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Basic concepts of oversampling converter models

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1. Introduction

Sigma-Delta modulation is an efficient technique to quantize an analog signal and is used in a growing number of applications. The major advantage of this type of quantizer is that there is no need for high-precision components or high-order analog filters. The three major concepts are oversampling, feedback (noise shaping) and digital decimation. The analog signal is sampled at a rate much higher than the Nyquist rate and after filtering, a coarse quantization takes place. The resulting bit stream is then fed back and subtracted from the original signal at the filter input. The filter usually consists of one or more integrators and this implies that any difference between the original input and the output stream will accumulate and eventually correct itself. The output bit-stream is then digitally low-pass filtered and decimated to Nyquist rate. The combination of oversampling and feedback creates a cheap and efficient way to perform A/D or D/A conversion. However, problems arise when one tries to model such a system because of the inherent strong non-linearity of the quantizer. In the past, several methods where employed to model the exact behavior of the Sigma-Delta modulator[1-3]. A drawback of those methods is that they are usually only valid for first- or second-order systems, or for some particular input pattern, so from a practical point of view, they are not of much use. There are several ways to deal which such a nonlinearity. A popular one is to replace it by an amplification factor and an additive noise source. Although this model can be used to get some insight in the frequency behavior, it fails to predict limit cycles and stability problems. This text is organized as follows: in chapter 2 the use of functional models is explained. In chapter 3, a way of dealing with the nonlinearity is explained, while in chapter 4 a more sophisticated form of this procedure is described. In chapter 5, the phase-shift introduced by the quantizer is explained. Chapter 6 deals with the origins of stability problems, while chapter 7 proposes a model based on recursive functions. This study then forms the basis for the further modeling work in the remainder of this project.

2. Difference equation approach

Because of the oversampling nature of the Delta-Sigma modulator, the clock frequency is much higher than the signal bandwidth. Because for most types of analysis several periods of the input signal are required, a large number of data points need to be simulated. This makes it very unpractical to use a standard spice-like transistor level simulation. A more efficient way to simulate the system is to evaluate the difference equations, i.e. simulating the system on a functional level. In Fig. 1a an example of a Sigma-Delta modulator is given. For the second integrator of this system the functional description would be:

$$v_2(k) = F_2\big(v_1(k-1), y(k-1), v_2(k-1)\big) \tag{1}$$

Using this functional description, nonidealities (linear as well as nonlinear) can be incorporated in the function $F_2(.)$. Because of the sampled nature of the system, this function needs only to be evaluated at the end of each clock phase.

This for instance implies that whenever the sampling period is much larger than the required settling time, the settling behavior of the opamps is of little concern, which means a significant gain in calculation speed. Furthermore, because of the delay in the loop, no iterations are needed to

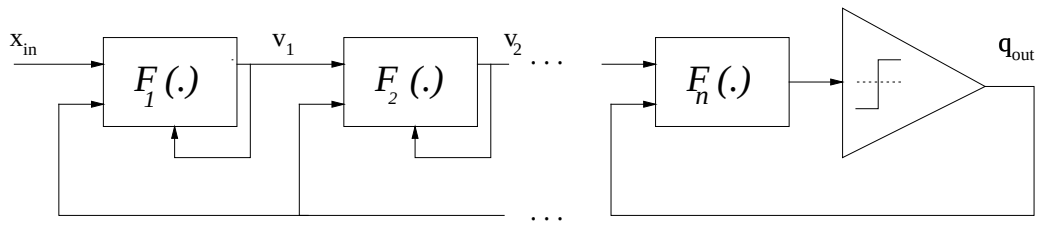

Figure 1 Functional model of Sigma-Delta modulator.

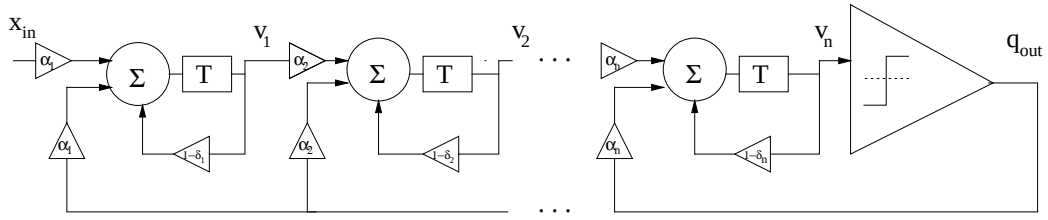

Figure 2 Functional model for Sigma-Delta modulator with leaky integrators

evaluate the different input signals to the functions. Fig. 2 shows an example of the circuit in the case the integrators are leaky, (1) then becomes:

$$v_2(k) = (1-\delta_2)\cdot v_2(k-1) + \alpha_2\left(v_1(k-1) - y(k-1)\right) \quad (2)$$

Here δ_2 is the integrator leakage of the second integrator. The main challenge when using this type of model is to find explicit functional descriptions for the major nonidealities in the integrator section. Examples are slew rate, settling behavior and nonlinearities of the opamp, thermal noise, nonlinearities of capacitors and so on. When these descriptions are known, the input signal of the quantizer v_n can be calculated using e.g. state space methods. This can easily be done using Matlab or standard C code. In the literature, several examples of implementations of this type of model can be found [4].

Although the functions describing the integrator's behavior

can be quite complex, the resulting simulation time for a given number of samples, is much lower than in the case of a standard transistor level simulation. For some purposes though, e.g. finding the input amplitude for which the circuit becomes unstable, a large number of samples is needed. In those particular cases, there is a need for an even faster model.

