Optimization and Survivability of Telecommunication Networks

PHAM Thi-Tuyet-Loan

***Abstract*— In this paper, we focus on the capacitated survivable network design problem when the survivability is expressed in terms of rerouting techniques. We propose an approach that permits to achieve optimal solution in reasonable time. Our approach uses both Benders decomposition and a cutting plane approach to reduce the high dimensionality of the problem. Some numerical results are also presented.**

Key words: Survivability, dimensioning problem, network topology planning, rerouting mechanism, link failures, multi-commodity flow networks, linear mixed-integer programming.

I. Introduction

Survivability, together with fault protection and restoration, is a growing area of concern with increasing interconnection of high-bandwidth networks. More traffic is concentrated on fewer routes, increasing the number of customers that can be potentially affected by a failure. The objective of the restoration method should be to reroute the affected traffic accurately and rapidly using the redundancy provided in the network. These operations are generally processed automatically either by a centralized process able to supervise and control the whole network, or by a local reroute procedure run by the neighbor nodes. Independently of the concerned layers (e.g. SDH/SONET, ATM, WDM, IP, MPLS), the main requirement is to maintain the survivability of networks in case of link or node failures. As failures are not very common, network designers often consider traffic protection strategies against only the destruction of one link or one node. Moreover one of the most efficient protection systems consists in rerouting traffic in the case of a single link cut. In this paper, we deal with the survivability problem when a single link failure occurs. We can use the same approach solution for the problem with the node failures and the multiple link failures.

When a failure occurs, the perturbed traffic is restored by rerouting techniques. There are various traffic rerouting strategies against single link failures. The *local rerouting*, which is the classical one, permits to reroute traffic between the extremities of the failed link [11]. A more flexible rerouting strategy, called the *end-to-end* rerouting, consists in rerouting traffic between the extremities of the demands. This flexibility is particularly appreciated for the upper layers of networks such as IP or ATM. Several variants of end-to-end rerouting, depending on the set of demands that can be rerouted after a failure (global or partial) and the capacities that can be used during failure (with or without recuperation) may be considered [2]. Our main objective in this paper is to consider the survivability based on the partial end-to-end rerouting without recuperation of the capacities, that is when the network capacities will be completely dedicated either to the nominal state or to restoration and only the interrupted demands have to be rerouted.

The network's survivability requires that its topology and capacity must allow rerouting around a fault condition. This issue falls under the category of network design and planning while minimizing the overall cost. Ideally, we would like to optimize the network cost when we determine both the links and the capacity installed on them with respect to the survivability requirements. Unfortunately, this global problem has the drawback to be quite complicated. Hence, in practice, the overall optimization problem is decomposed into two stages of optimization to simplify the planning task. In the first one, the topology of the network as well as the dimensioning for the nominal state is computed in minimizing the network cost. Moreover, this first stage is also a NP-hard problem and thus, a lot of researchers were interested either in the topology problem [14] or in the dimensioning one [1]. The second stage permits to determine the capacity that has to be added to some links for rerouting traffic during a failure [15]. This problem decomposition has some inconveniences (*e.g.* may produce a sparse topology, [2]).

The objective of this paper is to develop modeling and solution approaches for the capacitated survivable network design problem. Notice that in the telecommunication networks, the capacities are modulated. A capacity module is called capacity *facility*. We may be allowed to install either a single type of facility with constant capacity [7] or different kinds of facilities with varying capacities on the links of the network. More precisely, our problem may be defined as follows. Given a network with a set of potential links, the set of traffic demands and the value of capacity facilities, one must determine the links and the minimum necessary capacity to be installed in the network in order to route all traffic demands even in case of link failures. This problem is known to be NP-hard. In fact, it includes as special cases a number of well known NP-hard combinatorial optimization problems such as the *survivable network design* [14] and the *capacity allocation and traffic management* [2] [11].

The rest of the paper is organized as follows. Section II

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presents the formal description and formulation of the capacitated survivable network. In section 3, we discuss the solution approach for the problem. Some numerical results are analyzed in the section 4. The last section presents our conclusions and some future research direction.

