Performance Evaluation of TH-PPM UWB Systems in the Presence of Multiuser Interference

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***Abstract*—This letter presents a new method for the evaluation of the bit error probability of a time hopping binary pulse position ultra-wideband modulation scheme, in the presence of multiuser interference. The technique permits to predict the system performance with high accuracy and reasonable complexity. A perfect agreement with simulation results is shown, and a comparison with the Gaussian approximation is presented.**

***Index Terms*—Gaussian quadrature rule, multiple access, ultra-wideband (UWB) communications.**

I. Introduction

ULTRA-WIDEBAND (UWB) technology has been recently proposed as a viable solution for high-speed indoor short range wireless communication systems, because of its robustness to severe multipath conditions and low cost and low power implementation.

Time Hopping combined with pulse position modulation (TH-PPM) has been the original proposal for UWB systems [1]. An analysis of this modulation and multiaccess scheme performance in terms of bit error rate (BER) and capacity has been proposed in [1] and [2] for AWGN channel. In both papers the multiuser interference (MUI) is modeled as a zero-mean Gaussian random variable; this assumption, usually called "Gaussian approximation" or "standard approximation," is not accurate enough to predict DS-CDMA system performance [3]. In [4] it is shown, by comparison with simulation results, that this technique may significantly underestimate the BER also for TH-PPM systems.

In this letter, we propose a method to evaluate the BER performance of TH-PPM in the presence of MUI and AWGN channel, based on Gaussian quadrature rules (GQR). This technique was successfully employed for the performance evaluation of multilevel digital signals with intersymbol interference [5], and of noncoherent receivers in optical communication [6]. If applied to UWB system performance analysis, its main advantage is to overcome the problem of exactly evaluating the MUI probability density function (pdf), a cumbersome problem as shown in [7], by requiring only its moments estimation. Furthermore, it is able to predict the BER with the necessary accuracy in most of the practical cases, with limited computational complexity.

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In Section II the problem of the BER evaluation in an asynchronous UWB system employing random hopping sequences is formulated and GQR technique is presented; in Section III a method for evaluating the MUI moments is proposed. Finally, in Section IV some results obtained adopting this technique are illustrated together with a comparison with curves obtained by simulation and using the Gaussian approximation.

II. BER Evaluation in the Presence of MUI

An UWB TH-PPM signal can be described as [1]:

$$s^{(k)}(t) = \sum_{j=-\infty}^{+\infty} \sqrt{E_x} x\left(t - jT_f - c_j^{(k)} T_c - \delta \alpha_{\lfloor j/N_s \rfloor}^{(k)}\right) \quad (1)$$

where $s^{(k)}(t)$ is the random process modeling the signal transmitted by the kth user, and $x(t)$ is the reference pulse, normalized so that $\int_{-\infty}^{+\infty} [x(t)]^2\, dt = 1$, and with autocorrelation function $\gamma(t)$. The energy transmitted for each pulse is therefore E_x.

The TH code $\left\{c_j^{(k)}\right\}$, $0 \le c_j^{(k)} < N_h$, $\forall\, k$, provides an additional shift (in multiples of the chip time T_c), in order to reduce collisions effects in multiple access. As the system is not synchronous, the choice of orthogonal hopping patterns does not guarantee collision-free transmission. In our analysis, hopping sequences will be modeled as independent random variables (RVs). $c_m^{(k)}$ will be then a discrete RV, taking values in the interval $[0, N_h - 1]$ with equal probability.

The binary information stream $\left\{\alpha_i^{(k)}\right\}$ is transmitted employing a PPM modulation format, introducing an additional shift (δ), used to distinguish between pulses carrying the bit 0 and the bit 1. Each information bit is transmitted using N_s consecutive pulses, leading to a $(N_s, 1)$ repetition code. The resulting information bit rate is thus $R_b = (N_s T_f)^{-1}$.

The frame time is assumed to be an integer multiple of the chip time, that is $T_f = N_h T_c$, with $N_h \in \mathbf{N}$. Denoting T_x the time extension of the transmitted waveform, it is also required that $T_x + \delta < T_c$.

