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Assessment of the Performance of Flow Measurement Instruments and Techniques

Project output 6 – Uncertainty analysis of discharge determination via various techniques

Task (a) Literature review of methods for estimating the uncertainty

A literature review of methods for estimating the uncertainty associated with stage-discharge relations

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Contents

1 Scope of this literature review

The expression of the uncertainty associated to stream discharge measurements or estimates is of paramount importance in issues related to water resources, flood frequency analysis, flood control, flood and drought forecast, compensation discharge and water use conflicts. The most common and most simple method for monitoring streamflow at a hydrometric station is to establish a stage-discharge relation (rating curve, e.g., Rantz, 1982, Schmidt, 2002) used to convert continuously recorded water level into discharge time series. In situations where a simple stage-discharge relation cannot be established with acceptable errors, more complex relations including additional parameters such as the water slope, the stage change rate, or an index flow velocity, may be developed. This literature review is restricted to 1) documents focusing on the most widespread relation between stage h and discharge Q; and 2) documents proposing methods for estimating the uncertainty associated with the rating curve. Works dealing with other discharge determination methods or with methods only used for establishing the rating curve without uncertainty analysis were not considered in this review.

The accuracy and stability of a rating curve depend on the hydraulic conditions prevailing at the site, on the knowledge of the physical processes linking water stage and discharge at the site, and on the availability and uncertainty of individual ratings, i.e. a set of direct stage-discharge observations. The methodology for assessing the uncertainty associated with a given stage-discharge relation, over a given period of time and for given hydraulic conditions, still appears to be an open scientific issue. Along with a quick reminder of the position of the problem, the following sections intend to provide a summary of relevant works reported in the literature. As a conclusion, general requirements for an ideal and widely accepted methodology are presented.

2 Position of the problem

2.1 Uncertainty associated with stage-discharge relations and derived quantities

First of all, the difference between uncertainty in the stage-discharge relation (in reference hydraulic conditions) and uncertainty in the derived instantaneous discharge Q_t (in the actual hydraulic conditions at time t) must be acknowledged. Following the first-order approximation proposed by the ISO GUM, and assuming independent error sources, the combined uncertainty in Q_t may be written as follows:

$$u^2(Q_t) = u^2_{RC}(Q) + u^2_{HC}(Q_t) + \left(\frac{\partial Q}{\partial h}\right)^2 u^2(h_t) \tag{1}$$

where:

1. $u_{RC}(Q)$ is the uncertainty in the stage-discharge relation established for reference (or usual) hydraulic conditions. Related errors mainly arise from improper modelling/knowledge of the hydraulic relation, typically the shape of the assumed functions and the calibration of the parameters. Even implicitly, a model of the hydraulic relation is always assumed and must be justified physically. This is particularly important in regions of the curve where individual ratings or other hydraulic data are scarce or missing, i.e. in the extrapolated parts of the stage-discharge relation for high flows, or towards the cease-to-flow level. $u_{RC}(Q)$ may become detrimental in these extrapolated regions of the rating curve.

2. $u_{HC}(Q_t)$ is the uncertainty related to errors due to real hydraulic conditions at time t, potentially differing from the reference flow regime. Sources of errors include transient flow effects (hysteresis) and variable hydraulic controls (backwater effects in non-uniform flows, seasonal vegetation changes, changes in reach or control section geometry).

3. $u(h_t)$ is the uncertainty in instantaneous water level measurement h_t at time t. According to the first-order approximation made here, it propagates proportionally to the sensitivity coefficient of the stage-discharge relation $\partial Q/\partial h$.

Finally, hydrologists often want to compute the uncertainty in the quantities derived from the hydrograph (instantaneous discharge time series) computed from the rating curve and the water level time series. Those quantities may be averaged hydrological data (mean discharges, volumes, etc.), hydrological statistics (flood quantiles), or other variables such as the cost of flood damages or the results of hydraulic models using the discharge as input data.

