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Hysteretic Behavior of Magnetorheological Fluid and Identification Using Preisach Model

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ABSTRACT: This study examines the field-dependent hysteretic behavior of a magnetorheological (MR) fluid and its identification using a Preisach model. The field-dependent shear stress of a commercially available MR fluid is measured using a parallel disk rheometer, and its shear stress versus shear rate hysteretic behavior is measured. To demonstrate the applicability of the Preisach model to MR fluid hysteresis modeling, two significant properties are empirically studied; the minor loop property and the wiping-out property. Using these data, the Preisach model for the MR fluid is identified using experimental first order descending curves. The effectiveness of the identified hysteresis model is verified in the time domain by comparing the reconstructed to the measured field-dependent shear stress. In addition, the identified hysteresis is compared to the conventional Bingham model.

Key Words: magnetorheological fluid, hysteresis identification, shear stress, Preisach model.

INTRODUCTION

HYSTERETIC behavior is encountered in many actuation systems that employ magnetic and ferroelectric materials, and the impact of this hysteresis on mechanical system performance can be profound in terms of actuation efficiency due to cyclic actuator losses induced by this hysteresis. Recently, smart materials such as magnetorheological (MR), electrorheological (ER) fluids, ferroelectric (such as PZT or PMN) and shape memory alloys have been adopted as the actuation materials in various active and semi-active control systems. These smart materials also exhibit hysteresis in their responses, which causes adverse effects on actuator performance (typically destabilization or reduced command following performance, as well as undesirable self heating). A key goal in control system design implementation and miniaturization is high performance and high efficiency, so that nonlinear hysteresis models (Song and Li, 1999; Mittal and Menq, 2000) and robust feedback control schemes (Choi and Lee, 1997) are being intensively investigated to mitigate the detrimental effects of, or compensate for, the hysteretic behaviors of actuators that employ smart materials.

MR fluids are currently being studied and implemented as actuating fluids for valve systems, shock absorbers, engine mounts, and other control systems. The rheological properties of MR fluids are reversibly and instantaneously changed by applying a magnetic field to the fluid domain. This is generally modeled as a Bingham fluid (Shames and Cozzarelli, 1992; Choi et al., 2000), which shows perfect Newtonian behavior after a yield point. However, MR fluids also exhibit hysteresis phenomenon caused by the change of dynamic conditions such as shear rate, which has been widely studied. This is because it can easily be considered together with the hysteresis of application devices. This phenomenon leads to many investigations on the hysteresis behaviors of application devices. Stanway et al. (1987) proposed an idealized mechanical model for the behavior of an ER damper. The mechanical model consisted of a Coulomb friction element placed in parallel with a viscous damper. Spencer et al. (1997) proposed a Bouc–Wen model to describe the behavior of an MR damper. The parameters of the Bouc–Wen model were estimated by fitting the force–displacement and the force–velocity behavior of the damper. Kamath and Wereley (1997) presented a nonlinear viscoelastic–plastic model which accounts for the behavior of ER fluids in both the pre-yield and the post-yield regimes for the applied shear. Wereley et al. (1998) also proposed a nonlinear biviscous model for MR/ER dampers.

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The above hysteresis models have proven to be successful in approximating the post-yield behavior of MR/ER devices. However, these models have not been nearly as successful in describing the hysteretic behavior of the field-dependent rheological fluid itself. Especially, the phenomenological hysteretic behavior of a MR fluid itself is still unclear while a few works for hysteresis of ER fluid itself were performed (Kamath and Wereley, 1997; Choi et al., 1999; Han et al., 2003). Furthermore, in the application devices, the control input to be applied to the MR fluid domain is a magnetic field. Therefore, the hysteresis behavior with respect to a control input can significantly affect the control performance of application devices. However, this type of hysteresis is a complex nonlinearity and research work on it is considerably rare, on the contrary to many research works on the hysteresis of PZT and SMA. In order to capture this characteristic of MR fluids, the classical Preisach independent domain model (Mayergoyz, 1991) which is well-known in the ferromagnetic field of study can be a potential candidate. It is noted that the Preisach model has been adopted as an effective hysteresis model of ER fluid (Han et al., 2003), PZT (Ge and Jouaneh, 1997) and SMA (Gorbet et al., 1998).

This work presents a key issue in the control of MR devices by identifying a general hysteresis model of the MR fluid itself with respect to an applied magnetic field using the Preisach Model. In order to achieve this goal, the commercial MR product (MRF-132LD, Lord Corporation) is employed. The field-dependent shear stress is then obtained using a rheometer with a MR cell. Two significant properties: the minor loop property and the wiping-out property (Mayergoyz, 1991; Hughes and Wen 1997) are examined to show that the Preisach model is well-matched to a physical hysteresis phenomenon of the MR fluid itself. A Preisach model for the MR fluid is then presented and numerically identified. Several first order descending (FOD) curves for the adopted MR fluid are experimentally achieved to determine the relationship between a shear stress and applied magnetic field. The verification of the identified Preisach model is performed by comparing the predicted shear stress with the measured one. In addition, the predicted shear stress is compared between the proposed Preisach model and conventional Bingham model.

HYSTERESIS PHENOMENON IN MR FLUID

MR fluid is a suspension consisting of magnetizable particles in a low-permeability carrier fluid. The nearly instantaneous change in rheological properties is effected by a magnetic field inducing polarization in the suspended particles. These polarized particles then form chains or networks, and the breaking of these chains is manifested by a yield stress in the suspension. This yield stress is strongly dependent on the applied magnetic field. MR fluids have been typically modeled as Bingham fluids whose constitutive equation is given by (Shames and Cozzarelli, 1992; Choi et al., 2000)

$$\tau = \eta\dot{\gamma} + \tau_y(H) \tag{1}$$

where τ is the shear stress, η is the dynamic viscosity, $\dot{\gamma}$ is the shear rate, and $\tau_y(H)$ is the dynamic yield stress of the MR fluid. It is generally accepted that τ_y is a function of the magnetic field H. Although the Bingham model is widely used to model and control MR devices because it accurately describes post-yield behavior of an MR device, the Bingham-plastic model is still incapable of describing the hysteretic behavior of the MR fluid itself.

