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INTUITIONISTIC FUZZY α-IDEALS IN BCI-ALGEBRAS

BY

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Abstract. In this paper, we introduce the concept of intuitionistic fuzzy α-ideals in BCI-algebras and investigate some of its properties.

1. Introduction and Preliminaries

After the introduction of the concept of fuzzy sets by Zadeh [12], several researchers were conducted on the generalization of the notion of fuzzy sets. The idea of "intuitionistic fuzzy set" was first published by Atanassov [2, 3], as a generalization of the notion of fuzzy sets. After that, many researchers considered the fuzzification of ideals and subalgebras in $BCK(BCI)$-algebras. In this paper, we introduce the concept of intuitionistic fuzzy α-ideals in BCI-algebras and investigate some of its properties.

Let us recall that an algebra $(X, *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(I) $((x * y) * (x * z)) * (z * y) = 0$,
(II) $(x * (x * y)) * y = 0$,
(III) $x * x = 0$,
(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$,

for all x, y, $z \in X$.

In a BCI-algebra, we can define a partial ordering "$\leq$" by $x \leq y$ if and only if $x * y = 0$.

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A mapping $f : X \to Y$ of BCI-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all x, $y \in X$. A nonempty subset I of a BCI-algebra X is called an ideal of X if (i) $0 \in I$, (ii) $x * y \in I$ and $y \in I$ imply that $x \in I$. By a fuzzy set μ in a nonempty set X we mean a function $\mu : X \to [0, 1]$, and the complement of μ, denote by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x) = 1 - \mu(x)$ for all $x \in X$. We shall write $a \wedge b$ for $\min\{a, b\}$ and $a \vee b$ for $\max\{a, b\}$, where a and b are any real numbers.

Definition 1.1.([11]) A fuzzy set μ of X is called a fuzzy ideal of X if it satisfies: (i) $\mu(0) \geq \mu(x)$, (ii) $\mu(x) \geq \mu(x * y) \wedge \mu(y)$ for all x, $y \in X$.

Definition 1.2.([10]) A nonempty subset I of X is called an α-ideal of X if it satisfies: (i) $0 \in I$, (ii) $(x * z) * (0 * y) \in I$ and $z \in I$ imply $y * x \in I$.

Definition 1.3.([9]) A fuzzy set μ of X is called a fuzzy α-ideal (briefly, Fα-ideal) of X if it satisfies:

(i) $\mu(0) \geq \mu(x)$,

(ii) $\mu(y * x) \geq \mu((x * z) * (0 * y)) \wedge \mu(z)$,

for all x, y, $z \in X$.

An intuitionistic fuzzy set A in a nonempty set X is an object having form IFSA$=\{(x, \alpha_A(x), \beta_A(x)) \mid x \in X\}$, where the functions $\alpha_A : X \to [0, 1]$ and $\beta_A : X \to [0, 1]$ denote the degree of membership and the degree of nonmembership, respectively, and $0 \leq \alpha_A(x) + \beta_A(x) \leq 1$, $\forall\ x \in X$.

An intuitionistic fuzzy set IFSA$=\{(x, \alpha_A(x), \beta_A(x)) \mid x \in X\}$ in X can be identified to an ordered pair (α_A, β_A) in $I^X \times I^X$. For the sake of simplicity, we shall use the symbol IFSA$=(\alpha_A, \beta_A)$ instead of the IFSA$=\{(x, \alpha_A(x), \beta_A(x)) \mid x \in X\}$.

Definition 1.4.([8]) An IFSA$=(\alpha_A, \beta_A)$ in X is called an intuitionistic fuzzy ideal (briefly, IF-ideal) of X if it satisfies:

(IF1) $\alpha_A(0) \geq \alpha_A(x)$ and $\beta_A(0) \leq \beta_A(x)$,

(IF2) $\alpha_A(x) \geq \alpha_A(x * y) \wedge \alpha_A(y)$,

(IF3) $\beta_A(x) \leq \beta_A(x * y) \vee \beta_A(y)$,

for all x, $y \in X$.