II. Mathematical modeling

In this section we describe the mathematical model of Capacitated Survivable Network Design Problem ($CSNDP$). We recall that the objective of ($CSNDP$) is to determine where and how much capacity to install in network while minimizing its overall cost. The installed capacity have to be enough to route all traffic demand in case without failure (*nominal state*) and to reroute the perturbed traffic during a failure occurs (*failure state*). We have used *edge-path formulation* that permits us to determine the routing of traffic demands by the elementary paths, (each path is presented by a set of edges) to model this problem.

The input of this problem is an undirected graph $G = (V, E)$ where V is the set of nodes and E is the set of potential links; a set of traffic demands on the network nodes indexed by $D = \{1, \ldots, K\}$, each traffic demand (called *commodity*) k is defined by the extremities nodes o_k, t_k and the traffic requirement d_k; a set of link failure indices $LF = \{1, \ldots, |LF|\}$, $|LF| \leq |E|$. Here, we consider all possible link failures, so $|LF|$ = $|E|$. The values of capacity facility is λ_1 for the lower facility and λ_2 for the higher facility. To route the traffic demands D in nominal state, it uses the network $G = (V, F)$ with all installed capacities. During the failure of link l, the set of demands routed on this edge have to be rerouted. To reroute these perturbed demands, it uses the network $G = (V, F \setminus l)$ and the spare capacity that is not used in nominal state. In this case the rerouting problem in failure state is considered as a multi-commodity flow problem in the graph $G = (V, F \setminus l)$ with the traffic demands are the perturbed traffic.

Before presenting the mathematical model, let us begin with some preliminary notation. We denote:

- a_e: the installation cost of lower facility on the edge e;
- c_e: the installation cost of higher facility on the edge e;
- x_e: the number of lower facilities installed on the edge e;
- y_e: the number of higher facilities installed on the edge e;
- $P(k)$: the set of elementary paths between the extremities o_k and t_k of the traffic demand k in the graph G;
- $Q(kl)$: the set of elementary paths between the extremities of traffic demand k in the graph $G \setminus l$;
- f_p^k: the flow belonging to the traffic demand k and passing on the path p in nominal state;
- $(g_q^k)_l$: the flow belonging to the traffic demand k and passing on the path q during the failure of link l;
- s_e: the spare capacity of the edge e that is used for rerouting the perturbed traffic:

$$s_e = \lambda_1.x_e + \lambda_2.y_e - \sum_{k \in K} \sum_{\substack{p \in P(k) \\ p \ni e}} f_p^k \qquad (1)$$

The problem ($CSNDP$) based on the end to end rerouting without recuperation the capacity used in the nominal state is then formulated as follows:

$$\begin{cases} Minimize \sum_{e \in E} (a_e x_e + c_e y_e) & \\ \text{subject to} & \\ \sum_{p \in P(k)} f_p^k = d_k \qquad \forall\, k \in K, & (2) \\ \sum_{g \in Q(kl)} (g_q^k)_l - \sum_{\substack{p \in P(k) \\ p \ni e_l}} f_p^k = 0 \qquad \forall\, l \in LF, & (3) \\ \sum_{k \in K} (\sum_{\substack{p \in P(k) \\ p \ni e}} f_p^k + \sum_{\substack{q \in Q(kl) \\ q \ni e}} (g_q^k)_l) \leq & \\ \lambda_1.x_e + \lambda_2.y_e \qquad \forall\, e \in E,\ l \in L,\ e \neq e_l, & (4) \\ x_e, y_e \in \mathbb{Z}_+ \qquad \forall\, e \in E, & (5) \\ f_p^k \geq 0 \qquad \forall\, p \in P(k),\ k \in K, & (6) \\ (g_q^k)_l \geq 0 \qquad \forall\, q \in Q(kl),\ l \in LF. & (7) \end{cases}$$

This formulation is a type of ***edge-path multi-commodity flow formulation***. The constraints (2) ensure that all traffic demands are routed in the case without failure. When a link $l \in LF$ fails, constraints (3) ensure that the affected traffic demand, given by the second term of left hand side, is rerouted between the extremities of perturbed traffic demand. Constraints (4) are capacity constraints: they express that the capacity installed on edge $e \in E$ must be enough to carry whole traffic in both nominal and failure situations. Constraints (5) force an integer number of installed capacity modules per link. Positivity of flow variables is expressed by the constraints (6), (7).

