Multi Target Tracking of Ground Targets in Clutter with LMIPDA-IMM*

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Abstract – *Tracking of ground targets presents a number of challenges. Target trajectories meet various motion constraints. Substantial non-homogenous clutter is usually present. In multi-target situations measurement assignment may be computationally challenging as the number of operations increases exponentially with number of tracks and number of measurements. LMIPDA-IMM aims to provide a solution to these issues. Use of the IMM approach allows tracking ground targets with motion constraints and/or maneuvers. LMIPDA calculates the probability of target existence for false track discrimination to enable automatic track initiation and termination. The robust data association properties of LMIPDA are further enhanced by the use of a clutter map. LMIPDA provides multi-target data association with number of operations linear in the number of tracks and the number of measurements. Simulation studies illustrate the effectiveness of this approach in an environment of heavy non-homogenous clutter.*

Keywords: Tracking, Probabilistic data association (PDA), Integrated PDA (IPDA), Linear-Multitarget, LMIPDA, Interacting Multiple Model (IMM).

1 Introduction

Tracking multiple targets in clutter is a well researched problem. A range of recursive Bayesian techniques like Joint Probabilistic Data Association (JPDA) [1, 2, 3, 4], Joint Integrated Probabilistic Data Association (JIPDA) [5, 6], and Linear Joint Probabilistic Data Association (LJIPDA) [7] techniques are proposed in literature to address this problem. The later techniques address automatic tracking initiation and maintenance, along with issues of multi target data association in clutter. LJIPDA is a multi-target tracking algorithm, first presented in [7] as a new approach to multi target tracking in clutter using the PDA approximation. JIPDA considers all feasible measurement-to-track allocations to achieve optimal data association performance. The number of operations for JIPDA grows exponentially with the number of tracks and measurements. For LJIPDA the number of operations is linear in the number of tracks and measurements, with apparently negligible performance penalty compared to JIPDA [7]. In LJIPDA, when a measurement can be allocated to multiple tracks,

it is "split", and each track uses a "fragment" of the measurement. Linear Multitarget (LM) tracking is a procedure for converting single-target tracking in clutter into multi-target tracking in clutter by modifying clutter measurement density according to predicted measurement density of the other tracks. In this paper, the LMIPDA extension to tracking multiple maneuvering targets in clutter is considered.

Recently, automatic track initiation and maintenance for a single maneuvering target in clutter was considered and an IPDA-IMM filter was proposed in [8]. Here we extend this approach to tackle automatic track initiation and maintenance of multiple maneuvering targets in clutter. By consistently combining LMIPDA with an IMM filter, a linearly scalable LMIPDA-IMM algorithm (the number of operations linear in the number of tracks and measurements) is derived. The non-parametric version, or LMIPDA-IMM, assumes no a priori clutter measurement density information. The parametric version, indicated by suffix MAP, as LMIPDA-IMM-MAP [9], uses a priori spacial clutter density knowledge. Both are presented in this paper.

This paper is organized as follows. Following the introduction, the problem definition and modelling issues are considered in section 2. The LMIPDA and the IMM approximations in the context of data association along with the LMIPDA-IMM specific equations are considered in section 3. Simulations are considered in section 4 and conclusions are drawn in section 5.

2 Problem Description and Notation

We assume that the trajectory of the target can be described at any time by one of M dynamic models. This allows tracking of maneuvering targets. The dynamic models and sensor measurement processes are described by the following equations

$$\begin{aligned} x_k &= F_k(M_k)x_{k-1} + \nu_k(M_k) \\ y_k &= H_k(M_k)x_k + \omega_k(M_k) \end{aligned} \tag{1}$$

where, at time k, $M_k = [1, 2, \cdots, M]$ is the dynamic/measurement model, x_k is the track state, y_k is the measurement, $F_k(M_k)$ is the state propagation matrix, and $H_k(M_k)$ is the measurement matrix. Process noise $\nu_k(M_k)$ and measurement noise $\omega_k(M_k)$ are zero mean white and

uncorrelated Gaussian noise sequences with covariance matrices $Q_k(M_k)$ and $R_k(M_k)$ respectively. In the target tracking applications, H_k and R_k are often model independent.