Lemma 1.5.([8]) *Let IFSA=(α_A, β_A) in X be an IF-ideal of X. If the inequality $x * y \leq z$ holds in X, then $\alpha_A(x) \geq \alpha_A(y) \wedge \alpha_A(z)$, $\beta_A(x) \leq \beta_A(y) \vee \beta_A(z)$.*

Lemma 1.6.([8]) *Let IFSA=(α_A, β_A) be an IF-ideal of X. If $x \leq y$ in X, then $\alpha_A(x) \geq \alpha_A(y)$, $\beta_A(x) \leq \beta_A(y)$.*

2. Main Results

Definition 2.1. An IFSA=(α_A, β_A) in a BCI-algebra X is called an intuitionistic fuzzy α-ideal (briefly, IFα-ideal) of X if it satisfies the following inequalities:
(IFα1) $\alpha_A(0) \geq \alpha_A(x)$ and $\beta_A(0) \leq \beta_A(x)$,
(IFα2) $\alpha_A(y * x) \geq \alpha_A((x * z) * (0 * y)) \wedge \alpha_A(z)$,
(IFα3) $\beta_A(y * x) \leq \beta_A((x * z) * (0 * y)) \vee \beta_A(z)$,
for all x, y, $z \in X$.

Example 2.2. Let $X = \{0, a, b, c\}$ be a BCI-algebra with Cayley table as follows:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.

Define an IFSA=(α_A, β_A) in X as follows $\alpha_A(0) = \alpha_A(a) = 1$ and $\alpha_A(b) = \alpha_A(c) = t$, $\beta_A(0) = \beta_A(a) = 0$ and $\beta_A(b) = \beta_A(c) = s$, where t, $s \in (0, 1)$ and $t + s \leq 1$.

By routine calculations, we know that IFSA=(α_A, β_A) is an IFα-ideal of X.

Theorem 2.3. *Any IFα-ideal of X is an IF-ideal of X, but the converse is not true.*

Proof. Suppose that IFSA=(α_A, β_A) is an IFα-ideal of X. Putting $y = z = 0$ in (IFα2) and (IFα3), it follows that
(1) $\alpha_A(0 * x) \geq \alpha_A(x)$,

(2) $\beta_A(0*x) \leq \beta_A(x)$ for all $x \in X$.

Setting $x = z = 0$ in (IFα2) and (IFα3), it follows that

(3) $\alpha_A(y) \geq \alpha_A(0*(0*y))$,

(4) $\beta_A(y) \leq \beta_A(0*(0*y))$ for all $y \in X$.

By (1), (2), (3) and (4), we get $\alpha_A(x) \geq \alpha_A(0*(0*x)) \geq \alpha_A(0*x)$ and $\beta_A(x) \leq \beta_A(0*(0*x)) \leq \beta_A(0*x)$ for all $x \in X$.

Thus for any x, y, $z \in X$, $\alpha_A(x) \geq \alpha_A(0*x) \geq \alpha_A((x*z)*(0*0)) \wedge \alpha_A(z) = \alpha_A(x*z) \wedge \alpha_A(z)$ and $\beta_A(x) \leq \beta_A(0*x) \leq \beta_A((x*z)*(0*0)) \vee \beta_A(z) = \beta_A(x*z) \vee \beta_A(z)$. Hence IFSA$=(\alpha_A, \beta_A)$ is an IF-ideal of X.

Example 2.4. Let $X = \{0, a, b\}$ be a BCI-algebra with Cayley table given by the following:

$*$	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

.

Define $\alpha_A(0) = 1$ and $\alpha_A(a) = \alpha_A(b) = t$, $\beta_A(0) = 0$ and $\beta_A(a) = \beta_A(b) = s$, where t, $s \in (0, 1)$ and $s + t \leq 1$.

