HST582J/6.555J/16.456J Biomedical Signal and Image Processing Spring 2005

Fetal & Maternal ECG

Blind Source Separation Lab

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April 7, 2005

1 Introduction

Many health-related problems that occur during a human's lifespan (particularly in the first few months of life) can be reduced if related signs and symptoms are noticed during pregnancy. Since heart rate is one of the most important physiological indicators of health, accurately determining a fetus' heart rate is an important task. One important low-cost non-invasive method for deriving the HR is by recording and processing the electrocardiogram (ECG). If ECG sensors are placed on the mother's abdomen (usually in a *kite* configuration - see Fig. 1), then it is possible to record multiple differential signals between each pair of electrodes. In this example there are three possible *leads*[1] between each of the positions labeled 2, 3 and 4. An example of a fetal ECG (fECG) is given in figure 2 (left). Note the lower amplitude and simplified nature of the signal in comparison to the maternal ECG (mECG) in Fig. 2 (right).

ECGs are sensitive to surrounding contaminants such as measurement noise, electrode movement (due to shifts in the mother and fetus) and the mother's ECG itself. Fig. 3(i) illustrates a typical signal recorded from the configuration in figure 1. This is combination of the maternal ECG [Fig. 3(ii)], observation noise [Fig. 3(iii)], and fetal ECG [Fig. 3(iv)]. Note that, apart from the morphology differences, the fECG has a smaller amplitude (about $\frac{1}{3}$ the height of the maternal QRS complex), which is nearly lost in the background noise. Furthermore, the fetus has a much higher heart rate (156 bpm) than the mother (60 bpm). However, the spectral content of the mECG and fECG are similar. This makes filtering in the frequency domain extremely difficult.

[1]The term *lead* refers to the signal that constitutes the P.D. fluctuations between two electrodes. Confusingly, the wire attached to each electrode is sometimes referred to as a lead, but this is not what is meant in a clinical sense

Conventional methods for separating the mECG from the fECG involve a combination of band pass filtering, maternal template matching and signal subtraction. Unfortunately this leads to severe distortion of the fetal ECG and only the derived heart rate is useful. In this lab we will attempt to use Wiener filtering, Single Value Decomposition (SVD), and Independent Component Analysis (ICA) to separate the fECG an mECG out from the mixed (noisy) signals.

