Empirical analysis to determine Weighted OWA orness

Vicenç Torra
Institut d'Investigació en Intel.ligència Artificial - CSIC
Campus UAB s/n, E-08193 Bellaterra (Catalunya, Spain)
e-mail: vtorra@iiia.csic.es, http://www.iiia.csic.es/˜vtorra

Abstract – *In this work we study the orness of the WOWA (Weighted Ordered Weighted Averaging) operator. This operator takes into account the importance of the sources (e.g. the reliability of a source or the confidence in an expert) and the importance of the values (e.g. whether the output should be sensitive to small values - to avoid collisions - or to large values). It generalizes the weighted mean and the OWA (Ordered Weighted Averaging) operator. The orness of an aggregation operator measures in which degree the method behaves as the maximum operator or as the minimum. This can be understood as being optimistic or not, or being compensative or not.*

Keywords: Keywords: Aggregation operators, Choquet integral, OWA operators, WOWA operators, function selection, fuzzy sets.

1 Introduction

Information fusion methods are currently in use in applications belonging to completely different areas. However, they are used mainly for two purposes: to make decisions and to have a good representation of the application domains. In both cases, techniques are used to combine information (i) corresponding to the same attribute that come either from different sources or from a single source but obtained at different instants of time or (ii) corresponding to different attributes (e.g. different criteria).

Due to the different situations in which information fusion methods apply, there exist nowadays a large quantity of operators. To help on the process of selecting a method, it is absolutely necessary to characterize them. This is, to determine intrinsic differences between the methods. This can be done from a formal point of view (which are the properties - e.g. equations - the methods satisfy and which are the methods that satisfy a given set of properties) or from a behavioral point of view (e.g. tolerance to error, degree of disjunction, etc).

Research has been focused in the last years in both aspects. For example, there exists already a characterization and classification of several numerical aggregation methods. For example, see [1], [2] and [3] for results related to fuzzy t-conorm integrals. In particular, it is well known that the weighted mean and the Ordered Weighted Aggregation (OWA) operator are generalized by the Weighted OWA (WOWA) and that all these operators are generalized by the Choquet integral. It is also known that the Choquet integral and the Sugeno integral are generalized by the Fuzzy integral. Similar results have also been obtained for fusion methods for hierarchical classifications (e.g. [4]).

Here, we understand generalization in the following sense:

Let $\mathbb{C}_1$ and $\mathbb{C}_2$ be two information fusion operators. We use $\mathbb{C}_1(p_1)$ to denote that $\mathbb{C}_1$ depends on a certain parameter p_1 and $\mathbb{C}_2(p_2)$ to denote that $\mathbb{C}_2$ depends on a certain parameter p_2. Then, it is said that $\mathbb{C}_1$ generalizes $\mathbb{C}_2$ if for all parameterization p_1 of $\mathbb{C}_1$ exists a parameterization p_2 of $\mathbb{C}_2$ such that both operators are equal. this can be expressed as:

$$\text{for all } p_1 \text{ there exists } p_2 \text{ such that } \mathbb{C}_1(p_1) = \mathbb{C}_2(p_2) \quad (1)$$

For methods that combine numerical values, behavioral analysis has also been studied (see [5] for a review). In particular, several indices have been defined. Importance and interaction among criteria (the Shapley and Banzhaf power indices measure importance of criteria) and the degree of disjunction (the orness measure). As these indices are computed for a particular operator and a particular parameterization, they are also adequate to analyze the behavior of the parameters in a given method.

In this work we study the orness of the Weighted OWA (WOWA) operator. This operator., introduced in [6], was defined to take into account in the same operator the importance of the sources (e.g. the reliability of a source or the confidence in an expert) and the importance of the values (e.g. whether the output should be sensitive to small values - to avoid collisions - or to large ones). The orness of an aggregation operator measures in which degree the method behaves as the maximum operator or as the minimum. This can be understood as being optimistic or not, or being compensative or not.

The structure of this work is as follows. We begin in Section 2 with some basic definitions of aggregation operators.

Then, we review the orness measures and finally we analyse the orness of the WOWA operator for several parameterizations. Conclusions finish this work.

2 Aggregation operators for numerical values

In this Section we review some aggregation operators for numerical values, outlining the meaning of their parameters and the relationship between operators. We define the weighted mean, the OWA and the WOWA operators. Also, some preliminary definitions (i.e., weighting vector and regular monotonically non-decreasing fuzzy quantifier) are given.