It is easy to check that IFSA$=(\alpha_A, \beta_A)$ is an IF-ideal of X, but not an IFα-ideal of X as follows:

$$\alpha_A(b*a) = \alpha_A(a) = t \not\geq \alpha_A((a*0)*(0*b)) \wedge \alpha_A(0) = 1.$$

Theorem 2.5. *Let IFSA$=(\alpha_A, \beta_A)$ be an IF-ideal of X. Then the following conditions are equivalent:*

(I) *IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X,*

(II) $\alpha_A(y*(y*x)) \geq \alpha_A((x*z)*(0*y)) \wedge \alpha_A(z)$ *and* $\beta_A(y*(y*x)) \leq \beta_A((x*z)*(0*y)) \vee \beta_A(z)$ *for all x, y, $z \in X$,*

(III) $\alpha_A(y*x) \geq \alpha_A(x*(0*y))$ *and* $\beta_A(y*x) \leq \beta_A(x*(0*y))$ *for all x, $y \in X$.*

Proof. (I) $\Rightarrow$ (II) Assume that IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X, then $\alpha_A(y*(x*z)) \geq \alpha_A(((x*z)*0)*(0*y)) \wedge \alpha_A(0) = \alpha_A((x*z)*(0*y))$ and $\beta_A(y*(x*z)) \leq \beta_A(((x*z)*0)*(0*y)) \vee \beta_A(0) = \beta_A((x*z)*(0*y))$ for all x, $y \in X$.

(II) $\Rightarrow$ (III) Putting $z = 0$ in (II), we get (III).
(III) $\Rightarrow$ (I) Since $(x * (0 * y)) * ((x * z) * (0 * y)) \leq x * (x * z) \leq z$, we have $\alpha_A(x*(0*y)) \geq ((x*z)*(x*y)) \wedge \alpha_A(z)$ and $\beta_A(x*(0*y)) \leq \beta_A((x*z)*(0*y)) \vee \beta_A(z)$ by Lemma 1.5.

Therefore $\alpha_A(y * x) \geq \alpha_A(x * (0 * y)) \geq \alpha_A((x * z) * (0 * y)) \wedge \alpha_A(z)$ and $\beta_A(y * x) \leq \beta_A(x * (0 * y)) \leq \beta_A((x * z) * (0 * y)) \vee \beta_A(z)$. Hence IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X. The proof is complete.

Theorem 2.6. *Let IFSA$=(\alpha_A, \beta_A)$ be an IF-ideal of X, then IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X if and only if it satisfies the following conditions:*

(I) $\alpha_A(x) \geq \alpha_A(0 * (0 * x))$ *and* $\beta_A(x) \leq \beta_A(0 * (0 * x))$ *for all* $x \in X$,
(II) $\alpha_A(x * y) \geq \alpha_A(x * (0 * y))$ *and* $\beta_A(x * y) \leq \beta_A(x * (0 * y))$ *for all* $x, y \in X$.

Proof. Assume that IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X, setting $x = 0$ in Theorem 2.5 (III), we get $\alpha_A(y) \geq \alpha_A(0*(0*y))$ and $\beta_A(y) \leq \beta_A(0*(0*y))$. Since $(0*(0*(y*(0*x))))*(x*(0*y)) = ((0*(0*y))*(0*(0*(0*x))))*(x*(0*y)) = ((0*(0*y))*(0*x))*(x*(0*y)) \leq (x*(0*y))*(x*(0*y)) = 0$. By Lemma 1.6, $\alpha_A(0*(0*(y*(0*x)))) \geq \alpha_A(x*(0*y)) \wedge \alpha_A(0) = \alpha_A(x*(0*y))$ and $\beta_A(0*(0*(y*(0*x)))) \leq \beta_A(x*(0*y)) \vee \beta_A(0) = \beta_A(x*(0*y))$.

Hence $\alpha_A(y*(0*x)) \geq \alpha_A(0*(0*(y*(0*x)))) \geq \alpha_A(x*(0*y))$ and $\beta_A(y*(0*x)) \leq \beta_A(0*(0*(y*(0*x)))) \leq \beta_A(x*(0*y))$. Applying Theorem 2.5 (III), we have $\alpha_A(x*y) \geq \alpha_A(y*(0*x)) \geq \alpha_A(x*(0*y))$ and $\beta_A(x*y) \leq \beta_A(y*(0*x)) \leq \beta_A(x*(0*y))$.