3. Linear model with additive white noise

In the past [5], the model that is shown in Fig. 3 was used to explain the frequency behavior of the Delta-Sigma converter. The idea here is that the quantizer can be replaced by an amplifier with amplification factor K and an additive white noise source. The so-called noise-shaping behavior of a Delta-Sigma modulator can easily be explained using such a model. If the additive noise is uniform over the quantizer output range (±Δ/2), its rms value can be calculated using:

$$e_{rms} = \frac{1}{\Delta}\int_{-\Delta/2}^{+\Delta/2} e^2 de = \frac{\Delta^2}{12} \tag{3}$$

The noise at the output can be calculated using:

$$Q_{out}(\omega) = \frac{e}{1 + H(\omega)\cdot K} \tag{4}$$

Where H(ω) is the transfer function of the linear system preceding the quantizer. Because this linear system is basically a low-pass filter and the noise is white, the spectrum of Q_{out} will contain relatively low noise power at low frequencies (the process of noise shaping). For oversampling ratios larger than two, the rms-value of the in-band noise can be approximated by:

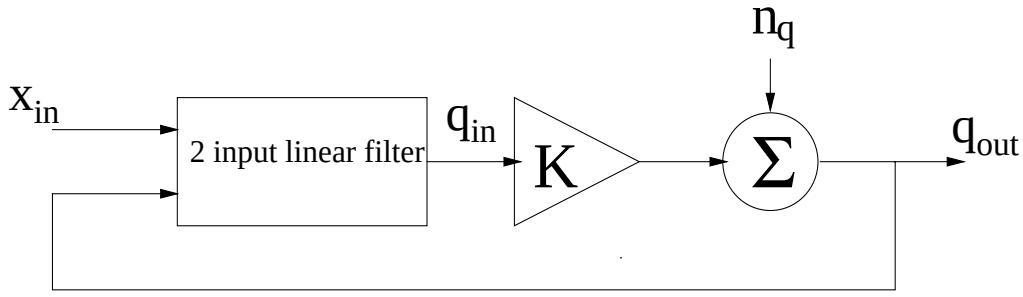

Figure 3 Linear model with additive white noise source

$$n_0 = e_{rms} \frac{\pi^L}{\sqrt{2L+1}} (OR)^{L+1/2} \tag{5}$$

Here L is the order of the filter (or the number of feedback loops) and OR denotes the oversampling ratio, which is given by:

$$OR = \frac{f_s}{2 f_n} \tag{6}$$

Where f_n denotes the signal Nyquist frequency and f_s the actual sample frequency.

Although this model seems to predict the correct result, when compared to a measurement on a real system, it still does not make the proposed substitution valid. The conditions under which it is valid are [6]:

1. The quantizer does not overload,
2. The number of levels of the quantizer is large,
3. The distance between the levels is small,
4. The probability distribution of pairs of input samples is given by a smooth probability density function.

While these conditions may be true for a normal A/D converter, this is certainly not the case for an oversampling A/D converter:

1. The internal levels of the system are not known in general, so it is not certain whether a certain input may cause overload at the quantizer input,
2. Most quantizers are single-bit, this means that there exist only two levels. Even for multibit-quantizers the number of levels is small,
3. The distance between the levels is typically large, due to the limited number of bits that is used,
4. The probability density function of pairs of input samples is not smooth in general due to the feedback of a digital signal.

This means that in general the quantization noise will not be white and that the spectrum of the output signal will not be given by (5).

There are several ways to make the spectrum of the quantizer input approach white noise. One of them is adding dither with a white spectrum at the quantizer input. If a higher-order sigma-delta modulator is used, the output is put through a number of different filters, which all add to the quantizer input signal, this also tends to make a more white spectrum. The problem with such higher-order systems is that they are not unconditionally stable, i.e. certain (bounded) inputs will cause the output of the linear system to go out of bounds. However, this unstable behavior is not predicted by this linear model, proving that it cannot be correct for such systems.

4. Modeling the nonlinearity using two interlocked linear systems

In the last chapter, it was mentioned that the normal linearized model does not predict the unstable behavior. However it is possible to do this by using separate models [7] both for the wanted signal as for the quantization noise. This is shown in Fig. 4. Now two amplification factors K_y and K_x have to be determined and this must be done such that the mean square error is minimized. The signal at the quantizer output q_{out}(k) can be considered a stationary random variable.

