Decision Making for Robust Resilient Systems

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Abstract

Robust and resilient interconnected structures rely on decision procedures, both under uncertainty and multicriteria. In decision under uncertainty, we aim at finding a scoring procedure to determine an optimal decision without prior knowledge on the actual state of the world. In multicriteria decision making, the state of the world is known but we aim at ranking alternatives defined over a multidimensional set. Therefore, the problem is to find an appropriate aggregation procedure. In practical applications, we have to deal with decision problems where the state of the world is not known, and the alternatives are multidimensional. It is well known that the probability approach to these problem leads to paradoxes, that are related to independence properties required on the preferences. This naturally leads us to drop additivity and therefore, replace probabilities with non-additive measures.

The aim of this paper is to present several benefit functions to identify optimal actions, in the fuzzy measure and fuzzy logic perspective.

1 Introduction

One of the goals of decision making is to come up with a strategy to determine an optimal course of action. In decision under uncertainty, the decision maker does not know the a priori true state of the world, and in many cases, does not even have a probability available on the set of states of the world S.

A possible approach to solve such problems is to assume some rationality axioms (that are understandable from a decision maker's behavior perspective) on the preference relation defined over the actions, and derive the existence of a probability measure and a utility function such that, if f and g are two actions and $f \succeq g$ means f is preferred to g,

$$f \succeq g \Leftrightarrow \int_S u(f(s))dP(s) \geq \int_S u(g(s))dP(s) \quad (1)$$

where the integrals represent the subjective expected utility of the actions. We encounter two problems here. To have the additivity of the probability measure, we require an independence hypothesis, the sure-thing principle [20], that is unlikely to be satisfied in most practical applications, as the Ellsberg's paradox [9] shows.

Therefore, the addivity property needs to be weakened, leading to the same type of representation with a fuzzy expected utility with respect to a fuzzy measure.

Second, Savage's theorem requires the states of the world to be an infinite set, which is not convenient in practice. Through the concept of lotteries (finite support probability distributions), Anscombe and Aumann [1] allows an additive representation of the preferences in the finite setting, that still requires an independence hypothesis which appears to be too strong.

In multicriteria decision making, the state of the world is known, but the decision maker has to choose the best alternative in a multidimensional set $X = X_1 \times \cdots \times X_n$ where X_i represent the ith set of values of attributes and $I = \{1, \cdots, n\}$ is the set of criteria or attributes over which the alternatives are defined.

The classical approach to solve this problem is to assume that the decision maker has clear preferences over a set X_i, and use a linear combination of these preferences with respect to a set of weights associated to the importance of each attribute. That is, if $x, y_1 X$ and α_i is the weight of criterion i, then

$$x \succeq y \Leftrightarrow \sum_i \alpha_i u_i(x_i) \geq \sum_i \alpha_i u_i(y_i)$$

where u_i represent the local preference over X_i. This approach presents the same problems as the additive approach in decision under uncertainty, that is, assuming that the global preference can be represented by a linear combination of local preferences is equivalent to assuming some kind of independence between the criteria (see [16]). In practical applications, this is simply not the case.

However, we will focus on decision under uncertainty in this paper. The reason is that as was shown in [7], there is a formal equivalence between the two paradigms. Therefore, we can use the richer axiomatization of decision under uncertainty to transfer the results to multicriteria decision making in a rather automatic way. Only some interpretation work is needed to give a semantic to the results in multicriteria decision making. Therefore, we will focus on decision under uncertainty, and particularly lotteries and see how this applies to robust and resilient systems.

These lotteries require the existence of a benefit function which is always assumed to be accurately known. A first contribution of this paper is to allow this function to be fuzzy valued. Second, we can relaxed the hypotheses even more by allowing uncertainty over the lotteries, and we propose two criteria to select an optimal act in this situation (either maximizing an extended Choquet integral or maximizing an entropy function). Eventually, we present a selection criterion for set-valued actions.

