$$\frac{\Delta J_{n,f}}{\Delta J_{n,ps}} \longrightarrow \frac{1}{3}\left(\frac{\Delta R_u}{\Delta R_t}\right)^{3/2} \tag{16}$$

For a fair comparison it has here been assumed that the terminal film thickness $h_{\min,t}$ reached in plain squeezing is equal to the constant thickness $h_{\min}$ in the present particular case of flapping. At the upper bound, $(\Delta R_u/\Delta R_t) = 10$, of the representative range (11*a*) the relative gain in impulse capacity, $\Delta J_{n,f}/\Delta J_{n,ps}$, would thus amount to as much as about 10.5.

Admittedly, a certain amount of idealization and approximation is involved in the present illustrative particular case of flapping.[5] Nevertheless it is thought to have sufficiently served the author's main purpose, namely to at least bring out qualitatively the potentialities and certain trends inherent in flapping action when it comes to promoting impulse capacity as compared with plain squeezing.

Acknowledgment

The author is grateful to his then scientific staff members, Messrs. K. Herrebrugh and H. Moes, for their computations of basic data, such as the impulse level chart [3] and the orbits corresponding with several duty cycles. Further, he expresses his appreciation of the efforts ably put by his technical staff member, Mr. G. M. L. A. Bernsen, into the graphical construction of such orbits.

References

1 Booker, J. F., "Dynamically Loaded Journal Bearings: Mobility Method of Solution," TRANS. ASME, *Journal of Basic Engineering,* Series D, Vol. 87, 1965, pp. 534–537. See also the author's more recent paper, "Dynamically Loaded Journal Bearings: Numerical Application of the Mobility Method," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 83, p. 1181 for errata (see April 1971 issue, p. 315).

2 Campbell, J., Love, P. P., Martin, F. A., and Rafique, S. O., "Bearings for Reciprocating Machinery—A Review of the Present State of Theoretical, Experimental and Service Knowledge," 1967 Conference of Lubrication and Wear, Proceedings of the Institution of Mechanical Engineers, London, England, Vol. 182, Part 3A, 1967, pp. 51–74.

3 Blok, H., and Herrebrugh, K., "Discussion on Paper of J. F. Booker," TRANS. ASME, JOURNAL OF LUBRICATION TECHNOLOGY, Series F, Vol. 91, 1969, pp. 538–539; author's closure, pp. 541–542.

4 Blok, H., "The Orbital Hydrodynamics of Dynamically Loaded Self-Acting Journal Bearings, as Simplified by the Impulse/Whirl Angle Method," English abstract of series of lectures to students of Delft University of Technology in 1964/1965; 1967, 34 pp.

5 Blok, H., "The Impulse Capacity as a Design Criterion for Full Journal Bearings under Severe Dynamic Duty," Report WTHD 45, Lab. Machine Elements and Tribology, Univ. Tech., Delft, Holland, 1973, 36 pp.

6 Hays, D. F., "Squeeze Films: A Finite Journal Bearing with a Fluctuating Load," TRANS. ASME, *Journal of Basic Engineering,* Series D, Vol. 83, 1961, pp. 579–588.

7 Booker, J. F., "Dynamically Loaded Journal Bearings: Maximum Film Pressure"; TRANS. ASME, JOURNAL OF LUBRICATION TECHNOLOGY, Series F, Vol. 91, 1969, pp. 534–537.

8 Shaw, M. C., and Macks, E. F., "Analysis and Lubrication of Bearings," McGraw-Hill Book Co., Inc., New York, 1949, 618 pp.

9 Bremi, P., "Calculation of the Stresses and Most Important Deformations on a Connecting Rod Big End with the Help of a Computer," Sulzer Tech. Review, Switzerland, No. 1, 1971, pp. 59–64.

10 Oh, K. P., and Huebner, K. H., "Solution of the Elastohydrodynamic Finite Journal Bearing Problem," TRANS. ASME, JOURNAL OF LUBRICATION TECHNOLOGY, Series F, Vol. 85, No. 3, 1973, pp. 342–352.