In this formulation, the number of variables of flow is exponential in the number of nodes of G. We cannot explicit all elementary paths. Moreover, it is obvious that it is very difficult to express all the constraints. Furthermore, our problem modeled as above is a linear mixed-integer program, and thus it is difficult to obtain some optimal solution. The solution approach used to solve our problem is presented in the following section.

III. Solution approach

Since the elementary paths cannot be all expressed in the formulation, we have used the column generation method to generate step by step the path for each demand, this method is presented in [7]. To generate the constraints of type (3) and to facilitate the resolution of the mixed-integer program, we used the Benders decomposition as follows:

In our formulation, there are the integer and continuous variables, we decomposed our problem into the integer linear problem, called *Super Master Problem* (determine the topology and the capacities of obtained network) and a continuous linear problem, called *Capacity Feasibility Problem* that verify the sufficiency of capacities calculated by the first one. The sufficiency of capacity is expressed by feasibility of the multi-commodity flow problem. A feasible multi-commodity flow is a non-negative real valued function satisfying the capacity constraints and the traffic demands requirements. In the survivable network, the sufficiency of capacity must be verified in nominal state and the failure states. The *Capacity Feasibility Problem* thus includes the routing problem in nominal state, called *Master Problem*, and $|LF|$ routing problems in failure state, called *Satellite problem*. So, the framework of our algorithm based on the two-level Benders decomposition is in the Figure 1.

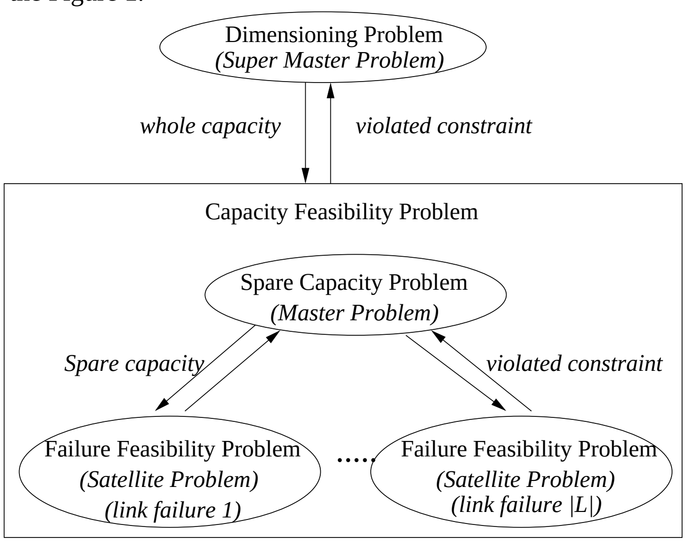


Fig. 1. Sketch of our solution algorithm

To summarize our algorithm, we solve the super master problem to determine the capacity and the topology; then the master problem and the satellite problems are solved to verify the sufficiency of these resources. If they are not sufficient, we generate a corresponding constraint to the super master problem to strengthen the resource constraints, we go back to the solution of *(SMP)*; otherwise the master problem and the satellite problems are feasible and the optimal solution is obtained.

Now let us detail the above linear problems. We begin with the satellite problem.

A. Satellite problem

Given the spare capacities ($s_e, \forall e \in E$) calculated by *master problem*, satellite problem permits to verify the sufficiency of these spare capacities in the correspond failure state with respect to a given routing. Each satellite problem is associated to a link failure. We have considered all possible link failures, so $|LF| = |E|$. It is obvious that the routing of the perturbed traffic in a failure state is considered as a multi-commodity flow problem in the network $G \setminus l$ with the set of traffic demands are the perturbed traffic.