Figure 1 presents the experimental apparatus used in this study to measure shear stress induced in the MR fluid as a function of field. The field-dependent shear stress was measured using a Paar-Physica MCR-300 parallel disk rheometer with an MR cell (TEK 70MR, Paar-Physica). An air bearing is used to minimize hysteresis effects caused by mechanical friction, and the operating temperature of the MR fluid is maintained at 25°C. The diameter of the rotating disk is 20 mm. The gap between the rotating disk and the stationary platen is 1 mm, and this gap contains the MR fluid. Both the shear rate and input field are controlled by the personal computer using A/D (analog/digital) and D/A (digital/analog) converters.

Figure 2 shows the measured hysteretic behaviors of the MR fluid (MRF-132LD, Lord Corporation) used in this study. Two different hysteretic behaviors were measured because shear stress in Equation (1) is a function of both the dynamic condition (shear rate) and control input (magnetic field). The shear stress shown in Figure 2(a) was measured under constant magnetic field as shear rate was varied. The shear stress shown in Figure 2(b) is measured under change of the magnetic field. The measured hysteresis loop is traced in the counter-clockwise direction. It is clearly observed that there are hysteresis loops in the measured shear stress associated with both shear rate (Figure 2a) and magnetic field intensity (Figure 2b). However, hysteresis effects associated with magnetic field intensity are much more significant than the hysteresis effects due to shear rate. In the application devices, the magnetic field intensity acts as the control input. Therefore, the large hysteretic behavior with respect to the control input can significantly affect the control performance of application devices.

In this work, the measured shear stress at the shear rate of $1\,s^{-1}$ is considered as an approximated yield stress in order to establish the hysteresis model between the yield stress (output) and magnetic field (input).

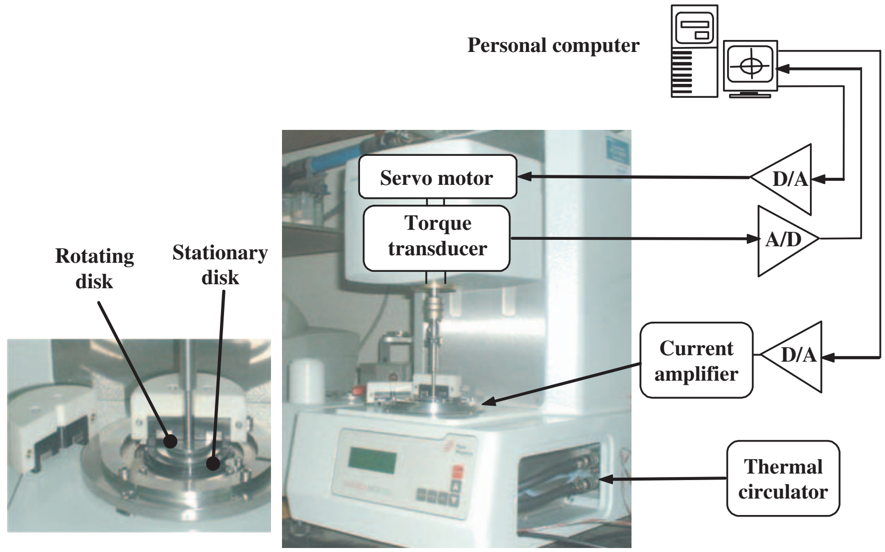

Figure 1. *Schematic configuration of experimental apparatus.*

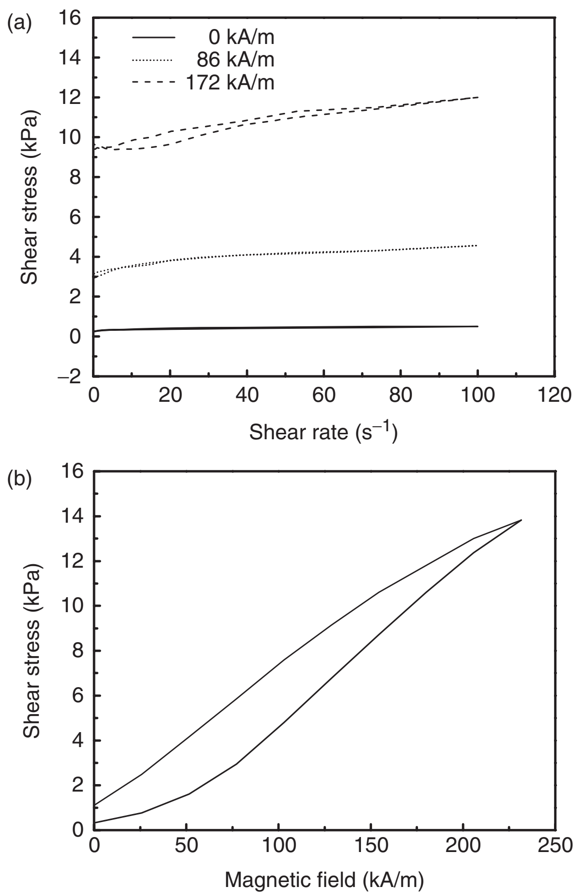

Figure 2. *Hysteretic behavior of the MR fluid. (a) Shear rate vs shear stress and (b) magnetic field vs shear stress.*