2.2 Physical basis: hydraulic control and reference regime

A simple rating curve is a one-to-one relation between water stage and discharge assumed to prevail at a cross-section of the flow in the reference hydraulic conditions. This reference hydraulic regime is seldom explicitly defined. Nevertheless, any time the flow deviates from the reference regime, significant errors in the discharge estimate may appear. Such errors must be distinguished from the errors directly related to the reference stage-discharge relation. Most often, the reference regime refers to the mean hydraulic conditions usually prevailing in the considered flow [Schmidt, 2002], i.e. steady flow (no transient effects) and usual hydraulic controls (e.g. no varying backwater effects, no change in channel roughness or the geometry of the controlling cross-section). When the reference regime is permanently changed, e.g., in case of changed channel geometry after a flood, the rating curve is no longer valid, and a new one must be established corresponding to the new reference regime. Temporary changes of the reference regime, e.g., in case of seasonal vegetation growth or variable downstream boundary condition, may impose the use of different rating curves according to time periods.

The physical characteristics of the channel which govern the link between stage h and discharge Q at a section constitute the hydraulic control. Basically, two kinds of hydraulic controls may be distinguished: channel versus section controls (Fig. 1ab). When channel control occurs, the flow is mainly regulated by the geometry and roughness of a certain extent of the channel. In non-uniform flow cases, the downstream boundary condition may also influence the stage-discharge relation (backwater effects). When section control occurs, the flow is mainly regulated by the geometry and surface state of a cross-section or hydraulic work where the flow becomes critical due to a water fall (e.g. riffle, weir, sill) or due to constriction (e.g., Venturi, Parshall flumes). Since this control cross-section is located in the downstream vicinity of the stage measurement point, head losses related to the flow approach conditions are accounted for with a discharge coefficient.

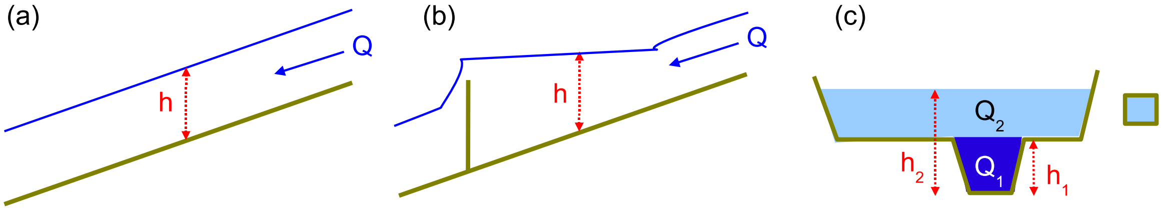

Figure 1: Channel (a) versus section (b) hydraulic control along an idealized channel reach. Variable hydraulic control according to discharge range (c), in a compound channel cross-section with a flood evacuation orifice.

Hydraulic controls might change the role they play commensurate with the stage in the river, i.e., active for low flows, negligible for high flows. Fig. 1c shows the example of a compound channel cross-section where flood flows are controlled not only by the main channel, but also by the floodplain and a flood evacuation orifice. In this case, channel and section controls are activated simultaneously.

Usual hydraulic formulas for uniform flows (Chézy, Manning-Strickler) and for usual hydraulic works (weirs, gauging flumes. . .) can be approximated by simple power functions of the following general expression:

$$Q = \alpha(h - h_0)^n \tag{2}$$

where α is a coefficient, h_0 is a cease-to-flow reference level, and n is an exponent with typical theoretical values 5/3 (Manning-Strickler), 3/2 (rectangular weir), 5/2 (triangular weir).

As soon as hydraulic controls are identified and some hydraulic data are gathered, it is therefore possible to model most stage-discharge relations using sums or piecewise combinations of power functions like Eq. 2, suited for different segments of the stage-discharge relation. Of course, numerical hydraulic models can be built and calibrated to compute the stage-discharge relation. Since Q and h are regarded as cross-section averaged parameters, a one-dimensional model can be used, as well as more sophisticated models.

Whatever the complexity of the model, the quality of the results will be poor if the relevant hydraulic controls are not taken into account properly.

2.3 Uncertainty in individual stage-discharge ratings

Stage-discharge ratings conducted by hydrometry staff consist of direct measurements of stage and discharge over a reduced period of time, in given hydraulic conditions. It should be noted that most authors recognize that uncertainty in stage measurements of individual ratings are negligible compared with uncertainty in discharge measurements. It could also be argued that an individual rating is an estimation of discharge knowing a given stage. Thus, all the uncertainty of the rating can be expressed in the discharge term.