2 Fuzzy Integration and Decision Making

First, we recall some definitions of fuzzy measures and the Choquet integral and basics of decision under uncertainty. For more details on fuzzy integration, see for example [5] ,[4] and for application to decision making, see [13].

2.1 Fuzzy integration

In this section, Ω is a set and $\mathcal{P}(\Omega)$ is a set of subsets of Ω such that $\emptyset$, $X \in \mathcal{P}(\Omega)$.

Definition 1 *A fuzzy measure (or non-additive measure) on $(\Omega, \mathcal{P}(\Omega))$ is a set function $\mu : \mathcal{P}(\Omega) \to [0,1]$ such that*

(1) $\mu(\emptyset) = 0$, $\mu(X) = 1$,

(2) if $A, B \in \mathcal{P}(\Omega)$, $A \subset B$, then $\mu(A) \leq \mu(B)$, that is, μ is a non decreasing set function w.r.t inclusion.

Note that fuzzy measures encompass probability measures, possibility measures, belief functions, Choquet capacities...

Definition 2 *Let μ be a fuzzy measure on $(\Omega, \mathcal{P}(\Omega))$ and an application $f : \Omega \to [0, +\infty]$. The Choquet integral of f w.r.t μ is defined by:*

$$(C)\int_\Omega f d\mu = \int_0^{+\infty} \mu(\{x : f(x) > t\})dt$$

provided $\{x : f(x) > t\} \in \mathcal{P}(\Omega)$, $\forall t \in \mathbb{R}_+$. If Ω is a finite set $\Omega = \{\omega_1, \cdots, \omega_n\}$, the Choquet integral of f can be written as follows:

$$(C)\int_\Omega f d\mu = \sum_{i=1}^{n} (f(\omega_{(i)}) - f(\omega_{(i-1)}))\mu(A_{(i)})$$

where the subscript $(.)$ indicates that the indices have been permuted in order to have $f(\omega_{(1)}) \leq \cdots \leq f(\omega_{(n)})$, $A_{(i)} = \{\omega_{(i)}, \ldots, \omega_{(n)}\}$ and $f(\omega_{(0)}) = 0$, by convention.

In the finite case, it is easy to see that the Choquet integral is a classical Lebesgue integral up to a reordering of the indices. Besides, if the fuzzy measure μ is additive, then the Choquet integral reduces to a Lebesgue integral.

2.2 Decision under uncertainty

A decision problem under uncertainty can be formalized as a 4-tuple $(S, A, X, \succeq)$ where S is the set of states of the world (on which the uncertainty is), X is the set of consequences or results and $\succeq$ is a preference relation over the actions A that are functions from S to X.

For example, we consider a computer network under attack. Our decision model can be represented with two possible states of the world: under attack, not under attack, and two (very basic) courses of action: disconnect the network or not. We may assume that in case of a doubt, we should disconnect the network. However, for critical infrastructures such as military computer networks the cost of a non-legitimate disconnection, may be too high to justify such a course of action.

The utility approach consists in finding a set of axioms, that guarantee the existence of a function U such that f is preferred to g if and only if $U(f) \geq$

$U(g)$. In the uncertainty case, we could assume the existence of a probability measure on S and a natural choice for U would be the expected utility, as seen in the previous section. From a computer security point of view, this would lead to finding an appropriate probability measure over the states (attacked or not) and the cost associated with the considered actions.

Though, we know that additive representations are not sufficient to model every facet of human behavior and therefore a non-additive representation is necessary. Schmeidler [21], [22] has proved that under a set of conditions on the preference relation $\succeq$ (continuity and independence in some sense), there exists a Choquet representation of the preference relation, that is, there exist a function $u : X \to \mathbb{R}$, unique up to an affinity, and a unique fuzzy measure μ on S such that: for all acts $f, g \in A$ we have:

$$f \succeq g \Leftrightarrow (C)\int_S u(f(\cdot))d\mu \geq (C)\int_S u(g(\cdot))d\mu. \quad (2)$$

This result has been refined later by Gilboa [10], then by Wakker [24].