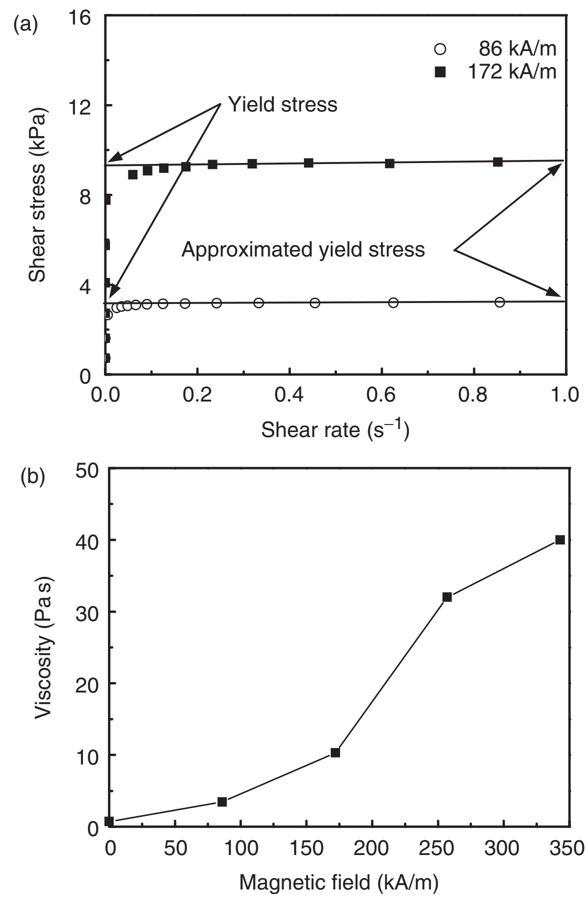

Figure 3. *Approximated yield stress of the MR fluid. (a) Yield point and (b) plastic viscosity.*

By measuring the shear stress at the constant shear rate of 1 s^{-1}, it can be sufficiently assumed to be close to the dynamic yield stress as shown in Figure 3(a), while the field-dependent viscosity is kept below 50 Pa s as shown in Figure 3(b). It is remarked that the ordinates of figures titled by shear stress represents the dynamic yield stress.

The Preisach model has been developed for modeling hysteresis in ferromagnetic materials. This model is a promising candidate to capture the hysteretic behavior of a MR fluid, since the chain clusters of a MR fluid are formed by induced magnetic dipoles via electromagnetization. The Preisach model is phenomenological in nature, and it is characterized by two significant properties: the minor loop property and the wiping-out property (Mayergoyz, 1991; Hughes and Wen, 1997). The minor loop property specifies that two comparable minor loops that are generated by moving between two same pairs of input maximum and minimum are to be congruent if one exactly overlaps the other after some shift in the output parameter. The wiping-out property specifies which values of the preceding input trajectory affect the current output, that is, which dominant maximum and minimum can wipe out the effects of preceding smaller dominant maxima and minima. In order to determine whether a physical hysteresis phenomenon is matched by the Preisach model, one can examine both the minor loop and wiping-out properties.

Figure 4(a) shows the applied magnetic field trajectory for the minor loop experiment. The imposed trajectory is composed of triangular input signals. The achieved shear stress is shown in Figure 4(b). A favorable congruency is seen from the results. Figure 5(a) shows the applied magnetic field trajectory for the wiping-out experiment, which is composed of two sets of dominant maxima and minima. Figure 5(b) shows a good agreement between the first set of dominant maxima (a_1, a_2, a_3) and the second set $(\bar{a}_1, \bar{a}_2, \bar{a}_3)$. This implies that the effect of the preceding former (a_1, a_2, a_3) has been wiped out by the larger maxima $(\bar{a}_1)$ of the latter $(\bar{a}_1, \bar{a}_2, \bar{a}_3)$, and thus the preceding input values (a_1, a_2, a_3) do not affect current outputs. Therefore, it is clearly seen that the wiping-out property is sufficiently satisfied for the MR fluid used in this study.

The results, shown in Figures 4 and 5, demonstrate a favorable congruency of the comparable minor loops and a satisfactory wiping-out property for the MR fluid. Thus, the Preisach model can describe the hysteresis behavior of the MR fluid used in this study. On the basis of these results, the hysteresis identification of the MR fluid was performed using the Preisach model.

