Regionalisation of Rainfall-Runoff Models

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Abstract: A new approach to regionalisation for prediction of flow characteristics in ungauged catchments is presented. If flows in ungauged catchments are to be predicted using calibrated rainfall-runoff models, regional relationships between the parameters of such models and catchment attributes must be determined. This is only possible with parsimonious models (fewer than about 7 parameters), and even then there are considerable uncertainties in the prediction of ungauged catchment response, due to the accumulation of uncertainties in the regionalization process for estimating parameter values. The uncertainties in the catchment attributes and climate data used for the ungauged catchment combine with the uncertainty in the relationships between catchment attributes and model parameters for the gauged catchments. Moreover, these relationships are subject to uncertainty in both catchment attributes and gauged-catchment model parameter values. The latter are influenced by model structure, parameter identifiability, subjective selection of "optimal" parameter values and errors in the climate data. The uncertainty for the ungauged catchment can be minimised either by minimising each contributing uncertainty or by altering the approach so as to bypass some of them. An example of the second option is to regionalize catchment response characteristics rather than model parameters, then relate the ungauged-catchment model parameter values to these response characteristics. This bypasses the influence of model structure, parameter identifiability and subjective selection of "optimal" parameter values, thereby reducing uncertainty in the estimates of the response characteristics at the catchment scale. Possible response characteristics are the mean annual runoff coefficient, slope and shape of the flow duration curve, fraction of time with no flow, and the average-event flow-response curve. Initial regionalisation of these characteristics will be illustrated for a selection of catchments in Australia.

Keywords: Regionalisation; Rainfall-Runoff Model

1. INTRODUCTION

The ability to predict flows in ungauged catchments and to predict the effect of land-use change on catchment response remains an important goal of hydrology, as success indicates a useful understanding of the principal drivers of hydrologic response. This paper will discuss approaches using lumped, parametrically parsimonious conceptual models for prediction in ungauged basins, though some of the techniques described here may be helpful in the application of physics-based models.

Regionalisation of conceptual rainfall-runoff models is a popular approach to estimating flows in ungauged catchments [*e.g.* Post and Jakeman 1996; 1999, Post *et al.* 1998, Sefton and Howarth 1998, Kokkonen *et al.* 2003, Merz and Blöschl, 2004]. The technique involves calibrating models on gauged catchments, and determining relationships between model parameter values and catchment attributes (such as topography, geology, vegetation cover). These relationships can then be used to estimate the parameter values for the ungauged catchments from their attributes. Ideally, the technique used should yield an "optimal" parameter set (i.*e.* that which minimises a selected measure of error in the parameter values). Unfortunately, the approach outlined above will yield near-optimal parameter values for the ungauged catchment only if the scatter in the relationships between catchment attributes and model parameters is sufficiently small. This is highlighted by Merz and Blöschl [2004], who explored various methods of regionalisation of a rainfall-runoff model for 308 catchments in Austria, concluding that the best method was to use average parameter values of the immediate upstream and downstream nested, gauged catchments, followed by kriging. Both these techniques are biased towards selecting a set of parameter values from neighbouring, and thus broadly similar, catchments.

As nested gauged catchments are not always available, an alternative approach is to scale the model parameters from similar, but spatially distinct, gauged reference catchments. This approach requires a method of estimating the difference in response between the reference catchments and the study catchment. It has been applied in the Mae Chaem catchment in Northern Thailand [Schreider *et al.* 2002], where a crop model was used to estimate the difference in hydrologic response between trees and grass on each land unit within the reference and study catchments. The difference in hydrologic response was then used to estimate the parameter values of the IHACRES rainfall-runoff model [Jakeman *et al.* 1990, Jakeman and Hornberger 1993] for the ungauged catchment, by adjusting the calibrated parameter values for the reference catchment to take into account the difference in the land units in each catchment, as well as the fractional forest cover on each land unit.

2. REGIONALISATION AND UNCERTAINTY

While regionalisation of catchment model parameter values is a straightforward approach to the problem of prediction in ungauged basins, the uncertainty in the model predictions depends on the accuracy and precision of the catchment attributes, the relationships (both functional form and parameter values) between catchment attributes and model parameters, and on the choice and appropriateness of the rainfall-runoff model structure. The uncertainty in the relationships between catchment attributes and model parameters depends on the uncertainty in the catchment attributes, and the calibrated values of the model parameters for the gauged catchments used to derive the relationships.