The sequel of this paper offers several extensions of fuzzy expected utility-like functions.

2.3 Multicriteria decision making

For the sake of completeness, we briefly present multicriteria decision making and the role of fuzzy measures.

In a multicriteria framework, let us assume we have a cartesian product set $X = X_1 \times \cdots \times X_n$, with a preference relation $\succeq$ on X. Let $u_i : X_i \to \mathbb{R}$ be the i-th monodimensional utility function. Then, we need to find an aggregation operator $\mathcal{H} : \mathbb{R}^n \to \mathbb{R}$ such that for all $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_n)$ we have:

$$\begin{aligned} x \succeq y \Leftrightarrow \quad & \mathcal{H}(u_1(x_1), \cdots, u_n(x_n)) \geq \\ & \mathcal{H}(u_1(y_1), \cdots, u_n(y_n)). \end{aligned} \quad (3)$$

It is known that many aggregation usual operators can be written as a Choquet integral, that is, there exists a fuzzy measure μ on the set of criteria $I = \{1, \cdots, n\}$ such that $\mathcal{H}(u_1(x_1), \cdots, u_n(x_n)) = \sum_{i=1}^{n}[(u_{(i)}(x_{(i)}) - u_{(i-1)}(x_{(i-1)})]\mu(A_{(i)})$ with the same notations as before (see [13] pp.203-205 for more details).

In this case, the fuzzy measure is defined on the criteria, and acts like a weighting operator, that is, an operator defining the importance of a coalition of criteria. This is not the only interpretation of fuzzy measures in multicriteria decision making. Indeed, fuzzy measures can be used to define the interaction between two criteria $i, j \in I$ in the following way [19]:

$$\begin{aligned} I_{ij} = \textstyle\sum_{K \subset X \setminus \{i,j\}} \xi(|K|)[\mu(K \cup \{i,j\}) \\ -\mu(K \cup i) - \mu(K \cup j) + \mu(\{i,j\})] \end{aligned} \quad (4)$$

with $\xi(k) = \frac{(n-k-2)!k!}{(n-1)!}$. Recently, Grabisch [11], [12] extended this definition to any number of criteria, leading to what was called an interaction representation of fuzzy measures. This representation through interaction indices happens to be much closer to the decision maker's mind than the usual measure representation. This approach was used to develop a distributed web-based group-multicriteria decision making engine (see [18]). It is a typical example of an application where we cannot assume the independence between the criteria. This also pertains to the independence of the individual decisions made by each member of a group. Note that this can be easily transposed to a network architecture where each node "makes a decision" and these decisions have to be combined into a global decision for the considered system. Each decision may be influenced by the decision on an other node of the architecture and therefore, we have to aggregate partial decisions using a fuzzy integral rather than a simple weighted sum.

These facts show the usefulness of fuzzy measures and the Choquet integral in decision making under uncertainty and multicriteria decision making.

3 A fuzzy-valued benefit function

In the Anscombe-Aumann perspective [1], we assume that the consequences and the results are not the same. They assume the set of consequences to be the set of lotteries, that is the probabilities with finite support written $(p_1; x_1, \cdots p_n; x_n)$ where p_i is the probability that x_i happens. For example, the probability of an attack on a computer network, the probability of a sleeping virus sent to the network, or the probability of change of behavior by a "normal" user. This last remark is important. It is one of the reasons why attacks can be hard to detect.

We will denote the outcomes with X and the consequences with C in this section.

Therefore, the actions will be functions from S to C. Then, for every act f, state s, and outcome x, there is a benefit function associated that is denoted

by $B_f(s,x)$. In the Anscombe-Aumman approach, or in the non-additive case case proposed by one of the authors in [17], it is assumed that the lotteries yield an accurately known benefit function. This can be a strong assumption, and we relax it here, to allow fuzzy-valued benefit functions. Therefore, $\forall f \in A, x \in X, s \in S$, $B_f(s,x)$ is a fuzzy subset of some set W.