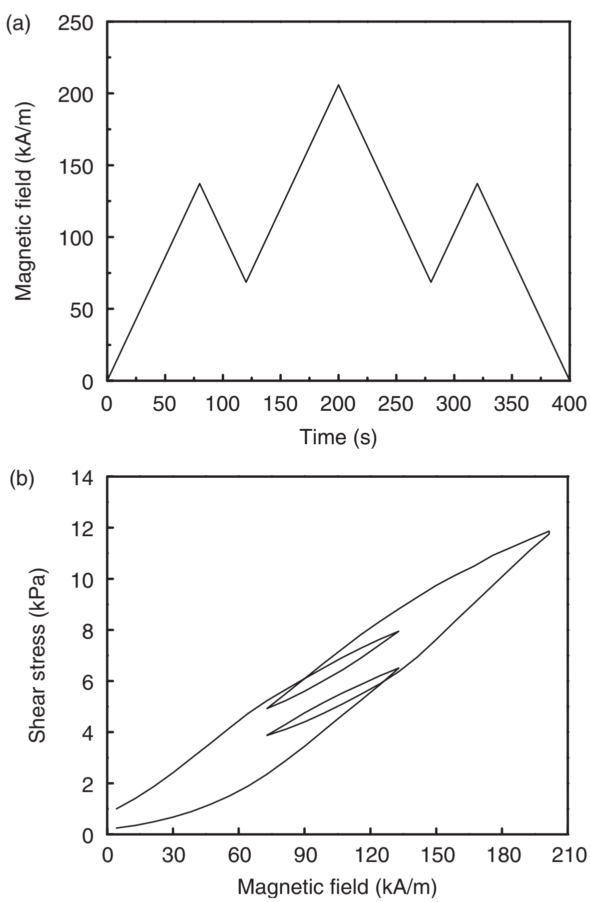

***Figure 4.** Minor loop property of the MR fluid. (a) Field trajectory and (b) minor loop response.*

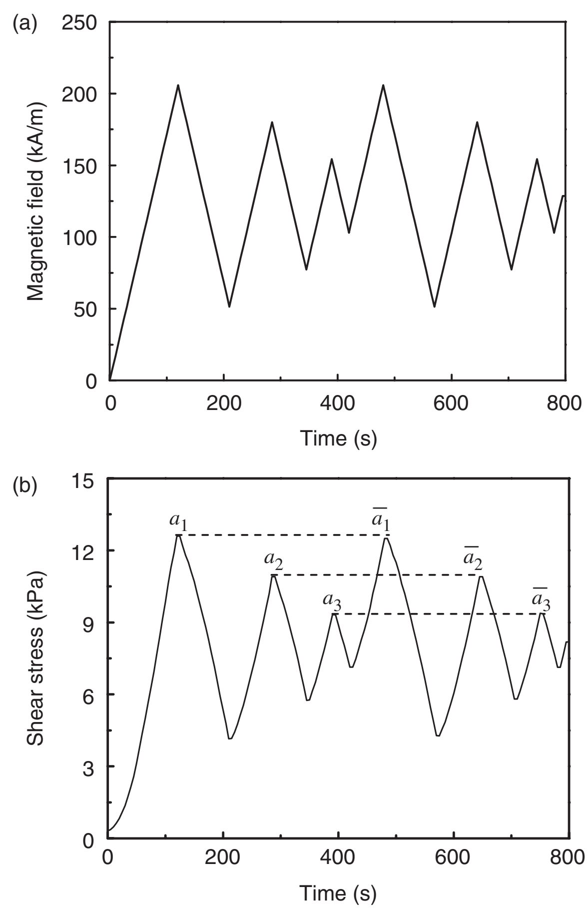

***Figure 5.** Wiping-out property of the MR fluid. (a) Field trajectory and (b) wiping-out response.*

PREISACH MODEL

The Preisach model that we adopt to describe the hysteretic behavior of the MR fluid can be expressed as follows (Mayergoyz, 1991):

$$\tau_y(t) = \iint_\Gamma \mu(\alpha,\beta)\gamma_{\alpha\beta}[H(t)]\mathrm{d}\alpha\mathrm{d}\beta \tag{2}$$

where Γ is the Preisach plane, $\gamma_{\alpha\beta}[\cdot]$ is the hysteresis relay, $H(t)$ is the magnetic field, $\tau_y(t)$ is the yield stress (or shear stress), and $\mu(\alpha, \beta)$ is the weighting function that describes the relative contribution of each relay to the overall hysteresis. Each relay is characterized by the pair of switching values (α, β) with $\alpha \geq \beta$. As the input varies with time, each individual relay adjusts its output according to the magnetic field input value, and the weighted sum of all relay outputs provides the overall system output as shown in Figure 6(a). The simplest possible hysteresis relay for the MR fluid is shown in Figure 6(b). It is a modification of a classical relay which has two states, -1 and 1, corresponding to the opposite polarization of a ferromagnetic material. The output of the adopted relay is either 0 or 1. This is suitable for modeling hysteresis of the MR fluid where the output (yield stress) varies between zero and a maximum value. In this case, a hysteresis loop is located in the first quadrant of the input-output (magnetic field-yield stress) plane. The Preisach plane can be geometrically interpreted as one-to-one mapping between relays and switching values of (α, β) as shown in Figure 6(c). The maximum yield stress restricts the plane within the triangle in Figure 6(c), where α_0 and β_0 represent the upper and lower limit of the magnetic field input. In this work, they are chosen by 0 kA/m and 257.25 kA/m, respectively. The Preisach plane provides the state of individual relay, and thus the plane is divided into two time-varying regions as follows:

$$\begin{aligned} \Gamma_- &= \{(\alpha,\beta) \in \Gamma \,|\, \text{output of } \gamma_{\alpha\beta} \text{ is } 0\} \\ \Gamma_+ &= \{(\alpha,\beta) \in \Gamma \,|\, \text{output of } \gamma_{\alpha\beta} \text{ is } 1\}. \end{aligned} \tag{3}$$

The two regions represent that relays are on 0 and 1 positions, respectively. Therefore, Equation (2) can be reduced to

$$\tau_y(t) = \iint_{\Gamma_+} \mu(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta. \tag{4}$$

The use of a numerical technique for the Preisach model identification has been normally proved as an effective way for smart materials (Ge and Jouaneh, 1997; Mittal and Menq, 2000). Therefore, in this study, the Preisach model for the MR fluid is numerically implemented by employing a numerical function for each mesh values of (α, β) within the Preisach plane. The mesh values are identical with the data in the FOD curves, which are experimentally obtained. Figure 7(a) shows one of the mesh values of (α_1, β_1), and its corresponding