Definition 1 *A vector* $\mathbf{v} = (v_1...v_n)$ *is a* weighting vector *of dimension n iff* $v_i \in [0,1]$ *and* $\sum_i v_i = 1$.

Definition 2 *A function* $Q : [0,1] \to [0,1]$ *is a* regular monotonically non-decreasing fuzzy quantifier *(non-decreasing fuzzy quantifiers for short) if it satisfies: (i)* $Q(0) = 0$*; (ii)* $Q(1) = 1$*; (iii)* $x > y$ *implies* $Q(x) \geq Q(y)$.

A fuzzy quantifier generalizes the quantifiers existing in propositional logic. In this case, not only "there exists" and "for all" can be modeled but also e.g. "most", "some" and "about a half".

Two particular quantifiers are of interest: (i) $Q_x(0) = 0$ and $Q_x(x) = 1$ for $x \neq 0$; (ii) $Q_n(1) = 1$ and $Q_n(x) = 0$ for $x \neq 1$. The former quantifier corresponds to "there exists" and the latter corresponds to "for all". When combined with the OWA operator (see definition below), we obtain, respectively, the maximum and the minimum of the values we are aggregating.

Definition 3 *Let* $\mathbf{p}$ *be a weighting vector of dimension* n*, then a mapping WM:* $\mathbb{R}^n \to \mathbb{R}$ *is a* weighted mean *of dimension* n *if* $WM_{\mathbf{p}}(a_1,...,a_n) = \sum_i p_i a_i$.

We consider two alternative definitions of OWA (Ordered Weighted Averaging) operators [7] [8]. First using a weighting vector and then using a non-decreasing fuzzy quantifier.

Definition 4 *Let* $\mathbf{w}$ *be a weighting vector of dimension* n*, then a mapping* $OWA_w : \mathbb{R}^n \to \mathbb{R}$ *is an* Ordered Weighting Averaging (OWA) operator *of dimension* n *if*

$$OWA_w(a_1,...,a_n) = \sum_{i=1}^{n} w_i a_{\sigma(i)} \tag{2}$$

where $\{\sigma(1),...,\sigma(n)\}$ *is a permutation of* $\{1,...,n\}$ *such that* $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ *for all* $i = \{2,...,n\}$ *(i.e.* $a_{\sigma(i)}$ *is the i-th largest element in the collection* $a_1,...,a_n$*).*

Definition 5 *Let* Q *be a non-decreasing fuzzy quantifier, then a mapping* $OWA_Q : \mathbb{R}^n \to \mathbb{R}$ *is an* Ordered Weighting Averaging (OWA) operator *of dimension* n *if*

$$OWA_Q(a_1,...,a_n) = \sum_{i=1}^{n} (Q(i/n) - Q((i-1)/n)) a_{\sigma(i)} \tag{3}$$

where σ *is defined as above.*

Both definitions are equivalent as w_i can be defined from Q as $w_i = Q(i/n) - Q((i-1)/n$ and Q can be defined as a function that interpolates the points $\{(i/n, Q(i/n))\}$ for $i \in \{0,1,\cdots,n\}$. Observe that the quantifier Q_x defined above lead to the maximum when used with definition 5. Thus, the OWA operator generalizes the maximum operator (in a similar way the OWA operator generalizes the minimum with Q_n).

The weighted mean and the OWA operator have similar expressions. However, in spite of their similarity, the meaning of the weights is radically different due to the presence of the (ordering) permutation σ in the OWA operator. It is well known that in the weighted mean, weighting vectors are used to express the reliability of the information sources that have supplied a particular value. I.e., p_i corresponds to a measure of the reliability of the i-th sensor or of the expertise of the i-th expert. This is not the case of the OWA operator, where weights assign importance to elements according to their position with respect to the others. In this way, a system can reduce the importance of extreme values, or to give larger importance to small values than to larger ones (e.g. to avoid collisions in a mobile system). So, this corresponds to weight the values themselves instead of weighting the sources.

According to this interpretation, weighting vectors in these operators are complementary (we will refer to them as $\mathbf{p}$ and $\mathbf{w}$ as in the definitions above) and in some circumstances both are of interest in a single application (e.g., robotics, multicriteria decision making). See, for example, in the following situations:

1. *Robot sensing (all data corresponding to the same time instant):* In this case, $\mathbf{p}$ would be used to express the reliability of each sensor, instead $\mathbf{w}$ would be used to determine in which degree small values are important (to avoid collisions).

