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A Computation Method for Symmetry

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Abstract—This paper puts forward a mathematical method to measure the symmetry property of two image elements—a specific instance of the Gestalt law of symmetry. The method makes an abstract of image elements by introducing vector points and experimental results show that the computation results is consistent with the human perception interpretation in most cases.

Introduction

The Gestalt theory started in 1921 with the Max Wertheimer founding paper [1]. It is a theory of mind and brain positing that the operational principle of the brain is holistic and parallel with self-organizing tendencies. The Gestalt effect refers to the form-forming capability of our senses, particularly with respect to the visual recognition of figures and whole forms instead of just a collection of simple lines and curves.

The fundamental principle of gestalt perception is the law of prägnanz (German for pithiness) which says that we tend to order our experience in a manner that is regular, orderly, symmetric, and simple. Gestalt psychologists attempt to discover refinements of the law of prägnanz, and this involves writing down laws which hypothetically allow us to predict the interpretation of sensation, what are often called "gestalt laws"[2]. Yet there is still no quantification model for Gestalt laws. Among previous works only Agnes Desolneux etal.[3] pointed out the problem of how to compute the "Gestalt properties" and investigated the rule for image pixels forming lines from the statistics point of view.

In this paper we design a mathematical method for computing to what extent two lines or image edges are symmetric to each other, which is a part of the computation model for Gestalt law of symmetry. The paper is organized as follows: In section Ⅱ the basic idea for computation model design is described; the quantification model for law of symmetry is presented by section Ⅲ; section Ⅳ gives some experimental verifications before concluding in section Ⅴ.

The Mathematical model

Let QM_{Ges} denote the quantification method of a certain Gestalt law. Firstly a parameter space P is selected, then a function $m_{\mathrm{Ges}}(.)$ is constructed so that $\forall p,q \in P$, $m_{\mathrm{Ges}}(p,q)$ can get the measure for p and q's "Gestalt property" . Sometimes $m_{\mathrm{Ges}}(p,q)$ needs to compute the integral of $m^2{}_{\mathrm{Ges}}(p,q)$. over P ,so it is required that $\int m_{Ges}^2(p,q)dpdq$ exist and be limitary. We can use $L^2(P)$ to represent a function space with such requirements.

In $L^2(P), \forall m,n \in L^2(P)$, if the inner-product is defined as:

$$\langle m,n \rangle = \int_P m(\xi)\overline{n}(\xi)d\xi \tag{1}$$

Then the norm is

$$\|m\| = \sqrt{\langle m, m \rangle} \tag{2}$$

Correspondingly the measure for distance can be defined as

$$d(m(p), m(q)) = \|m(p) - m(q)\| \tag{3}$$

In summary, the key to the quantification model is the construction of $m^2_{\text{Ges}}(p,q)$. Once it is determined, we can use equation (2)(3) to computer the measure for Gestalt property where $m^2_{\text{Ges}}(p,q)$ satisfies the following three conditions:

$$m_{Ges}(\cdot) \in L^2(P) \tag{4}$$

$$\|m_{Ges}(\cdot)\| \le 1 \tag{5}$$

$$m^2_{\text{Ges}}(p,q) \geqslant 0 \tag{6}$$

The computation approach for Symmetry

Symmetry exists all around us and many people see it as being a thing of beauty. The most familiar type of symmetry for many people is geometrical symmetry. Formally, this means symmetry under a sub-group of the Euclidean group of isometries in two or three dimensional Euclidean space. These isometries consist of reflections, rotations, translations and combinations of these basic operations. An object or figure which is indistinguishable from its transformed image is called mirror symmetric or reflection symmetry: in 1D, there is a point of symmetry; in 2D there is an axis of symmetry; in 3D a plane of symmetry. Rotational symmetry is symmetry with respect to some or all rotations in *n*-dimensional Euclidean space[4].

From the related works on symmetry detection, we can see the importance of symmetry axis or symmetry center. Therefore our computation method begins with these elements of symmetry. To simplify the description of following paragraphs, we give some definitions as below:

***Definition 1 vector point*:** A point is called a vector point when it possess two properties: position and direction. We denote a vector point by $vp((x,y),o)$ or vp (simplified version), where (x,y) describe the position and o is the direction.

Note: Here we introduce the vector point to simplify the discussion about symmetry computation.