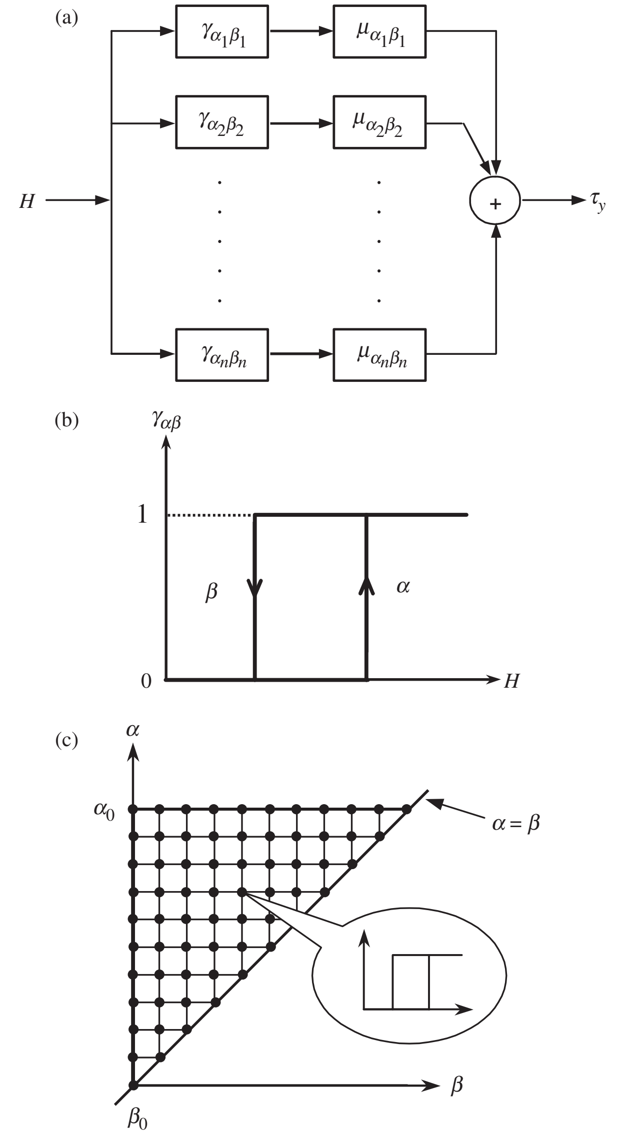

Figure 6. *Configuration of the Preisach model. (a) Block diagram; (b) hysteresis relay for the MR fluid and (c) Preisach plane.*

FOD curve is a monotonic increase to a value α_1, then a monotonic decrease to β_1. After the input peaks at α_1, the decrease sweeps out area Ω, generating the descending branch inside the major loop. A numerical function $T(\alpha_1, \beta_1)$ is then defined as the output change along the descending branch as follows:

$$T(\alpha_1,\beta_1) = \iint_\Omega \mu(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta = \tau_y^{\alpha_1} - \tau_y^{\alpha_1\beta_1}. \tag{5}$$

From Equations (4) and (5), we can determine an explicit formula for the hysteresis in terms of experimental data.

Figure 7(b) shows the increasing and decreasing series of an input magnetic field, where Γ_+ can be subdivided into n trapezoids Q_k. Geometrically, the area of each trapezoid Q_k can be expressed as a difference of two triangle areas concerned with $T(\alpha_k, \beta_{k-1})$ and

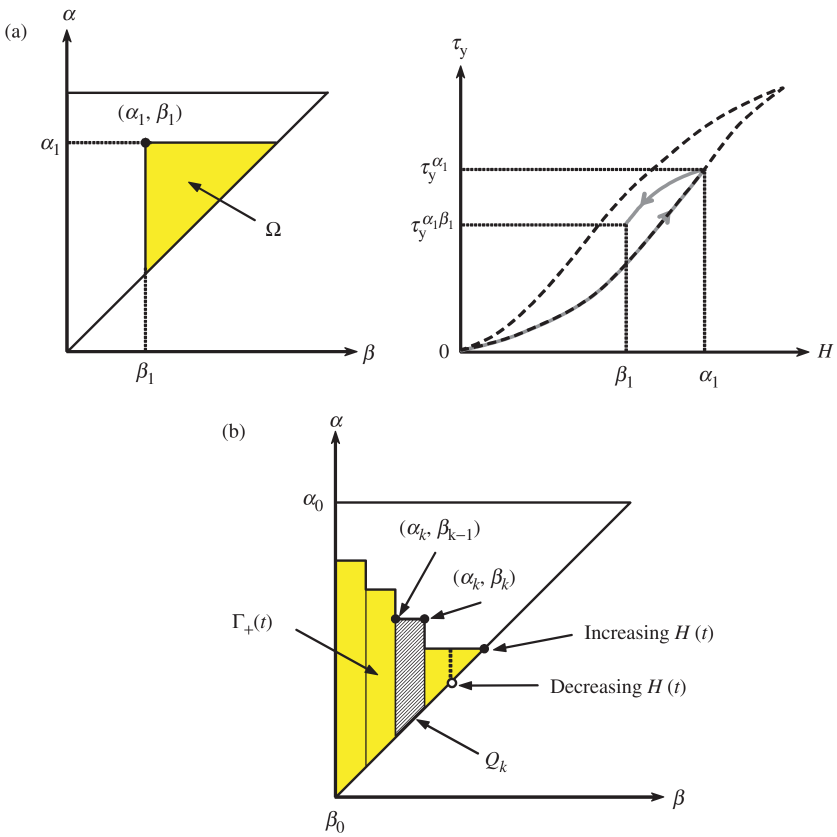

***Figure 7.** Numerical identification and implementation of the Preisach model. (a) Experimental identification and (b) numerical implementation.*

$T(\alpha_k, \beta_k)$, respectively. Therefore, the output for each Q_k is derived by

$$\iint_{Q_k} \mu(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta = T(\alpha_k,\beta_{k-1}) - T(\alpha_k,\beta_k). \tag{6}$$

By summing the area of trapezoids Q_k over the entire area Γ_+, the output is expressed by

$$\begin{aligned}\tau_y(t) &= \sum_{k=1}^{n(t)} \iint_{Q_k} \mu(\alpha,\beta)\mathrm{d}\alpha\mathrm{d}\beta \\ &= \sum_{k=1}^{n(t)} [T(\alpha_k,\beta_{k-1}) - T(\alpha_k,\beta_k)].\end{aligned} \tag{7}$$