The satellite problem based on ϵ-*formulation*, denoted (P_S), is formulated as:

$$\begin{cases} Minimize \quad \epsilon \\ \text{subject to} \\ \sum\limits_{q \in Q(kl)} (g_q^k)_l = \sum\limits_{\substack{p \in P(K) \\ p \ni l}} f_p^k \quad \forall\, k \in K, \qquad (\pi_k) \\ \sum\limits_{k \in K} \sum\limits_{\substack{q \in Q(kl) \\ q \ni e}} (g_q^k)_l) \leq s_e + \epsilon \\ \qquad \forall\, e \in E,\ l \in LF,\ e \neq l, \qquad (\mu_e) \\ (g_q^k)_l \geq 0 \qquad \forall\, q \in Q(kl),\ l \in LF. \end{cases} \tag{8}$$

where f_p^k are the given flow value. With this formulation, the satellite problem permits us to determine the rerouting paths of the perturbed traffic, but we want only to check the sufficiency of the spare capacities. We thus have considered the corresponding dual problem. Now, let us present the constraint generated from the satellite problem to master problem. The condition (8) expressing that the spare capacities are sufficient for the considered failure problem if $\epsilon \leq 0$. But we can not add the constraint $\epsilon \leq 0$ to super master problem, we must express it in the other way. A fundamental result of linear programming duality [4], [12] states that if both primal and dual problems are feasible (*i.e.* the linear problem have a solution), their optimal values are equal. The linear dual problem for satellite problem, denoted (D_S), that is:

$$\begin{cases} Maximize \ \sum\limits_{k \in K} (\sum\limits_{\substack{p \in P(k) \\ p \ni l}} f_p^k \pi_k) - \sum\limits_{e \in E, e \neq l} s_e \mu_e \\ \text{subject to} \\ \pi_k - \sum\limits_{e \in E, e \neq l} \mu_e \leq 0, \qquad \forall k \in K & (9) \\ \sum\limits_{e \in E, e \neq l} \mu_e = 1 & (10) \\ \mu_e \geq 0 & (11) \end{cases}$$

As P_S and D_S have the same optimal value, D_S can be substituted for P_S in the above necessary and sufficient condition. Denote by (π^r, μ^r), $r = 1, .., R,$, the extreme points of the polyhedron specified by the constraints of D_S. As the optimal value of D_S is reached at one extreme point, the above necessary and sufficiency condition for sufficiency of the spare capacity becomes: s_e for $\forall e \in E$ is feasible solution to satellite problem if and only if:

$$\sum_{k \in K} (\sum_{\substack{p \in P(k) \\ p \ni l}} f_p^k \pi_k) - \sum_{e \in E, e \neq l} s_e \mu_e \leq 0 \tag{12}$$

where (π, μ) is a extreme point of the domain, *D*, defined by the constraints (9), (10) and (11). Constraints (9) and (10) define a cone, *C*. As each extreme point (π^r, μ^r) of *D* is an extreme ray of *C* and each extreme ray, (π, μ) of *C* corresponds to an extreme point (π, μ) of *D*. The constraint (11) may be viewed as a normalization constraint. The constraint of type (12) is called *spare capacity feasibility* constraint. When the spare capacity is not sufficient, it generates a corresponding constraint of type (12) and add it to master problem to force the spare capacity.