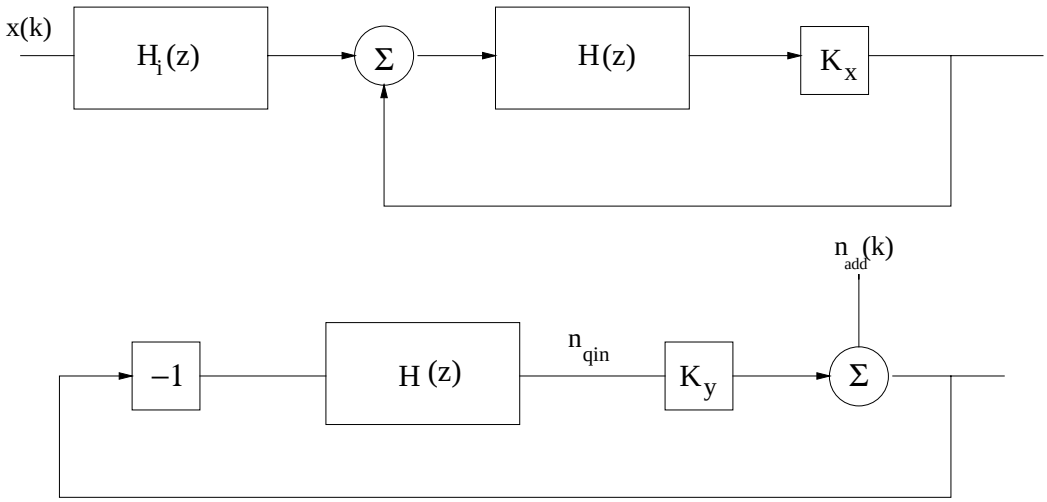

Figure 4 Splitting u p signal and noise components

Since, due to the feedback loop, the signal at the quantizer input q_{in} is an integrated version of q_{out} and integration causes the distribution to approach a Gaussian one, it can be modeled using a zero-mean Gaussian noise source $n_q(k)$ and a DC signal m_{qin}. This means

$$E\left[\left(q_{out}(k) - K_y n_q(k) - K_x m_{qin}\right)^2\right] \tag{7}$$

has to be minimized. The two amplification factors can be calculated as follows:

$$K_y = \frac{E\left(q_{out}(k)n_q(k)\right)}{E\left(n_q^2(k)\right)}$$
$$K_x = \frac{E\left(q_{out}(k)\right)}{m_{qin}} \qquad (8)$$

From this equations it can be made clear that the system is not unconditionally stable. The numerator of the first equation is the cross-correlation of the quantizer-input noise and the output signal. Because the mean value of $q_{out}(k)$ has to follow the DC-input signal and this signal is limited to plus and minus one, it is clear that the cross-correlation will decrease when the DC-input signal is larger. When the root-locus is examined, it becomes clear that there exists a critical minimal value of K_y below which the system is unstable: this is the value where the locus crosses the unity-circle [8].

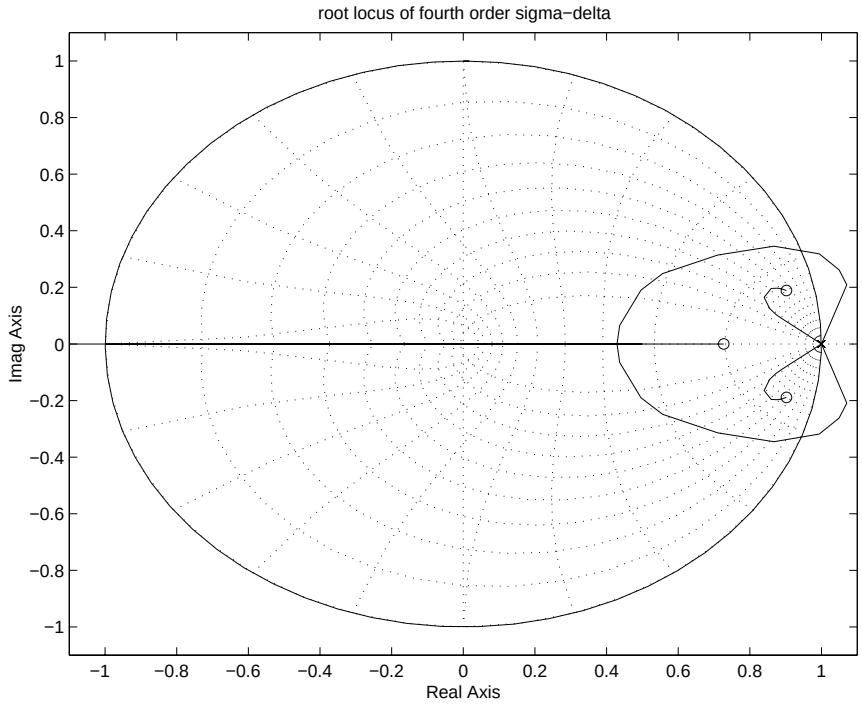

Figure 5 Rootlocus of a 4-th order Sigma-Delta

Although this method gives result that are accurate, still there are some ways to improve the result. This is because this approach takes the complete noise-signal into account to calculate the amplification factor. However, it is the amplification factor at the frequency of the limit-cycle that is of importance for the stability-analysis.

5. Quantizer induced phase shift

In the last chapters, efforts where made to model the quantizer as an amplification factor. Recent studies show however that the quantizer introduces an additional phase shift [9]. This is because the quantizer output is not changed at the exact moment a zero crossing occurs, but rather after a delay. This is caused by the uniform sampling speed. Let the input signal to the quantizer be a sinusoid with a frequency which is an rational fraction of the sampling period:

$$\frac{f}{f_s} = \frac{m}{M} \tag{9}$$

Where the greatest common divider of (m,M) equals 1. If this is the case, an extra phase shift will be introduced by the quantizer. This is shown in Fig. 6 for a frequency of f_s /4. The arrows are the instances at which a sample is taken.

