyz and Lesniak (1967) and Bassel and Wilkin (1968) using the independent particle model Gaussian density for the target nucleus. By using their expression, we obtain a closed expression for $S_{el}(b)$ as:

$S_{el}(b)$ =

$$1-A\alpha_m^2\sum_{r=1}^{A}(-1)^{r+1}\binom{A}{r}X^r\frac{1}{(A\alpha_m^2-r\,w)}e^{-\left(\frac{r\,w\,A\alpha_m^2}{A\alpha_m^2-r\,w}\right)b^2} \qquad (16)$$

with

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$$w = \frac{\alpha_m^2}{1+2\alpha_m^2\beta^2} \quad \text{and}$$

$$X = \frac{\sigma(1-i\gamma)}{2\pi} w,$$

where all the symbols have already been explained except α_m^2 which is the size parameter of the independent particle model Gaussian density. This α_m^2 is directly related to the size parameter of the intrinsic density α_0^2 through the relation $\alpha_m^2 = \left(\frac{A-1}{A}\right)\alpha_0^2$. The difference between the two size parameters is due to the c.m. constraint. Finally, σ_R can now be evaluated numerically by substituting the closed expression (16) for $S_{el}(b)$ in Eq. (1).

It has been shown earlier (Charagi & Gupta, 1990; Madani, 2002), Glauber model though based on high energy approximation, is successfully accounts the reaction cross-section data down to energy about 20 MeV/nucleon provided that the model is suitably modified to account for the deviation of projectile trajectory due to Coulomb field. To account this, we follow the method of Faldt and Pilkuhn (1973) which replaces the impact parameter *b* by *b'*, the distance of closest approach on the Coulomb trajectory. We, therefore, calculate the elastic *S*-matrix in Eq. (1) at *b'* instead of *b* where *b'* is related
to *b* as:

$$b' = \eta / k_{cm} + \sqrt{(\eta / k_{cm})^2 + b^2}\,, \qquad (17)$$

with $\eta = \frac{Ze^2}{\hbar v}$ with *v* as the projectile velocity in unit of c.

Calculation of σ_R for p-deuteron, -^{3}He and -^{4}He

To calculate σ_R in the present formalism with Eq. (1) using expression (16), we need the parameters of *NN* scattering amplitude namely σ, γ and β^2; and the size parameter α_m^2 of the independent particle model Gaussian densities of the deuteron, *^{3}He* and *^{4}He* nuclei. The parameters σ and γ at the desired energy are determined by suitably averaging the corresponding *pp* and *pn* forward scattering amplitudes. For this we use the following parameterizations (Charagi & Gupta, 1990) for σ_{pp} and σ_{pn} which reproduce the observed values in the energy range 10 to 1000 MeV quite well:

$$\sigma_{pp} = 13.73 - \frac{15.04}{v} + \frac{8.76}{v^2} + 68.67\,v^4 \qquad (18)$$

$$\sigma_{pn} = -70.67 - \frac{18.18}{v} + \frac{25.26}{v^2} + 113.85\,v\,, \qquad (19)$$

where σ_{pp} and σ_{pn} are in mb. To calculate γ_{pp} and γ_{pn}, we use the parameterizations of Ahmad *et al.* (2001):

$$\gamma_{pp} = -0.386 + 1.224\, e^{-\frac{1}{2}\left(\frac{k-0.427}{0.178}\right)^2} + 1.01\, e^{-\frac{1}{2}\left(\frac{k-0.592}{0.638}\right)^2} \qquad (20)$$

$$\gamma_{pn} = -0.666 + 1.437\, e^{-\frac{1}{2}\left(\frac{k-0.412}{0.196}\right)^2} + 0.617\, e^{-\frac{1}{2}\left(\frac{k-0.797}{0.291}\right)^2} \qquad (21)$$

where *k* is the incident nucleon lab momentum in GeV/c. As regards to the value of β^2, it is well known that it is an energy dependent quantity. Very different values ranging from β^2 = 0.0 to about 1.24 fm^2 have been used by different authors (Charagi & Gupta, 1990; El-Gogary *et al.*, 2009). Therefore, in the absence of any energy dependent parameterization of β^2, we consider it appropriate to choose the iso-spin averaged value β^2 = 0.423 fm^2, which has been found (Charagi & Gupta, 1990) to give quite satisfactory result for σ_R in the energy range considered in this work.

Fig. 1. Proton-deuteron total reaction cross-section as a function of energy. Full curve shows the result of our microscopic method. The dashed and dotted curves are the results of optical limit approximation and effective profile function expansion (Ahmad et al., 1997) approach respectively. Open circles with error bars represent the experimental data from Carlson (1996).