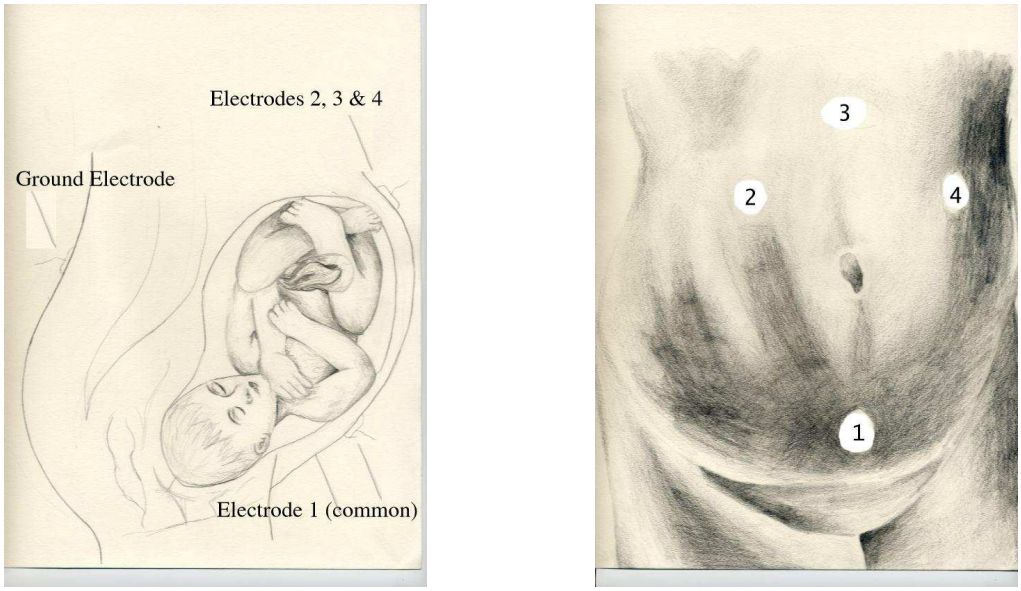

Figure 1: Five standard electrode positions on the mother's abdomen for measuring three channels of fetal ECG (three over the fetus plus one common electrode just above the pubic bone and one ground electrode on the mother's lower back). Each observed ECG is a record of the mV fluctuations on the surface of the mother's abdomen between each pair of electrode positions (2, 3,and 4). Images drawn by B. Campbell.

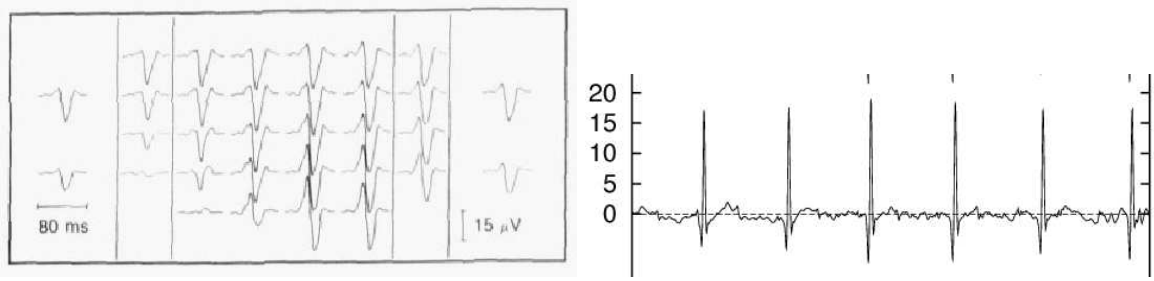

Figure 2: Typical fetal ECG trace (left) recorded from a 26 week old subject and a typical maternal ECG (right). Note that the fECG morphology often changes significantly from the above trace after about 30 weeks. Adapted from Oostendorp *et al* [1] and [2].