It is natural to propose the following fuzzy expected value for the benefit function

$$E_R[B_f(s,\cdot)](w) = \quad (5)$$
$$sup_{\{w_i|\sum_i R(x_i)B_f(s,x_i)(w_i)=w}\}min_i(B_f(s,x_i)(w_i)$$

where R is a distribution over X. It is very easy to see that if the benefit function is not fuzzy-valued, then the above expression reduces to the classical expected utility.

Let us denote by $P_{s,f}$ the lottery associated with the act f and the state s. If there exists a probability Q on S giving the probability to be in a given state, then the value of the act f can be defined as

$$V(f) = \sum_s Q(s)[\sum_i P_{s,f}(x_i)B_f(s,x_i)] \quad (6)$$

We can see that

$$V(f) = E_Q[E_{P_{s,f}}[B_f(s,\cdot)]]$$

Hence, $V(f)$ is a fuzzy subset of W following the definition of B_f, such that

$$V(f)(w) = \sum_s Q(s)[E_{P_{s,f}}[B_f(s,\cdot)](w)]$$

Hence, the best act is an act $f^* \in A$ that maximizes $V(f)$ in a sense to define as $V(f)$ is a fuzzy set. One obvious way is to defuzzify the value of f. An other possibility is to use Jain's maximizing sets.

4 Uncertainty over lotteries

In some instances, we do not have a probability over the states of the world S. This is obviously the case in computer security, where we do not have an exact probability for an attack of a given type. At best, we have a frequency type probability, which is likely to change in the future. Instead, we have a partially known probability Q. In some sense, we can understand this as a second order uncertainty over S. Assume $\mathcal{P}$ is a set of distributions over X and $Q \in \mathcal{P}$, then we define the (fuzzy) measure

$$B(A) = \wedge_{Q\in\mathcal{P}} Q(A)$$

4.1 Extended Choquet integral

We are particularly interested in the case when such measures are belief functions that are a particular type of fuzzy measures. (see for example [23], [13], [6]) where B is a super-additive fuzzy measure satisfying

$$B(\cup_i A_i) \leq \sum_{J\subset Q, J\neq\emptyset} (-1)^{card(J)+1} B(\cap_{j\in J} A_j)$$

for every finite family A_i of subsets of S. It is well known that if B is a belief function (see [13]) then

$$(C)\int u(.)d\mu = inf_{Q\in\mathcal{P}} \int u(.)dQ$$

that is the inferior envelop of lebesgue integrals with respect to a family of probabilities yielding a belief function, is a Choquet integral.

If u is a function from S into fuzzy subsets of W that is $u(s) : W \to [0,1]$, then a natural extension of the Choquet integral is

$$(CE)\int u(s)dB(s) = inf_{Q\in\mathcal{P}} E_Q(u)$$

where E_Q is defined as in the previous section. Therefore, if we assume B is a belief function, a strategy to pick the best act f^* is to maximize the extended Choquet integral

$$(CE)\int E_{P_{s,f}}[B_f(s,\cdot)]dB(s)$$

with respect to f where B is a belief function generated by $Q \in \mathcal{P}$. We can show that the above expression is equivalent to

$$\wedge_{Q\in\mathcal{P}} E_Q[E_{P_{s,f}}[B_f(s,\cdot)]]$$

4.2 Maximizing the Entropy

An alternate way to pick the best act is to choose the appropriate distribution Q^*. Indeed, each $Q \in \mathcal{P}$ has a density function f_Q given by $f_Q(s) = Q(\{s\})$ as S is a finite set. The idea is to pick Q^* in such a way that its density maximizes the entropy

$$H(f) = -\sum_s f(s)log(f(s))$$

over $f \in \{f_Q | Q \in \mathcal{P}\}$.