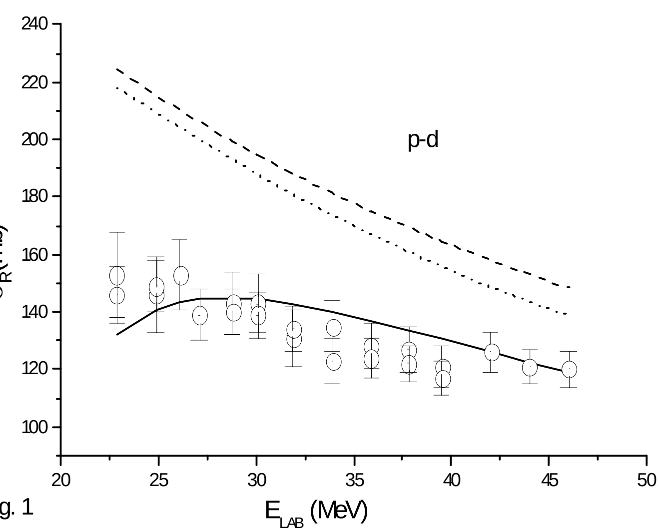

Fig. 1

The model size parameters α_m^2 for deuteron and *^{3}He* have been determined from their charge rms radius (de Vries *et al.*, 1987) after correcting for the finite proton charge distributions. For the deuteron it is 0.2fm^{-2} and for *^{3}He* it comes out to be 0.462fm^{-2}. And finally, for *^{4}He* we use α_m^2 = 0.535 fm^{-2} as given by Bassel and Wilkin (1968).

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Results and discussion

Proton total reaction cross-sections on the target nuclei deuteron, ^3He and ^4He have been calculated with the method described in the last section and the results are illustrated in the Figs. 1-3 with full curves. The open circles with statistical error bars represent the available experimental data (Carlson, 1996) in the energy range about 20 to 50 MeV. It is seen from the Figs. 1 and 2 that in general, the descriptions of *p- d* and -^3He data are quite satisfactory throughout the energy range. In particular, the *p- d* reaction cross-section data are nicely reproduced. It is because of deuteron which is just a bound nucleus; the assumption of independent single particle Gaussian model density is as usual a good approximation for the target density.

Fig. 2. Proton-^3He total reaction cross-section as a function of energy. Descriptions of the curves are same as in Fig. 1.

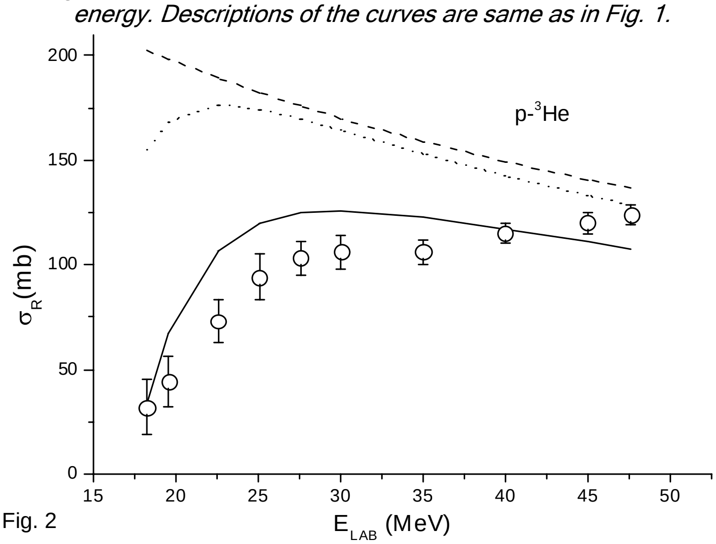

Fig. 2

For comparison, in all the Figures we also show the results of our calculations for σ_R : (1) the dotted curves, in the effective profile function expansion approach (Ahmad *et al.*, 1997) using only the first term of Eq. (8) for $S_{el}(b)$ and (2) the dashed curves, in the usual optical limit approximation using Eq. (9). It can be seen from the Figures that both of these approaches fail to reproduce the σ_R data. The failure of optical limit approximation is understandable, because it is valid for large mass nuclei whereas in the effective profile function expansion approach, it seems that the contribution of the neglected higher order terms of the multiple scattering series of Glauber model is quite significant.