2. *Robot sensing (data obtained at different time instants):* In this case, $\mathbf{p}$ would be used to give more importance to recent data than to *old* one, while $\mathbf{w}$ would be used, as before, to express the importance of small values.

To allow both weighting vectors in a single problem, [6] introduced the Weighted OWA (WOWA) operator. This operator aggregates a set of values using two weighting vectors: one corresponding to the vector $\mathbf{p}$ in the weighted

mean and the other corresponding to $\mathbf{w}$ in the OWA operator.

Definition 6 *Let* $\mathbf{p}$ *and* $\mathbf{w}$ *be two weighting vectors of dimension* n*, then a mapping WOWA:* $\mathbb{R}^n \to \mathbb{R}$ *is a* Weighted Ordered Weighted Averaging (WOWA) operator *of dimension* n *if*

$$WOWA_{\mathbf{p},\mathbf{w}}(a_1, ..., a_n) = \sum_i \omega_i a_{\sigma(i)} \tag{4}$$

where, as before, (i.e. $a_{\sigma(i)}$ *is the i-th largest element in the collection* $a_1, ..., a_n$*), and the weight* ω_i *is defined by*

$$\omega_i = w^*(\sum_{j \le i} p_{\sigma(i)}) - w^*(\sum_{j \le i} p_{\sigma(i)}) \tag{5}$$

with w^* *being a monotonic increasing function that interpolates the points* $(i/n, \sum_{j \le i} w_j)$ *together with the point* $(0, 0)$*. The function* w^* *is required to be a straight line when the points can be interpolated in this way (see [9], [10] and [11] for interpolation).*

In a similar way that for the OWA operator, we can define the WOWA using a fuzzy quantifier (instead of having the weighting vector $\mathbf{w}$). This definition is equivalent to Yager's definition of OWA using importances [8].

Definition 7 *Let* Q *be a non-decreasing fuzzy quantifier, let* $\mathbf{p}$ *be a weighting vector of dimension* n*, then a mapping WOWA:* $\mathbb{R}^n \to \mathbb{R}$ *is a* WOWA operator *of dimension* n *if*

$$WOWA_{\mathbf{p},Q}(a_1, ..., a_n) = \sum_i \omega_i a_{\sigma(i)} \tag{6}$$

where, σ *is defined as before and*

$$\omega_i = Q(\sum_{j \le i} p_{\sigma(i)}) - Q(\sum_{j \le i} p_{\sigma(i)}) \tag{7}$$

Note that the WOWA operator is also a linear combination of the values with respect to a vector (in this case ω). When studied from this point of view, the operator determines a weight $\omega_{\sigma(i)}$ for each value a_i in terms of the two weighting vectors $\mathbf{p}$ and $\mathbf{w}$ in such a way that the initial weight p_i for a_i is increased (i.e. $\omega_{\sigma(i)} \ge p_i$) if the value a_i is small and small values have more importance than larger ones (the same if the value is large and importance is given to large values). Instead, when importance is given to large values and a_i is small, then $\omega_{\sigma(i)} \le p_i$ (the same if importance is given to small valuess and a_i is large). Here, small and large should not be understood as an absolute term in the domain but relative to the other values in $a = (a_1, ..., a_n)$. In particular, quantifiers similar to Q_x give impoerance to large values while quantifiers similar to Q_n give importance to small values.

In fact, the shape of the function w^*, or the fuzzy quantifier Q, (specially its derivative) show the importance of the elements in relation to their position.

This operator, see [12] for details, generalizes the weighted mean and the OWA operator: when $\mathbf{p} = (1/n \ ... \ 1/n)$ the operator reduces to the OWA operator and when $\mathbf{w} = (1/n \ ... \ 1/n)$ the operator reduces to the weighted mean.

All these operators can be expressed as Choquet integrals when appropriate fuzzy measures are defined. As this property will be used to compute the orness of the WOWA operator we define Choquet integral below (see [1]):

Definition 8 *A fuzzy measure* μ *on a set* X *is a set function* $\mu : \wp(X) \to [0, 1]$ *satisfying the following axioms:*

(i) $\mu(\emptyset) = 0, \mu(X) = 1$ *(boundary conditions)*

(ii) $A \subseteq B$ *implies* $\mu(A) \le \mu(B)$ *(monotonicity)*

Fuzzy measures replace the axiom of additivity in probability measures ($\mu(A \cup B) = \mu(A) + \mu(B)$ when $A \cap B = \emptyset$) by a more general one: monotonicity. Thus, probability measures are also fuzzy measures. Fuzzy measures are used in Choquet integrals to express the importance of a set of information sources. When additivity is not satisfied, it means that the importance of a set is not the addition of the importance of the elements by themselves. In this setting, weighting vectors for weighted means corresponds to additive fuzzy measures.