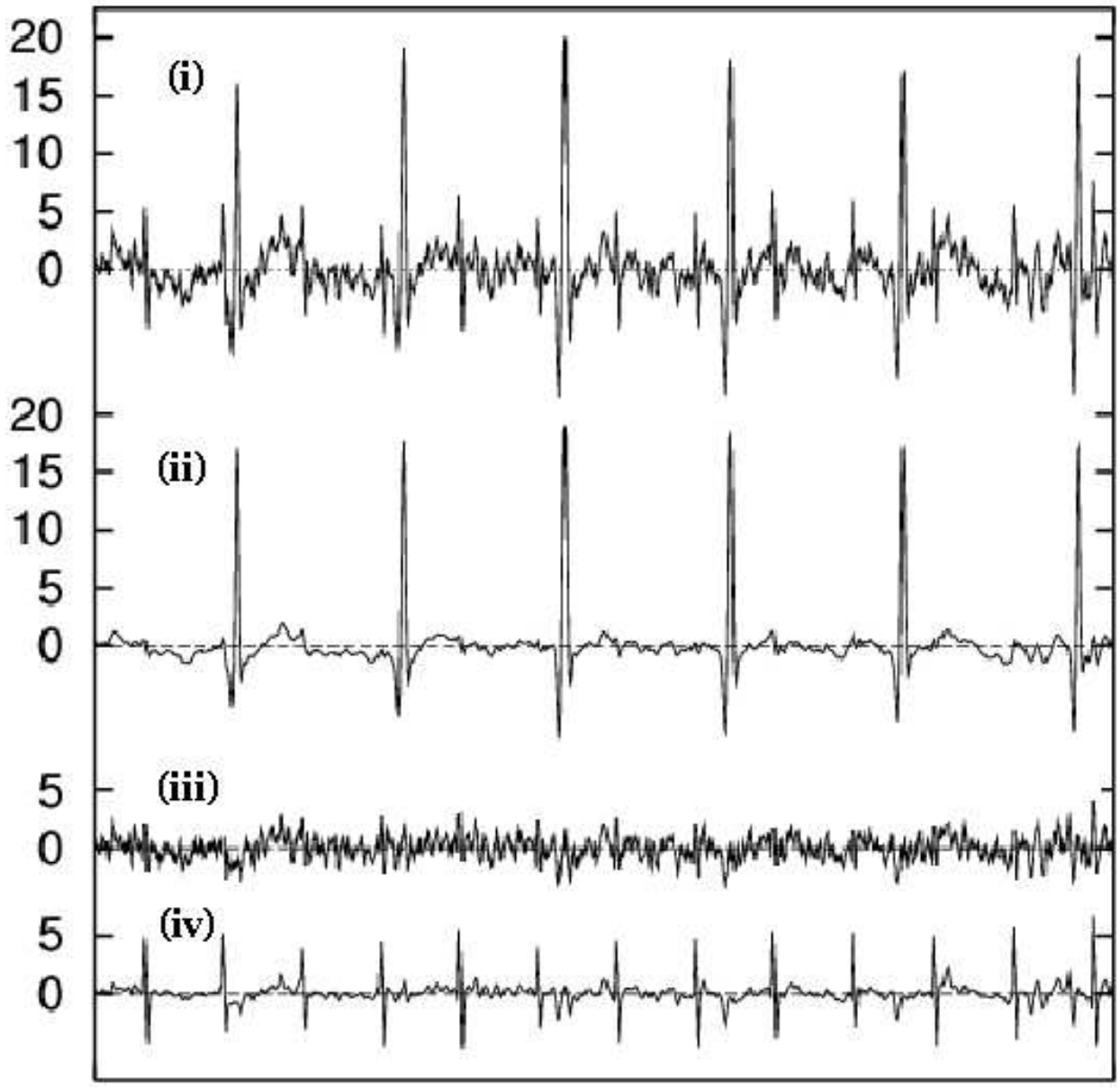

Figure 3: From top: (i) Mixture of maternal and fetal ECG, (ii) maternal ECG only (iii) noise (iv) fetal ECG. The window is 5 seconds long for this trace and therefore the maternal heart rate is 72bpm and the fetal heart rate is 156bpm. Adapted from [2].

2 Laboratory exercises

In this laboratory, you will be attempting to separate a maternal waveform (like that in Fig. 3(ii)) and observation noise (Fig. 3(iii)) from a fetal ECG recording (Fig. 3(i)). Throughout this lab, you will be examining three methods of signal separation: Wiener Filtering, Singular Value Decomposition, and Independent Component Analysis. However, one major obstacle to assessing how well our separation techniques have performed is the inability to truly know what the underlying maternal and fetal ECGs actually look like. In this laboratory we will use artificial ECG and noise signals to provide us with an exact evaluation of how well our chosen techniques perform in separating out the source signals.

2.1 Preliminary Observations

Begin by examining the maternal and fetal ECG signals as well as an example of measurement noise. The data is stored in **mecg1.dat**, **fecg1.dat**, and **noise1.dat** and can be read into Matlab using the **load** command. Each file consists of a vector of the recorded ECG signal (in mVs), sampled at a rate of 256 Hz.

Question 1 *Include in your lab report separate time domain plots of the maternal, fetal, and noise signals. What is the heart rate of the maternal and fetal ECGs? Now, combine the three signals to create a new signal known as* ***mixture*** *and plot it in the time domain. Identify, by hand, the fetal spikes in the* ***mixture*** *plot.*

Question 2 *Calculate and plot the power spectrum of the maternal, fetal, and noise signals. You may find the Matlab commands* ***psd*** *or* ***pwelch*** *to be useful. How do the fetal and maternal ECGs compare? Describe the similarities and differences.*

2.1.1 Statistical distributions of the signals; calculating their moments

Question 3 *Calculate the first two moments (mean and variance) of the maternal, fetal, and noise signals. How do they compare? What does the variance tell you about the signals' power spectra?*