At least three kinds of errors should be considered when such measurements are to be used for establishing or evaluating a stage-discharge relation:

1. errors related to the measurement process itself, combining instrumental errors, environmental errors, human errors, and spatial integration errors
2. time integration error due to flow non-stationarity during the measurement
3. systematic error due to hydraulic conditions deviating from the reference flow regime

The literature provides some methods for quantifying discharge uncertainty according to the measurement technique (e.g. for current-meters Herschy, 2002, Pelletier, 1988). However, the task remains challenging for many practitioners facing a range of situations and techniques, and some research work is still ongoing in order to improve the uncertainty analysis of individual discharge measurements. It is remarkable that none of the works referenced in this review on rating curve uncertainty analysis was based on a thorough uncertainty analysis of individual ratings. In many cases indeed, an arbitrary relative uncertainty value was fixed for all discharge data.

When the flow significantly varies during the measurement process, time integration error e_1 may occur (cf. Fig. 2). Additionally, a systematic hysteresis error e_2 due to transient flow effects may also occur. Both errors are different in essence, and one may be negligible compared with the other, according to the flow unsteadiness and to the measurement duration. For instance, the Jones [1916] formula, using the stage change rate, may be applied to estimate the overestimation (rising flow) or underestimation (falling flow) of the reference discharge due to hysteresis for a given water stage (error e_2 in Fig. 2). Deviations from the reference flow regime due to other processes than hysteresis may be difficult to quantify if the physical cause of the deviation (e.g. backwater effect) is not precisely identified.

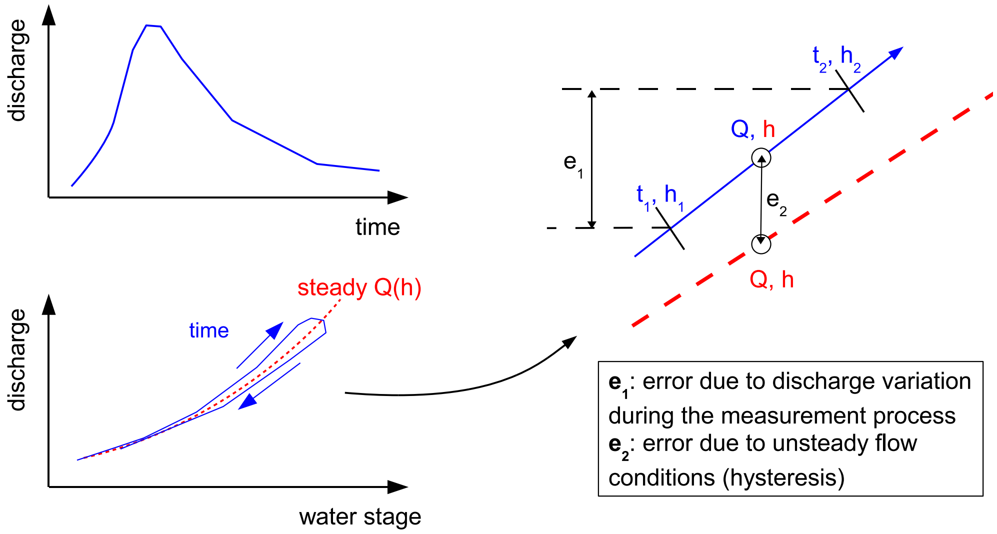

Figure 2: Time integration error e_1 due to variable flow during individual discharge measurement versus error e_2 due to transient flow effects (hysteresis).

3 Methods for the expression of uncertainty in stage-discharge relations

3.1 Hydraulic modelling and sensitivity analysis

A first approach based mainly on hydraulic analysis of the stage-discharge relation can produce a valuable quantification of errors, since the physical basis of such errors is explicitly defined. Sensitivity analysis of the hydraulic model parameters provides an estimation of realistic error bounds. However, translating these worst-case errors into probabilistic distributions from which uncertainty may be derived and combined is usually not a straightforward task.