Conversely, suppose IFSA$=(\alpha_A, \beta_A)$ satisfies (I) and (II). In order to prove that IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X, from Theorem 2.5, we want to show that $\alpha_A(y*x) \geq \alpha_A(x*(0*y))$ and $\beta_A(y*x) \leq \beta_A(x*(0*y))$ for all $x, y \in X$.

By (II), we have $\alpha_A(x*y) \geq \alpha_A(x*(0*y))$ for all $x, y \in X$. Since $0*(y*x) \leq x*y$, by Lemma 1.6, we get

$$\alpha_A(0*(y*x)) \geq \alpha_A(x*y) \geq \alpha_A(x*(0*y)).$$

Thus $\alpha_A(0*(0*(y*x))) \geq \alpha_A(0*(y*x)) \geq \alpha_A(x*(0*y))$. Applying (I), we get $\alpha_A(y*x) \geq \alpha_A(0*(0*(y*x))) \geq \alpha_A(x*(0*y))$. Similarly, we can get $\beta_A(y*x) \leq \beta_A(x*(0*y))$. Therefore IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of X. This completes the proof.

Lemma 2.7. *An IFSA=(α_A, β_A) is an IFα-ideal of X if and only if α_A and $\overline{\beta}_A$ are Fα-ideals of X.*

Proof. Let IFSA=(α_A, β_A) be an IFα-ideals of X. Clearly α_A is a Fα-ideal of X. For every x, $y \in X$, we have $\overline{\beta}_A(0) = 1 - \beta_A(0) \geq 1 - \beta_A(x) = \overline{\beta}_A(x)$ and $\overline{\beta}_A(y*x) = 1-\beta_A(y*x) \geq 1-\beta_A((x*z)*(0*y)) \vee \beta_A(z) = (1-\beta_A((x*z)*(0*y))) \wedge (1-\beta_A(z)) = \overline{\beta}_A((x*z)*(0*y)) \wedge \overline{\beta}_A(z)$. Hence $\overline{\beta}_A$ is a Fα-ideal of X.

Conversely, assume that α_A and $\overline{\beta}_A$ are Fα-ideals of X. For every x, $y \in X$, we get $\alpha_A(0) \geq \alpha_A(x)$, $1 - \beta_A(0) = \overline{\beta}_A(0) \geq \overline{\beta}_A(x) = 1 - \beta_A(x)$, that is, $\beta_A(0) \leq \beta_A(x)$. Moreover, $\alpha_A(y*x) \geq \alpha_A((x*z)*(0*y)) \wedge \alpha_A(z)$ and $1-\beta_A(y*x) = \overline{\beta}_A(y*x) \geq \overline{\beta}_A((x*z)*(0*y)) \wedge \overline{\beta}_A(z) = (1-\beta_A((x*z)*(0*y))) \wedge (1-\beta_A(z)) = 1 - \beta_A((x*z)*(0*y)) \vee \beta_A(z)$, that is, $\beta_A(y*x) \leq \beta_A((x*z)*(0*y)) \vee \beta_A(z)$. Hence IFSA=$(\alpha_A, \beta_A)$ is an IFα-ideal of X. This completes the proof.

Theorem 2.8. *An IFSA=(α_A, β_A) is an IFα-ideal of X if and only if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\diamond A = (\overline{\beta}_A, \beta_A)$ are IFα-ideals of X.*

Proof. If IFSA=(α_A, β_A) is an IFα-ideal of X, then $\alpha_A = \overline{\overline{\alpha}}_A$ and β_A are Fα-ideals of X from Lemma 2.7, hence $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\diamond A = (\overline{\beta}_A, \beta_A)$ are IFα-ideals of X.

Conversely, if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\diamond A = (\overline{\beta}_A, \beta_A)$ are IFα-ideals of X. Then the fuzzy sets α_A and $\overline{\beta}_A$ are Fα-ideals of X, hence IFSA=(α_A, β_A) is an IFα-ideal of X. This completes the proof.

For $t \in [0, 1]$ and a fuzzy set μ in a nonempty set X, the set $U(\mu; t) = \{x \in X \mid \mu(x) \geq t\}$ is called an upper t-level cut of μ and the set $L(\mu; t) = \{x \in X \mid \mu(x) \leq t\}$ is called a lower t-level cut of μ.