The construction of f_{Q^*} is very natural using the following algorithm:

while $S_i \neq \emptyset$ *do*

$S_0 \leftarrow S$

$b_i \leftarrow Max_{\emptyset \neq K \subset S_i} \frac{[B(K \cup \overline{S}_i) - B(\overline{S}_i)]}{card(K)}$

Let K_i be the largest subset of S_i such that

$$B(K_i \cup \overline{S}_i) - B(K_i) = b_i card(K_i)$$

$S_{i+1} \leftarrow S_i - K_i$

end while

We then define $f_{Q^*}(x) = b_i$ for $x \in K_i$.

4.3 Maximizing without Belief functions

The above results are valid when the fuzzy measure over the states of the world is a belief function. Hence, we need to come up with a strategy to deal with the case where B is a general fuzzy measure.

In this section, we assume that the probability over S is not known, and we have a set of families $\mathcal{P}_i$, $i = 1, \cdots, t$ to choose from. Each family $\mathcal{P}_i$ represents a family of possibile probability distributions over S, for an expert e_i. Once the distribution $\mathcal{P}_i$ has been chosen based on expert e_i, an appropriate distribution within $\mathcal{P}_i$ is chosen using the results presented in the above section, or using possible additional information.

Thus, we have a two-step selection mechanism. First, we need to choose the "right" family of distributions $\mathcal{P}_i$, then choose an appropriate distribution within the selected family.

The choice of the family of distributions is based on expert credibility. We assume, we have a preference relation (a weak-order, that is a reflexive, connected and transitive binary relation) over the set of experts, generating a preference relation $\succeq$ over the family $(\mathcal{P}_i)_i$ where

$$\mathcal{P}_i \succeq \mathcal{P}_j$$

indicates that $\mathcal{P}_i$ is preferred to $\mathcal{P}_j$. Of course, we have that

$$\mathcal{P}_j \subset \mathcal{P}_i \Rightarrow \mathcal{P}_i \succeq \mathcal{P}_j$$

as a larger set gives more freedom to select an optimal distribution in the second stage of the selection mechanism. We will also make the following assumption:

$$\mathcal{P} \sim \mathcal{P} \cup \mathcal{P}' \Rightarrow \mathcal{P} \cup \mathcal{P}'' \sim \mathcal{P} \cup \mathcal{P}' \cup \mathcal{P}''$$

We then follow [15] to select an optimal act. Let us denote by $\mathcal{A}$ the set of families of distributions over S, under consideration. We define a function f over $\mathcal{A}$ by

$$f(\mathcal{P}) = \cup_{\mathcal{P} \succeq \mathcal{P}'} \mathcal{P}'$$

and let us denote $\hat{S}$ by

$$\hat{S} = \{\mathcal{P} \in \mathcal{A} | \mathcal{P} = f(\mathcal{P})\}$$

Let G denote all the distributuions over S. Then we can define $U : G \times \hat{A} \rightarrow \mathbb{R}$ in the following way. We have that

$$\hat{s}_2 \subset \hat{s}_1 \text{ and } \hat{s}_1 \neq \hat{s}_2 \Rightarrow \neg(\hat{s}_2 \succeq \hat{s}_1)$$

for $\hat{s}_i \in \hat{S}$. Hence, there exists a negative-valued function n on $\hat{S}$ such that

$$\hat{s}_1 \succeq \hat{s}_2 \Leftrightarrow \sum_{\hat{s} | \hat{s}_1 \subset \hat{s}} n(\hat{s}) \geq \sum_{\hat{s} | \hat{s}_2 \subset \hat{s}} n(\hat{s})$$

Then, we define

$$U(g, \hat{s}) = n(\hat{s}) \text{ for } g \in \hat{s}$$

and 0 otherwise. Then, following [15],

$$V(\mathcal{P}) = \sum_{\hat{s} \in \hat{S}} \{max_{g \in \mathcal{P}} U(g, \hat{s})\}$$

is a preference relation on $\mathcal{A}$. Hence, we have a (constructive, following lemma 3 in [15]) way to select an element in $\mathcal{A}$, and the above section, provided a way to select within the chosen element.