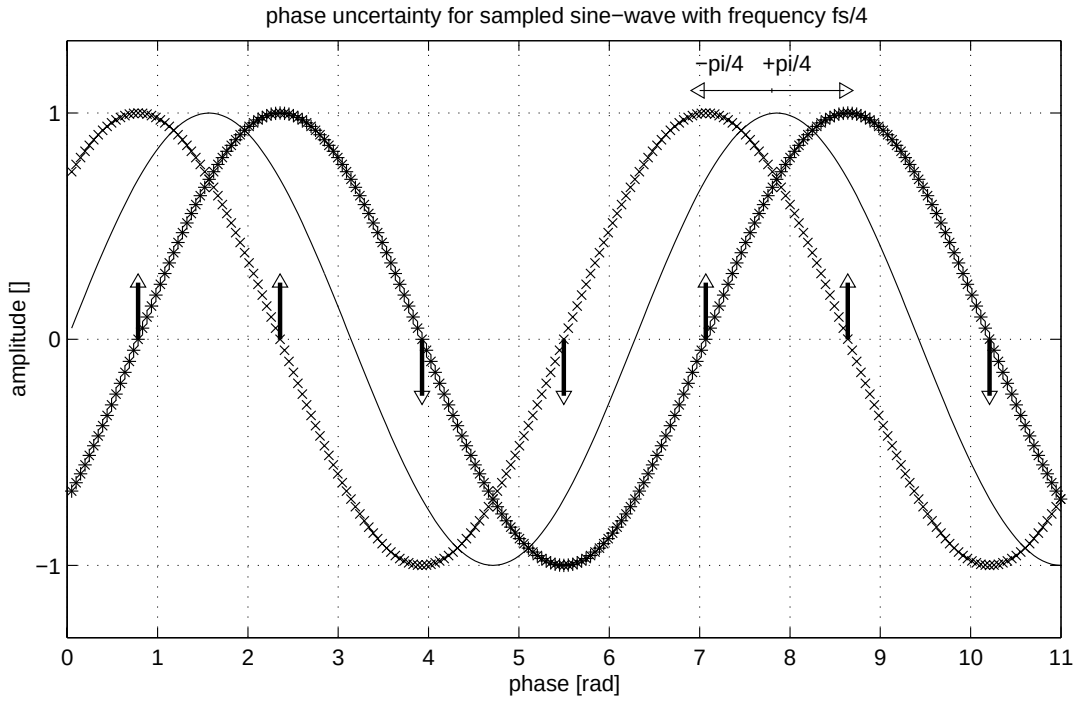

Figure 6 Phase uncertainty of a (single bit) sampled sine-wave for a frequency f_s /4

For a quantizer with quantization levels $q_n = q \cdot p$, with q the quantization step size and p an integer

$$\Phi:\phi=\begin{cases}\arcsin(\frac{pq}{A})+\frac{2\pi}{M}k & M=2l\\ \arcsin(\frac{pq}{A})+\frac{\pi}{M}k & M=2l+1\end{cases} \tag{10}$$

Where Φ is the set of phases for which the uncertainty equals zero. If ϕ_2 and ϕ_1 are adjacent solutions in Φ, the maximum absolute phase uncertainty (or possible phase shift) equals

$$\Delta\phi_{\max}=\tfrac{1}{2}\max(\phi_2-\phi_1) \tag{11}$$

For a single bit quantizer there is only one quantization level (p=0), this means that (3) becomes:

$$\Phi:\phi=\begin{cases}\frac{2\pi}{M}k & M=2l\\ \frac{\pi}{M}k & M=2l+1\end{cases} \tag{12}$$

Calculating the maximum phase shift is straightforward now:

$$\Delta\phi_{\max}=\begin{cases}\frac{\pi}{M} & M=2l\\ \frac{\pi}{2M} & M=2l+1\end{cases} \tag{13}$$

The maximum phase shift for a number of frequencies is depicted in Fig. 7.

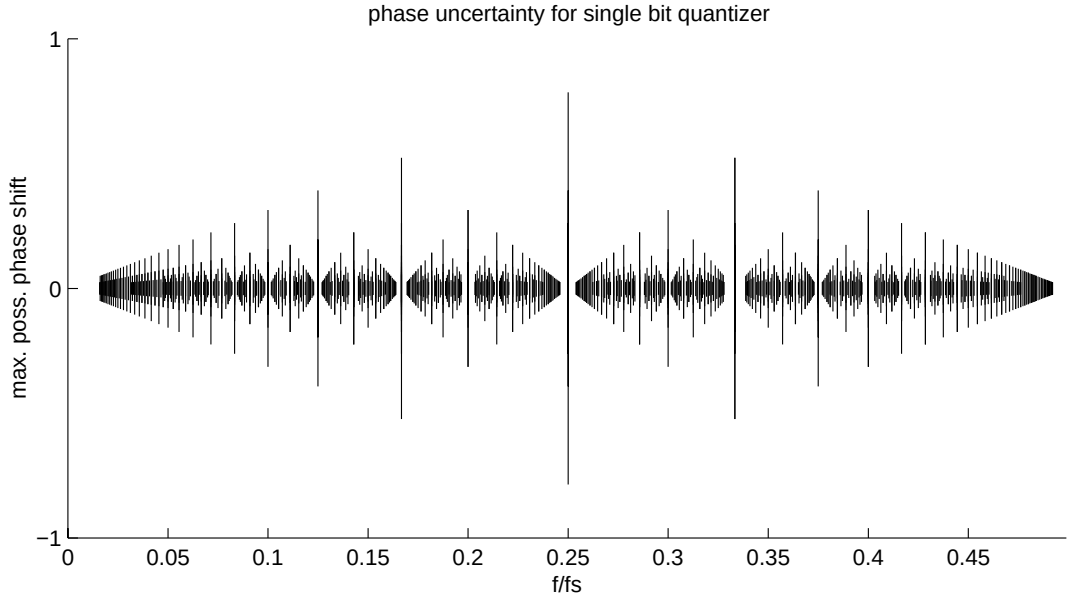

Figure 7 Phase uncertainty for rational input frequencies

This phase shift should be taken into account when modeling the Delta-Sigma modulator because it provides a way of predicting limit cycles, which have great impact on the stability of the system.