Question 4 *Now calculate the* 3^{rd} *and* 4^{th} *moments of the signal distributions using* ***skewness.m*** *and* ***kurtosis.m****. Note that* MATLAB *adopts the convention that a Gaussian signal has a kurtosis of 3. The normal convention is that a Gaussian signal has a kurtosis of 0. Include in your lab writeup a plot of the empirical PDFs of each signal. You will find the Matlab command* ***hist*** *to be useful. What do you notice about the gaussianity of the three signals? Compare and contrast both the skewness and kurtosis of the signals.*

The first two methods we will examine (Wiener Filtering and Singular Value Decomposition) only take into account 2^{nd} order statistics. Independent Component Analysis, however, uses 3^{rd} and 4^{th} order statistics to perform signal/noise separation. Later in the lab, we will revisit how the statistical distribution of the signals impacts the effectiveness of the three signal separation techniques.

2.2 Wiener filtering of the Maternal ECG

Recall from Chapter 12 in the lecture notes, that the optimal (non-causal) Wiener filter is given by $H(f) = \frac{S_y(f)}{S_y(f)+S_d(f)}$ where $S_y(f)$ is the power spectrum of the model of the true signal, *y*, and $S_d(f)$ is the spectrum estimate of the noise component, *d*. The estimate of the signal, $\hat{Y}$, (in the Fourier domain), from the observation $S_x(f)$, is therefore

$$\hat{Y}(f) = S_x(f)H(f) \tag{1}$$

Your next task is to build a Wiener filter for this data and to comment on how well this extracts out the fetal component from the observations.

Note: *Remember to save the output of the filter, as you will compare it with the other techniques we use later in this lab.*

Hints:

- Edit the function **wienerFilter.m** to make it work correctly. The function is of the form [yhat H] = wienerFilter(ideal, observation) where **ideal** is the ideal model of the signal we wish to extract.
- The function will be applied using **mixture** as the observation and the fetal ECG as the ideal signal.
- Remember that since we want to extract *only* the fetal components, the noise is now really the sum of the maternal and noise components ... we are treating the maternal ECG as a noise component in this problem!

Question 5 *Use the Wiener filter on the **mixture** data to extract the fetal ECG components. Look at the filtered signal in both the time and frequency domain and comment on the general shape. Compare the ideal fetal ECG signal to the filtered data. Explain the source of any differences you see.*

We have already seen that the signals are zero mean with unit variance. However, as we have seen, their differing distributions over the frequencies allows us to filter the components out (to some extent) if we know the approximate form of these distributions (and they are different). Another powerful method of achieving this, without using any prior knowledge of the spectral components of the signals is Singular Value Decomposition (SVD), which we will now explore.

2.3 Signal separation using SVD

SVD is a powerful, yet standard, 2^{nd} order technique for filtering data based upon the projection of the data onto orthogonal axes corresponding to vectors of maximal variance. Although this means we are projecting data onto a set of orthogonal axes which correspond to the variance (and hence power) in a particular direction in the data, each projection (or component) does not correspond to a discrete power band in the frequency domain. In fact, the components of the *eigen*spectrum formed from an SVD are not completely set before hand, (as is the case for an FFT-based analysis where each component corresponds to a particular frequency interval) but are **learned from the data**. In general, each component will overlap in the frequency domain to some extent. We will now explore SVD using a similar data set to that which we have just analyzed.

First, we will load and plot a new set of ECG observations using the code below.

```
load X.dat          % load the three ECG channels
ld3channel(X)       % creates plots
```

This time you will not be provided with the sources (mECG, fECG and noise), but rather three channels of (observed) ECG, all of which have a component of fetal ECG, maternal ECG and observation noise. The data $\mathbf{X}$ is composed of 3 channels of 10 second ECG sampled at 256 Hz:

Running the function **ld3channel** will produce two figures. The first figure plots the three ECGs in the time domain. In the second figure, the three ECG channels(X) are decomposed to form at most 3 (orthogonal) eigenvectors, which lie along the three axis of maximal variance. The plot demonstrates how the data varies between each channel (as a function of the other channels). Note that most of the data is located near the origin (which is the mean for a zero-mean data set). Data outside of the central cluster are mostly associated with the QRS peaks.