5 Set-valued functions

In this section, we will deal with the case where for an act f and a state s, we have a family of possible distributions over the set of outcomes X. We denote this family by $\mathcal{P}_{s,f}$. That, we assume the acts to be set-valued. We should first note that despite an apparent complexity, assuming set-valued functions is very natural. In general, in decision making, we assume, we can choose the granularity of the description of the problem, to avoid dealing with set-valued function. Though, set-valued analysis is well developped as [2] shows. In fact, it is very easy to relate set-valued analysis with fuzzy sets.

From a computer security point of view (or any other critical infrastructure), this is a natural extension to be considered. For example, if the state of the world is "combined attack on several computers of the network", the action f, "disconnect the network and initiate back-ups" generate several outcomes such as the

cost of the back-ups and the disconnection, the effects on some other infracstrutures linked to the considered system, etc...Therefore, we have a set of distributions rather than single distribution over the outcomes.

One way to find the best set-valued act would be to use optimization technique from set-valued analysis (e.g finding the Tangent cone), but we will see how we can adapt more traditional techniques, in line with what we have presented so far to deal with this particular case.

We could define a preference relation over the distributions at state s as follows:

$$P_{s,f_i} \succeq P_{s,f_2} \Leftrightarrow E_{P_{s,f_1}}[B_{f_1}(s,\cdot)] \succeq E_{P_{s,f_2}}[B_{f_2}(s,\cdot)]$$

where the right preference s defined by comparing two fuzzy sets. but, we would like these preferences to be complete and transitive, hence, we choose an other approach. To avoid technical details, we will assume that W the space of possible benefits is finite, but this is not restrictive at all here.

$$W = \{b_1, \cdots, b_m\}$$

where $b_1 \leq b_2 \leq \cdots \leq b_m$. Then, we define our maximizing set $\mathcal{M}$ by

$$\mathcal{M} = \sum_i (\frac{b_i}{b_m})/b_i$$

Hence, the membership of b_i in $\mathcal{M}$ is the fraction to which b_i is the maximal element of $\mathcal{M}$ and naturally, the membership of b_m in $\mathcal{M}$ is 1.

We now define $\succeq$ by $P_{s,f_1} \succeq P_{s,f_2}$ if and only if

$$S(E_{P_{s,f_1}}[B_{f_1}(s,\cdot)], \mathcal{M}) \geq S(E_{P_{s,f_2}}[B_{f_2}(s,\cdot)], \mathcal{M})$$

where

$$S(A,B) = \frac{\sum A(b_i) \wedge B(b_i)}{\sum A(b_i)}$$

$S(A,B)$ represents the fraction of A falling into B.

We now make the following assumptions, over the preference relation that we have defined.

(1). We assume that if $P_{s,f_1} \succ P_{s,f_2}$ then the preference attached to the set $\{P_{s,f_1}, P_{s,f_2}\}$ falls in between the preference attached to P_{s,f_1} and P_{s,f_2} that is

$$P_{s,f_1} \succ \{P_{s,f_1}, P_{s,f_2}\} \succ P_{s,f_2}$$

assuming that non of the preferences are trivial. We refer to this property as the weak dominance principle.

(2). If $\{P_{s,g}|g \in A\} \succ \{P_{s,g}|g \in B\}$ and $\{P_{s,g}|g \in A\} \cap \{P_{s,g}|g \in C\}$ is equal to $\{P_{s,g}|g \in B\} \cap \{P_{s,g}|g \in C\}$ is equal to the empty set, then

$$\{P_{s,g}|g \in A \cup C\} \succeq \{P_{s,g}|g \in B \cup C\}$$

We will refer to this property as the weak independence property. We can see easily that it is an extension of the sure-thing principle of Savage. See also [8] for a discussion of a similar independence property in the single-valued case, but in a qualitative setting (i.e Sugeno integrals).