B. Master problem

Given the installed capacities (*i.e.* x_e and y_e for all $e \in E$) computed by the *super master problem*, the master problem permits to verify the feasibility of these capacities for both nominal and failure states. It is an extended multi-commodity flow problem. It includes the capacity constraints, the routing constraints for nominal state and the *spare capacity feasibility* constraints. The latter constraints are generated by the satellite problems with respect to the rerouting strategy. It follows that the master problem can be formulated as a linear program based on the following extended ϵ-*formulations*:

$$\begin{cases} Minimize\ \epsilon \\ \text{subject to} \\ \sum_{p \in P(k)} f_p^k = d_k & \forall\, k \in K, (\alpha_k) \\ \epsilon - \sum_{k \in K} \sum_{\substack{p \in P(k) \\ p \ni e}} f_p^k \geq s_e - \lambda_1.x_e - \lambda_2.y_e & \\ & \forall\, e \in E,\ (\beta_e) \\ \sum_{e \in E} s_e \mu_e^r - \sum_{k \in K} (\sum_{\substack{p \in P(k) \\ p \ni l}} f_p^k \pi_k^r) \geq 0 & \\ & \forall\, r \in R,\ (\gamma_r) \\ f_p^k \geq 0 & \forall\, p \in P(k), \\ s_e \geq 0 & \forall\, e \in E, \\ \epsilon \lesseqgtr 0, \end{cases}$$

where R represents the set of generated *spare-capacity feasibility* constraints provided by the satellite problems.

If the master problem has a nonnegative solution (*i.e.* the supplementary capacity $\epsilon > 0$ or the capacity calculated by super master problem is not sufficient), then we add a violated constraint to the super master problem in order to increase the amount of installed capacities. As we explain above for the satellite problem, this constraint is obtained by duality theory. More precisely the condition of the feasibility of solution (x, y) for master problem is

$$\sum_{k \in K} d_k \alpha_k - \sum_{e \in E} (\lambda_1.x_e + \lambda_2.y_e) \beta_e \leq 0. \tag{13}$$

That is nothing else that the objective function of the dual formulation of the initial Master problem. They are called *capacity feasibility* constraints.

C. Super master problem

This problem chooses the necessary edges and calculates enough the capacity in respecting the constraints generated from the *Capacity Feasibility Problem* that expressed by the feasibility of routing problem of traffic demands in nominal state and in failure states.

$$\begin{cases} Minimize\ \sum_{e \in E} (a_e.x_e + c_e.y_e) \\ \text{subject to} \\ \sum_{k \in K} d_k \alpha_k^t - \sum_{e \in E} (\lambda_1.x_e + \lambda_2.y_e) \beta_e^t \leq 0 \\ \qquad\qquad\qquad\qquad\qquad \forall\, t \in T, \\ x_e \in \mathbb{Z}_+ \qquad\qquad\qquad\qquad \forall\, e \in E, \end{cases}$$

where T represents the set of *capacity feasibility* constraints (13) that may be provided by the master problem.
Since the cardinality of T can be very high, we only consider a subset $\overline{T}$ of these constraints (initially $\overline{T}$ may be empty), and we solve the reduced super master problem by MIP of CPLEX. Once the optimal solution is obtained we check the feasibility of these capacities with the master problem. If no constraints are generated by this latter, we have already reached the optimum. Otherwise we add the violated constraints (13) to $\overline{T}$ and we solve again the reduced super master problem.

Notice that it is a integer linear problem that is expensive to obtain a solution. We will improve the convergence of super master problem by adding some supplementary constraints. A well-known necessary condition for the existence of a multi-commodity flow is cut conditions which is stated as follows. For any set of nodes $W \subset V$, let $\delta(W)$ be the set of edges with one extreme in W and the other in $V \setminus W$. The set $\delta(W)$ is called a *cut*. Let $D(W, \overline{W}) = \sum_{o_k \in W} \sum_{t_k \in \overline{W}} d_k$ be the traffic passing through the cut. The cut conditions for the existence of multi-commodity flow is

$$\sum_{e \in \delta(W)} (\lambda_1.x_e + \lambda_2.y_e) \geq D(W, \overline{W})$$
$$\forall\, W \subset V, W \neq \emptyset, W \neq V$$

Without loss of generality, we assume that $\lambda = \lambda_2/\lambda_1$ is a integer value. It implies that

$$\sum_{e \in \delta(W)} (x_e + \lambda.y_e) \geq \frac{D(W, \overline{W})}{\lambda_1}$$
$$\forall\, W \subset V, W \neq \emptyset, W \neq V \tag{14}$$