Consequently, for the cases of the increasing and decreasing input, the output $\tau y(t)$ of the Preisach model is expressed by the experimentally defined $T(\alpha_k, \beta_k)$ (Ge and Wen, 1997; Mittal and Menq, 2000):

$$\begin{aligned}\tau_y(t) &= \sum_{k=1}^{n(t)-1} [T(\alpha_k,\beta_{k-1}) - T(\alpha_k,\beta_k)] \\ &\quad + T(H(t),\beta_{n(t)-1}), \text{ for increasing input and} \\ \tau_y(t) &= \sum_{k=1}^{n(t)-1} [T(\alpha_k,\beta_{k-1}) - T(\alpha_k,\beta_k)] \\ &\quad + \left[T(\alpha_{n(t)},\beta_{n(t)-1}) - T(\alpha_{n(t)},H(t))\right], \\ &\quad \text{for decreasing input.}\end{aligned} \tag{8}$$

Numerical implementation of the Preisach model requires experimental determination of $T(\alpha_k, \beta_k)$ at a finite number of grid points within the Preisach plane. Practically, the finite number of grid points and the corresponding measured output values cause a certain problem that some values of the magnetic field input do not lie at the grid point. Furthermore, it may not have the measured output values. In this work, the following interpolation functions are employed to determine the corresponding $T(\alpha_k, \beta_k)$ when the value of a magnetic field input does not lie at the grid point:

$$\begin{aligned}\tau_y^{\alpha_k\beta_k} &= c_0 + c_1\alpha_k + c_2\beta_k + c_3\alpha_k\beta_k, \text{ for square cells and} \\ \tau_y^{\alpha_k\beta_k} &= c_0 + c_1\alpha_k + c_2\beta_k, \text{ for triangular cells.}\end{aligned} \tag{9}$$

HYSTERESIS IDENTIFICATION OF THE MR FLUID

As a first step, FOD data sets were constructed to calculate the numerical function of the Preisach model. Figure 8(a) shows the magnetic field input used to collect FOD curves. The magnetic field ranged from 0 kA/m to 257.25 kA/m, and was partitioned into 10 sub-ranges. The magnetic field was applied in a stepwise manner, and each step of the magnetic field was

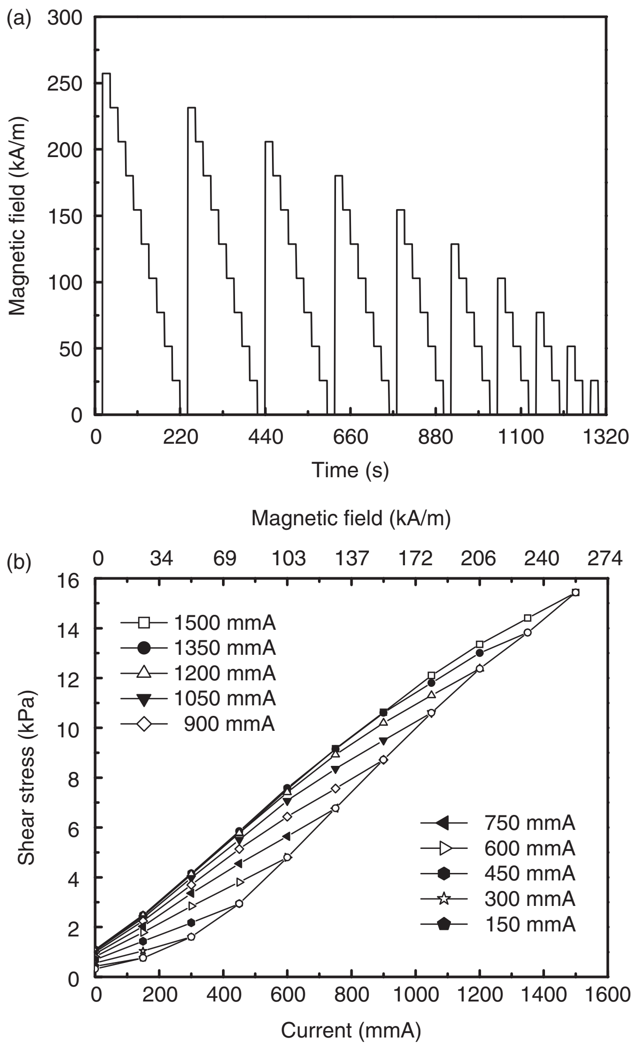

Figure 8. *Measured FOD curves. (a) Magnetic field input and (b) FOD curves.*

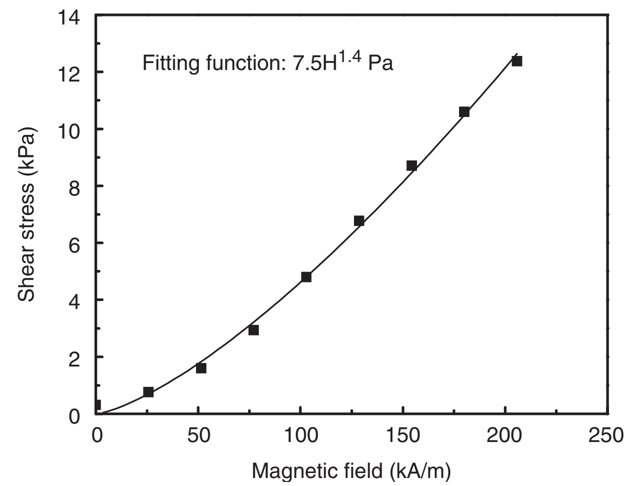

Figure 9. *Bingham characteristics of the MR fluid.*

maintained for 20 s to satisfy a steady state condition. From the measured shear stress using the rheometer, the FOD curves shown in Figure 8(b) were determined. At this point, the Preisach model for the MR fluid was numerically identified using these FOD data sets.