Assuming ideal free-space propagation, AWGN channel and perfect power control [1],when N_u users are transmitting, the received signal is

$$y(t) = s^{(1)}(t - \tau_1) + \sum_{k=2}^{N_u} s^{(k)}(t - \tau_k) + n(t) \quad (2)$$

where τ_k, for $k = 1, \ldots, N_u$, is the delay associated to each user in an asynchronous multiple access system, and $n(t)$ is a white Gaussian noise process with two-sided power spectral density $N_0/2$.

The receiver is assumed to be a single-user correlator that adopts $v(t) = x(t) - x(t-\delta)$ as correlation waveform. This receiver is obviously not optimum in presence of MUI interference. User 1 is the "useful" user that we want to receive, and a perfect knowledge of its time hopping pattern $\left\{c_j^{(1)}\right\}$, together with an ideal synchronism between transmitter and receiver, is assumed. Without loss of generality we will further impose $\tau_1 = 0$ and $c_j^{(1)} = 0, \forall\, j$. The repetition decoder output, when $\alpha_0^{(1)}$ is transmitted, can be written as

$$\begin{aligned}\lambda_0^{(1)} =& \sum_{m=0}^{N_s-1} \int_{mT_f}^{(m+1)T_f} y(t) v\left(t - mT_f\right) dt \\ =& s_0^{(1)} + n_{\mathrm{MUI},0} + n_{G,0}\end{aligned} \tag{3}$$

where

$$n_{\mathrm{MUI},0} = \sqrt{E_x} \sum_{k=2}^{N_u} n_{\mathrm{MUI},0}^{(k)} \tag{4}$$

$$n_{\mathrm{MUI},0}^{(k)} = \sum_{m=0}^{N_s-1} n_{\mathrm{MUI},0}^{(m,k)} \tag{5}$$

$$\begin{aligned}n_{\mathrm{MUI},0}^{(m,k)} =& \int_{mT_f}^{(m+1)T_f} v\left(t - mT_f\right) \\ & \cdot \sum_{j=-\infty}^{+\infty} x\left(t - jT_f - c_j^{(k)} T_c - \delta \alpha_{\lfloor j/N_s \rfloor}^{(k)} - \tau_k\right) dt\end{aligned} \tag{6}$$

is the multiuser interference term we want to characterize, while $s_0^{(1)}$ and $n_{G,0}$ are the contributions to $\lambda_0^{(1)}$ coming from user 1's useful signal and Gaussian thermal noise. Conditioning on MUI we obtain

$$P(e) = E_{n_{\mathrm{MUI},0}}\left[\frac{1}{2}\mathrm{erfc}\left(\frac{s_0^{(1)} + n_{\mathrm{MUI},0}}{\sqrt{2}\sigma_{n_{G,0}}}\right)\right]. \tag{7}$$

The exact computation of (7) requires the evaluation of the MUI pdf and the computation of the average, by solving the associated integral. The feasibility of the first step was analyzed in [7], where a closed-form expression for the MUI pdf was evaluated, based on an approximation of the $n_{\mathrm{MUI},0}^{(m,k)}$ pdf. However, the result is still unwieldy for a numerical evaluation of the integral in (7).

A more practical approach is given by the GQR method proposed here. This technique can be adopted since the BER computation involves averaging a generic function $g(x)$ with respect to the RV X, whose pdf is unknown. According to the GQR technique, the integral we are dealing with can be computed in the following way:

$$\int_a^b g(x) f_X(x) dx = \sum_{i=1}^{N} w_i g(x_i) + E_N \tag{8}$$

where $\{x_i\}$ and $\{w_i\}$ are the abscissas and the weights associated with the quadrature rule, whose values depend on the given interval of integration and on the pdf $f_X(x)$, and E_N is the truncation error. The abscissae and weights can be computed by finding a set of orthonormal polynomials with respect to $f_X(x)$. These polynomials can be shown to satisfy a three terms recurrence relation [9], whose coefficients are used to construct a three-diagonal matrix. The quadrature rule can be finally generated by computing the eigenvalues and the first component of the normalized eigenvectors of the matrix. Adopting the algorithm proposed in [8], only the knowledge of the first $2N-1$ moments of X is required in order to generate a GQR formula based on N points. In Section III we present a technique for evaluating the moments of the MUI.