In his seminal PhD work, Schmidt [2002] explored an exhaustive range of hydraulic formulations for stage-discharge relations, and considered all sources of error. However, the proposed comprehensive error analysis does not constitute a proper uncertainty analysis, since no basic statistical considerations are developed. He distinguished three kinds of errors due to natural uncertainties, knowledge uncertainty and data uncertainties. These three error sources are respectively linked with non-stationarity or fluctuations of natural processes, with limited and imperfect knowledge of the real hydraulic controls, and uncertainty in available hydraulic data, primarily individual ratings but also all data used to assess the hydraulic controls. As pointed out by Westerberg et al. [2011], *there is an overlap between the two categories knowledge and natural uncertainty, which Schmidt uses because natural variability results in non-random and non-stationary structure that should be treated as knowledge or epistemic uncertainty.* Actually, the definition of the three error categories may appear more philosophical than physical, potentially producing misleading controversy in worst cases. If we relate the knowledge uncertainty to $u_{RC}(Q)$ as defined in Section 2.1 and the natural uncertainties to $u_{HC}(Q_t)$, including potential variability of the reference hydraulic regime for which the rating curve is established, there is no longer overlap and both uncertainty types are meaningful. Anyway, it is useful to clearly distinguish the data (or individual ratings) uncertainties from the rating curve uncertainty. In several works, some confusion arises from the use of the data uncertainties to estimate the two other types of uncertainty.

Using a 1D hydraulic model of a section of the Po river, Italy, Di Baldassarre and Montanari [2009] investigated stage-discharge errors considering discharge data errors, interpolation and extrapolation rating curve errors, unsteady flow effects, and vegetation impact on channel roughness. The errors estimated in this case study turned out to be far from negligible. The uncertainty analysis method is limited to sensitivity analysis performed with steady flow 1D hydraulic simulations, and not directly using the individual ratings. Conclusions are site-specific. Another important point is the undiscerning use of the uncertainty analysis method for velocity-area discharge measurements given in ISO 748 standard.

Lang et al. [2010] and Neppel et al. [2010] also used a 1D hydraulic model to derive stage-discharge errors from sensitivity analysis. First, the 1D hydraulic model is calibrated using available discharge and water level data. Assuming a 5 cm uncertainty in water levels and a 10% uncertainty in discharges, lowest and highest values of the Manning flow resistance parameters are derived corresponding to worst cases. In the floodplain where no measurements are available usually, extreme Manning coefficient values of 1/10-1/20 are considered from expert knowledge. The model is also used to study the reference hydraulic regime at the site for different flood discharges. Here again, the probabilistic meaning of the stage-discharge error bounds for further uncertainty analysis is not straightforward.

Following another approach, Freestone [1983] conducted sensitivity analysis on fitted rating curves for a set of gauging stations, to assess the propagation of stage errors in the discharge uncertainty. He found that basins with small flows are the most sensitive to stage errors, which could be expected. He also concluded that gains from weirs/flumes are not enough to offset the decreased sensitivity due to small flows.

3.2 Regression and likelihood techniques

With the development of computers, usual non-linear regression techniques replaced graphical or analytical techniques previously used to establish stage-discharge relations. More versatile likelihood-based techniques were also proposed and demonstrated to be better suited to stage-discharge relations. Such regression and likelihood techniques provide mathematically sounded estimation of errors that can be used for the uncertainty analysis of stage-discharge relations. However, some assumptions have to be made, sometimes implicitly, on the probability density functions associated with parameters and errors. Besides, expert knowledge about the physical processes is usually not taken into account properly. It is not always clear if $u_{RC}(Q)$ and/or $u_{HC}(Q_t)$ are considered, and how the data uncertainties are managed.

The work by Venetis [1970] seems to be the first published statistically sounded method for computing uncertainty associated to rating curves, based on nonlinear regression of a single segment power function (Eq. 2). Errors are estimated from the variances of the parameter estimates. In the same way, Dymond and Christian [1982] suggested a new method accounting not only for rating curve error and stage error, but also for error caused by ignoring all physical parameters other than stage. Unfortunately, the methodology application seems not straightforward and leads to likely underestimated uncertainty levels.

In works by Herschy [1999], Clarke [1999], Clarke et al. [2000], the rating curve uncertainty analysis is based on the residual variance from regression of a power function like Eq. 2, and possibly on the standard error of the parameter estimates.