Definition 9 *Let* μ *be a fuzzy measure on X. The* Choquet integral *of a function* $f : X \to \mathbb{R}$ *with respect to* μ *is defined by:*

$$\sum_{i=1}^{n} (f(x_{s(i)}) - f(x_{s(i-1)})) \mu(A_{s(i)}) \tag{8}$$

where $f(x_{s(i)})$ *indicates that the indices have been permuted so that* $0 \le f(x_{s(1)}) \le ... \le f(x_{s(N)}) \le 1$*,* $A_{s(i)} = \{x_{x(i)}, ..., x_{s(N)}\}$ *and* $f(x_{\sigma(0)}) = \emptyset$*.*

A WOWA operator over a non-decreasing fuzzy quantifier Q and a weighting vector $\mathbf{w}$ is a Choquet integral over a fuzzy measure μ defined by [13]:

$$\mu(A) = Q(\sum_{x \in A} p(x)) \tag{9}$$

3 Orness measures

One way to evaluate the behaviour of an aggregation operator with a particular parameter is to use the so-called orness measure. It is used to account for the similarity of the operator with the maximum when a particular parameterization is used. Marichal [5] points out that the concept of orness was introduced in 1974 by Dujmovic in the particular case of root-mean powers. In an independent way it was introduced (together with a dual measure called *andness*)

by Yager [7] to account for the similarity of the OWA operator with the maximum (and minimum, respectively) when a particular weighting vector is used.

In [5], Marichal derives, from the concept of average value, an orness measure for the Choquet integral with a particular fuzzy measure μ (we denote this Choquet integral by C_μ). It is based on the average value of the Choquet integral $m(C_\mu)$:

$$m(C_\mu) = (C)\int_{[0,1]^n} f d\mu \tag{10}$$

and it is defined as:

$$orness(C_\mu) = \frac{m(C_\mu) - m(min)}{m(max) - m(min)} \tag{11}$$

In the case of the Choquet integral this measure can be rewritten as:

$$orness(C_\mu) = \frac{1}{n-1}\sum_{T\subset X}\frac{(n-t)!t!}{n!}\mu(T) \tag{12}$$

where $t = |T|$ and $n = |X|$

Yager's definition of orness for the OWA operator is a particular case of this measure and it is defined as:

Definition 10 *[7] Given a weighting vector w, the orness of the OWA operator for w is:*

$$orness(w) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i \tag{13}$$

This definition is in such a way that when applied to a weighting vector w the orness is maximal (i.e., orness (w) = 1) when the OWA operator behaves like the maximum operator (i.e., this corresponds to have an OWA with the maximum quantifier – the "there exists" quantifier). Instead, orness is minimal (i.e., orness(w) = 0) when the OWA operator behaves like the minimum operator (i.e., this corresponds to have the OWA with the minimum quantifier – the "for all" quantifier). Note that when considering aggregation in decision making process, orness can be understood as the degree of compensation [14].

Yager's definition clearly shows that the definition of the orness depends on the weighting vector. However, when several weighting vectors of different dimensions are extracted from the same quantifier, the orness of the weighting vectors are different. To make comparison among quantifiers easier and to make evaluation of quantifiers possible, the orness of a quantifier was defined in [15]. The measure is defined in such a way that when weighting vectors of increasing dimension are extracted, their orness tend to be the orness of the quantifier. So, orness of weighting vectors approximate the orness of the quantifier.

Definition for the quantifier is based on a rewritting of expression (13). Note that the following expressions are equivalent to the one given above:

$$\begin{aligned} orness(w) &= \sum_{i=1}^{n}(\frac{n-i}{n-1})w_i \\ &= \frac{n-1}{n-1}w_1 + \frac{n-2}{n-1}w_2 + ... + \frac{n-n}{n-1}w_n \\ &= \sum_{i=1}^{n-1}(1/n-1)Q(i/n) \end{aligned} \tag{14}$$

where Q is the quantifier that interpolates the points $\{(i/n, \sum_{j<=i} w_j)\}$ (or, alternatively, it is the quantifier from which the weights w_i are extracted). Now, instead of considering the last expression we use:

$$\sum_{i=1}^{n}(1/n)Q(i/n) \tag{15}$$

It is clear that $\sum_{i=1}^{n}(1/n)Q(i/n)$ and $\sum_{i=1}^{n-1}(1/n-1)Q(i/n)$ tend to be equal for large values of n. The latter expression is easily generalized to the continuous case by means of an integral. We will refer to this generalization as the continuous orness (or orness of the quantifier Q).