Theorem 2.9. *An IFSA=(α_A, β_A) is an IFα-ideal of X if and only if for all s, $t \in [0, 1]$, the sets $U(\alpha_A; t)$ and $L(\beta_A; s)$ are either empty or α-ideals of X.*

Proof. Let IFSA=(α_A, β_A) be an IFα-ideal of X and $U(\alpha_A; t) \neq \varnothing \neq L(\beta_A; s)$ since $\alpha_A(0) \geq t$ and $\beta_A(0) \leq s$. Let x, y, $z \in X$ be such that $(x * z) * (0 * y) \in U(\alpha_A; t)$ and $z \in U(\alpha_A; t)$. Then $\alpha_A((x * z) * (0 * y)) \geq t$ and $\alpha_A(z) \geq t$. It follows that $\alpha_A(y * x) \geq \alpha_A((x * z) * (0 * y)) \wedge \alpha_A(z) \geq t$, so that $y * x \in U(\alpha_A; t)$. Hence $U(\alpha_A; t)$ is an α-ideal of X. Now let x, y, $z \in X$ be such

that $(x*z)*(0*y) \in L(\beta_A; s)$ and $z \in L(\beta_A; s)$. Then $\beta_A((x*z)*(0*y)) \leq s$ and $\beta_A(z) \leq s$, which imply that $\beta_A(y*x) \leq \beta_A((x*z)*(0*y)) \vee \beta_A(z) \leq s$. Thus $y*x \in L(\beta_A; s)$, and therefore $L(\beta_A; s)$ is an α-ideal of X.

Conversely, assume that for each s, $t \in [0,1]$, the sets $U(\alpha_A; t)$ and $L(\beta_A; s)$ are empty or α-ideals of X. For any $x \in X$, let $\alpha_A(x) = t$ and $\beta_A(x) = s$. Then $x \in U(\alpha_A; t) \bigcap L(\beta_A; s)$, and so $U(\alpha_A; t) \neq \varnothing \neq L(\beta_A; s)$. Since $U(\alpha_A; t)$ and $L(\beta_A; s)$ are α-ideals of X, therefore $0 \in U(\alpha_A; s) \cap (\beta_A; s)$. Hence $\alpha_A(0) \geq t = \alpha_A(x)$ and $\beta_A(0) \leq s = \beta_A(x)$ for all $x \in X$. If there exist x', y', $z' \in X$ such that $\alpha_A(y'*x') > \alpha_A((x'*z')*(0*y')) \wedge \alpha_A(z')$. Putting $t_0 = \frac{1}{2}(\alpha_A(y'*x') + \alpha_A((x'*z')*(0*y')) \wedge \alpha_A(z'))$, we have $\alpha_A(y'*x') < t_0 < \alpha_A((x'*z')*(0*y')) \wedge \alpha_A(z')$. Hence $y'*x' \notin U(\alpha_A; t_0)$, $(x'*z')*(0*y') \in U(\alpha_A; t_0)$ and $z' \in U(\alpha_A; t_0)$, that is, $U(\alpha_A; t_0)$ is not an α-ideal of X, which is a contradiction. Finally, assume that there exist a, b, $c \in X$ such that $\beta_A(b*a) > \beta_A((a*c)*(0*b)) \vee \beta_A(c)$. Putting $s_0 = \frac{1}{2}(\beta_A(b*a) + \beta_A((a*c)*(0*b)) \vee \beta_A(c))$, then $\beta_A((a*c)*(0*b)) \vee \beta_A(c) < s_0 < \beta_A(b*a)$. Therefore $(a*c)*(0*b) \in L(\beta_A, s_0)$ and $c \in L(\beta_A, s_0)$, but $b*a \notin L(\beta_A; s_0)$, which is a contradiction. This completes the proof.

Let Λ be a nonempty subset of $[0,1]$.