Since the purpose of the regionalisation is to estimate some characteristics of the flows at an ungauged site rather than estimating the model parameters, the performance of the regionalisation should be assessed by comparing the predicted and observed response characteristics for gauged test catchments. For most models, response characteristics (such as mean annual flow, flow duration curve (FDC), and baseflow) depend on combinations of several model parameters. Optimality of the model parameter estimates does not generally imply optimality of response characteristics obtained from them. There are exceptions, such as the minimum error variance and zero bias obtained for any linear function of minimum-covariance, unbiased parameter estimates. However, none of the response characteristics considered here fall into that category, as all are non-linear in some of the model parameters. As a result, the uncertainty in the predicted response characteristics obtained from nominally optimal model parameter estimates can be considerably larger than the minimum uncertainty achievable in principle.

A further source of uncertainty in predicted response characteristics for ungauged catchments is that the criterion used in fitting model parameters to gauged catchments may be insensitive to the aspect of the rainfall-runoff behaviour determining the response characteristic. For example, parameter estimates in models of rainfall-runoff dynamics are often fitted by minimising mean-square error in runoff. This tends to be dominated by a relatively small number of large errors near flow peaks, and those errors are mainly due to model error in representing the most rapid part of the dynamics. By contrast, the mean annual flow (or the runoff coefficient) depends on the area under the unit hydrograph, which is the *zero-frequency* gain of the rainfall-runoff model. Moreover, a response characteristic may be highly sensitive to error in the parameter estimates from which it is derived. For instance, a 1% error in a slow-flow autoregressive parameter value of 0.9 in an IHACRES model translates into a 9% error in total runoff due to that component. Conversely, however, a moderately well estimated response characteristic may constrain one or more model parameters quite tightly.

Another contributing factor is how well the model structure represents the key processes; a model structure good for a restricted purpose may over-simplify or omit processes which have an important influence on another response characteristic.

Finally, gauged catchments may display similar behaviour in a given response characteristic while differing markedly in others which affect model parameter estimates. This may severely limit our ability to identify relationships between parameter values and catchment attributes.

3. ALTERNATIVE REGIONALISATION APPROACH

A new approach to regionalisation is proposed, aimed at reducing the uncertainty in the prediction of flows in ungauged catchments. Rather than regionalising the model parameters, key catchment response characteristics are regionalised directly and used to constrain the model parameters. If sufficient constraints can be applied, they will define a sufficiently precise set of parameter values. If not, additional constraints, possibly even partial regionalisation of model parameters, will be needed.

The catchment response characteristics considered here are the mean annual runoff coefficient, slope of the flow duration curve (in log-normal form), the fraction of time without flow and the form of the unit hydrograph.

3.1 Mean Annual Runoff Coefficient

The use of the mean annual runoff coefficient in regionalisation builds on the conclusion of Post *et al.* [1998] that the prediction of flows at ungauged sites can be improved by constraining the model to have the same runoff coefficient as a nearby catchment with similar attributes. The mean annual runoff coefficient r has been estimated at selected gauges in the Goulburn-Broken catchment in Victoria, the Upper Murrumbidgee catchment in New South Wales and the Upper Condamine catchment in Southern Queensland. Initially, the standard approach of using multiple linear regression to derive the regional relationships between r and catchment attributes was applied. This approach does not attempt to understand the processes involved, merely to identify the characteristics with the strongest correlation with r.