Let us denoted by $g(\mathcal{P}_{s,f})$ and $l(\mathcal{P}_{s,f})$ the greatest and lowest element of $\mathcal{P}_{s,f}$. Then, if we have the weak dominance principle and the weak independence property, we have that

$$\mathcal{P}_{s,f} \sim \{g(\mathcal{P}_{s,f}), l(\mathcal{P}_{s,f})\}$$

following [3]. It is assumed that the order generated on the $P_{s,f}$ is a linear order.

Let us now define the convex combination of l and g by:

$$U[\mathcal{P}_{s,f} = \alpha l(\mathcal{P}_{s,f}) + (1-\alpha) g(\mathcal{P}_{s,f})$$

where $\alpha \in [0,1]$ is a coefficient depending on the decision maker. A value close to 0 indicates a very optimistic behavior from the decision maker whereas a value close to 1 indicates a pessimistic behavior as the decision maker would be expecting the worst possible case.

This expression can be rewritten:

$$U[\mathcal{P}_{s,f} = \alpha P_{s,f_l} + (1-\alpha) P_{s,f_g}$$

where

$$S(E_{P_{s,f_l}}[B_f(s,\cdot)], \mathcal{M}) = inf_{P_s \in \mathcal{P}_{s,f}}(E_{P_s}[B_f(s,\cdot)], \mathcal{M})$$

Therefore, U is a utility function defined on $\mathcal{P}_{s,f}$ and its value is a distribution on the set of outcomes X. Thus, for a given state s a natural way to compare two acts f_1 and f_2 is to maximize

$$S(E_{\cup_{[\mathcal{P}_{s,f}]}}[B_f(s,\cdot)], \mathcal{M})$$

If Q is a distribution over S, we pick f in order to maximize

$$E_Q\{S(E_{\cup_{[\mathcal{P}_{s,f}]}}[B_f(s,\cdot)], \mathcal{M})\}$$

We can make the following remark. Let us define the notion of positive responsivness (PR) by $P_{s,f} \succ \{P_{s,g}|g \in A\}$ then

$$\{P_{s,g}|g \in A\} \cup \{P_{s,f}\} \succ \{P_{s,g}|g \in A\}$$

and if $P_{s,g} \succ P_{s,f}$ for all g in A then

$$\{P_{s,g}|g \in A\} \cup \{P_{s,f}\} \prec \{P_{s,g}|g \in A\}$$

Then, if we modify the conclusion of the weak dominace principle to $\{P_{s,g}|g \in A \cup C\} \succ \{P_{s,g}|g \in B \cup C\}$ we can show that id there is at least 6 distributions, there does not exist and ordering on $\mathcal{P}_{s,f}$ satisfying (PR) and (WIND) (see [14]). If the number of distributions is at least 3, there does not exist a binary relation on $\mathcal{P}_{s,f}$ satisfying (WDP) and (WIND).

Let us say that $l(P_{s,f_i})$ for $i = 1, \cdots, k$ are tied and similarly for $g(P_{s,f_i})$ for $i = 1, \cdots, t$ with respect to subsethood. Then, using the function

$$U[\mathcal{P}_{s,f} = \frac{\alpha k}{k+t} \sum_{i=1}^{k} l(P_{s,f_i}) + \frac{t + (1-\alpha)k}{k+t} \sum_{j=1}^{t} g(P_{s,f_j})$$

can be used in lieu of the previous function, in order to avoid the drawbacks presented in [14].

6 Conclusions

In this paper, we have presented several strategies to pick the best act in a decision under uncertainty problem in various situations. We have seen that it was not restrictive to consider only the uncertainty paradigm as it is formally equivalent to the multicriteria case. Therefore, these stategies can be very conveniently used in the decision problems involved in robust and resilient infrastructures. First, we have generalized a result given by one of the author to allow fuzzy-valued benefit functions. Then, we have shown how it was possible to introduce a second-order uncertainty over the lotteries, and have proposed two criteria to find the best act. One was to define an extended Choquet integral, the other was to maximize an Entropy function. This result was the extended when we do not have a belief function. Eventually, we have given a selection mechanism of the best act when the value of an act is set-valued, under mild assumptions.