Hysteresis prediction using the Preisach model was tested under three different types of input trajectories of the magnetic field: step, triangular, and arbitrary signals. Predictions using the Preisach model were compared to the results predicted by the Bingham model from Equation (1). Figure 9 shows the Bingham characteristics of the MR fluid adopted in this study, from which the field-dependent yield stress was measured and was fitted to the function: $\tau_y(H) = 7.5\,H^{1.4}$ Pa.

Figure 10(b) presents the actual and predicted hysteresis responses under the step input shown in Figure 10(a). In an increasing step, the predicted responses by the Preisach model and the Bingham model show similar results, but the hysteresis effect distinctly manifested in the next decreasing step. It is observed from the results that the Preisach model accurately predicted the measured responses with great accuracy. The Preisach model was more accurate than the Bingham model in reconstructing the hysteresis behavior, which can be verified from the prediction error shown in Figure 10(c). The maximum prediction errors were found to be 0.72 and 2.34 kPa in a steady state condition for the Preisach model and the Bingham model, respectively.

Figure 11(b) presents the hysteresis behavior in response to the triangular input trajectory shown in Figure 11(a). As seen in Figure 11(b), the Preisach model accurately reconstructs the hysteresis behavior, and the prediction error is small as shown in Figure 11(c). The Preisach model clearly outperforms the Bingham model: the maximum prediction errors of Preisach and Bingham models were found to be 0.43 and 2.49 kPa, respectively. Figure 12 compares the measured field-dependent hysteresis loops to those reconstructed using the Preisach model. From this comparison, the Preisach model well-represents the field-dependent hysteretic shear stress versus magnetic field behavior for the MR fluid used in this study.

Figure 13 presents the predicted behavior of the MR fluid in response to an arbitrary magnetic field input time history, which is composed of a piecewise continuous sequence of constant inputs and ramp inputs of varying slope (Figure 13a). The Preisach model accurately predicts the response of the MR fluid to this arbitrary magnetic field, whereas the Bingham model does not perform well (Figure 13b). Because the prediction error of the Preisach model is much less than that for the Bingham model (Figure 13c), it is clear that the Preisach model more accurately captures the hysteresis behavior of the MR fluid from the magnetic field input to the yield stress output.

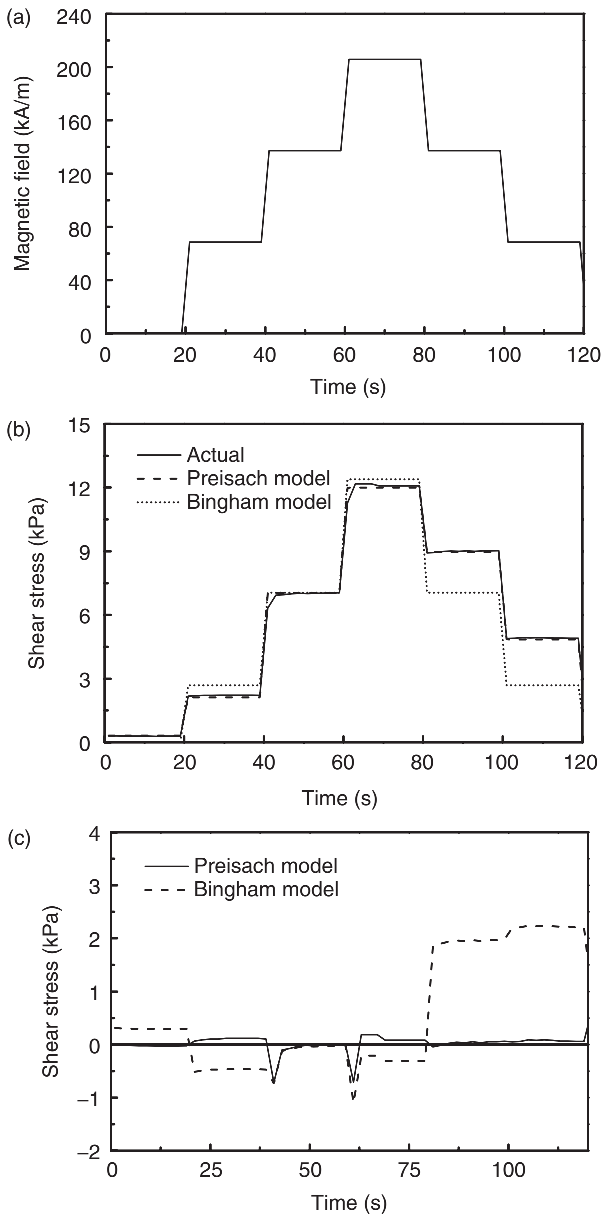

Figure 10. *Actual and predicted hysteresis responses under a step input. (a) Step input trajectory; (b) hysteresis response and (c) prediction error.*