Olivier et al. [2008] built a fully GUM-compliant uncertainty analysis of stage-discharge relations, based on a variance propagation equation similar to Eq. 1, but with $u_{HC}(Q_t)$ replaced by the mean individual ratings uncertainty. As this assumption is not explicitly justified, some rules are proposed for cases in which this term may be withdrawn of the equation. The method is applied successfully to study cases with stationary stage-discharge relations, leading to combined uncertainty of the order of 10%, at 95% confidence level. The advantage of the method is to clearly separate uncertainty in the stage-discharge relation and uncertainty in the instantaneous discharge estimate. However, the inclusion of data uncertainty in the computation of the global stage-discharge uncertainty remains unsatisfactory.

Petersen-Overleir [2004] proposed a likelihood-based method to take into account the usually observed heteroscedasticity of stage-discharge relations. Indeed, he showed that classical non-linear least squares method leads to unaccounted heteroscedasticity in the stage-discharge relations for several study cases. A heteroscedastic maximum likelihood model was introduced. However, it did not take into account variable relative uncertainties on individual rating data, which would be useful to separate the heteroscedasticity of the stage-discharge relation and the increasing relative uncertainty in low flow discharge measurements, for instance. The same author extended the nonlinear regression approach to more complex stage-discharge relation cases, including multi-segment (or piecewise) power functions [Petersen-Overleir and Reitan, 2005], hysteresis [Petersen-Overleir, 2006], and overbank flow in rivers with floodplains [Petersen-Overleir, 2008]. The continuity imposed to piecewise power functions produced numerical problems [Petersen-Overleir and Reitan, 2005]. Smooth transitions between segments over a short discharge range might make numerical resolution easier. The physical basis of the assumptions seems too loose since unexpected hydraulic exponents (> 3, > 4) are obtained. To account for hysteresis, Petersen-Overleir [2006] used stage-fall-discharge relations based on the Jones [1916] formula, which is out of the scope of the present review, focused on stage-discharge relations only. Petersen-Overleir [2008] presented a simple uniform flow depth-discharge model for a two-stage main channel-floodplain river section. The piecewise regression problem is solved using least-squares regression of rating curve parameters, including the main channel-floodplain change-point. Uncertainty of all parameters and discharge estimates is approximated by bootstrap techniques.

3.3 Bayesian inference and MCMC simulation

In recent years, the application of Bayesian inference and Markov chain Monte-Carlo (MCMC) simulations has brought new solutions to the problem of rating curve uncertainty analysis. While computational costs remain low, this approach shows decisive advantages for tackling the issue. First of all, hydraulic knowledge of the stage-discharge can be explicitly translated in prior distributions of the parameters of the assumed functions. This is a direct way to formalize the knowledge uncertainty. Second, it is possible to derive a likelihood function that accounts for the uncertainty in individual ratings, leading to heteroscedastic models similar to that of Petersen-Overleir [2004], for instance. This is also a satisfactory way to handle the data uncertainty. Based on the corresponding uncertainties, the best information from observations (likelihood functions) and model (hydraulic priors) will be used to produce the results in the form of a posterior distribution. Also, as the technique is directly based on probability density functions, uncertainty analysis can be achieved in an easy and clear way using percentiles of the posterior distribution.

MCMC simulation is an efficient way to estimate the probability density function of a random vector, by sampling realizations from the target distribution (here, the posterior distribution). the parameters of the variable according to their own probability density functions. This technique is very efficient in connection with Bayesian inference, but it can be used outside the Bayesian framework: McMillan et al. [2010] performed MCMC simulations of the stage-discharge relation (single segment power function) in a braided river in New-Zealand. They assessed the uncertainty in individual ratings, in the assumed form of the stage-discharge relation, in the extrapolation of the stage-discharge relation, and the uncertainty due to cross-section changes due to vegetation growth and/or bed movement.