If a multi-bit quantizer is used instead of a single-bit the situation gets more complicated, because then the possible phase shift is dependent on the number of bits that are actually used.

6. Analysis of the stability problem

6.1. Analysis of the quantizer in- and output signals

The signal at the quantizer output q_{out} can be considered a stationary random variable. Since, due to the feedback loop, the signal at the quantizer input q_{in} is an integrated version of q_{out} and integration causes the distribution to approach a Gaussian one, it can be modeled using a zero-mean Gaussian noise source $n_q(t)$ and a DC signal m_{qin}. Because the signal at the quantizer output is just the sign of the input signal, its mean m_{qout} can be calculated using :

$$\begin{aligned} E\left(q_{out}(t)\right) &= P\left(n_q(t) + m_{qin}(t) > 0 \mid m_{qin}(t) > 0\right) \\ &- P\left(n_q(t) + m_{qin}(t) < 0 \mid m_{qin}(t) > 0\right) = erf\left[\frac{m_{qin}}{\sigma_n\sqrt{2}}\right] \end{aligned} \tag{14}$$

Here σ_n denotes the standard deviation of noise source $n_q(t)$. When the filter consists of a chain of n integrators (Fig. 2), the value of σ_n can be calculated as follows:

$$\sigma_n = \sum_{i=1}^{n} \left(E(n_c)\cdot 0.5\right)^i \cdot \prod_{j=n-i+1}^{n} \alpha_j \tag{15}$$

Here $E(n_c)$ is the mean value of consecutive positive or negative signals and α_j are the integrator constants of the filter. When m_{qout} and σ_n are known, the probability that the number of consecutive positive and negative signals exceeds a certain number p can be calculated using:

$$P(n_c > p) = erf\left[\frac{p\cdot\alpha_n(1-m_{qout}) - m_{qin}}{\sigma_n\sqrt{2}}\right] \tag{16}$$

Fig. 8 gives a histogram of the occurrences of consecutive signals for a 4-th order modulator with a negative DC input of 50% full scale. Because $E(n_c)$ is only known after σ_n is calculated and vice versa, some iteration is necessary. From (3), the probability density distribution can be calculated. Now the autocorrelation function of q_{out} can be estimated using:

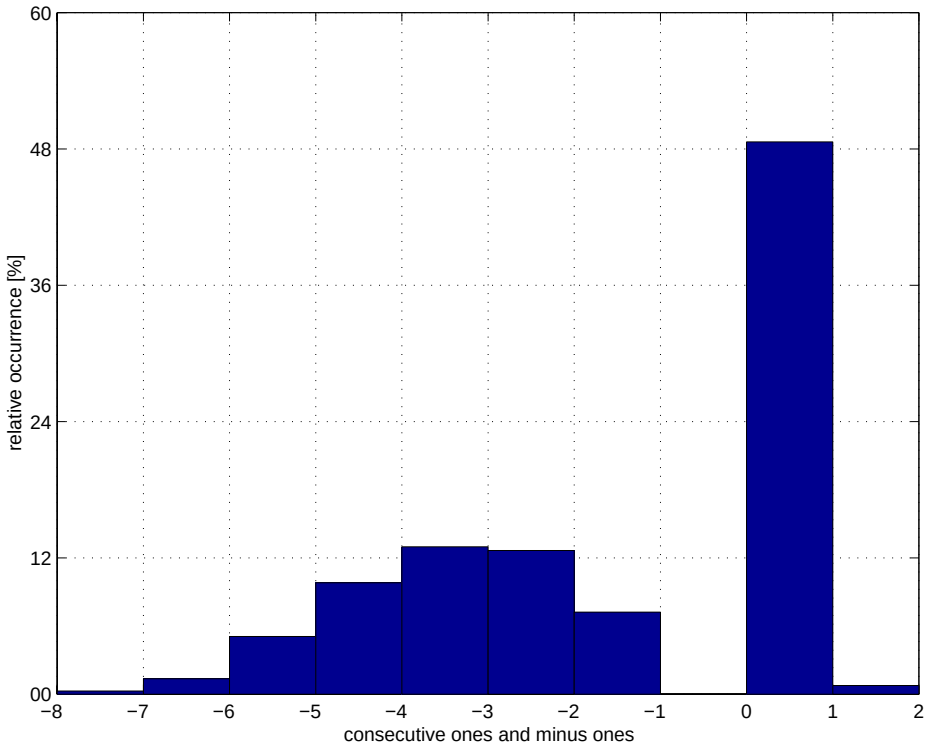

Figure 8 Consecutive positive and negative signals

$$R_{qout}(l) = E(q_{out}(k) \cdot q_{out}(k+l)) \approx$$
$$P(q_{out}(k) \cdot q_{out}(k+l) = 1) - P(q_{out}(k) \cdot q_{out}(k+l) = -1) \quad (17)$$