Now we will attempt to separate the maternal and fetal ECG using SVD. Recall Eq. 5 ($\mathbf{Y} = \mathbf{U} * \mathbf{S} * \mathbf{V}^T$), where $\mathbf{Y}$ (a real $M \times N$ matrix) can be decomposed into three other matrices, $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}^T$. $\mathbf{S}$ is a non-square diagonal matrix of singular values whose elements are arranged in descending order of magnitude and represent the eigenvalues of $\mathbf{C} = \mathbf{Y} * \mathbf{Y^T}$.

```
S =[   s1      0      0
       0       s2     0
       0       0      s3
       0       0      0
       ...     ...    ...
       0       0      0 ]
```

A plot of these diagonal elements is known as an **eigenspectrum**. The columns of **V** are the eigenvectors of $\mathbf{C} = \mathbf{Y} * \mathbf{Y}^T$ (with the lengths of the eigenvectors normalized to unity) and the matrix **U** is the matrix of projections of **Y** onto the eigenvectors of **C**. Perform SVD on the matrix of observations (***X***) using the Matlab command ***svd*** to create the matrices, $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}$.

Question 6 *Plot the eigenvectors onto the matrix of observations **X** using the Matlab function **plot3dv**. (**plot3dv** takes as input a column of the matrix **V**). Please save this plot for later comparisons. (No text response is required)*

Recall that the matrix **U** is the matrix of projections of $\mathbf{Y}$ onto the eigenvectors of $\mathbf{C}$ [3]. It has a size of $M \times M$ but only the first N projections are nonzero because the dimensionality of your singular value decomposition subspace cannot be larger than the dimensionality of the data.

Question 7 *: Plot the first three columns of the matrix **U**. For each of these projections, identify the dominant features (maternal, fetal, noise, or mixture). Indicate which projection is most representative of the fetal ECG. Also plot the eigenspectrum using **stem** and include this in your lab writeup.*

It should be noted however, that this projection is no longer clinically meaningful. Therefore if we wish to preserve the clinical information, we must invert the SVD transformation to extract the fetal ECG.

We can extract the fetal ECG by modifying the **S** matrix. Keep the eigenvalue corresponding to the output channel most representative of fetal ECG and set all other eigenvalues to 0. (The matrix is now singular and noninvertible, so we have a lossy transformation). Use the original **U** and **V** matrices along with the modified **S** to reconstruct the 3 channels of fetal ECG.

Question 8 *Describe the effects of the SVD reconstruction. Was this successful in extracting the fetal ECG? Please save and include a plot of the three columns of the SVD reconstruction in your lab writeup. This plot will also be used later in a comparison of Wiener filtering, SVD, & ICA.*

2.4 Separating sources using ICA

In this section we will use a 4^{th} order technique; *Independent Component Analysis* (ICA) [4] to perform the maternal-fetal-noise signal separation. Recall the mixing paradigm **observation** = **mixing matrix** $\times$ **underlying sources**, or

$$\mathbf{X^T} = \mathbf{AZ^T}. \tag{2}$$

Note that for dimensional consistency we have transposed the three channels (M data points by N channels) of observations $\mathbf{X}$, and the three source column vectors $\mathbf{Z}$ (also $M \times N$).

The de-mixed estimate of the sources $\mathbf{Z}$ is therefore

$$\hat{\mathbf{Z}}^{\mathbf{T}} = \mathbf{WX^T}. \tag{3}$$

where $\mathbf{W}$ is the de-mixing matrix and $\mathbf{W}^{-1}$ is the estimate of the mixing matrix $\mathbf{A}$.

Perform ICA on the matrix of observations ($\mathbf{X}^T$) using the Matlab command ***ica*** to create the matrices ***W*** and $\hat{\mathbf{Z}}$. Save these matrices as they will be used in later computations.