We model changes in target trajectory as a Markov Chain with given transitional probabilities, denoted by

$$\pi_{\eta,\theta} = P\{M_k = \theta | M_{k-1} = \eta\}; \eta, \theta \in [1, \cdots, M]. \quad (2)$$

In the cluttered environment, the sensor will return measurements created by zero or more targets as well as zero or more clutter measurements at each scan. The target and the clutter measurements are referred to as the true and false measurements respectively. True measurements are unknown and each is present in the measurement set with probability of detection P_D, which may vary from target to target.

Denote by z_k the set of validated [4] measurements at time k, and by $z_{k,i}; (i = 1, \cdots, m_k; m_k \geq 0)$ the i-th measurement of z_k, where m_k is the number of validated measurements at time k. Denote by $Z^k = z_k \bigcup Z^{k-1}$ the set of all validated measurements, i.e those candidate true measurements, up to and including time k.

Given the dynamics Eq. (1) and measurements Z^k, we aim to estimate recursively the *a posteriori* probability of target existence $\psi_{k|k}$, and the state estimate and its error covariance, $\hat{x}_{k|k}$ and $P_{k|k}$ respectively.

3 LMIPDA-IMM Algorithm Description

The LMIPDA-IMM is a recursive algorithm which combines multi–target data association algorithm (LMIPDA) with maneuvering target state estimation implemented using IMM algorithm. The IMM consists of a filter (usually Kalman or extended Kalman) bank, one for each possible target trajectory model.

LMIPDA–IMM algorithm description is presented below, denoting current time by k, defined as sensor measurement sampling time. For reasons of clarity and simplicity, the following expressions will assume that each track has the same gating P_G and detection P_D probabilities. It is a trivial exercise to add track indices to these quantities.

3.1 Filter Input

For each track t, filter input consists of:

- Predicted probability of target existence $\psi_{k|k-1}^t$, delivered by the LMIPDA part of the algorithm in the previous recursion.

- Predicted state for each IMM model θ, delivered by the IMM part of the algorithm in the previous recursion:

  – state prediction probability density function (pdf) $p^t(x_k|M_k = \theta, Z^{k-1})$, described by its mean $\hat{x}_{k|k-1}^t(\theta)$ and its error covariance $P_{k|k-1}^t(\theta)$,

  – predicted model state probability $\mu_{k|k-1}^t(\theta) \triangleq P^t\{M_k = \theta | Z^{k-1}\}$ and

  – the a priori measurement pdf function $p^t(z_k|M_k = \theta, Z^{k-1})$.

- Measurement set delivered by the sensor at time k. The measurement set may be empty.

3.2 Filter Output

For each track t, filter output at time k consists of:

- A posteriori probability of target existence $\psi_{k|k}^t$, delivered by the LMIPDA part of the algorithm. It is then used for confirmation or termination of tracks.

- Track state estimate and estimate covariance, $\hat{x}_{k|k}^t$ and $P_{k|k}^t$, delivered by the IMM part of the algorithm.

- Filter inputs for time $k+1$, enumerated in Section 3.1, $\psi_{k+1|k}^t$, and, for each IMM model θ, $\hat{x}_{k+1|k}^t(\theta)$, $P_{k+1|k}^t(\theta)$, $\mu_{k+1|k}^t(\theta)$ and $p^t(z_{k+1}|M_{k+1} = \theta, Z^k)$.

3.3 Validation of Measurements

The validation of measurements is referred to as "gating". A subset of sensor measurements is selected in order that, if the target exists and is detected, the target measurement will be selected with gating probability, P_G. Gating is done for each track separately; thus in the equations of this Section, the track index t will be omitted.