The catchment water balance is given by:

$$P = Q + E_T + Q_s + \Delta S \qquad (1)$$

where P is the rainfall, Q is the streamflow, E_T is the evapotranspiration, Q_s is the subsurface outflow (which may be negative if there is a subsurface inflow) and ΔS is the change in water stored within the catchment (as either soil moisture, groundwater or surface water) between the start and end of the period in question. The last two terms are assumed to be negligible, giving:

$$r = Q/P \cong 1 - \left(E_T/P\right) \qquad (2)$$

and the "observed" E_T is given by P-Q. The empirical relationships between rainfall and evapotranspiration E_f for forest and E_g for grass developed by Zhang *et al*. [2001] were adopted as a starting point for the regionalisation of the runoff coefficient r. The modelled evapotranspiration E_m for a catchment is then the linear combination of these terms using an estimate of the fraction f of woody cover:

$$E_m = fE_f + (1 - f)E_g \qquad (3)$$

Analysis of the E_T residuals (observed - modelled) for the gauges in the Goulburn-Broken and Upper Condamine catchments showed a correlation with potential evaporation E_p obtained from the National Land and Water Resources Audit [http://www.nlwra.gov.au/] as well as a correlation with catchment mean slope m. While the observed variation with E_p in Figure 1 seems plausible, the derived relationship may not be physically correct as other drivers correlated with E_p in the catchments studied may also be contributing (*e.g.* seasonal distribution of rainfall, rainfall intensity and vegetation cover). This highlights the care needed when using such empirical relationships.

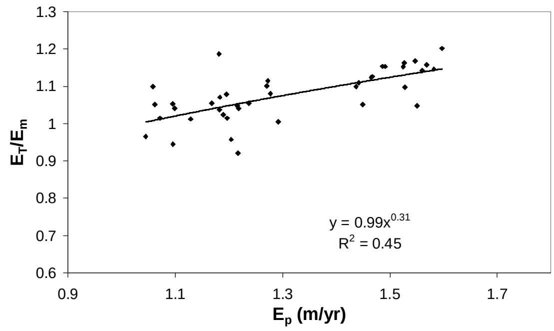

Figure 1. E_T/E_m versus E_p for catchments in the Goulburn-Broken and Condamine basins with more than 3000 days of recorded streamflow and slope less than 8.5°. [Three catchments with abnormally high runoff coefficients were removed from the sample].

The scatter in the E_T / E_m values in Figure 1 are due to the errors in the long-term mean streamflow and rainfall values, the forest fraction f and the assumption that the subsurface flux and change in storage can be ignored. Errors of 15% in rainfall, streamflow and forest cover for a catchment with a mean annual rainfall of 1 m, runoff coefficient of 0.3 and woody cover of 0.5, would result in an error in E_T / E_m of 11%. The strongest contributor to this error is the rainfall (10%), followed by streamflow (5%) and woody cover (1.6%). The estimated value of 11% for the error in E_T / E_m explains the scatter in values in Figure 1. Note that the standard deviation about fit is 5%, suggesting the assumed errors above are slightly overestimated, and that the subsurface flows and changes in storage are not contributing significantly to the scatter.

Investigation of catchments with slopes *m* greater than 8.5° shows a possible relationship between E_T and *m*, steep catchments having a higher runoff coefficient (see Figure 2). The value of *r* is then given by:

$$r_m = 1 - E_m f(E_P) g(m) \quad (4)$$

where $f(E_p)$ is given in Figure 1, $g(m)$ is given in Figure 2 and r_m is the modelled runoff coefficient. [Note that for m<~7.15, $g(m)$ =1].

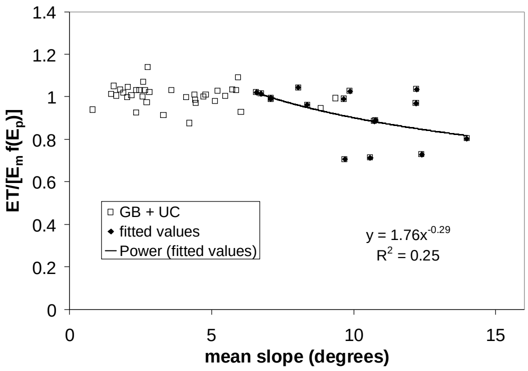

Figure 2. Influence of catchment mean slope on ET. Solid diamonds (overlaid on open squares) are the data values used in deriving the fit shown and comprise catchments in the Goulburn-Broken basin with high catchment mean slope. Open squares are the catchments used in deriving the influence of PE on ET.

