Improving the accuracy of dynamic localization systems using RTK GPS by identifying the GPS latency

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Abstract

For precise localization of outdoor mobile robots, Real-Time Kinematic Global Positioning System (RTK GPS) has obvious advantages: position data are given with centimeter-accuracy, and the required infrastructure is reduced to a sole fixed reference station. Yet, the use of this solution arises a number of issues, such as satellite maskings, or the existence of the so-called GPS latency which delays the output of the localization data. This paper deals with the latter problem, and proposes a method to identify this parameter without using other external sensors. Experimental results carried out with the robot MELODY are presented so as to validate our solution.

1 Introduction

By using correction data broadcast from a ground-based reference station at a known location, the Real-Time Kinematic technique allows to obtain centimeter-accuracy GPS positions while stationary or moving. Such positioning systems are studied for high-precision guidance of open-cut mining or agricultural vehicles [6]. Civil engineering, and particularly road construction, is also a very active research field, as exemplified by the european projet CIRC (Computer Integrated Road Construction) which concentrates on the automation of road construction machines and identifies localization as the keystone of field automation [7].

Despite its high performances, GPS has some drawbacks which have to be taken into account if one wants the localization system to be reliable. The most obvious problem is satellite masking: when the line of sight from the mobile receiver to one or several satellites is obscured by trees or buildings, GPS localization may not be possible. That is why internal sensors are often used to maintain position data using short term dead-reckoning navigation [2]. Another difficulty is that centimeter-accuracy GPS positions are always available to the user with a delay called *latency*.

This latency has several origins. One of them is of course the calculation of position data itself, but we must also consider the time for transmission to the user, usually by a serial link, and of course the correction data transmission which occurs between the fixed and moving receivers, via a radio serial link. The latency depends on the baudrate of this link, which is adjusted according to the power of the emitter and the distance between the two receivers, but it also depends on the number of visible satellites (since a correction message is sent for each of them). As a result, the total latency can be as large as two seconds [4]. Should the system be used in the control loop of a moving vehicle, this time must be known and taken into account. But there are also cases in which only off-line results are required and, yet, the parameter should be correctly identified. Such a situation arises when the localization algorithm fuses measures of internal sensors and GPS position data. As a consequence, a proper identification of the GPS latency depending on the number of visible satellites is essential when high-precision tasks are considered.

The paper is organised as follows. Section 2 presents a typical multi-sensor localization algorithm which uses RTK GPS and internal sensors. The problems caused by a bad estimation of GPS latency are illustrated using experimental data obtained with tests carried out with the mobile robot MELODY. In the third section, a solution to identify the latency is proposed. Conditions for the parameter to be identifiable are derived and experimental identification results are presented, together with the illustration of their positive effects on the localization algorithm. Conclusions are presented in section 4.

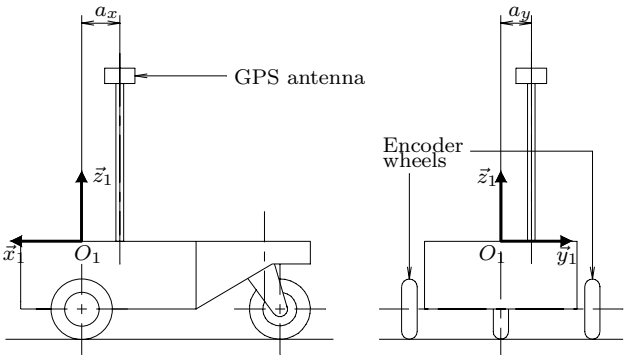

Figure 1: Robot and sensors geometry

2 2-D localization by EKF

2.1 The vehicle and its sensors

The mobile robot MELODY we used for our tests is a three-wheeled vehicle whose 2-D posture is given by the (x, y) co-ordinates of O_1 (see figure 1) and the robot heading ψ in the reference frame.
The wheel encoders yield, at each sampling period T_s, the elementary translation δ of O_1 along axis $\vec{x}_1$, and the elementary rotation θ of the vehicle around vertical axis $\vec{z}_1$, and allow us to write the following equations, assuming the robot does not slip laterally:

$$\left\{ \begin{array}{lcl} x(i) & = & x(i-1) + \delta(\mathbf{u}(i))\cos(\psi(i-1)) \\ y(i) & = & y(i-1) + \delta(\mathbf{u}(i))\sin(\psi(i-1)) \\ \psi(i) & = & \psi(i-1) + \theta(\mathbf{u}(i)) \end{array} \right. \quad (1)$$

where $\mathbf{u}(i)$ is the vector of the elementary rotations of the encoder wheels at discrete time instant t_i.
The GPS antenna is fixed on a pole whose co-ordinates in the vehicle frame (R_1) are (a_x, a_y). Since the amount of data sent by the fixed GPS depends on the number of satellites, we assume that the latency also does. In all cases, the latency is supposed to be a multiple of the sampling period T_s. If T_s is small enough, this is not a problem. Thus, the latency at observation instant t_j is written $T(j) = n(j) \times T_s$ and the GPS position data are given by:

$$\left\{ \begin{array}{lcl} x_{GPS}(j) & = & x(j-n(j)) + a_x\cos(\psi(j-n(j))) \\ & & -a_y\sin(\psi(j-n(j))) \\ y_{GPS}(j) & = & y(j-n(j)) + a_x\sin(\psi(j-n(j))) \\ & & +a_y\cos(\psi(j-n(j))) \end{array} \right. \quad (2)$$

with

$$x(j-n(j)) = x(j) - \sum_{k=j-n(j)+1}^{j} \delta(\mathbf{u}(k))\cos(\psi(j) - \Theta(j,k)) \quad (3)$$

$$y(j-n(j)) = y(j) - \sum_{k=j-n(j)+1}^{j} \delta(\mathbf{u}(k))\sin(\psi(j) - \Theta(j,k)) \quad (4)$$

$$\psi(j-n(j)) = \psi(j) - \Theta(j, j-n(j)+1) \quad (5)$$

$$\Theta(j,k) = \sum_{l=k}^{j} \theta(\mathbf{u}(l)) \quad (6)$$

The use of the new time index j instead of i in equation (1) is meant as a reminder of the fact that GPS measurements are not available at each sampling instant. Typical frequency for equation (2) is 1 Hz, as opposed to 100 Hz for equation (1) in our system.

2.2 The localization algorithm

Equations (1) and (2) can be summarised by the following non-linear system:

$$\left\{ \begin{array}{l} \mathbf{x}(i) = \mathbf{x}(i-1) + \mathbf{f}(\mathbf{x}(i-1), \mathbf{u}^a(i)) \\ \mathbf{z}(j) = \mathbf{g}(\mathbf{x}(j), \mathbf{u}^a(j)) \end{array} \right. \quad (7)$$

where $\mathbf{x}$ is the vehicle 2-D posture, $\mathbf{z}$ is the GPS observation, and $\mathbf{u}^a(k)$ is the vector of the proprioceptive measures performed between time instants $k-N+1$ and k, where N is the maximum value of n, which depends on the number of visible satellites.
Considering that the vehicle model and the sensor measures are corrupted by white, gaussian, zero-mean and uncorrelated noises $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ (associated covariance matrices: $\mathbf{Q}_\alpha$, $\mathbf{Q}_\beta$, and $\mathbf{Q}_\gamma$), system (7) becomes:

$$\left\{ \begin{array}{l} \mathbf{x}(i) = \mathbf{x}(i-1) + \mathbf{f}(\mathbf{x}(i-1), \mathbf{u}^a(i)) + \alpha(i) \\ \mathbf{u}^{a*}(i) = \mathbf{u}^a(i) + \boldsymbol{\beta}(i) \\ \mathbf{z}(j) = \mathbf{g}(\mathbf{x}(j), \mathbf{u}^a(j)) + \boldsymbol{\gamma}(j) \end{array} \right. \quad (8)$$