Gating is initially done for each IMM model separately, then the results are combined into a single validation gate [6, 10, 11]. The predicted measurement and innovation covariance matrix for each model is

$$\hat{z}_k(\theta) = H_k(\theta)\hat{x}_{k|k-1}(\theta);$$
$$S_k(\theta) = H_k(\theta)P_{k|k-1}(\theta)H_k(\theta)^T + R_k(\theta). \quad (3)$$

A validation gate is constructed around the predicted measurement $\hat{z}_k(\theta)$, so that the probability of the true measurement (if the target exists and is detected) falling in the gate is P_G. We select the validated measurements (i.e those inside the validation gate) $z_k(\theta)$ and calculate the volume $V_k(\theta)$ of the validation gate for each $\theta = 1, \cdots, M$. The track validation gate is the union of validation gates for separate models. The validated set of measurements z_k is the union of sets of validated measurements for all M models, i.e $z_k = \bigcup_{\eta=1}^{M} z_k(\eta)$. One possible approach to calculating the volume of the combined validation gate is presented in [11]. In this paper we follow the approach first presented in [5]. For non-empty set z_k we use the following approximation:

$$V_k = \max \left\{ \frac{m_k \sum_{\eta=1}^{M} V_k(\eta)}{\sum_{\eta=1}^{M} m_k(\eta)}, V_k(1), \cdots, V_k(M) \right\} \quad (4)$$

where $m_k(\eta)$ is the number of measurements in $z_k(\eta)$, and m_k is the number of measurements in z_k. When z_k is empty ($m_k = 0$) the volume of the validation gate is not used and

therefore its computation is not considered. The a priori measurement pdf of each selected measurement i is

$$p_i(\theta) = \begin{cases} \frac{1}{P_G} p(z_{k,i}|M_k = \theta, Z^{k-1}) & z_{k,i} \in z_k(\theta) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

for each model θ where

$$p(z_k|M_k = \theta, Z^{k-1}) = \mathcal{N}(z_k; \hat{z}_k(\theta), S_k(\theta))$$

are Gaussian with mean $\hat{z}_k(\theta)$ and covariance $S_k(\theta)$ calculated in Eq. (3). For any track a priori measurement pdf is

$$p_i \triangleq p(z_{k,i}|Z^{k-1}) = \sum_{\eta=1}^{M} \mu_{k|k-1}(\eta) p_i(\eta), \quad (6)$$

3.4 Linear Multitarget IPDA (LMIPDA) Filter

LMIPDA is a multi-target tracking filter, submitted to [12] as a new approach to multi-target tracking in clutter. Optimal approach to multi-target tracking in clutter considers all feasible measurement-to-track allocations to achieve optimal data association performance; under IPDA assumptions it is JIPDA [6]. The number of JIPDA operations grows exponentially with the number of tracks and measurements. LMIPDA has the number of operations which is linear in the number of tracks and the number of measurements, with apparently negligible performance penalty compared to JIPDA [12].

LMIPDA is an IPDA filter to which Linear Multitarget (LM) procedure has been applied. LM reduces the computational complexity of multi target tracking in clutter by eliminating the measurement to target assignment step entirely. Instead, when a single target tracking filter, in this case IPDA, is applied to a track, the clutter density at each measurement point is modified by the pdf of measurements originating from the neighboring tracks. The LM procedure permits use of other measurement features, such as amplitude, [12]; this is omitted here for reasons of clarity and is trivial to add in the fashion of [12]. In other words, other tracks are treated as additional clutter sources and LM achieves multi–target tracking capabilities using single–target tracking computational resources. Denote by ρ_i^t the clutter density in the validation gate of track t at coordinate $z_{k,i}$, then the a priori probability that i-th measurement is the true measurement for track t, given single track t is

$$P_i^t = P_D P_G \psi_{k|k-1}^t \frac{p_i^t/\rho_i^t}{\sum\limits_{i=1}^{m_k^t} p_i^t/\rho_i^t}, \quad (7)$$

where p_i^t is defined in (eq. 6). The modified clutter density for track t at the point of measurement $z_{k,i}$ is

$$\Omega_i^t = \rho_i^t + \sum_{\substack{s=1 \\ s \neq t}}^{T} p_i^s \frac{P_i^s}{1 - P_i^s}, \quad (8)$$

where T is the number of tracks. Ω_i^t is used instead of clutter density ρ_i^t when calculating data association probabilities for track t [14].