This cut-set constraint for W express that the capacity of this cut should be at least the sum of the traffic passing on this cut. Since the number of capacity modules installed on the cut must take integer values, the constraints of type (14) can be strengthened by rounding up the maximum of the two right-hand-sides:

$$\sum_{e \in \delta(W)} (x_e + \lambda.y_e) \geq \left\lceil \frac{D(W, \overline{W})}{\lambda_1} \right\rceil$$
$$\forall\, W \subset V, \emptyset \neq W \neq V \tag{15}$$

It can easily be seen that this cut condition does not take into account the survivability requirement. For this, the capacity installed on the cut must be sufficient to face link failures. While a link belonging this cut fails, the capacity installed on the rest edge of cut must be able to route whole traffic passing on this cut, that is:

$$\sum_{e\in\delta(W)} (x_e + \lambda.y_e) - x_f - \lambda.y_f \geq \left\lceil \frac{D(W,\overline{W})}{\lambda_1} \right\rceil$$
$$\forall\, W \in V, \quad \emptyset \neq W \neq V, \forall f \in \delta(W) \qquad (16)$$

We can add (16) to super master problem to improve the convergence of the algorithm.

IV. Numerical results

We have implemented ours algorithm in C++ using procedures from the callable library of CPLEX 7.1 to solve the linear programs as the super master, master and satellite problems.

In our experiments, to achieve the survivability, the initial network topology must be 2-connected. Main parameters characterizing of the networks used to perform the numerical experiments are described in Table 1, where $|V|$ is the number of nodes, $|E|$ is the number of potential links, $|K|$ is the number of traffic demands and $d(G)$ (resp. $t(K)$) represents the average nodal degree (resp. the average demand traffic). In this study, we considered all traffic demands possible, so the number of traffic demands $|K| = \frac{|V|*(|V|-1)}{2}$. We have run all tests with one type of capacity (600 unit) and two types of capacity modules ($\lambda_1 = 150$ unit, $\lambda_2 = 600$ unit). Here, we make the assumption that all links are subject to failure (*i.e.* $L = E$).

G	$\|V\|$	$\|E\|$	$d(G)$	$\|K\|$	$t(K)$
net1	10	30	6	45	75
net2	17	25	2.94	136	50
net3	17	35	4.12	136	50
net4	26	35	2.69	264	50
net5	26	43	3.31	264	50

TABLE 1: Test instances

The tables 2,3 report some results for the capacitated survivable network design problem based on end-to-end restoration without recuperation the capacities used in nominal state.

G	Installed Capacity		Idle Capacity	
	$\lambda = 600$	$\lambda_1=150$ $\lambda_2=600$	$\lambda = 600$	$\lambda_1=150$ $\lambda_2=600$
net1	9000	-33%	63.72	0
net2	30000	-4%	2334	584.64
net3	22200	-14.86%	1378.62	154.38
net4	75000	-7.4%	3247.50	743.11
net5	49800	-3.9%	2151.36	142.20

TABLE 2: Used capacities

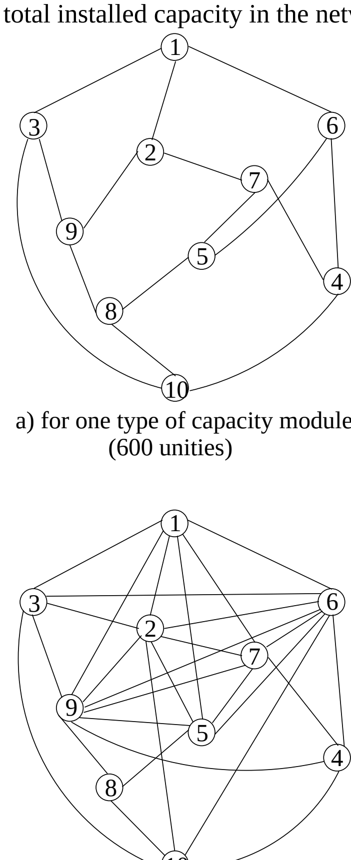
G	Network cost		Network Density	
	$\lambda = 600$	$\lambda_1=150$ $\lambda_2=600$	$\lambda = 600$	$\lambda_1=150$ $\lambda_2=600$
net1	150	-21.33%	3	5.8
net2	14840	-4.32%	2.94	2.94
net3	9353	-4.13%	3.53	3.7
net4	30188	-5.42%	2.69	2.69
net5	22763	-3.13%	3.15	3.31