To our best knowledge, Moyeed and Clarke [2005] reported the first published Bayesian analysis of stage-discharge relations, based on a power function, and Box-Cox transformation. They conclude that *Bayesian methods are appropriate for rating-curve calculation because their inherent flexibility (a) allows the incorporation of prior information about the nature of a rating curve; (b) yields credible intervals for predicted discharges and quantities derived from them; (c) can be extended to allow for uncertainty in stage measurements.* Some assumptions of this pioneering work appear questionable: no real analysis of the hydraulic relations and of the discharge measurements is performed, uncertainty on discharge data is quite arbitrarily assumed to be proportional to $\sqrt{h - h_0}$, and unrealistic hydraulic exponents are obtained.

Other Bayesian/MCMC studies of stage-discharge relations focused on the uncertainty due to the lack of individual ratings for a single-segment power function [Reitan and Petersen-Overleir, 2008], on the establishment, including extrapolation, of a piecewise power function [Reitan and Petersen- 2009], and on the rating procedures for gauging stations that are subject to variable backwater [Petersen-Overleir and Reitan, 2009a], using stage-fall-discharge relations for twin gauges, which are out of the scope of this document.

3.4 Stationarity analysis

Occasional changes of the reference hydraulic regime, due to changes in the hydraulic controls related to scour/deposition processes, vegetation cycles, debris/ice jams, dike break, or other processes, cause non-stationarity in the stage-discharge relation. Hydrometric station managers define periods of validity for established rating curves, depending on the detection of non-stationarity based on the deviation of individual ratings to the current curve, essentially. Until recently, little attention was paid in the literature to the stationarity analysis of stage-discharge relations. Most of the proposed uncertainty analysis methods assume that changes in the reference hydraulic regime are negligible or exceptional. For instance, McMillan et al. [2010] arbitrarily selected the 0.5-year return period discharge as a threshold to identify periods of stationarity of the stage-discharge relation between major flood events. However, changes in the hydraulic controls may affect certain discharge range and not other: individual ratings for large floods are often kept in the analysis even if the lowest part of the rating curve was changed due to bed geometry evolution.

To quantify uncertainty related to a mobile-bed river in Honduras, Westerberg et al. [2011] developed a weighted fuzzy regression technique within a moving time window. The stage-discharge model is a simple power function with Box-Cox transformation. A large temporal variability in the stage-discharge relation was found especially for low flows. The final estimated uncertainty limits in discharge varied from -43% to +73% of the best discharge estimate. This seems to be a valuable approach for assessing non-stationarity uncertainty in stage-discharge relations, though affected by a number of questionable assumptions: 1) assumed uncertainty levels in discharge ratings are unrealisticly high 25-30% and should be justified more precisely; 2) the form of the stage-discharge function is not justified hydraulically; 3) the definition of 30-point moving windows is arbitrary and independent of morphogenic events or vegetation growth periods.

Almost simultaneously, Jalbert et al. [2011] proposed a variographic analysis to account for the non-stationarity of single segment (power function) stage-discharge relations. At the beginning of the rating curve validity period, the initial uncertainty is quite arbitrarily fixed less than 5% according to a Gaussian distribution. Then a temporal variance term is derived from the temporal semi-variogram. The variographic approach to assess non-stationarity of the stage-discharge relation is elegant and efficient, however 1) instead of assessing continuous non-stationarity during expert-defined periods of validity, it would be interesting to detect non-stationary events, when sudden changes are linked to morphogenic floods for instance; 2) uncertainty in discharge ratings is not clearly separated from uncertainty in the stage-discharge relation; 3) a constant relative uncertainty instead of an in-depth analysis of individual ratings errors; 4) no hydraulic analysis of the stage-discharge relation is provided. The authors state that hydraulic knowledge of their sites is too poor to establish Bayesian priors, whereas very simple analysis from available information on the river (width, depth, slope, grain size, planform, hydraulic works. . .) would be possible and helpful.

Recently, Reitan and Petersen-Overleir [2011] proposed a Bayesian analysis including time evolution of the stage-discharge relations for unstable rivers.

It should be noted that stationarity analysis and uncertainty analysis under steady conditions are orthogonal approaches for rating curve uncertainty analysis, which should ideally be conducted in connection with each other. It is thus required to discriminate the uncertainty due to identified non-stationary processes from the uncertainty produced by other error sources, e.g., by the larger relative uncertainty on low-flow discharge ratings.