where $\mathbf{u}^{a*}$ is the measure of the deterministic input $\mathbf{u}^a$.
We briefly recall the Extended Kalman Filter equations allowing to combine internal and external sensors measures: at each sampling period, the state estimation $\hat{\mathbf{x}}$ and the associated error covariance matrix $\mathbf{P}$ are computed from the proprioceptive measures, and they are updated each time a GPS measure occurs (i.e. every second with the Trimble 7400 MSi we use). For further information, the reader is referred to [5].
The prediction step is computed as follows:

$$\left\{ \begin{array}{ll} \hat{\mathbf{x}}(i|i-1) = & \mathbf{f}(\hat{\mathbf{x}}(i-1|i-1), \mathbf{u}^{a*}(i)) \\ \mathbf{P}(i|i-1) = & \mathbf{A}(i)\mathbf{P}(i-1|i-1)\mathbf{A}(i)^T \\ & +\mathbf{B}(i)\mathbf{Q}_\beta\mathbf{B}(i)^T + \mathbf{Q}_\alpha \end{array} \right. \quad (9)$$

with the following Jacobian matrices:

$$\mathbf{A}(i) = \left.\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(i-1|i-1), \mathbf{u}^{a*}(i)} \tag{10}$$

$$\mathbf{B}(i) = \left.\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{u}^a}\right|_{\hat{\mathbf{x}}(i-1|i-1), \mathbf{u}^{a*}(i)} \tag{11}$$

And the updating step is given by the following equations:

$$\begin{cases} \hat{\mathbf{x}}(j|j) = & \hat{\mathbf{x}}(j|j-1) \\ & +\mathbf{K}(j)\left(\mathbf{z}(j) - \mathbf{g}(\hat{\mathbf{x}}(j|j-1), \mathbf{u}^{a*}(j))\right) \\ \mathbf{P}(j|j) = & \mathbf{P}(j|j-1) \\ & -\mathbf{K}(j)\left(\mathbf{C}(j)\mathbf{P}(j|j-1) + \mathbf{S}(j)^T\right) \end{cases} \tag{12}$$

where, knowing the correlation term:

$$\mathbf{S}(j) = \mathbf{B}(j)\mathbf{Q}_\beta \mathbf{D}(j)^T \tag{13}$$

and the Jacobian matrices:

$$\mathbf{C}(j) = \left.\frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(j|j-1), \mathbf{u}^{a*}(j)} \tag{14}$$

$$\mathbf{D}(j) = \left.\frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{u}^a}\right|_{\hat{\mathbf{x}}(j|j-1), \mathbf{u}^{a*}(j)} \tag{15}$$

the Kalman gain $\mathbf{K}(j)$ is given by:

$$\begin{aligned} \mathbf{K}(j) = & \left[\mathbf{P}(j|j-1)\mathbf{C}(j)^T + \mathbf{S}(j)^T\right] \\ & \left[\mathbf{C}(j)\mathbf{P}(j|j-1)\mathbf{C}(j)^T + \mathbf{D}(j)\mathbf{Q}_\beta\mathbf{D}(j)^T \right. \\ & \left. + \mathbf{Q}_\gamma + \mathbf{C}(j)\mathbf{S}(j) + \mathbf{S}(j)^T\mathbf{C}(j)^T\right]^{-1} \end{aligned} \tag{16}$$

The estimator defined in this section works well provided that the GPS latency is correctly identified. We show thereafter how to check the validity of this value.

Remark 1
Another algorithm fusing RTK GPS and internal sensors, and taking into account the GPS latency is proposed in [1] to localise an autonomous mower. But the way to obtain the value of the delay is not discussed.