Theorem 2.10. *Let $\{I_t \mid t \in \Lambda\}$ be a collection of α-ideals of X such that*

(i) $X = \bigcup_{t\in\Lambda} I_t$,

(ii) *$s > t$ if and only if $I_s \subset I_t$ for all s, $t \in \Lambda$.*

Then an IFSA$=(\alpha_A, \beta_A)$ in X defined by $\alpha_A(x) = \sup\{t \in \Lambda \mid x \in I_t\}$, $\beta_A(x) = \inf\{t \in \Lambda \mid x \in I_t\}$ for all $x \in X$ is an IFα-ideal of X.

Proof. According to Theorem 2.9, it is sufficient to show that $U(\alpha_A; t)$ and $L(\beta_A; s)$ are α-ideals of X for every $t \in [0, \alpha_A(0)]$ and $s \in [\beta_A(0), 1]$. In order to prove that $U(\alpha_A; t)$ is an α-ideal of X, we divide the proof into the following two cases:

(i) $t = \sup\{q \in \Lambda \mid q < t\}$,

(ii) $t \neq \sup\{q \in \Lambda \mid q < t\}$.

The case (i) implies that $x \in U(\alpha_A; t) \Leftrightarrow x \in I_q$, $\forall q < t \Leftrightarrow x \in \bigcap_{q<t} I_q$ so that $U(\alpha_A; t) = \bigcap_{q<t} I_q$, which is an α-ideal of X. For the case (ii), we claim that $U(\alpha_A; t) = \bigcup_{q\geq t} I_q$, If $x \in \bigcup_{q\geq t} I_q$, then $x \in I_q$ for some $q \geq t$. It follows that

$\alpha_A(x) \geq q \geq t$, so that $x \in U(\alpha_A; t)$. This shows that $\bigcup_{q\geq t} I_q \subseteq U(\alpha_A; t)$. Now assume $x \notin \bigcup_{q\geq t} I_q$. Then $x \notin I_q$ for all $q \geq t$. Since $t \neq \sup\{q \in \Lambda \mid q < t\}$, there exists $\varepsilon > 0$ such that $(t-\varepsilon, t) \bigcap \Lambda = \varnothing$. Hence $x \notin I_q$ for all $q > t - \varepsilon$, which means that $x \in I_q$, then $q \leq t - \varepsilon < t$. Thus $\alpha_A(x) \leq t - \varepsilon$, and so $x \notin U(\alpha_A; t)$. Therefore $U(\alpha_A; t) \subseteq \bigcup_{q\geq t} I_q$, and that $U(\alpha_A; t) = \bigcup_{q\geq t} I_q$, which is an α-ideal of X. Next, we prove that $L(\beta_A; s)$ is an α-ideal of X. We consider the following two cases:

(iii) $s = \inf\{r \in \Lambda \mid s < r\}$,
(iv) $s \neq \inf\{r \in \Lambda \mid s < r\}$.

For the case (iii), we have

$$x \in L(\beta_A; s) \Leftrightarrow x \in I_r, \quad \forall s < r \Leftrightarrow x \in \bigcap_{s<r} I_r,$$

and hence $L(\beta_A, s) = \bigcap_{s<r} I_r$, which is an α-ideal of X. For the case(iv), there exists $\varepsilon > 0$ such that $(s, s+\varepsilon) \bigcap \Lambda = \varnothing$, we will show that $L(\beta_A; s) = \bigcup_{s\geq r} I_r$. If $x \in \bigcup_{s\geq r} I_r$, then $x \in I_r$ for some $r \leq s$. It follows that $\beta_A(x) \leq r \leq s$. So that $x \in L(\beta_A; s)$. Hence $\bigcup_{s\geq r} I_r \subseteq L(\beta_A; s)$. Conversely, if $x \notin \bigcup_{s\geq r} I_r$, then $x \notin I_r$ for all $r \leq s$, which implies that $x \notin I_r$ for all $r < s + \varepsilon$, that is, if $x \in I_r$, then $r \geq s + \varepsilon$. Thus $\beta_A(x) \geq s + \varepsilon > s$, that is, $x \notin L(\beta_A; s)$. Therefore $L(\beta_A; s) \subseteq U_{s\geq r} I_r$ and consequently $L(\beta_A; s) = \bigcup_{s\geq r} I_r$, which is an α-ideal of X. This completes the proof.