If the estimate of clutter density ρ_i^t is known a priori, e.g when using a clutter map [9], we consider the parametric versions of tracking algorithm, which we denote by the suffix "MAP" to obtain for example LMIPDA-MAP and LMIPDA-IMM-MAP.

In the absence of the a priori clutter density knowledge, we consider the non-parametric version of the tracking algorithm by assuming homogenous clutter density within the validation gate of track t and calculate

$$\rho_i^t = \frac{\hat{m}_k^t}{V_k^t}$$
$$\hat{m}_k^t = \sum_{i=1}^{m_k^t} (\prod_{s=1}^{T} (1 - P_i^s)) \quad (9)$$

where $\hat{m}_k^t$ denotes the mean number of selected clutter measurements. If the tracks are far apart, i.e their validation gates do not intersect, $\Omega_i^t = \rho_i^t$ for all i and t, and LMIPDA reverts to IPDA.

LMIPDA input from previous scan consists of the predicted probability of target existence, $\psi_{k|k-1}^t$.

LMIPDA input from IMM of previous scan, for each IMM model, consists of the predicted model probabilities, $\mu_{k|k-1}^t(\theta)$, and predicted state pdf, parameterized by $\hat{x}_{k|k-1}^t(\theta)$ and $P_{k|k-1}^t(\theta)$, from which measurement prediction pdf $p^t(z_k|M_k = \theta, Z^{k-1})$, parameterized by $\hat{z}_k^t(\theta)$ and $S_k^t(\theta)$ are calculated.

LMIPDA Step 1 is the measurement selection (gating).

LMIPDA Step 2 is the probability of target existence and model probabilities update:

$$\delta_k^t(\theta) = P_D P_G \big(1 - \sum_{i=1}^{m_k^t} \frac{p_i^t(\theta)}{\Omega_i^t}\big);$$
$$\delta_k^t = P_D P_G \big(1 - \sum_{i=1}^{m_k^t} \frac{p_i^t}{\Omega_i^t}\big);$$
$$\psi_{k|k}^t = \frac{(1 - \delta_k^t)\psi_{k|k-1}^t}{1 - \delta_k^t \psi_{k|k-1}^t}; \quad (10)$$
$$\beta_{k,0}^t(\theta) = \frac{1 - P_D P_G}{1 - \delta_k^t(\theta)};$$
$$\beta_{k,i}^t(\theta) = \frac{P_D P_G}{1 - \delta_k^t(\theta)} \cdot \frac{p_i^t(\theta)}{\Omega_i^t}; \quad i > 0;$$
$$\mu_{k|k}^t(\theta) = \mu_{k|k-1}^t(\theta) \frac{1 - \delta_k^t(\theta)}{1 - \delta_k^t}.$$

LMIPDA output for IMM consists of data association probabilities for each model, $\beta_{k,i}^t(\theta), i = 0, \cdots, m_k$, and model probabilities $\mu_{k|k}^t(\theta)$.

LMIPDA output for next recursion consists of the predicted probability of target existence at time $k + 1$, denoted by $\psi_{k+1|k}^t$. We model target existence as a Markov process with known transition probabilities, and distinguish between *Markov Chain One* and *Markov Chain Two* [13, 14, 15, 16, 17]. Markov Chain One models the situation in which whenever the target exists it is detected with the probability of detection P_D. Markov Chain Two models

the situation in which if the target exists it may be temporarily invisible (undetected). In this paper we use the Markov Chain One model for propagation of the target existence, i.e the predicted probability of target existence $\psi_{k+1|k}^t$ is calculated as

$$\psi_{k+1|k}^t = p_{11}^t \psi_{k|k}^t + p_{21}^t(1 - \psi_{k|k}^t), \qquad (11)$$

where p_{11}^t and p_{21}^t are given transition probabilities. The extension to Markov Chain Two is straightforward and is not presented in this paper. The superscript t was convenient to differentiate between different tracks, but we drop it hereafter.