Because there is no information of the order in which the different slots of consecutive positive and negative signals occur, only the first few values of R_{qout} can be trusted. The feedback path between q_{out} and q_{in} is a linear filter. The corresponding autocorrelation of q_{in} can be calculated using :

$$R_{qin}(l) = R_{qout}(l) * h(l) * h(-l) \quad (18)$$

Where * denotes the convolution and h(l) the impulse response of the linear filter. Fig. 9 gives an example of simulated autocorrelation functions for q_{in} and q_{out}. The applied input signals are DC signals on 10 and 50 percent full scale and the filter is assumed ideal. To be able to make a better comparison, the mean values are subtracted. From Fig. 9 it can clearly be seen that for the determination of $q_{in}(k)$ only $q_{in}(k-1)$is of importance, while for larger inputs also $q_{in}(k-2)$, $q_{in}(k-3)$…

should be taken into account. For $q_{out}(k)$, this is not the case. The implication of this can be found by looking at the entropy of the noise source. The entropy $H_\infty(u)$ of a stationary source with memory is given by ($H(u)$ denotes entropy of u):

$$H_\infty(u) = \lim_{L\to\infty} \frac{1}{L}\left[H(u_1) + H(u_2 \mid u_1)...H(u_L \mid u_1 u_2..u_L)\right] \tag{19}$$

From elementary information theory it follows:

$$\frac{1}{L}\left[H(u_1)+(L-1)H(u_2 \mid u_1)\right] \geq \frac{1}{L}\left[H(u_1)+H(u_2 \mid u_1)+(L-2)H(u_3 \mid u_2 u_1)\right] \tag{20}$$

This basically means that for a noise source with larger memory, the entropy and thus the randomness is lower. Because normally, randomness of the input signal tends to smooth the non-linearity, a decrease in randomness will make the system less stable [10]. This will eventually cause the limit-cycle to grow and the input signal will go out of bounds.

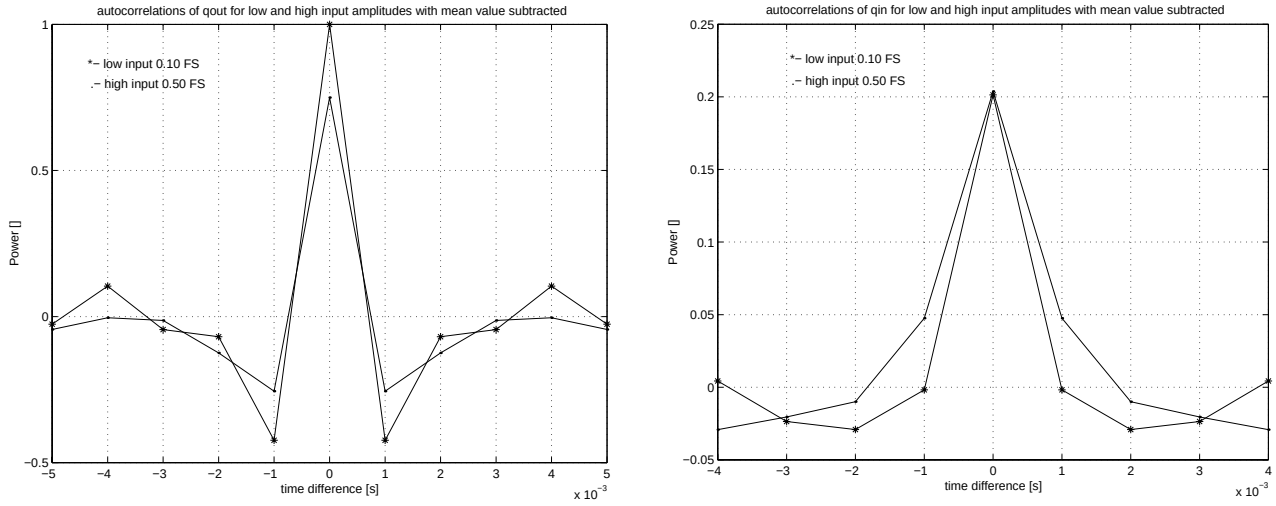

Figure 9 Autocorrelations for q_{in} and q_{out}

The problem now is to predict for which DC-input the quantizer input will go out of bounds. This can be found by extracting the effective K_y at the frequency of the limit cycle. The limit cycle frequency is given by the root-locus, taking into account the additional phase shift of the quantizer. However, it is not possible to do this using a Fourier transform due to the nonlinear character of the

quantizer. Fourier transformation gives the amplitude of a certain frequency component over the total signal, while the interest here is to see what is the effective gain for a short signal, which has some frequency content near the limit cycle. A tool that does this is the so-called wavelet transformation.

6.2. Introduction of wavelets

Wavelets are mathematical functions that cut up data in different frequency portions. The original signal is recursively split in a high-frequency (detail) and a low- frequency part (average). The different portions can then be studied according to scale [11-12]. Here the simplest wavelet is used, the Haar wavelet, because its properties fit nicely with the properties of the modulator's output signal. The Haar wavelet is a step function with value 1 for [0,0.5) and value –1 for (0.5,1]. A data signal (which will be denoted as $a_n(k)$) is cut up in two frequency portions by means of the operators *A* and *D*. Operator A generates the low-frequency part of the signal and is defined by:

$$a_{n+1,k} = A(a_n(k)) = \frac{a_n(2k) + a_n(2k-1)}{2} \tag{21}$$

The signal a_{n+1} has only half as many data points as the original signal a_n. Because the original data represented a (real) signal with a length of T seconds, the sample period should be doubled. Operator D is defined as:

$$d_{n+1,k} = D\big(a_n(k)\big) = \frac{a_n(2k) - a_n(2k-1)}{2} \tag{22}$$

This operator outputs a coefficient $d_{n+1,k}$ that acts as multiplication factor for the Haar function. The decrease of the number of data points is now resolved by scaling the corresponding Haar functions. The n-th order wavelet decomposition can then be obtained by applying the sequence of operators A^n, $A^{n-1}D$,..., A^1D, D to the original signal (which is denoted as a_0). Any function *f(x)*

with length 2^n, that is an element of $L^2(R)$, can be wavelet decomposed by order m (m>0) and represented by :

$$f(x)=\sum_{k=1}^{2^{n-m}} a_{mk} p_k(x)+\sum_{j=1}^{m}\sum_{k=1}^{2^{n-j}} d_{jk}\psi_{jk}(x) \qquad (23)$$