***Definition 2 Reflection symmetry of two vector points*:** Given two vector points vp_1 and vp_2, and N is the middle point of vp_1 and vp_2, passing though N there is a line l perpendicular to the line connecting the two vector points. if vp_1 and vp_2 were to be folded in half over the axis l, the two points would be identical, Then we call vp_1 and vp_2 take on reflection symmetry against the axis l.

***Definition 3 Rotation symmetry of two vector points*:** Given two vector points vp_1 and vp_2, let P_0 denote the midpoint of vp_1 and vp_2, r is the distance from P_0 to vp_1 (vp_2), if vp_1 (vp_2) rotates 180° around P_0 at a radius r and become a superposition of vp_2 (vp_1), Then we call vp_1 and vp_2 take on rotation symmetry.

***Definition 4 Circle rotation symmetry of multiple vector points*:** Given three or more vector points $vp_1, vp_2, \cdots vp_n$ ($n \ge 3$), if there exists a center P_o of these points with equal distance r_o to $vp_1, vp_2, \cdots vp_n$, if anyone of these vector points rotate clockwisely around P_o by the radius and can become a superposition of vp_2 (vp_1), then we will call $vp_1, vp_2, \cdots vp_n$ construct the circle rotation symmetry.

***Definition 5 Clockwise/anti-clockwise direction of a vector point*:** Given two vector points vp_1 and vp_2, vp_1 turns its direction at angle θ_1 clockwisely or at angle θ_2 anticlockwisely to points to vp_2. If $|\theta_1| > |\theta_2|$, then we call vp_1 is at the clockwise direction of vp_2; otherwise vp_1 is at the anticlockwise direction of vp_2.

***Definition 6 Clockwise/anti-clockwise angle between two vector points*:** Given two vector points vp_1 and vp_2, if vp_1 is at the clockwise/anticlockwise direction of vp_2 and the corresponding rotation angle is θ_{cw}/θ_{Acw}, then θ_{cw}/θ_{Acw} is defined as the clockwise/anti-clockwise angle between two vector points. Negative values are set for θ_{cw} and positive values for θ_{Acw}.

Definition 7 Preparative symmetry center--association points—preparative symmetry axis*:** Given a set of vector points S, if $\forall vp_i, vp_j \in S$, there is a point ***O as the midpoint of vp_i and vp_j,then ***O*** is called the preparative symmetry center of vp_i and vp_j; vp_i and vp_j are named as association points of ***O***; if there are more than one pair association points of ***O***, these association points will form a set of ***O*** 's association points, denoted by S_O; neighbor preparative symmetry centers will form a line segment named as preparative symmetry axis labeled by l_O.

Based on the preliminary definitions, we can draw lemma 1:

Lemma 1 When two vector points possess the reflection symmetry or rotation symmetry, their clockwise and anti-clockwise angle sum to zero.

Proof: let vp_1 and vp_2 represent two vector points, the clockwise angle from vp_1 to vp_2 is θ_{cw} and the anticlockwise angle is θ'_{Acw}. If we connect vp_1 with vp_2, forming a line *l1*. then θ_{cw} is the angle between vp_1 and *l1*, θ'_{Acw} is the angle between vp_2 and *l1*.

1) Suppose vp_1 and vp_2 take on the reflection symmetry, since the retroflexion does not change the angle between the vector and *l1*, we can draw from the definition of reflection symmetry that $|\theta_{cw}| = \theta'_{Acw}$, therefore $\theta_{cw} = -\theta'_{Acw}$, that is $\theta_{cw} + \theta'_{Acw} = 0$.

2) Suppose vp_1 and vp_2 take on the rotation symmetry, since the rotation around the symmetry center does not change the angle between the vector and *l1*, we can draw from the definition of rotation symmetry that $|\theta_{cw}| = \theta'_{Acw}$, therefore $\theta_{cw} = -\theta'_{Acw}$, that is $\theta_{cw} + \theta'_{Acw} = 0$. #

According to lemma 1, the symmetry extent of two vector points can be associated with the angle between them. The sum holds an inverse ratio to the symmetry extent. The measure for symmetry is defined as:

Definition 8 The symmetry measure of association points*:** The symmetry measure of association points is determined by their contribution to the symmetry center ***O by $m_{sym}(vp_i, vp_j)$:

$$m_{sym}(vp_i, vp_j) = \begin{cases} \exp\left(-\dfrac{(\theta_{cw} + \theta'_{Acw})^2}{2\sigma_\theta^2}\right), & |\theta_{cw}| + |\theta'_{Acw}| \neq \pi \\ \exp\left(-\dfrac{(\pi + \theta_{cw} - \theta'_{Acw})^2}{2\sigma_\theta^2}\right) \cdot (1 - \delta(\theta_{cw} \cdot \theta'_{Acw})), & |\theta_{cw}| + |\theta'_{Acw}| = \pi \end{cases} \tag{7}$$

Where σ_θ is the variance of Gaussian function, We can draw a conclusion from ref. [7][8] that two lines maybe parallel or collinear when their orientation difference is between 0 degree to 9 degree. Therefore we set σ_θ=7.65 so that when the orientation difference is larger than 9 degree, the similarity measure will be less than 0.5. If we choose radian as the unit, then the parameter σ_θ=0.14. When $|\theta_{cw}| + |\theta'_{Acw}| \neq \pi$, $\exp\left(-\dfrac{(\theta_{cw} + \theta'_{Acw})^2}{2\sigma_\theta^2}\right)$ holds a direct ratio to the symmetry extent of association points and reach an extremum at $\theta_{cw} + \theta'_{Acw} = 0$; when $|\theta_{cw}| + |\theta'_{Acw}| = \pi$, $\exp\left(-\dfrac{(\pi + \theta_{cw} - \theta'_{Acw})^2}{2\sigma_\theta^2}\right)$ is used to compute the symmetry extent of two association points, and $(1 - \delta(\theta_{cw} \cdot \theta'_{Acw}))$ makes a complementarity to exclude the collinear situation.

The geometry meaning of definition 8 is illustrated by Figure 1.

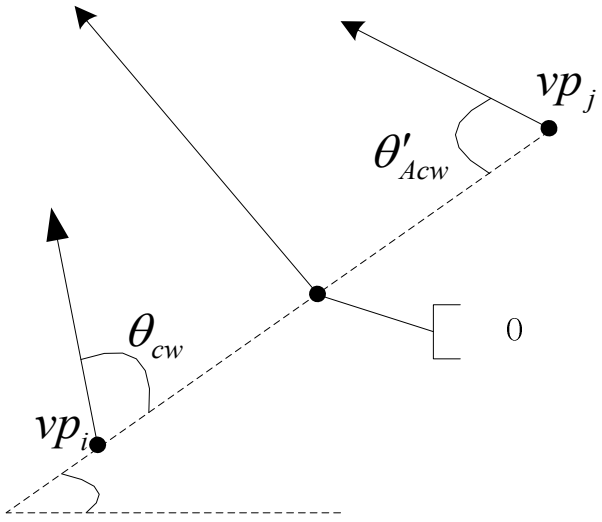

Figure 1. The geometry meaning of symmetry measure for association points

Experiments

We test the quantification function represented by equation (7) by comparing the computation measure with the human perception results. The human perception results are obtained from the questionnaires which are filled by one hundred students of our department.

The test data are drawn by computer and demonstrated in Figure 2-3. The first group is composed of fifteen vector point pairs and the second group consists three line fragments. The quizzes see the images in each group at a random sequence and interpret their sensation of symmetry.

In Figure 2, the quizzes are required to find the symmetry pairs according to the orientation and position of the vector points. They all find symmetry pairs in image (e), (g), (i), (n); 87% of the quizzes consider the vector points in (f) and (m) are approximately symmetric; 65% of them hold on that parallel is a special case of symmetry , so the vector points in (a) are symmetric too.

According to the computation method described by equation (7), we set $\sigma_\theta = 6$ and get the symmetry measure for the vector points in Figure 2 as 0.9862, 7.0550e-021, 0, 0.0164, 0.9994, 0.3008, 0.9511, 0.9912, 0.8751, 9.4070e-004, 0.0228, 0.0012, 0.1543, 0.9651 and 0.0033 sequentially from (a) to (o). Compared with the perceptual results of the quizzes, we can see that the symmetry mesure is consistent with the human perception in (a), (e), (g), (i), (n); but diverge for the case (f), (h) and (m). There are two possible explanations for these differnces: one is the perceptual error which is unavoidable; the other is that the quizzes take into account the lenth of the vector arrows although we stress the position and orientation as the two factors for symmetry judgement. What's more, we can not control the length of the vector arrow drawn by Matlab programme so that the length differs from each other when direction changes.