Table 1. Residuals for modelled runoff coefficient

set	$(r - r_m)/r$		$r - r_m$	
	mean	st dev	mean	st dev
fitted, all	-0.14	0.52	-0.003	0.054
fitted, r>0.11	-0.028	0.27	-0.001	0.061
test, all	-0.42	0.69	-0.025	0.080
test, r>0.11	-0.028	0.49	-0.014	0.078

The resulting formulation gives a runoff coefficient estimate that depends on the catchment-averaged values of rainfall, land use, potential evaporation and slope. Testing of the regionalisation of the runoff coefficient for selected gauges in the Upper Murrumbidgee and Wingecarribee catchments suggests that the approach is able to represent the differences between these catchments, but the predictions of *r* for individual gauges has considerably higher uncertainty. This is due to the influence of other drivers for which no clear correlation with *r* can be identified. An example is the set of 5 catchments in Figure 3, where the model significantly underestimates the runoff coefficients (modelled value ~ 0.15-0.2, observed value ~0.4). The underestimation may be due to deficiencies in the model or to errors in the data used to derive the observed and modelled runoff coefficient.

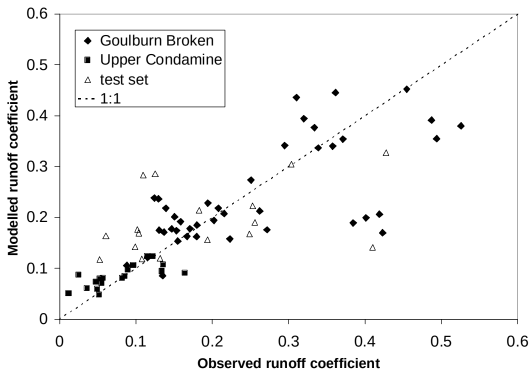

Figure 3. Modelled versus observed runoff coefficient.

The proportional and absolute residuals $(r - r_m)/r$ and $r - r_m$ in the runoff coefficient for the catchments fitted in deriving the relationship and the test set are shown in Table 1. In both cases, results are given for the full set (except for the 5 catchment noted above), and for only those catchments with runoff coefficient *r* above 0.11.

Assumed uncertainty of 15% in rainfall and streamflow leads to an error of 21% in the observed runoff coefficient. For the modelled runoff coefficient, there is additional uncertainty in the estimation of the evaporation losses for the catchment. Uncertainty of 25% in the potential evaporation estimate would result in 8% error in the correction term $f(E_p)$, so the runoff coefficient model is fairly robust with respect to E_p. Similarly, the model is not highly sensitive to errors in the catchment mean slope. The error in E_m/P for a 15% error in rainfall (P=1000 mm/yr, f=0.5) is 5% (with larger uncertainty in drier catchments), leading to a combined error in the modelled runoff coefficient, ignoring parameter uncertainty, of approximately 10%.

3.2 Shape of the Flow Duration Curve

The next most significant response characteristic after the mean annual runoff coefficient is the shape of the flow duration curve (FDC). Best *et al.* [2003] used paired catchments to investigate the variation of a normalised version of the FDC between similar catchments with different land use. They found that a 5-parameter model for the FDC (plus an additional parameter for the cease-to-flow point) gave the best compromise between number of parameters and fit to the FDC. With such a large number of parameters, regionalisation is difficult, though a paired catchment approach may yield useful information.

In comparison, for the selected catchments in the Murray-Darling Basin, the observed FDC (ignoring periods without flow) is nearly linear when plotted on log-normal axes, suggesting that a simpler model than that found by Best *et al.* may be used with only a slight degradation in the fit. Using the linear form gives a 2-parameter representation of the FDC (slope of FDC and fraction of time without flow), and has a higher potential for regionalisation. An attempt to regionalise the slope of the FDC was made (see Figure 4), and while reasonable relationships (R^2 ~ 0.7) could be identified (strongest signal with rainfall and forest cover), the correlation between catchment attributes prevented derivation of a unique relationship.

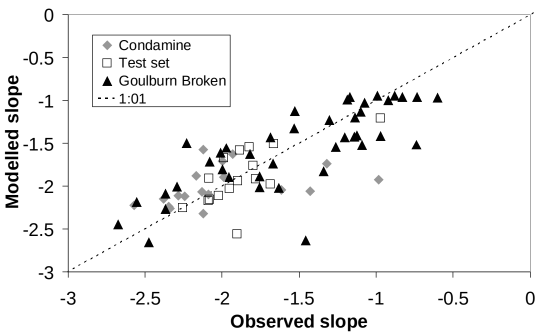

Figure 4. Modelled slope of flow duration curve against observed slope. Fit based on 40 catchments in the Goulburn-Broken basin.