3.5 The IMM Filter

The IMM filter [18] consists of a bank of (usually Kalman/extended Kalman) filters, one for each model. It updates a posteriori state estimates and their covariances for each model, as well as the relative model probabilities. Mixing of estimates of models approximates random switching between models. IMM calculates the track state estimate and its covariance $\hat{x}_{k|k}$ and $P_{k|k}$ as the combined state estimate and the covariance for each model. The IMM part of LMIPDA-IMM has the following steps.

IMM Step1: initialization starts with the state prediction $\hat{x}_{k|k-1}(\theta)$ and error covariance for each model, $P_{k|k-1}(\theta)$. From LMIPDA we obtain the set of validated measurements z_k, the set of model data association probabilities $\beta_{k,i}(\theta)$ and the a posteriori model probabilities $\mu_{k|k}(\theta)$.

IMM Step 2: data association update of state estimate $\hat{x}_{k|k}(\theta)$ and error covariance $P_{k|k}(\theta)$ for each IMM model using data association estimates. The state estimate is approximated by a single Gaussian probability density function [14, 19] with mean $\hat{x}_{k|k}(\theta)$ and covariance $P_{k|k}(\theta)$:

$$\hat{x}_{k|k}(\theta) = \sum_{i=0}^{m_k} \beta_{k,i}(\theta)\hat{x}_{k,i|k}(\theta)$$
$$P_{k|k}(\theta) = \sum_{i=0}^{m_k} \beta_{k,i}(\theta)\big(P_{k,i|k}(\theta)+ \qquad (12)$$
$$+ \hat{x}_{k,i|k}(\theta)\hat{x}_{k,i|k}(\theta)^T\big) - \hat{x}_{k|k}(\theta)\hat{x}_{k|k}(\theta)^T$$

where $\hat{x}_{k,i|k}(\theta)$ and $P_{k,i|k}(\theta)$ for $i = 1, \cdots, m_k$ are state means and covariances calculated assuming that the i-th measurement is true. For $i = 0$, i.e when there are no measurements, we have

$$\hat{x}_{k,0|k}(\theta) = \hat{x}_{k|k-1}(\theta)$$
$$P_{k,0|k}(\theta) = P_{k|k-1}^\star(\theta), \qquad (13)$$

where $\hat{x}_{k|k-1}(\theta)$ is the state prediction and $P_{k|k-1}^\star(\theta)$ is the corrected state prediction error covariance matrix. For a gating probability $P_G > 0.99$ this is approximately equal to the state prediction error covariance matrix $P_{k|k-1}(\theta)$[10].

IMM Step 3: output combination estimates the state of the tracker by combining the state estimates for each model. The result is used as an output of the tracking filter at time k.

$$\hat{x}_{k|k} = \sum_{\eta=1}^M \mu_{k|k}(\eta)\hat{x}_{k|k}(\eta)$$
$$P_{k|k} = \sum_{\eta=1}^M \mu_{k|k}(\eta)\big(P_{k|k}(\eta)+ \qquad (14)$$
$$+ \hat{x}_{k|k}(\eta)\hat{x}_{k|k}(\eta)^T\big) - \hat{x}_{k|k}\hat{x}_{k|k}^T$$

IMM Step 4: mixing calculates state estimates at time k, given the model at time $k+1$, $\hat{x}_{k|k}(\theta^m)$ and $P_{k|k}(\theta^m)$.

$$\mu_{k+1|k}(\theta) = \sum_{\eta=1}^M \pi_{\eta,\theta}\mu_{k|k}(\eta)$$
$$\mu_{k|k+1}(\eta,\theta) = \frac{\pi_{\eta,\theta}\mu_{k|k}(\eta)}{\mu_{k+1|k}(\theta)}$$
$$\hat{x}_{k|k}(\theta^m) = \sum_{\eta=1}^M \mu_{k|k+1}(\eta,\theta)\hat{x}_{k|k}(\eta) \qquad (15)$$
$$P_{k|k}(\theta^m) = \sum_{\eta=1}^M \mu_{k|k+1}(\eta,\theta)\big(P_{k|k}(\eta)+$$
$$+ \hat{x}_{k|k}(\eta)\hat{x}_{k|k}(\eta)^T\big) - \hat{x}_{k|k}(\theta^m)\hat{x}_{k|k}(\theta^m)^T$$

where $\mu_{k+1|k}(\theta)$ is the predicted model probability, and

$$\mu_{k|k+1}(\eta,\theta) \triangleq P\{M_k = \eta|M_{k+1} = \theta, Z^k\}$$

is the probability that the model at time k is η, given that the model at time $k + 1$ is θ.