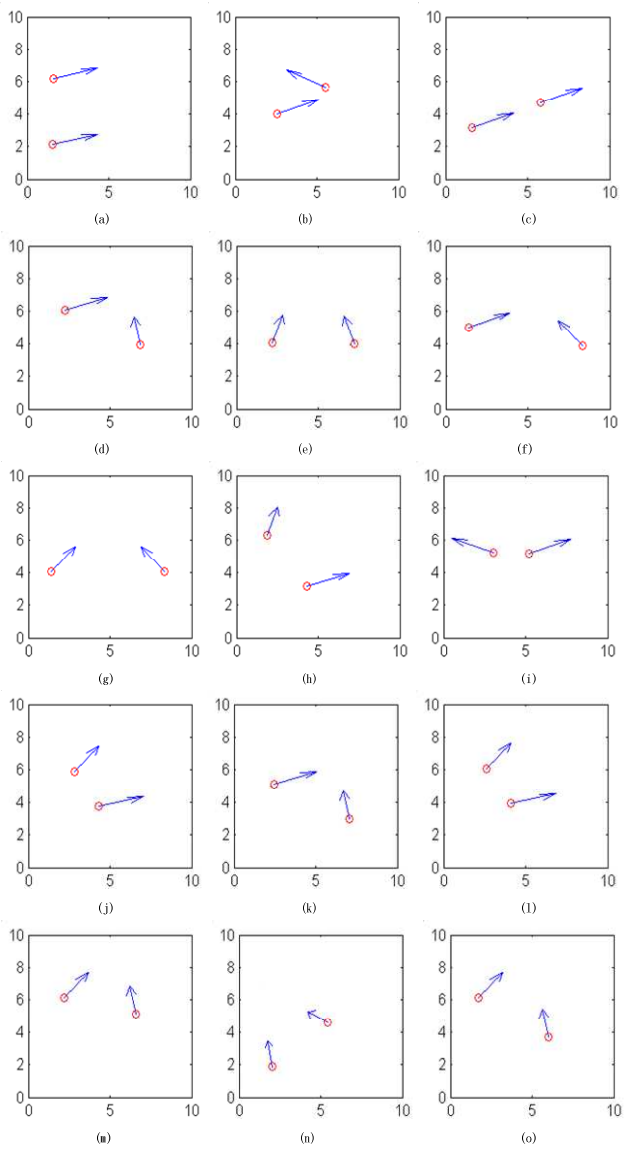

Figure 2. No.1 group test data(vector points)

In Figure 3 we extend our computation method to line segments. The equation (7) is replaced by the following function:

$$m_{sym}(EF_k, EF_l) = \begin{cases} 0 & n_k - n \geq n_k/3 \\ \frac{1}{n_l}\sum_{i=1}^{n} m_{sym}(vp_i, vp_j) & otherwise \end{cases} \tag{8}$$

Where EF_k and EF_l denote two line segments, n_k and n_l represent their length separately($n_k \leq n_l$). Since a line segment can be seen as a set of association points , suppose vp_i and vp_j are the partners such that

$$\{(vp_i, vp_j) | vp_i \in EF_k, vp_j \in EF_l, i = 1 \cdots n\} \tag{3}$$

Then $m_{sym}(vp_i, vp_j)$ is the symmetry measure of vp_i and vp_j .

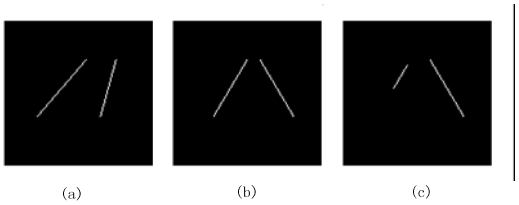

Figure 3. No.2 group test data (lines)

The quizzes think the line segments in (a) are not symmetric; image (b) contains two symmetric lines and image (c) describes a partial symmetric case. Using our method, the symmetry measure is 0, 1 and 0.42 respectively. The computation values are consistent with the human perceptual results.

Conclusion

We can see from the experiments that the computation results gotten by our quantification model are consistent with the human perception in most cases and can acquire a more fine partition. The discrimination can be explained by the psychology experiments executed in ref.[9][10]: on the one hand the human being can not recognize the difference within five degrees; on the other hand, our brain prefers to perceive symmetry in meaningful figures. Our computation method use vector points and line segments to make an abstract of symmetric elements. How to compute the symmetry extent of complex figures will be our further research work.

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