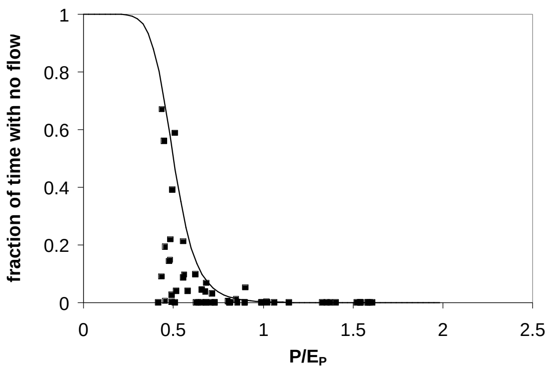

Figure 5. Fraction of time with no flow plotted against dryness index (P/E_p). The curve is an estimated upper limit with functional form (see equation 5) designed to have a value of 1 at P/E_p=0.

The fraction of time without flow will depend on both the frequency of rainfall events that produce effective rainfall and the duration of flow from a particular event. While an upper bound on the fraction of time without flow was also obtained (dependent on a dryness indicator, given by rainfall P divided by potential evaporation E_p), no clear relationship defining the actual value could be obtained (see Figure 5). The form of the upper bound shown is:

$$y = \frac{1}{1 + \left(2\, P/E_p\right)^8} \tag{5}$$

This form was adopted as it fits the observed envelope and gives a value of 1 for the limiting case P/E_p=0.

While this is a promising indication that regionalisation of the probability distribution of flows may be possible, further studies are needed.

3.3 Unit Hydrograph

The shape of the FDC depends on the temporal distribution of effective rainfall as well as the dynamic response characteristics of the catchment. Therefore, the FDC will only constrain the parameter values and cannot provide a unique set of model parameter values unless either the distribution of effective rainfall or the unit hydrograph can be determined by other means. One technique that has been used for estimating runoff response in ungauged catchments [e.g. Hall *et al.*, 2001] is the geomorphological instantaneous unit hydrograph (GIUH). A functional form is assumed for the IUH (*e.g.* the Clark [1945] or Nash [1957] models) and topographic and stream channel data are used to estimate the model parameters at ungauged sites. When coupled with a technique for estimating the baseflow response, this approach has the potential to predict the parameters of the unit hydrograph for ungauged sites, so long as the assumed functional form adequately represents the actual response, and the geomorphological data adequately represent the spatial variation in response function.

An alternative is to regionalise the parameters of the IUH derived from calibration of rainfall-runoff models. This has been successfully applied to small catchments with only daily rainfall data [e.g. Post and Jakeman 1996 and 1999, Post *et al.* 1998, Sefton and Howarth 1998, Kokkonen *et al.* 2003]. However, deriving the IUH directly from the observed hydrograph without reference to rainfall [Croke, 2004] can reduce the uncertainty, by removing the effect of the difference between the temporal resolution needed to resolve the unit hydrograph and the modelling time step employed, and by avoiding the uncertainty in the estimated effective rainfall.

4. CONCLUSIONS

The work presented here has shown that it is possible to develop a regionalisation of runoff coefficient that spans a wide range of climates and is sensitive to rainfall, land use, slope and potential evaporation. Thus, the long-term water budget for ungauged sites can be estimated without use of a rainfall-runoff model (assuming subsurface inflow/outflow and change in storage within the catchment are negligible). There is potential for estimating the FDC at ungauged sites, though the correlation between catchment attributes makes this problematic.

Maximising the agreement between what is modelled and observed (or estimated for ungauged catchments) is a necessary condition for selecting a set of rainfall-runoff model parameter values, but not always sufficient. However, the FDC can constrain parameter values considerably, and thus aid in their estimation for ungauged catchments. It is possible that a hybrid parameter estimation technique, combining regionalisation of catchment response characteristics and model parameters, will improve prediction of flows at ungauged sites.

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