Discussion

J. F. Booker[29]

In the present paper Professor Blok makes two separate but related contributions to the literature of dynamically-loaded journal bearings: namely, a review of his well-known "impulse method" for the analysis of rigid bearings, and the identification and examination of the "flapping action" of flexible bearings. The first is particularly welcome, since descriptions of this work have previously been available to a general audience only through such secondary references as the review paper of Campbell, et al. [2]. The second represents to this writer a new and timely departure of great potential, about which a bit more will be said later.

In his first contribution Professor Blok has very graciously made clear the close relation between his own "impulse method" and the writer's "mobility method." Nevertheless, it may be useful to review briefly and heuristically the formal connection between the two methods once again.

As discussed elsewhere [1, 2, 7, 11],[30] the mobility formulation provides an equation of motion of the form.

$$\frac{d\underline{\epsilon}}{dt} = \frac{F(C/R)^2}{LD\mu}\underline{\mathbf{M}} + \underline{\bar{\omega}} \times \underline{\epsilon}$$

If the second (rotation) term can be neglected, formal integration gives

$$\Delta J_n \equiv \int_{\underline{\epsilon}_1}^{\underline{\epsilon}_2} \underline{\mathbf{M}}^{-1} \cdot d\underline{\epsilon} = \frac{(C/R)^2}{LD\mu}\int_{t_1}^{t_2} F dt, \quad \begin{aligned}\underline{\mathbf{M}} \cdot \underline{\mathbf{M}}^{-1} &= 1\\ \underline{\mathbf{M}} \times \underline{\mathbf{M}}^{-1} &= \underline{0}\end{aligned}$$

where spatial integration along squeeze path (direction) lines is understood. Given some arbitrary[31] datum, such (numerical) integration can be carried out once and for all, resulting in specification of a scalar field of impulse values over the clearance space. This allows a very simple and accurate prediction of motion along squeeze paths in terms of load *integrals* (rather than instantaneous *values*).

Conversely, if the first (squeeze) term can be ignored, formal integration shows the journal center moving at fixed eccentricity (along a circular arc) through angular displacements given simply by

$$\Delta\bar{\gamma} \equiv \int_{t_1}^{t_2} \bar{\omega}\, dt$$

If neither term can be neglected, simultaneous formal integration is not generally possible. However, over small time intervals it is appropriate to superpose *sequentially* the results of the integrations just discussed, treating a general displacement step as the sum of successive steps along a squeeze path and a circular arc (as suggested by Fig. 3). This approximation seems to be an essential and altogether acceptable feature of the impulse formulation.

Thus, in the absence of finite rotations the impulse approach introduces no new approximations not already inherent in the mobility formulation. Since the presence of *small* finite rotations causes no practical difficulties either, the two methods may be regarded as equivalent for most computational purposes, the one being essentially the integral of the other, and the choice resting with the user. Each has its conceptual and computational advantages, depending on the problem at hand. If this writer's preference is generally towards the mobility formulation, particularly for digital computation, this can surely be understood and forgiven as simple personal bias.

In introducing secondly the concept of "flapping action" Professor Blok provides a new attack on the outstanding weakness of the impulse/mobility methods—the assumption of perfect bearing rigidity. It is hoped that in time this semi-analytical treatment of bearing flexibility can be elaborated further for a practical case to

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[30] Number 11 in brackets designates an Additional Reference at end of discussion.

[31] Note for example that the writer's 1961 thesis cited elsewhere [1, 2] uses a datum different from that employed in the similar Fig. 5 of the present paper.

demonstrate its relation to the very promising but cumbersome finite element approaches presently coming into use [10].

Additional Reference

J. F. Booker, "Dynamically-Loaded Journal Bearings: Numerical Application of the Mobility Method," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 93, No. 1, Jan. 1971, p. 168; Errata: JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 93, No. 2, Apr. 1971, p. 315.