IMM Step 5: forward prediction calculates the predicted state and error covariance for each model

$$\hat{x}_{k+1|k}(\theta) = F_{k+1}(\theta)\hat{x}_{k|k}(\theta^m)$$
$$P_{k+1|k}(\theta) = F_{k+1}(\theta)P_{k|k}(\theta^m)F_{k+1}(\theta)^T+ \qquad (16)$$
$$+ Q_{k+1}(\theta)$$

IMM output for LMIPDA consists of the state prediction mean and covariance and the prediction of the model probability for each model:

$$\hat{x}_{k+1|k}(\theta), \quad P_{k+1|k}(\theta), \quad \mu_{k+1|k}(\theta) \qquad (17)$$

4 Simulations

Simulations are performed to compare three algorithms: IPDA-IMM presented in [8], LJIPDA-IMM and LMIPDA-IMM presented here, in the environment of non-homogenous clutter and multiple maneuvering targets with crossing trajectories. Two targets are simulated as shown in Fig. 1. Both trajectories consist of 8 segments, 10 seconds each:

1. uniform motion with constant velocity of 18 m/s for target one and 17.12 m/s for target two,

2. exponential acceleration motion, with acceleration $a = v_0\alpha \exp(\alpha t)$, where v_0 is velocity at the start of the segment, t denotes time since segment start, and $\alpha = 0.05$ s^{-1} for the first target, $\alpha = 0.04$ s^{-1} for the second,

3. exponential deceleration, with acceleration $a = v_0\alpha\exp(\alpha t)$, where v_0 is velocity at the start of the segment, t denotes time since segment start and $\alpha = -0.05\,\text{s}^{-1}$ for the first target, $\alpha = -0.04\,\text{s}^{-1}$ for the second,

4. right turn with the angular velocity of $\pi/9\,\text{rad/s}$ for the first target, $\pi/8.8\,\text{rad/s}$ for the second,

5. exponential acceleration with $\alpha = 0.05\,\text{s}^{-1}$ for the first target, $\alpha = 0.06\,\text{s}^{-1}$ for the second,

6. exponential deceleration with $\alpha = -0.05\,\text{s}^{-1}$ for the first target, $\alpha = -0.06\,\text{s}^{-1}$ for the second,

7. left turn with angular velocity $\pi/9\,\text{rad/s}$ for the first target, $\pi/10\,\text{rad/s}$ for the second, and

8. uniform motion.

Targets start moving at the same time and approach each other at ≈ 21 degrees as shown in Fig. 1. For this scenario the target trajectories intersect at the same time twice.
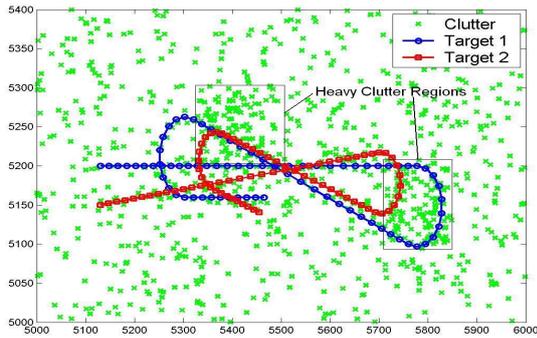

Fig. 1: Test Tracks and Clutter Distribution

Clutter measurements are generated in clusters as shown in Fig. 1. We simulated two rectangular regions of heavy clutter. The rest of the clutter is light and is distributed uniformly in the surveillance area. Every second the number of clutter measurements is selected from a Poisson distribution to maintain average clutter density $2\text{e}^{-5}\,\text{m}^{-2}$ in the light clutter regions and $1\text{e}^{-4}\,\text{m}^{-2}$ in the heavy clutter regions. The algorithms had no a priori knowledge of clutter densities; i.e the non- parametric versions were applied.