III. Evaluation of the MUI Moments

In order to simplify the approach, the evaluation of the $P(e)$ will be carried out in two step:

- evaluation of the conditional probability $P(e|\underline{\tau})$, $\underline{\tau} = (\tau_2, \ldots, \tau_{N_u})$
- computation of $P(e)$ with a semi-analytic method, assuming the RV τ_j, $j = 2, \ldots, N_u$, uniformly distributed over the bit interval $T_b = N_s T_f$.

For the first step the knowledge of the first $2N-1$ conditional moments of $n_{\mathrm{MUI},0}$ is required. We will now illustrate a method for performing this task.

Replacing $t - mT_f$ with z in (6) leads to

$$\begin{aligned}n_{\mathrm{MUI},0}^{(m,k)} = \sum_{j=-\infty}^{+\infty} \int_0^{T_f} v(z)\, x\Big(z + (m-j)T_f - c_j^{(k)} T_c \\ \pm \delta \alpha_{\lfloor j/N_s \rfloor}^{(k)} - \tau_k\Big)\, dz.\end{aligned} \tag{9}$$

Then, defining

$$\begin{aligned}\xi_j^{(k)} =& -c_j^{(k)} T_c - \delta \alpha_{\lfloor j/N_s \rfloor}^{(k)} - \tau_k \\ \xi_j^{(k)} \in& [-(N_h - 1)T_c - \delta - N_s T_f, 0]\end{aligned}$$

and noting that the region where the integrand in (9) is nonzero is fully contained inside the region of integration, the sum over j can be limited to $N_s + 1$ contributing terms

$$\begin{aligned}n_{\mathrm{MUI},0}^{(m,k)} =& \sum_{j=m-N_s} \Gamma\left(\xi_j^{(k)} + (m-j)T_f\right) \\ =& \sum_{j=0}^{N_s} \Gamma\left(\xi_{m-j}^{(k)} + jT_f\right)\end{aligned} \tag{10}$$

where $\Gamma(\xi) = \gamma(\xi) - \gamma(\xi + \delta)$. The $\xi_j^{(k)}$'s, given τ_k, are identically distributed RVs, correlated because of the repetition code. Defining

$$\Lambda_i\left(\xi_i^{(k)}\right) = \begin{cases} \sum_{j=0}^{N_s-i-1} \Gamma\left(\xi_i^{(k)} + jT_f\right), & i \geq 0 \\ \sum_{j=|i|}^{N_s} \Gamma\left(\xi_i^{(k)} + jT_f\right), & i < 0 \end{cases} \tag{11}$$

then

$$\begin{aligned}n_{\mathrm{MUI},0}^{(k)} =& \sum_{i=-N_s}^{-1} \Lambda_i\left(\xi_i^{(k)}\right) + \sum_{i=0}^{N_s-1} \Lambda_i\left(\xi_i^{(k)}\right) \\ =& n_{\mathrm{MUI},0}^{(k)}\left(\alpha_{-1}^{(k)}\right) + n_{\mathrm{MUI},0}^{(k)}\left(\alpha_0^{(k)}\right)\end{aligned} \tag{12}$$

TABLE I
VALUE OF THE PRINCIPAL PARAMETERS FOR THE ANALYZED SYSTEM

Parameter	Symbol	Value
Time decay constant	t_n	0.2877 ns
PPM delay	δ	0.15 ns
Frame width	T_f	7.2 ns
Chip width	T_c	0.9 ns
Number of chip per frame	N_h	8
Number of users	N_u	8
Rep. code length	N_s	2, 4
Number of GQR points	N	7

where we have separated the terms characterized by the same transmitted bit and grouped them in two independent RVs. The evaluation of the first $2N-1$ moments of $n_{\mathrm{MUI},0}^{(k)}\left(\alpha_i^{(k)}\right)$, for $i=-1,0$, can be obtained by conditioning on $\alpha_i^{(k)}$

$$E\left[\left(n_{\mathrm{MUI},0}^{(k)}(\alpha)\right)^l\right] = \frac{1}{2}E_{\xi|\alpha}\left[\left(n_{\mathrm{MUI},0}^{(k)}(\alpha)\right)^l \middle| (\alpha=0)\right] + \frac{1}{2}E_{\xi|\alpha}\left[\left(n_{\mathrm{MUI},0}^{(k)}(\alpha)\right)^l \middle| (\alpha=1)\right]. \tag{13}$$