Author's Closure

The author agrees with most of the points raised in Professor J. F. Booker's discussion. For instance, it is agreed that when it comes to digitally computing a complete orbit of the journal center, there is not much of an advantage in the impulse method as compared with Booker's mobility method. On the other hand, when it comes to the graphical construction of such an orbit the former method is both more practicable and more accurate, particularly when the topological characteristics of Section IV are accounted for.

Furthermore, it would appear that the impulse method is better adapted to certain important needs of those responsible for the design and/or the development of the bearings concerned. As has been shown in the paper, by virtue of the concepts of the impulse capacity, the dominant quasi-squeeze interval and the pertinent dominant impulse increment, both the digital computation and the graphical construction of the complete orbit can often be avoided altogether. For instance, this can be achieved whenever the "quick," or even the rather trivial "express," estimate indicates a smallest film thickness that is distinctly unsafe from the point of view of metallic contact. Then, any efforts that might nevertheless be put into the computation or graphical construction would be futile, the theory of full hydrodynamic lubrication no longer being applicable.

Last but not least, there are additional potentialities in the impulse method that, owing to lack of space, could not be disclosed in the paper. For instance, from a cross-plot of Fig. 5 one may readily determine the optimal clearance of a journal bearing of which the dynamic duty is characterized by a known normalized specific impulse increment, ΔJ_{spec}. A cross-plot suiting this purpose can be established by replotting the curves of Fig. 5 point by point as follows. The original coordinates of any chosen point in Fig. 5, $x = h_{\min}/\Delta R$ and $y = \Delta J_n$, are transformed to the new ones of a corresponding point, $xy^{1/2} = (\Delta J_{\text{spec}}/\mu)^{1/2} \cdot h_{\min}/R$ and $y^{1/2} = (\Delta J_{\text{spec}}/\mu)^{1/2} \cdot \Delta R/R$. The new curves thus obtained for the various L/D-ratios indeed suit the present optimizing problem in that the quantity to be optimized, ΔR, appears only in the new coordinate $y^{1/2}$ and the quantity for which the maximum attainable is sought, $h_{\min}$, appears only in the other new coordinate, $xy^{1/2}$. So, the trend of $h_{\min}$ as a function of ΔR can be viewed at a glance from the cross-plotted curves Further, a maximum of $h_{\min}$, as well as the corresponding optimal clearance for the L/D-ratio concerned, can be established forthwith.

It is worth noting that the cross-plot concerned will usually indicate an optimal clearance that, at least at the first blush, might appear impracticably small. For instance, with the journal bearing under the dynamic duty depicted in Fig. 2, the optimal radial clearance thus obtained amounts to a mere 9.9 μm = 3.8×10^{-4} in.[1]) Such a clearance might be considered much too small only if, erroneously enough, no distinction should be made between the clearance to be specified to, and to be made by, the workshop and the one to be aimed at in actual operation. After all, any optimal clearance calculated from some theory of hydrodynamic lubrication cannot but represent the latter, the operational clearance. More precisely (see Section V), this clearance may be conceived as the optimal difference between the local radius of curvature of the surfaces of the bearing and that of the journal to be aimed at in actual operation (compare definition (9d) for the radius of conformity), albeit still under the neglect of the flapping action. In fact, it is the consistent distinction between the constant nominal or workshop clearance and the fluctuating actual or operational one that led the author up to the notion of flapping action in Section V. Lastly, a "clearance," in the sense of a difference between the local radii concerned, is still far from unrealistic if it attains values as small as the aforementioned one of 9.9 μm = 3.8×10^{-4} in.

[32] The corresponding relative gain in minimum film thickness, amounting to a factor of about 3, is quite substantial in view of the thickness occurring with the much greater radial clearance of 82.6 μm = 32.5×10^{-4} in. that was specified for constructing the orbit in Fig. 4.