TABLE 3: Obtained network

The capacity module plays a significant part in the solution of $(CSNDP)$. For example, the traffic passing on the edge e is 280 unit, if there is only a higher capacity module $\lambda = 600$, it must install on the edge e at least one capacity module to ensure this traffic, so the installed capacity on the edge e is highly idle; if there are one lower and one higher modules $\lambda_1 = 150$ and $\lambda_2 = 600$ are considered, then the capacity installed is at least $2.\lambda_1 = 300$. So we can use the lower capacity modules to install on the edge to assure the small traffic passing on the edge.

If we consider only one higher capacity module, the installed capacity can be idle, see in [7]. Nevertheless, with a lower capacity module and a higher one, the idle installed capacity can be improved, (see columns 4 and 5 in TABLE 2). Moreover, it is obvious to see that the total installed capacity for $(CSNDP)$ with two types of capacity modules is less than the one with one type of capacity modules. This difference can be 33% (see the columns 2 and 3 of TABLE 2).

Notice that our algorithm may produce a non-optimal routing strategy since the considered paths in nominal state are just necessary to establish the spare capacity in order to minimize the total installed capacity in the network.



a) for one type of capacity module
(600 unities)

b) for two types of capacity modules
(150 unities and 600 unities)

Fig. 2. Example of obtained network topology

Now we turn to the obtained network topology. If we consider only one type of capacity module, and the value of module is sufficiently high, the obtained topology tends to a Hamiltonian cycle, but the installed capacity can be idle, see in [7]. It appears that the average nodal degree for the obtained network depends on the values of capacity modules. More precisely a small value of capacity module implies that the traffic is often distributed on many paths and thus the obtained networks is highly meshed. Therefore, when we consider two types of capacity modules, the lower one is preferably used to optimize the capacity installation cost if the traffic passing on the edges are less than the value of higher capacity module. To minimize the capacity installation cost by using the lower capacity module, it must use more paths than the one using only one type of capacity module (the higher one). So the obtained network topology with two types of module (a lower and a higher modules) is always meshed than the one with one type of module (a higher module), see the columns 4 and 5 in TABLE 3. For example, with the network *net1*(10 nodes, 30 potential edges), the obtained network topology is as in the Figure 2.

In order to compare ($CSNDP$) with one higher capacity module or with two types of capacity modules, we study the obtained cost. Here, we considered only the capacity installation cost. You can see the columns 2 and 3 in the TABLE 2. With the two types of capacity module, the installation network cost is less than the one with a higher capacity module, but this difference is not very large (8% in average).

We want to remark that the installed capacity cost for ($CSNDP$) with one lower and one higher capacity module is lightly less than the one with one higher capacity module, but the obtained network topology of the second one is meshed for the first one. So if we consider the network cost including the edge installation cost and the capacity installation cost, the results concerning obtained network cost can be different.

V. CONCLUSION

We have considered the capacitated survivable network design problem is expressed in terms of end-to-end rerouting without recuperation the capacity used in nominal state. This problem has been formulated as a large linear mixed-integer program. Since it is well structured, we have presented an algorithm based on Benders' decomposition and Branch-and-Bound method to solve it.

As the previous remarks, we must integrate the edge installation cost in the mathematical model. From a mathematical point of view, we have to study more our algorithm in order to solve the larger instance and to improve the convergence of our program.

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