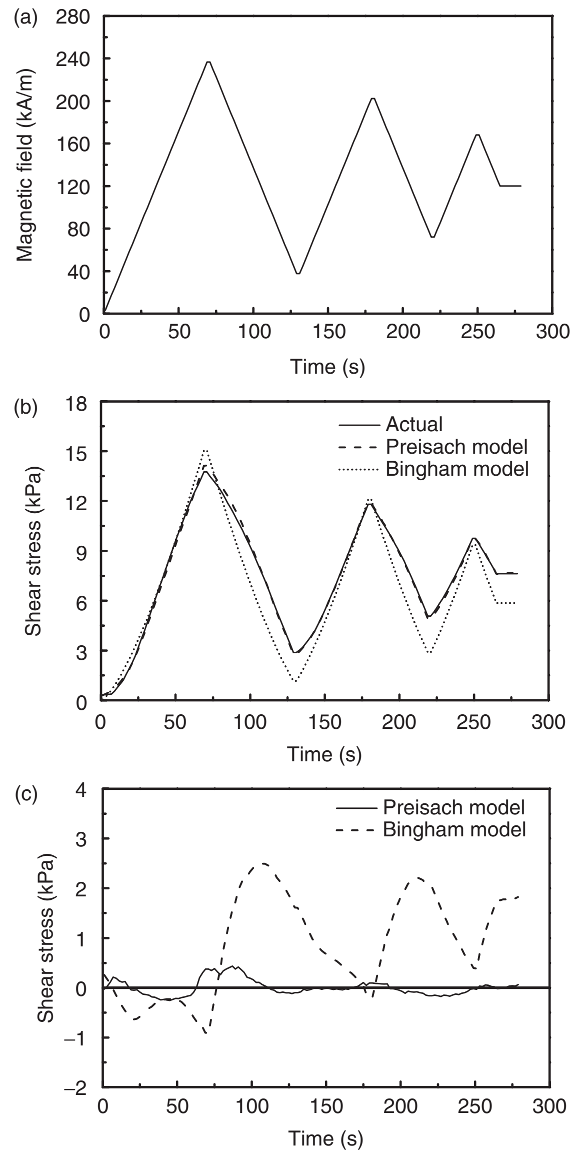

Figure 11. *Actual and predicted hysteresis responses under a triangular input. (a) Triangular input trajectory; (b) hysteresis response and (c) prediction error.*

CONCLUSION

The field-dependent hysteretic behavior of the shear stress in the MR fluid has been investigated and identified using a Preisach model. The hysteretic characteristics were experimentally observed to be functions of both dynamic condition (shear rate) and control input (magnetic field). It was shown that the effect of the magnetic field on the hysteresis behavior is more significant than that of shear rate. The applicability of the Preisach model to the MR fluid was demonstrated through tests for the minor loop and wiping-out properties. The Preisach model was then established using FOD curves and numerical techniques.

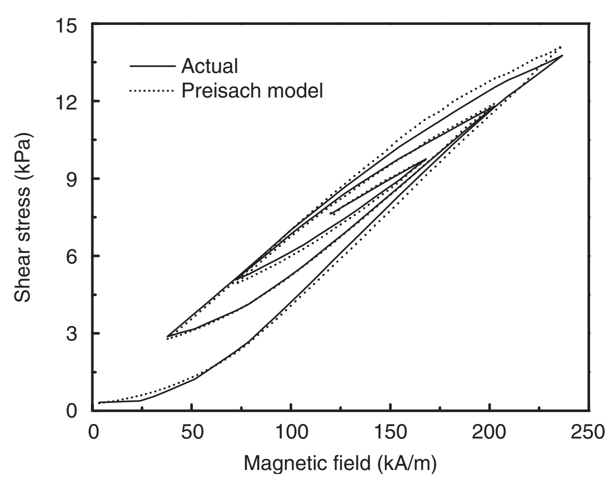

Figure 12. *Hysteresis loops of the MR fluid.*

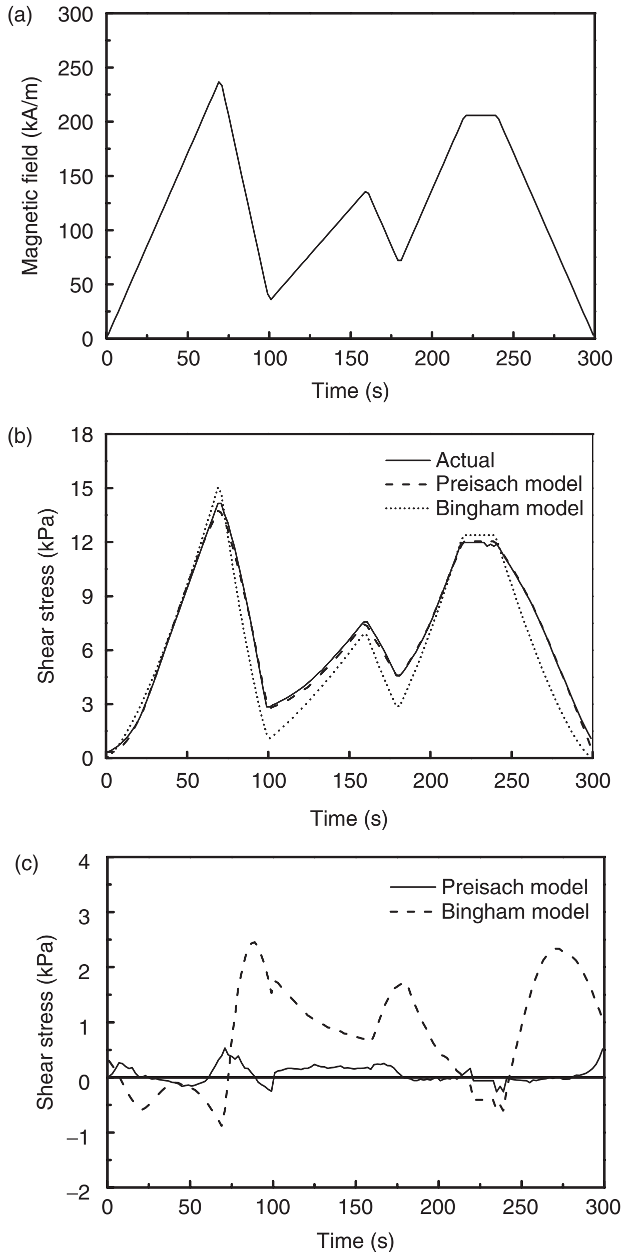

Figure 13. *Actual and predicted hysteresis responses under an arbitrary input. (a) Arbitrary input trajectory; (b) hysteresis response and (c) prediction error.*

Its effectiveness was verified by comparing the measured (or actual) and predicted field-dependent shear stresses. It has been demonstrated that the Preisach model can substantially reduce the prediction error under various input trajectories in comparison with the conventional Bingham model. It is finally remarked that the control response of MR application devices such as shock absorber integrated with the Preisach model be explored as a second phase of this study.

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