IMM filter consists of four models of target motion:

1. Uniform motion: target moves on a straight line with constant velocity.

2. Acceleration: target moves with constant acceleration

3. Target is executing a left coordinated turn with constant angular velocity $\omega = \pi/9\,\text{rad/s}$.

4. Target is executing a right coordinated turn with constant angular velocity $\omega = \pi/9\,\text{rad/s}$.

Transition probability matrix of IMM models is:

$$\Pi = \begin{pmatrix} 0.91 & 0.03 & 0.03 & 0.03 \\ 0.03 & 0.91 & 0.03 & 0.03 \\ 0.05 & 0.05 & 0.9 & 0 \\ 0.05 & 0.05 & 0 & 0.9 \end{pmatrix} \tag{18}$$

The state vector is modelled as

$$x = \begin{pmatrix} \xi & \dot{\xi} & \zeta & \dot{\zeta} & \ddot{\xi} & \ddot{\zeta} \end{pmatrix}^T \tag{19}$$

where (ξ, ζ) denote Cartesian coordinates. All models have white plant noise with covariance matrix:

$$Q = \frac{3}{4} \begin{pmatrix} \frac{1}{4}T^4 & \frac{1}{2}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 \\ \frac{1}{2}T^3 & T^2 & 0 & 0 & T & 0 \\ 0 & 0 & \frac{1}{4}T^4 & \frac{1}{2}T^3 & 0 & \frac{1}{2}T^2 \\ 0 & 0 & \frac{1}{2}T^3 & T^2 & 0 & T \\ \frac{1}{2}T^2 & T & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2}T^2 & T & 0 & 1 \end{pmatrix} \tag{20}$$

where sampling interval $T = 1$ s.

Track existence is modelled by Markov Chain One with transition probabilities

$$p_{11} = 0.98 \text{ and } p_{21} = 0. \tag{21}$$

A two dimensional radar surveillance system is modelled. The simulated measurement noise had standard deviation of 5 m in range and 1 mrad in bearing at the range 5 km. Probability of target detection is chosen to be $P_D = 0.9$.

Tracks are initiated using measurements at each time step, using the two-point differencing track initialization algorithm [13, 9]. The initial probability of target existence for each track is calculated using algorithm in [9]. Both true and false tracks were initiated and the probability of target existence with fixed track confirmation and track termination thresholds was used to confirm and terminate tracks. The thresholds and initial probability of target existence for each algorithm were calculated separately to optimize the performance.

Each simulation experiment consisted of 500 runs. The averaged position and velocity RMSE (root mean square error) are represented in Fig. 2 for target one and Fig. 3 for target two. The number of confirmed true tracks is represented in Fig. 4. The number of confirmed false track scans for each algorithm is < 24 for all runs.