Finally, observing that

$$f_{\xi|\alpha}(\xi|\alpha=a) = \frac{1}{N_h}\sum_{h=0}^{N_h-1}\delta_D(x+hT_c+\delta a+\tau_k) \tag{14}$$

(where $\delta_D(x)$ is the Dirac delta function) and that $\xi_j^{(k)}$ are i.i.d. RVs, if conditioned on $\alpha_{\lfloor j/N_s\rfloor}^{(k)}$, it is possible to apply the technique described in [9, Appendix E] for the evaluation of the conditional moments in (13). The same technique can be employed to compute the moments of $n_{\mathrm{MUI},0}$ because of the independence between $n_{\mathrm{MUI},0}^{(k)}\left(\alpha_{-1}^{(k)}\right)$ and $n_{\mathrm{MUI},0}^{(k)}\left(\alpha_0^{(k)}\right)$ and between the RVs $n_{\mathrm{MUI},0}^{(k)}\ \forall\ k$.

IV. EXAMPLES AND CONCLUSIONS

In order to validate the proposed technique, we compared the predicted BER with the one obtained by simulation and employing the Gaussian approximation (see [1]).

The transmitted waveform is the second derivative of a Gaussian pulse [1], with time decay constant t_n. We consider an indoor scenario, characterized by a limited number of user ($\leq$8) and a high bit rate per user ($>$50 Mbit/s). In Table I the value of the main parameters for the analyzed system are listed.

The choice of the number of points (N) in the GQR is a crucial aspect in order to obtain reliable results. In principle, the truncation error introduced in (8) can be made as small as desired, by increasing the number of abscissae and weights of the quadrature rule. However, this is not feasible in practice, since the computed moments are not known with infinite accuracy. In particular, round-off errors can make the matrix computed during the evaluation of $\{x_i, w_i\}$ ill-conditioned with increasing N [9].

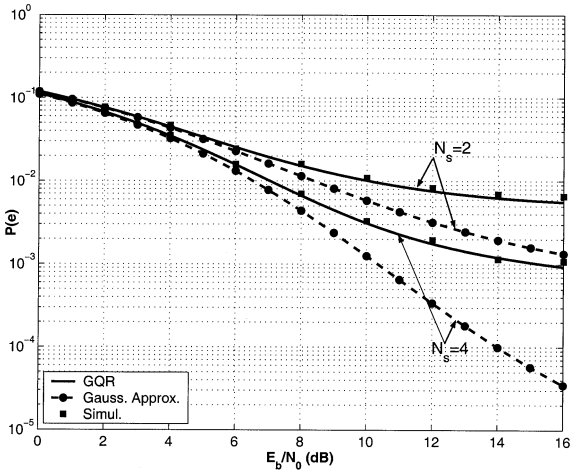

Fig. 1. Probability of error of a TH-PPM system. Repetition codes of length 2 and 4 are considered. $E_b = N_s E_x$.

In Fig. 1, BER curves of a TH-PPM system are presented, for two values of the repetition code length; the results obtained by simulation are compared with those employing GQR and the Gaussian approximation technique. Curves with GQR method are obtained averaging over 1000 realizations of the vector $\underline{\tau}$, and setting $N=7$, the maximum number of points we found before experimenting numerical problems due to ill-conditioned matrix. For the analyzed configurations, GQR shows an almost perfect agreement with simulated curves. Furthermore, it requires a limited computational complexity, if compared with simulation, an impractical approach for UWB systems performance evaluation, because of the large bandwidth of the transmitted signals. Finally, the technique described here can be appropriately generalized to also account for the effects of multipath channels and absence of power control.

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