Definition 11 *Given a fuzzy quantifier Q, the continuous orness measure for Q is defined as:*

$$orness(Q) = \int_0^1 Q(x)dx \tag{16}$$

4 Experiments and conclusions

Orness of WOWA operators have been evaluated using expression (12) with the fuzzy measure μ defined by expression (9) and compared with the orness of the corresponding quantifier using Definition 11. The experiments show that the orness of the former tends to be the orness of the latter (i.e., the value tend to be the orness of the fuzzy quantifier and does not depend much on the $\mathbf{p}$). Experiments have been implemented using three different quantifiers. They are the following ones:

$$Q_1(x) = \frac{e^{xlog(1+\lambda)} - 1.0}{\lambda} \tag{17}$$

$$Q_2(x) = x^\lambda \tag{18}$$

$$Q_3(x) = \begin{cases} 0 & \text{if } x \le \lambda \\ 1 & \text{if } x > \lambda \end{cases} \tag{19}$$

The first quantifier corresponds to the Sugeno quantifiers (the quantifiers that generate Sugeno λ-measures - defined in [16] and used e.g. in [17]), the second one is the family of Yager quantifiers (defined in [18]) and the third one is

the Heavyside family of quantifiers (these quantifiers correspond to "more than λ% of the individuals"). These three families have been selected because apart from the existing applications, parameterizations allow to cover the whole range of values between the minimum and the maximum of the values to be aggregated. Note that the ranges of λ are $(-1, +\infty)$ for the Sugeno family of λ-measures, $(0, +\infty)$ for the Yager family and $(0, 1)$ for the Heavyside family, and note also that in all three cases with $\lambda = 0$ the maximum is achieved and, instead, $\lambda = +\infty$ or $\lambda = 1$ lead to the minimum.

For each quantifier, each parameterization and dimension (where dimension is the number of elements to be taken into account), we have computed the orness for 50 different weighting vectors $\mathbf{p}$ generated at random. Then, we have averaged the corresponding orness. In all tested cases, the general tendency is the same: The greatest the dimension (i.e., more elements are considered), the more similar is the WOWA orness to the orness of the corresponding quantifier.

These tests have been done using the quantifier given above with the following parameterizations. For Q_1, the following λ values were used: $\{-0.9, -0.05, 0.05, 0.9, 9.9, 99.9\}$; for Q_2, the following λ were used: $\{0.1, 0.5, 1.0, 2.0, 10.0\}$ and for Q_3, the following λ were used $\{0.1, 0.25, 0.5, 0.75, 0.9\}$.

In Table 4, we give the orness for quantifier Q_1 for dimensions between 2 to 16. Also, in the last column, it is given the orness of the fuzzy quantifier computed using Definition 11. It can be seen that when the dimension increases the value tends to be the one of the fuzzy quantifier. Similar results have been obtained for the other quantifiers and parameters.

These results can be understood on the light of the relation between the operators. It has been said that the WOWA generalizes the weighted mean and the OWA operator. In this sense, it is meaningful to consider the orness of the WOWA as related (somehow *proportional*) to the orness of the weighted mean and the orness of the OWA. It can be observed that in this case the orness of the weighted mean (it is 0.5 [5]) behaves as a neutral value (recall tha the minimum and maximum values for orness are, respectively, 0 and 1 and 0.5 is the central value in this interval), and thus, the orness measure for the WOWA corresponds to the orness measure for the OWA.

Dim	-0.9	99.9
2	0.72756	0.16532
3	0.71166	0.18206
4	0.70346	0.18435
5	0.69763	0.18052
6	0.69533	0.18717
7	0.69303	0.18812
8	0.69027	0.19305
9	0.68807	0.18925
10	0.68723	0.18989
11	0.68661	0.19054
12	0.68605	0.19255
13	0.68546	0.19349
14	0.68430	0.19478
15	0.68431	0.19491
16	0.68354	0.19593
Q	0.67681	0.20671

Table 1: Orness for quantifier Q_1 computed for dimensions from 2 to 16 and $\lambda = -0.9$ and 99.9.

5 Acknowledgments

Partial support of the European Community (under the contract IST-2000-25069) and of the CICYT (under the contract SMASH: TIC-96-1038-C04-04) are acknowledged. Anonymous referees comments are also acknowledged.

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