2.3 Coherence test to check the latency value

The test we use consists in calculating the following squared distance (sometimes called "Mahalanobis distance") at each observation instant [8]:

$$d^2(j) = \mathbf{e}^T(j)\mathbf{P}_{ee}^{-1}(j)\mathbf{e}(j) \tag{17}$$

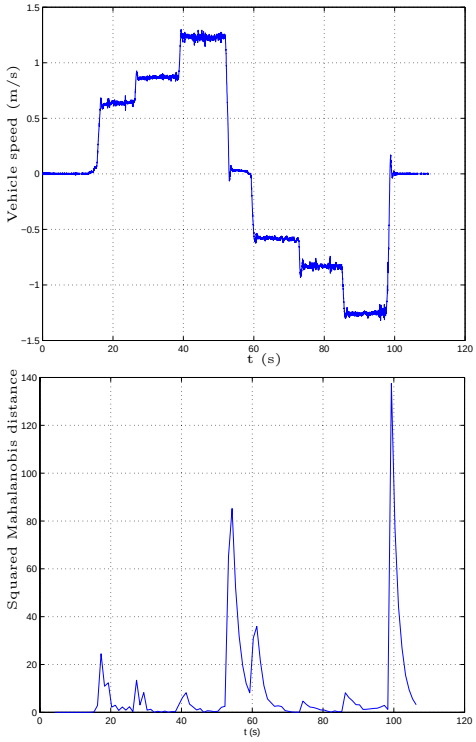

Figure 2: Squared Mahalanobis distance for an erroneous value of the GPS latency

where $\mathbf{e}$ is the innovation, i.e. the difference between the measured and expected outputs, and $\mathbf{P}_{ee}$ is its covariance matrix.

In the case of a linear system corrupted by zero-mean gaussian noises, and with a 2-dimensional innovation, the squared Mahalanobis distance d^2 has a 99.7% probability to be lower than $k = 11.820$ (provided the noise variances are properly identified) [5]. Thus, if $d^2(j)$ exceeds this value, it means that at instant t_j, it is highly probable that the observation is not coherent with the prediction process. Despite the fact that in our application, the system is nonlinear and the perturbations not necessarily gaussian, we have kept this value to check the coherence of our data.

Figure 2 displays, for a straight line motion of a civil-engineering machine, the squared Mahalanobis distance calculated with a latency set to the minimal value given by the manufacturer Trimble (i.e. $T = 0.4$ s). In the acceleration/deceleration phases, it is clear that GPS data are not coherent with the proprioceptive measures, and this because the speed changes are immediately reflected by the internal sensors, whereas GPS data are only affected by accelerations after a delay equal to the latency.

To estimate this parameter, a solution could be to tune T until the squared Mahalanobis distance is always smaller than 11.820. But this manual solution is not satisfying in practice. Instead, we show that it is possible to identify the latency using another Kalman filter.

3 Latency identification

3.1 Definition of a new estimator

On the basis of the estimator defined in section 2, we now want to evaluate not only the vehicle 2-D posture, but also the GPS latency T. The state vector of the system is chosen as:

$$\mathbf{x}(i) = [x(i), y(i), \psi(i), T(i)]^T \tag{18}$$

Since we do not know the real GPS latency, we must store the proprioceptive measures performed between time instants $t_i - T_{max}$ and t_i, where T_{max} is the maximum possible value of the latency. As a consequence, the input vector $\mathbf{u}^a$ contains the proprioceptive measures of the last two seconds:

$$\mathbf{u}^a(i) = \left[\mathbf{u}^T(i), \mathbf{u}^T(i-1), \dots, \mathbf{u}^T(i - I(2/T_s))\right]^T \tag{19}$$

Assuming that the number of visible satellites does not change during the vehicle displacements (i.e. the GPS latency is constant), we have the following evolution model (1):

$$\begin{cases} x(i) &= x(i-1) + \delta(\mathbf{u}(i))\cos(\psi(i-1)) \\ y(i) &= y(i-1) + \delta(\mathbf{u}(i))\sin(\psi(i-1)) \\ \psi(i) &= \psi(i-1) + \theta(\mathbf{u}(i)) \\ T(i) &= T(i-1) \end{cases} \tag{20}$$