One of the practically significant improvements achieved by LMIPDA-IMM is its ability to retain larger percentage of tracks than LJIPDA-IMM and IPDA-IMM at instances involving tracking targets with near or crossing trajectories in an environment of heavy and non-uniform clutter and significant maneuvers while maintaining the low confirmed false track statistics (Fig. 4). When the tracks are well separated, the RMSE differences are negligible between algorithms, as expected. LMIPDA-IMM shows a small RMSE increase in target crossing situations; IPDA-IMM on the other hand shows a large increase in RMSE during the target crossing situations. When targets (and tracks) are well separated, LMIPDA-IMM matches the performance of IPDA-IMM. During target crossing situations multi-target tracking capabilities of LMIPDA-IMM improve the tracking performance significantly.
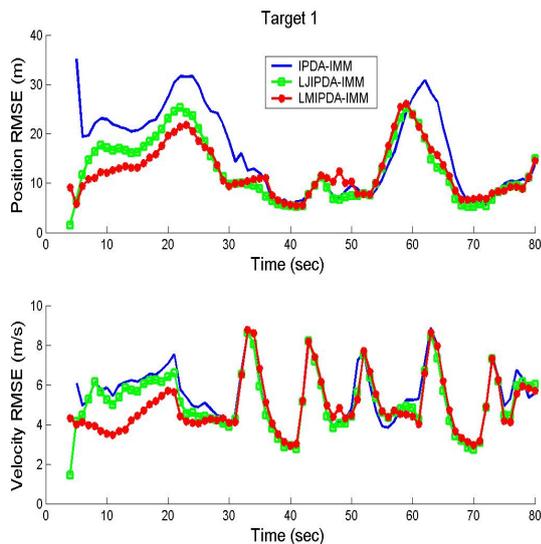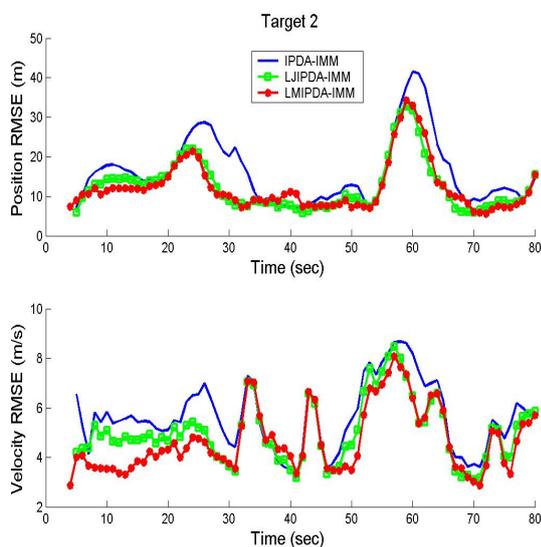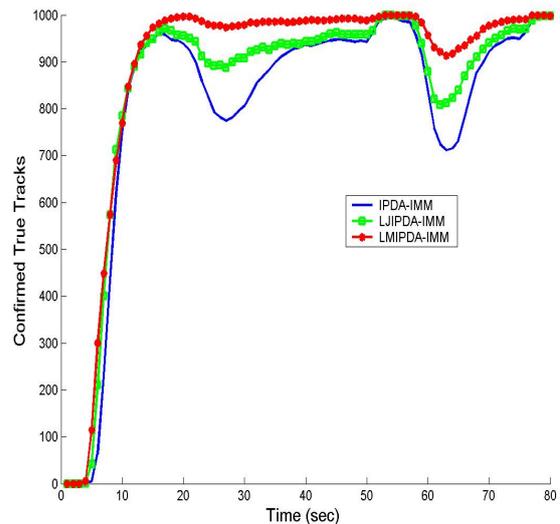
Fig. 2: Position and Velocity RMSE

Fig. 4: Number of Confirmed True Tracks

LMIPDA-IMM we have demonstrated that it is possible to improve the true track confirmation statistics while maintaining the false track confirmation statistics. In addition, RMSE performance of LMIPDA-IMM - especially in situations involving multiple crossing maneuvering targets in dense non-uniform clutter - is significantly better than both LJIPDA-IMM and IPDA-IMM.

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Fig. 3: Position and Velocity RMSE

5   Conclusion

A new recursive tracking filter, the LMIPDA-IMM, for automatic track initiation and maintenance of multiple maneuvering targets in clutter is proposed in this paper. It is an extension of the recently proposed IPDA-IMM filter to multiple target tracking scenarios. Conventional extension of IPDA-IMM would result in JIPDA-IMM filter that would optimally take all joint track-to-plot into account but computationally, it is infeasible in most practical problems of interest - as it scales exponentially with the number of tracks initiated by the filter. However, LMIPDA-IMM, by intelligently using its track existence probabilities, reduces the number of joint associations to be considered to the extent that it scales linearly in the number of tracks initiated by the filter without compromising the filter performance. In the context of automatic track initiation of multiple maneuvering targets in clutter, by using

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