Question 9 *Now make a scatter plot of the data and the three independent components along which the sources are projected using the Matlab function* ***plot3dv****. (Note that* ***plot3dv*** *will take as input a column of the matrix* ***W****). Compute* $\mathbf{W}^{-1}$ *which is the estimate of the matrix* ***A****. Please save this plot for later comparisons. (No text response is necessary.)*

Question 10 *Plot the three columns of the matrix* $\hat{\mathbf{Z}}$*. For each of the projections identify the dominant features (maternal, fetal, noise, or mixture). Indicate which projection is most representative of the fetal ECG.*

We can extract the fetal ECG by modifying the $\mathbf{W}^{-1}$ matrix. Keep the column of $\mathbf{W}^{-1}$ most representative of the fetal ECG and set the columns corresponding to the maternal and noise projections to 0. Use the $\hat{\mathbf{Z}}$ matrix along with the modified $\mathbf{W}^{-1}$ matrix to reconstruct the three channels of fetal ECG.

Question 11 *Describe the effects of the ICA reconstruction. Was this successful in extracting the fetal ECG? Please save and include a plot of the results of the ICA reconstruction in your lab writeup. This plot will also be used later in a comparison of Wiener filtering, SVD, & ICA.*

2.5 Comparisons

Question 12 *Load the figures created in Questions 6 & 10 showing the matrix of observations* ***X*** *plotted along two different sets of axes. Rotate and compare the two figures. Describe any differences you notice, in particular, with regards to the axes.*

Question 13 *Now quantitatively compare the two sets of axes. Use the Matlab function* ***dot*** *do pairwise comparisons on the columns of* $\mathbf{V}$*. Do the same with the columns of* $\mathbf{W}^{-1}$*. How do they compare and what do the numbers mean? Also calculate the lengths of the two sets of axes using 3-D pythagoras. Note that when you are done you will have three scalar values for each axis. How do the lengths compare?*

One essential question we must ask ourselves in all filtering procedures, is *how well did the signal/noise separation work?* Often this requires a good definition of what exactly is the signal and what exactly is the noise. When we use simulated data, then the distinction is simple; it is whatever we initially chose. However, in our example, we have further defined the maternal ECG to be a noise (or artifact) source, since we wish to extract the fetal ECG for further analysis. For the data matrix **X**, that you have been working with, the ideal reconstructed fetal ECG is given by **fecg2** which is a scaled version of **fecg1**.

Question 14 *Compare the plots of the results of Wiener filtering, SVD, and ICA (from questions 5, 8, & 11) with a plot of the ideal fetal ECG (****fecg2****). Note that there are three output channels of fetal ECG for SVD and ICA as opposed to only one output channel for Wiener filtering. Please describe qualitatively the relative effectiveness of each filtering technique. Which one did the best? Which one did the worst? Note that there are 3 channels of filtered fetal ECG for the SVD and ICA outputs in comparison to the single channel in the Wiener filter.*

Question 15 *Now quantitatively evaluate the effectiveness of each filtering technique using the correlation coefficient to assess which method produced the cleanest fECG. You will find the Matlab function* ***corrcoef*** *to be useful. Please include these results in your lab writeup. Which filtering technique did the better job? For SVD & ICA, you can choose one channel from each, however, please ensure that they are the same channels.*

Question 16 *Compare and contrast the three filtering techniques explored in this lab. Describe the advantages and limitations of each technique. Why did Wiener filtering perform better than SVD. Was it a fair comparison?*

Question 17 *What is the most important thing that you learned from this lab exercise?*

Question 18 *What did you like/dislike the most about this lab exercise?*

References

[1] Oostendorp T, vanOosterom A, H.W. J. The fetal ECG throughout the second half of gestation. Clin Phys Physiol Meas 1989;10(2):147–160.

[2] Richter M, Schreiber T, Kaplan DT. Fetal EEG extraction with nonlinear phase space projections. IEEE Trans Bio Med Eng Jan 1998;45:133–137.

[3] Golub GH, Van Loan CF. Matrix Computations. 2^{nd} edition. Oxford: North Oxford Academic, 1983.

[4] Cardoso JF. High-order contrasts for independent component analysis. Neural Comput 1999;11(1):157–192. ISSN 0899-7667.