And the new observation equations are given by:

$$\begin{cases} x_{GPS}(j) = & x(j) - \sum_{k=M(j)}^{j} \delta(\mathbf{u}(k))\cos\left(\psi(j) - \Theta(j,k)\right) \\ & + a_x \cos\left(\psi(j) - \Theta(j, M(j))\right) \\ & - a_y \sin\left(\psi(j) - \Theta(j, M(j))\right) \\ y_{GPS}(j) = & y(j) - \sum_{k=M(j)}^{j} \delta(\mathbf{u}(k))\sin\left(\psi(j) - \Theta(j,k)\right) \\ & + a_x \sin\left(\psi(j) - \Theta(j, M(j))\right) \\ & + a_y \cos\left(\psi(j) - \Theta(j, M(j))\right) \end{cases} \tag{21}$$

where

$$M(j) = j - I(T(j)/T_s) + 1 \tag{22}$$

The Kalman filter deduced from this system is initialized by setting T equal to a minimum value.
We now propose to determine in which conditions this estimator is usable by studying the system observability.

3.2 System observability

The property of observability for a dynamical system is related to the possibility to reconstruct the state information given the inputs and outputs of the system. More precisely, a system is *observable* if, for any pair of different initial states $\mathbf{x}'(0)$ and $\mathbf{x}''(0)$, there exists an input $\mathbf{u}(t)$ such that the corresponding output trajectories $\mathbf{z}(\mathbf{x}'(0), t)$ and $\mathbf{z}(\mathbf{x}''(0), t)$ are not identical. If this property is true only if $\mathbf{x}''(0)$ belongs to the neighbourhood of $\mathbf{x}'(0)$, the system is called *weakly observable*.
Let us consider the following continuous-time MIMO system:

$$\begin{cases} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{z} &= \mathbf{g}(\mathbf{x}, \mathbf{u}) \end{cases} \tag{23}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{z} \in \mathbb{R}^p$.
We associate it with the $(np \times n)$ *observability matrix* $\mathbf{O}$ calculated as follows:

$$\mathbf{O} = \begin{bmatrix} \dfrac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \\ \dfrac{\partial \mathcal{N}_f \mathbf{g}(\mathbf{x}, \bar{\mathbf{u}})}{\partial \mathbf{x}} \\ \vdots \\ \dfrac{\partial \mathcal{N}_f^{n-1} \mathbf{g}(\mathbf{x}, \bar{\mathbf{u}})}{\partial \mathbf{x}} \end{bmatrix} \tag{24}$$

with the differential operator $\mathcal{N}_f$ defined by:

$$\mathcal{N}_f \mathbf{g} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \mathbf{f} + \frac{\partial \mathbf{g}}{\partial \bar{\mathbf{u}}} \dot{\bar{\mathbf{u}}} \tag{25}$$

and the mp-dimensional vector $\bar{\mathbf{u}}$ given by:

$$\bar{\mathbf{u}} = \left[\mathbf{u}^T, \dot{\mathbf{u}}^T, \dots, \mathbf{u}^{(n-1)^T}\right]^T \tag{26}$$

In [3], the authors define the following rank condition for non-linear systems: *if, for all $\boldsymbol{x} \in \mathbb{R}^n$, there exists $\bar{\boldsymbol{u}} \in \mathbb{R}^{mn}$ such that the rank of the observability matrix $\boldsymbol{O}$ is equal to n, then system (23) is weakly observable.* This sufficient condition has been later applied to non-linear systems whose output $\mathbf{z}$ depends on the state $\mathbf{x}$ *and* the input $\mathbf{u}$ in [9]. But since this observability condition is given for continuous systems, we rewrite our system as:

$$\begin{cases} \dot{x} &= v(\mathbf{u}^a)\cos(\psi) \\ \dot{y} &= v(\mathbf{u}^a)\sin(\psi) \\ \dot{\psi} &= \omega(\mathbf{u}^a) \\ \dot{T} &= 0 \end{cases} \tag{27}$$

where ω is the rotation speed around $\vec{z}_1$, and v, the speed of O_1 along $\vec{x}_1$, and the observations are given by:

$$\begin{cases} g_1 = & x(t) + a_x \cos\left(\psi(t) - \int_{t-T}^{t} \omega(\mathbf{u}^a(\tau))d\tau\right) \\ & -a_y \sin\left(\psi(t) - \int_{t-T}^{t} \omega(\mathbf{u}^a(\tau))d\tau\right) \\ & - \int_{t-T}^{t} v(\mathbf{u}^a(\tau)) \cos\left(\psi(t) - \int_{\tau}^{t} \omega(\mathbf{u}^a(\tau'))d\tau'\right) d\tau \\ g_2 = & y(t) + a_x \sin\left(\psi(t) - \int_{t-T}^{t} \omega(\mathbf{u}^a(\tau))d\tau\right) \\ & +a_y \cos\left(\psi(t) - \int_{t-T}^{t} \omega(\mathbf{u}^a(\tau))d\tau\right) \\ & - \int_{t-T}^{t} v(\mathbf{u}^a(\tau)) \cos\left(\psi(t) - \int_{\tau}^{t} \omega(\mathbf{u}^a(\tau'))d\tau'\right) d\tau \end{cases} \tag{28}$$

For the rank of the observability matrix to be 3 or less, the determinants of *all* the 4×4 partial observability matrices obtained by selecting rows of $\mathbf{O}$ must be equal to zero. Conversely, if one of those determinants is not equal to zero, then the system is observable. Let us consider a first determinant Δ_1:

$$\Delta_1 = det\left(\begin{bmatrix} \dfrac{\partial g_1(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{x}} \\ \dfrac{\partial g_2(\mathbf{x}, \mathbf{u}^a)}{\partial \mathbf{x}} \\ \dfrac{\partial \mathcal{N}_f g_1(\mathbf{x}, \bar{\mathbf{u}}^a)}{\partial \mathbf{x}} \\ \dfrac{\partial \mathcal{N}_f g_2(\mathbf{x}, \bar{\mathbf{u}}^a)}{\partial \mathbf{x}} \end{bmatrix}\right) \tag{29}$$

After simplification, we get:

$$\begin{aligned} \Delta_1 = \quad & v(t-T)(\dot{v}(t-T) - a_y\dot{\omega}(t-T)) \\ & +\omega(t-T)((a_x^2 + a_y^2)\dot{\omega}(t-T) - a_y\dot{v}(t-T)) \end{aligned} \tag{30}$$

According to (30), by cancelling $\omega(t-T)$ and $\dot{\omega}(t-T)$, which means that the vehicle follows a straight line, the system is observable (and the latency identifiable) provided the vehicle tangential acceleration and speed are not zero. Moreover, when $\omega(t-T) = 0$, $\dot{\omega}(t-T) = 0$, and $\dot{v}(t-T) = 0$, we observe that whatever the partial observation matrix we consider, its determinant is always equal to zero.
Since a translation at constant speed is a very common situation, these results mean that the filter with online identification of the latency cannot be used as a routine localization algorithm. But since the latency is constant for a given tuning of the GPS parameters and is identifiable under simple conditions, it is possible to identify it in a proper initialisation phase and then switch to the localization algorithm of section 2.

3.3 Identification protocol and results

From the results given by equation (30), we have defined the vehicle movements so as to properly identify the GPS latency. The mobile robot follows straight lines, repeatedly accelerating and braking, and without cancelling its tangential speed (see figures 3a). The identifications carried out on several tests give for six visible satellites and a 4800-baud radio link an average value of $T_{6sat} = 0.8$ s (see figure 3b). For seven satellites, the average value is $T_{7sat} = 0.9$ s and for eight satellites, it reaches $T_{8sat} = 1$ s.
Thus, if we consider for instance the localization of compactors, which are civil-engineering machines moving at the average working speed $v = 1$ m/s, not taking into account the RTK GPS latency means that, without any satellite masking, the positioning system makes a systematic error of 80 cm at least, which is already higher than the ±20 cm tolerance defined by the end-users of an operator-aiding system such as the one described in [7] for compactors.