3.5 Uncertainty in instantaneous discharge and derived quantities

Only Olivier et al. [2008] and, in a more empirical way, Freestone [1983] estimated separately the uncertainty in the stage-discharge relation itself and in the discharge computed from the stage-discharge relation, accounting for uncertainty in the water stage continuous measurement. Uncertainty may be propagated proportionally to the stage-discharge sensibility $\partial Q/\partial h$ using the third variance term in Eq. 1, if one follows the first-order approximation proposed by the ISO GUM, or even more directly by MCMC simulations. This should be clarified in further works when the uncertainty on the instantaneous discharge time series is computed.

The uncertainty in other derived hydrological quantities was examined by several authors. Since a comprehensive review of this scientific issue would lead far beyond the initial metrologic purpose of this note, only a brief and incomplete overview is given hereafter, in relation with works dealing with rating curve uncertainty.

The propagated uncertainty in the average discharge over a given period of time was studied by Dymond and Christian [1982] (arbitrary period), Moyeed and Clarke [2005], Clarke [1999] (mean annual flood), Westerberg et al. [2011] (mean daily discharges). Amongst others, Westerberg et al. [2011] considered the error due to limited time resolution of water stage sampling.

As regards flood frequency analysis, Kuczera [1996] examined the errors induced in maximum likelihood estimators by correlated errors introduced by the extension of rating curves beyond the highest ratings. Clarke [1999] also took into account the correlations due to the rating curve for analysing mean annual flood and other percentiles. Petersen-Overleir and Reitan [2009b] also accounted for sample variability and rating curve uncertainty to compute significant wide confidence intervals around T-year flood estimates. Neppel et al. [2010] implemented a Bayesian flood frequency analysis accounting for random errors in water level records and systematic errors in the rating curve.

More generally, several works aim at establishing methods for a rigorous consideration of flow data uncertainty in hydrologic studies, ranging from hydrological model calibration [McMillan et al., 2010, Kuczera et al., 2010] to cost evaluation of flood damages, or hydraulic model results for instance.

4 Conclusions

Published works provide innovating solutions to tackle the issue of uncertainty related to stage-discharge relations and derived quantities. All acknowledged sources of errors seem to have been investigated, but a general uncertainty analysis framework is still missing in order to clarify some remaining confusions.

Further research work is therefore required for defining a versatile, practical and widely acknowledged method. Anyhow, some important requirements of an ideal method for expressing the uncertainty associated with instantaneous discharge derived from rating curves may be drawn as follows:

Expression of uncertainty should be compliant with the ISO GUM or any other standard methodology for uncertainty analysis. If possible, the best practice is to express uncertainty in the form of a probability density function.

Hydraulic analysis of the physical stage-discharge relation at a site must be quantified, even roughly, and taken into account in the rating curve analysis. This is particularly important for low-flow and high-flow ranges where the stage-discharge relation has to be extended outside of the more intensively rated region of the rating-curve.

Uncertainty analysis of individual ratings should be performed in a rigorous and individual way, including the deviation from the reference hydraulic regime. In the rating curve analysis, the uncertainty of each individual rating should be taken into account. Thus, *data uncertainty* sensu Schmidt [2002] should be clearly separated from the uncertainty in the stage-discharge relation itself (*knowledge uncertainty* sensu Schmidt [2002]).

Uncertainty analysis of stage-discharge relation in the reference hydraulic regime should be conducted using a sounded mathematical approach. To separate *knowledge uncertainty* from *natural uncertainty* sensu Schmidt [2002], the reference hydraulic regime must be elicited. Possible heteroscedasticity of the reference stage-discharge relation should be accounted for.

Non-stationarity of stage-discharge relation due to temporary/permanent changes of the reference hydraulic regime should be quantified in the uncertainty analysis, either continuously or periodically, according to the physical cause of the non-stationarity. Weighted fuzzy regression, variographic as well as Bayesian inference techniques are promising tools to assess non-stationarity of stage-discharge relations.

Uncertainty in the instantaneous discharge Q_t derived from the rating curve should be computed taking into account the uncertainty in the water level record and the potential deviation from the reference regime at time t, due to transient flow effects or backwater effects for instance.

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