References

[1] F.J. Anscombe and R.J. Aumann. A definition of subjective probability. *The Annals of Mathematical Statistics*, 34:199–205, 1963.

[2] J.P. Aubin and H. Frankowska. *Set-Valued Analysis*. Birkh

[3] S. Barbera, C.R. Barret, and P.K. Pattanaik. On some axioms for ranking sets of alternatives. *Journal of Economic Theory*, 33:301–308, 1984.

[4] G. Choquet. Theory of capacities. *Annales de l'Institut Fourier*, 5, 1953.

[5] D. Denneberg. *Non-Additive Measure and Integral*. Kluwer Academic Publisher, 1994.

[6] D. Denneberg. *Fuzzy measures and integrals. Theory and applications*, chapter Non-additive measure and integral, basic concepts and their rôle for applications. Physica Verlag, 2000.

[7] D. Dubois, M. Grabisch, F. Modave, and H. Prade. Relating decision under uncertainty and MCDM models: an AI perspective. In *IJCAI*, Nagoya, Japan, August 1997. poster.

[8] D. Dubois, H. Prade, and R. Sabbadin. A possibilistic logic machinery for qualitative decision. In *AAAI Workshop*, Stanford, Ca, March 1997.

[9] D. Ellsberg. Risk, ambiguity, and the Savage axioms. *Quart. J. Econom.*, 75:643–669, 1961.

[10] I. Gilboa. Expected utility with purely subective non-additive probabilities. *J. of Math. Economics*, 16(1):65–88, 1987.

[11] M. Grabisch. The representation of importance and interaction of features by fuzzy measures. *Pattern Recognition Letters*, 17:567–575, 1996.

[12] M. Grabisch. Alternative representations of discrete fuzzy measures for decision making. *Int. J. of Uncertainty, Fuzziness, and Knowledge Based Systems*, pages 587–607, 1997.

[13] M. Grabisch, H. T. Nguyen, and E. A. Walker. *Fundamentals of Uncertainty Calculi with Applications to Fuzzy Inference*. Kluwer Academic Publisher, Dordrecht, 1995.

[14] Y. Kannai and B. Peleg. A note on the extension to an order on a set to the power set. *Journal of Economic Theory*, 32:172–175, 1984.

[15] D. M. Keeps. A representation theorem for preference for flexibility. *Econometrica*, 47(3):565–577, 1979.

[16] F. Modave and M. Grabisch. Preference representation by the Choquet integral: the commensurability hypothesis. In *Proc. 7th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU)*, Paris, France, July 1998.

[17] F. Modave and M. Grabisch. Lotteries in multicriteria decision making framework. In *Proc. 8th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU)*, Madrid, Spain, July 2000.

[18] F. Modave and T. Tilley. Web-based collaborative multi-criteria decision making. In *1st Int. Conference on Information Technology*, Bangkok, Thailand, May 2001.

[19] T. Murofushi and S. Soneda. Techniques for reading fuzzy measures (iii): interaction index. In *Proc. 9th Fuzzy System Symposium*, pages 693–696, Sapporo, Japan, May 1993. In Japanese.

[20] L. J. Savage. *The Foundations of Statistics*. Dover, 2 edition, 1972.

[21] D. Schmeidler. Integral representation without additivity. *Proc. of the Amer. Math. Soc.*, 97(2):255–261, 1986.

[22] D. Schmeidler. Subjective probability and expected utility without additivity. *Econometrica*, 57(3):571–587, 1989.

[23] G. Shafer. *A Mathematical Theory of Evidence*. Princeton University Press, 1976.

[24] P. Wakker. A behavioral foundation for fuzzy measures. *Fuzzy Sets & Systems*, 37:327–350, 1990.