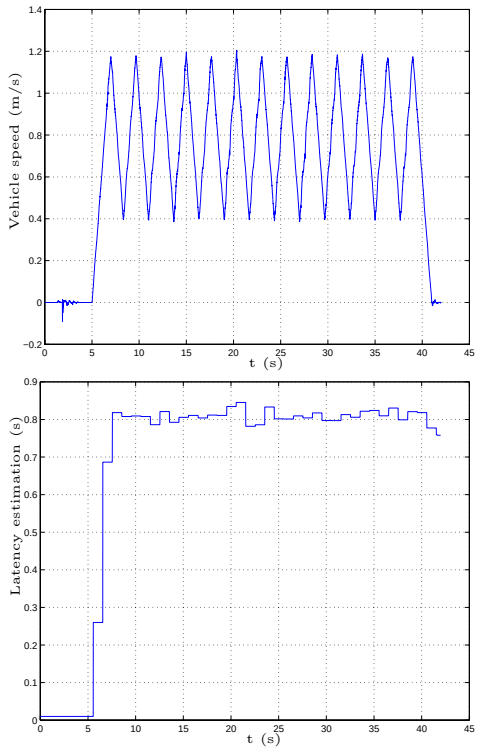

Figure 3: Example of latency identification with 6 visible satellites

3.4 Experimental validation

Once the GPS latency is identified for different values of the number of visible satellites (on our trial field, it never exceeded 8 satellites), the localization algorithm can be properly tuned: since the chosen sampling period is 100 Hz and the maximum value of the latency is

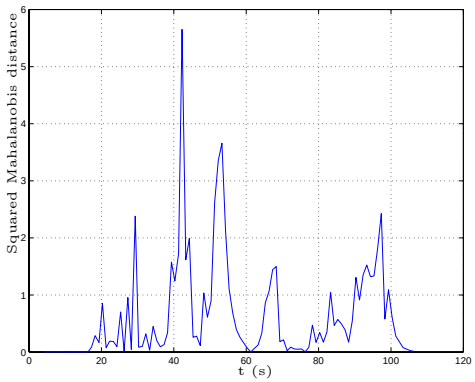

Figure 4: Squared Mahalanobis distance with properly identified latency

1 s, the last 100 proprioceptive measures will be stored and available at each time instant ($N = 100$). And, depending on the number of visible satellites issued by the mobile GPS station, the latency value used in the updating phase of the EKF will vary between 0.8 and 1 s.

Figure 4 presents the same test as the one in section 2.3 but this time, the latency value is correct: since only 6 satellites are visible, we have $T = 0.8$ s for all the updating steps and the resulting Mahalanobis distance is always lower than the chosen threshold.

4 Summary and Conclusions

Localization sensors such as RTK GPS deliver position data with a delay called latency, which depends on some setup parameters and on the number of visible satellites. In any dynamic application, or when such a sensor is used with proprioceptive sensors, the latency must be known. We have shown that, in the latter case, a wrong value of the latency results in GPS data being incoherent with position data obtained by a prediction based on proprioceptive sensors. This occurs when the vehicle accelerates or brakes and can be detected without additional position sensors using the Mahalanobis distance.

Moreover, we present an algorithm to identify the latency for a given number of visible satellites. Through observability analysis, we derive the conditions under which identification must take place to yield good results. Namely, the vehicle must perform a series of accelerations and brakings. Again, this identification does not require any additional exteroceptive position sensor. This makes it suitable for use on the vehicle solely equipped with its standard localization setup: GPS plus relative localization sensors. To our knowledge, this is the first time such an approach is proposed.

Our algorithms have been tested using experimental data obtained with a mobile robot and an instrumented civil-engineering machine. After identifying the latency, the coherence between GPS data and predicted positions obtained by relative localization is restored. The solution we present is not limited to our experimental setup. Any vehicle equipped with relative localization sensors and a GPS (or any other localization sensor with significant latency) would require to solve the same problem, and could use the principle of our solution.

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