Where ψ_{jk} represent the scaled and translated Haar functions:

$$\psi_{jk}=\psi(2^{n-j}x-(k-1)2^{j-n}) \qquad (24)$$

and p_k is the translated pulse function:

$$p_k(x)=\varepsilon(x-\frac{k-1}{2^{n-m}})-\varepsilon(x-\frac{k}{2^{n-m}}) \qquad (25)$$

6.3. Analysis of the stability

Wavelet decomposition can now be used on q_{out} and q_{in}. Performing the wavelet operation means calculating the local correlation between the wavelet and the signal. If now the order of the wavelet is chosen, such that the scaling factor is near the period of the limit cycle, the resulting correlation is proportional to the presence of the limit cycle. If this is done for q_{out} and q_{in}, for a number of samples, a linear relation (i.e. an amplification factor) between the two signals can be found using a least square fit. A problem here is that discrete wavelets scale by multiples of a factor two. This means that for periods of limit cycles that are not (near) a power of two, the results are not accurate enough. A way to avoid this is using a symmetric square wave of a period that matches the limit cycle instead of a regular Haar wavelet. If the quantizer is replaced by the calculated effective K, the system can be analyzed with linear methods. Because the amplification factor should be determined independently from the phase, the points where a sign change occurs due to the inherent phase shift, are corrected before the calculation of the K. The exact value of the critical K can be found by comparing the phase margin of the system with the calculated phase shift. Because the value of *K* tends to decrease with increasing input amplitude, a maximum value for the input

amplitude can be found at the point where the system has zero phase margin. This will be called the stability boundary point. The advantage of using this model for the stability analysis is that only a small number of data points are needed to be able to extract the amplification factors and phase shifts. This is much more efficient than the traditional sledgehammer approach of performing a long transient simulation to detect whether a ΣΔ-modulator is stable or not.

To verify the method, a number of long transient simulations (200k samples) are done on the same system (a 4^{th}-order Sigma-Delta modulator), but the value of the integrator constant α_1 is changed from 0.05 to 0.45 (see Fig.2). Values higher than 0.45 are not of practical use because the stability boundary is very close to zero there.

The system is decided to be unstable when the input to the quantizer gets out of bounds [3]. In this manner, a maximum value for the amplitude of the input signal can be determined as a function of α_1. The results are then compared to the stability boundary predicted by the described method. The input signal is always DC, but with different amplitudes. A binary search algorithm is used to calculate the stability boundary. The CPUtime needed is given by:

$$t_{cpu} = N \cdot I_{bs} \cdot P_{\alpha_1} \tag{26}$$

Where I_{bs} is the number of iterations used for the binary search and P_{α^1} is the number of values for which α_1 is evaluated. The values of these two parameters are the same, both for the linear model as for the long transient simulation. t_{cpu} is thus determined by the number of samples N. For the linear case, this value is 8192, for the long transient simulation 200k (the latter is chosen, because an increase does not yield a significant change in the result). Taking into account the time needed for the wavelet decomposition (about 15 percent of t_{cpu}), the method based on the linear model is forty times faster than the traditional approach of long transient simulation. The difference between the stability boundary value resulting from long transient simulations and the predicted value is less than 5 percent as shown in Fig. 10. This indicates that this approach can be used to predict the

stability of a Sigma-Delta modulator in a short CPU-time. During the design phase, it can now easily be checked whether the system meets the stability specification, which will speed up the design.

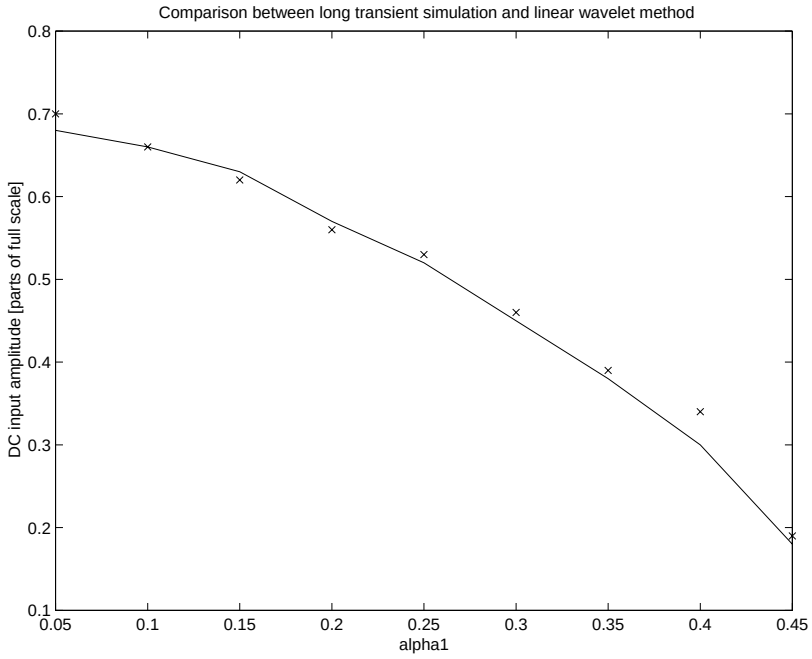

Figure 10 Comparison between long transient simulation and wavelet method

7. Model based on recursive functions

Looking at the autocorrelation function of q_{in} it is clear that there exists a relative large correlation between signals that are shifted one unit of time. This means that an (non-linear) operator G :R->R can be defined:

$$q_{in}(k) = Gq_{in}(k-1) + n_{add} \tag{27}$$

Where n_{add} is an additive noise source, necessary because $q_{in}(k)$ is not only dependent on its predecessor but also on the precise values stored on the other integrators (which are not taken into account). Fig. 11 shows a scatter plot of the different values for $q_{in}(k)$ and its successor. In this case, G can be approximated by a piecewise linear function (solid lines).

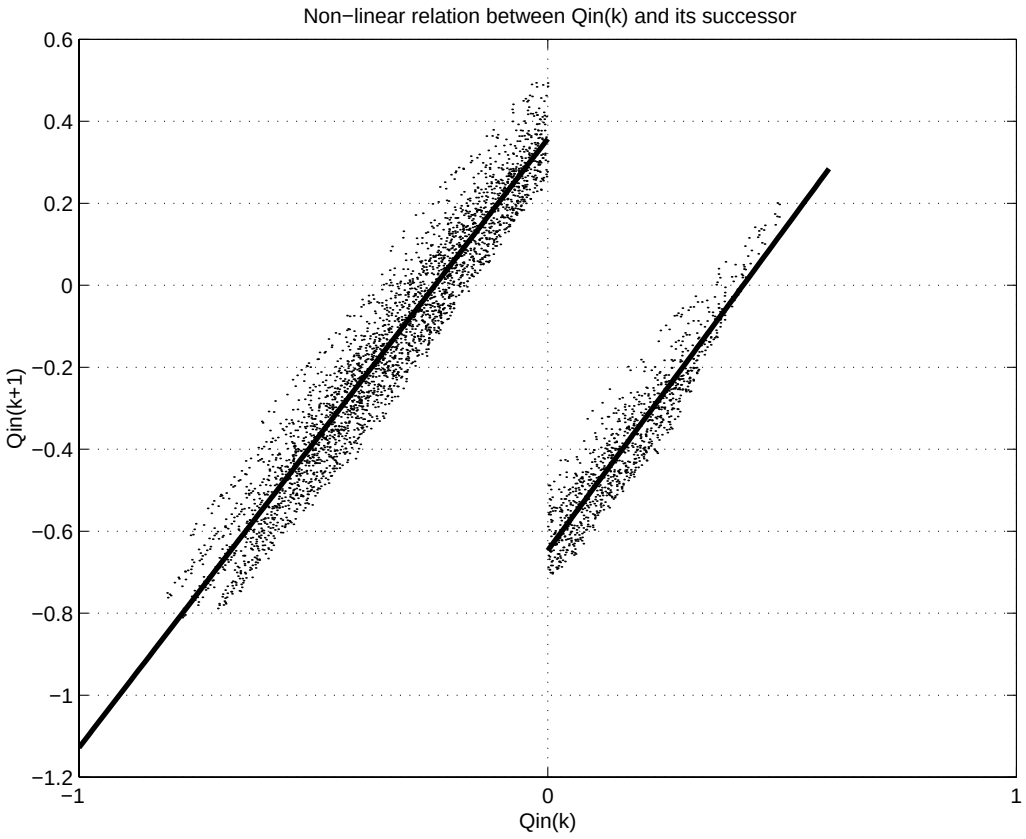

Figure 11 Recursive relation between $q_{in}(k)$ and $q_{in}(k+1)$

Using this PWL model, the Sigma-Delta modulator can be simulated very efficiently. However, for a realistic simulation it is not sufficient to only look at the predecessor, i.e. values further in the past should also be taken into account. This is certainly the case for large-range inputs, because of the already mentioned change in autocorrelation function. Future work will be focused on these types of models for a Sigma-Delta modulator.

8. Conclusions

Most of the existing models are not suited to be used in system-level simulations. Spice models and even functional descriptions of the system require usually to much calculation time so it is not possible to evaluate for instance the input-output stability. The models based on replacing the quantizer by a linear equivalent are fast, but are not very accurate in modeling stability and limit-cycles. Furthermore, recent work shows that any accurate model should incorporate a phase shift due to the phase uncertainty in the quantizer.

Recently, it was shown that information regarding the stability of sigma-delta modulator can be obtained by performing a functional simulation for a limited number of samples, followed by wavelet decomposition. This method can be advantageous during the design cycles, because accurate information on the stability of the system is available without the need for long transient simulations. However this method works by extracting information from a limited number of samples rather than speeding the simulation time for each sample. This means that whenever a large number of samples is needed, there is still need for a faster model.

One way of speeding up this process is by finding a relation between the current value of the quantizer input and its past values. In a first order approach, a piecewise linear function can be found between the current value and its predecessor. These types of models will be investigated in the remainder of the project.

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