Fig.3 for *p-*^4He shows a strange behavior (full curve) where the reaction cross-sections become negative at lower energies. This may be due to the large *p-p* and *p-n* total cross-sections at lower energies, that make the modulus of $S_{el}(b)$ greater than unity and thus violating the unitarity limit. But, beside the input model size parameter for ^4He, the method of our calculation for reaction cross-section is same as those on the deuteron and ^3He. The

Fig. 3. Proton-^4He total reaction cross-section as a function of energy. Descriptions of the curves are same as in Fig. 1.

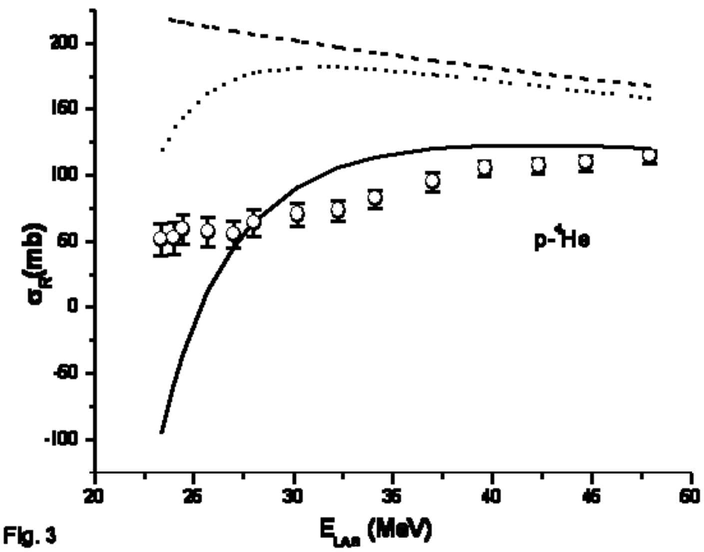

Fig. 3

helium nucleus, which is the most tightly bound light nucleus, we suspect this unphysical result at lower energies is due to its large model size parameter (the intrinsic size parameter $\alpha_0^2 = 1.5/\langle r_m^2 \rangle$ where $\langle r_m^2 \rangle$ is the mean square matter radius of ^4He). However, an over all agreement between theory and experiment could certainly be improved if one takes into account the two and higher order correlations in ^4He nucleus. Whatever may be the reason, in the absence of any other theoretical study of *p-*^4He reaction cross-sections we are unable to reach to a definite conclusion for unacceptable results at lower energies. Here, it is to be noticed that a slight increase in the value of range (slope) parameter β^2, which determines the fall-off of the angular distribution of the *NN* elastic scattering, a better representation of the data over the whole energy region is achieved. Fig. 4 shows the result of our calculation with β^2 = 0.48 fm^2.

Conclusion

Prior to this work, probably no microscopic study for proton reaction cross-sections on deuteron, ^3He and ^4He has ever been made. As mentioned earlier, the purpose of this work is to examine the predictive ability of our microscopic method of approach for proton reaction cross-sections on the above light nuclei. Therefore, working within the framework of Coulomb modified Glauber model and using independent single particle Gaussian model target density, we develop a simple method to calculate proton reaction cross-sections on light nuclei. It is found that our method is very effective in reproducing the *p-d* and also to some extent *p-*^3He reaction cross-section data in the available proton beam energy range 20 to 50 MeV. For *p-*^4He, the calculated σ_R becomes negative at lower energies which is unphysical.

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Fig. 4. Same as in Fig.3 but with β^2 *= 0.48 fm*^{2}*.*

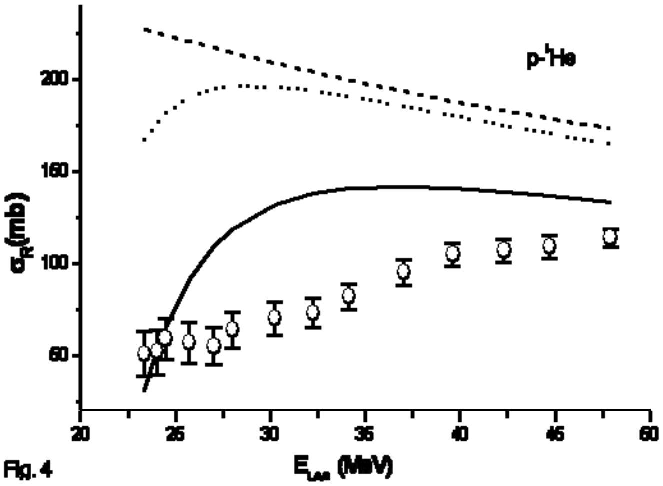

Fig.4

In view of the above results we suggest to explore some other, more sophisticated method to analyze at least *p-*^{4}*He* reaction cross-section without introducing any ad-hoc assumption.

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