Let $f : X \to Y$ be a homomorphism of BCI-algebras. For any IFSA $= (\alpha_A, \beta_A)$ in Y, we define a new IFSA$^f = (\alpha_A^f, \beta_A^f)$ in X by $\alpha_A^f(x) = \alpha_A(f(x))$, $\beta_A^f(x) = \beta_A(f(x))$ $\forall x \in X$.

Theorem 2.11. *Let $f : X \to Y$ be a homomorphism of BCI-algebras. If an IFSA$=(\alpha_A, \beta_A)$ in Y is an IFα-ideal of Y, then IFSA$^f = (\alpha_A^f, \beta_A^f)$ in X is an IFα-ideal of X.*

Proof. We first have that $\alpha_A^f(x) = \alpha_A(f(x)) \leq \alpha_A(0) = \alpha_A(f(0)) = \alpha_A^f(0)$ and $\beta_A^f(x) = \beta_A(f(x)) \geq \beta_A(0) = \beta_A^f(0)$ for all $x \in X$. Let x, y, $z \in X$. Then $\alpha_A^f((x * z) * (0 * y)) \wedge \alpha_A^f(z) = \alpha_A(f((x * z) * (0 * y))) \wedge \alpha_A^f(z) = \alpha_A((f(x) * f(z)) * (f(0) * f(y))) \wedge \alpha_A(f(z)) \leq \alpha_A(f(y) * f(x)) = \alpha_A(f(y * x)) = \alpha_A^f(y * x)$ and $\beta_A^f((x * z) * (0 * y)) \vee \beta_A^f(z) = \beta_A((f(x) * f(z)) * (f(0) * f(z))) \vee \beta_A(f(z)) \geq$

$\beta_A(f(y) * f(x)) = \beta_A(f(y * x)) = \beta_A^f(y * x)$. Hence $\text{IFSA}^f = (\alpha_A^f, \beta_A^f)$ is an IFα-ideal of X.

If we strengthen the condition of f, then we can construct the converse of Theorem 2.11 as follows:

Theorem 2.12. *Let $f : X \to Y$ be an epimorphism of BCI-algebras and IFSA$=(\alpha_A, \beta_A)$ be in Y. If IFSA$^f = (\alpha_A^f, \beta_A^f)$ is an IFα-ideal of X, then IFSA$=(\alpha_A, \beta_A)$ is an IFα-ideal of Y.*

Proof. For any $x \in Y$, there exists $a \in X$ such that $f(a) = x$. Then $\alpha_A(x) = \alpha_A(f(a)) = \alpha_A^f(a) \leq \alpha_A^f(0) = \alpha_A(f(0)) = \alpha_A(0)$ and $\beta_A(x) = \beta_A(f(a)) = \beta_A^f(a) \geq \beta_A^f(0) = \beta_A(f(0)) = \beta_A(0)$. Let x, y, $z \in Y$. Then $f(a) = x$, $f(b) = y$ and $f(c) = z$ for some a, b and $c \in X$. It follows that $\alpha_A(y*x) = \alpha_A(f(b)*f(a)) = \alpha_A(f(b*a)) = \alpha_A^f(b*a) \geq \alpha_A^f((a*c)*(0*b)) \wedge \alpha_A^f(c) = \alpha_A(f((a*c)*(0*b))) \wedge \alpha_A(f(c)) = \alpha_A((f(a)*f(c))*(f(0)*f(b))) \wedge \alpha_A(f(c)) = \alpha_A((x*z)*(0*y)) \wedge \alpha_A(z)$ and $\beta_A(y*x) = \beta_A(f(b)*f(a)) = \beta_A(f(b*a)) \leq \beta_A^f((a*c)*(0*b)) \vee \beta_A^f(c) = \beta_A(f((a*c)*(0*b))) \vee \beta_A(f(c)) = \beta_A((f(a)*f(c))*(f(0)*f(b))) \vee \beta_A(f(c)) = \beta_A((x*z)*(0*y)) \vee \beta_A(z)$. This completes the proof.

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