IEICE TRANS. FUNDAMENTALS, VOL.E89–A, NO.4 APRIL 2006

PAPER Special Section on Selected Papers from the 18th Workshop on Circuits and Systems in Karuizawa

An Active Noise Control System Based on Simultaneous Equations Method without Auxiliary Filters

Mitsuji MUNEYASU[†a)], ***Member***, **Osamu HISAYASU**[††*], ***Nonmember***, **Kensaku FUJII**[†††], ***Member***, ***and*** **Takao HINAMOTO**[††], ***Fellow***

SUMMARY A simultaneous equations method is one of active noise control algorithms without estimating an error path. This algorithm requires identification of a transfer function from a reference microphone to an error microphone containing the effect of a noise control filter. It is achieved by system identification of an auxiliary filter. However, the introduction of the auxiliary filter requires more number of samples to obtain the noise control filter and brings a requirement of some undesirable assumption in the multiple channel case. In this paper, a new simultaneous equations method without the identification of the auxiliary filter is proposed. By storing a small number of input signals and error signals, we avoid this identification. Therefore, we can reduce the number of samples to obtain the noise control filters and can avoid the undesirable assumption. From simulation examples, it is verified that the merits of the ordinary method is also retained in the proposed method.

key words: *active noise control, simultaneous equations method, auxiliary filter, multiple channel, adaptive filter*

1. Introduction

Since low frequency noise cannot easily be reduced with passive means, active noise control (ANC) techniques are an effective approach for such a case [1], [2]. Several techniques for the ANC system have been proposed. Often, filtered-X LMS (FXLMS) algorithm is used as an adaptive algorithm for the ANC system [3]. This algorithm requires prior estimation of impulse responses of a secondary path. The estimation error of the impulse responses affects stability of the system and performance of noise reduction. Recently, several ANC algorithms without the estimation of the error path in advance have been proposed [4]–[6].

A simultaneous equations method is one of such ANC algorithms [6]. In this algorithm, two different coefficients are substituted into a noise control filter and each impulse response of the primary path which means a transfer function from an output of a noise detection microphone to an output of an error detection microphone is estimated by using auxiliary filters. After that, using the solution of the simultaneous equations given by this procedure, the coefficients of the noise control filter can be updated without prior estimation of the error path. This algorithm has also been extended to a multiple channel case.

In this algorithm, the introduction of the auxiliary filter requires more number of samples to obtain the noise control filter. For example, 10,000 times of updating the coefficients of auxiliary filters should be required to reduce the estimation error of them up to 50 dB [6]. In the multiple channel case, some undesirable assumption should be introduced.

This paper proposes a new ANC system based on the simultaneous equations method without the identification of auxiliary filters. In this estimation technique, the memories of error signals and input signals within a given term are used. Therefore we can avoid introduction of such assumption and can achieve same performance of noise reduction with more reduced number of samples. Simulation examples also show that the proposed method has the same advantages in the ordinary algorithm.

2. Simultaneous Equations Method

In this section, the simultaneous equations method [6], [7] is briefly reviewed.

2.1 Single Channel Case [6]

Let the transfer functions of a primary path from a reference microphone to an error microphone and a secondary path from a canceling loudspeaker to the error microphone be $P(z)$ and $C(z)$, respectively. In this case, the transfer function of the whole system containing a noise control filter $H(z)$ from the reference microphone to the error microphone is represented as

$$S(z) = P(z) - H(z)C(z). \tag{1}$$

Note that coefficients of the noise control filter can be set arbitrarily if degradation of performance of noise control is allowed. Let $H_1(z)$ and $H_2(z)$ be the transfer functions of the noise control filters with different coefficients, respectively. Then, by using an auxiliary filter shown in Fig. 1, if $S(z)$ is identified, the following equations are obtained:

$$S_1(z) = P(z) - H_1(z)C(z) \tag{2}$$

and

$$S_2(z) = P(z) - H_2(z)C(z). \tag{3}$$

Manuscript received June 27, 2005.
Manuscript revised September 21, 2005.
Final manuscript received November 22, 2005.

[†]The author is with the Faculty of Engineering, Kansai University, Suita-shi, 564–8680 Japan.
[††]The authors are with the Faculty of Engineering, Hiroshima University, Higashi-Hiroshima-shi, 739–8527 Japan.
[†††]The author is with the Faculty of Engineering, University of Hyogo, Himeji-shi, 671–2201 Japan.
[*]Presently, with Mitsubishi Engineering, Co. Ltd.
a) E-mail: mm@m.ieice.org

DOI: 10.1093/ietfec/e89–a.4.960

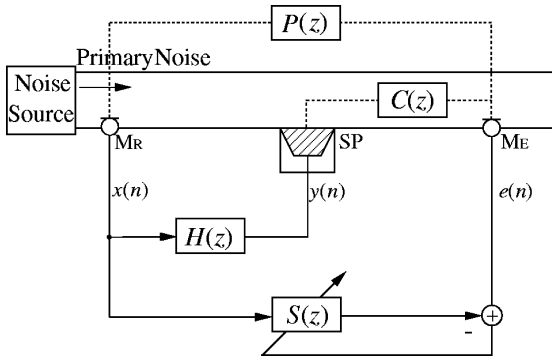


Fig. 1 Structure of the simultaneous equations method.

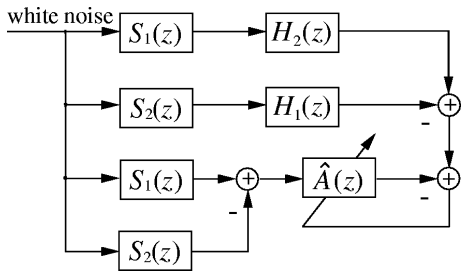


Fig. 2 Estimation of $\widehat{A}(z)$ by using system identification.

From (2) and (3), $P(z)$ can be given by

$$P(z) = \frac{S_1(z)H_2(z) - S_2(z)H_1(z)}{H_2(z) - H_1(z)}. \tag{4}$$

Let $H_{opt}(z)$ mean the optimal noise control filter, then

$$P(z) - H_{opt}(z)C(z) = 0 \tag{5}$$

is held.

Substituting (4) into (5),

$$H_{opt}(z) = \frac{S_1(z)H_2(z) - S_2(z)H_1(z)}{S_1(z) - S_2(z)} \tag{6}$$

is obtained. Since $S_1(z)$ and $S_2(z)$ are derived from the different $H_1(z)$ and $H_2(z)$, $S_1(z) \neq S_2(z)$ is held. This equation shows the principle of the simultaneous equations method to obtain the transfer function of the noise control filter.

Let us rewrite (6) to

$$\begin{aligned}&\{S_1(z) - S_2(z)\}\widehat{A}(z)\\ &\quad = S_1(z)H_2(z) - S_2(z)H_1(z)\end{aligned} \tag{7}$$

where $\widehat{A}(z)$ means the estimation of $H_{opt}(z)$. From this equation, the simultaneous equations can be solved by using the framework of the system identification shown in Fig. 2 [6]. In (7), $\widehat{A}(z)$ is considered as an adaptive filter and let $S_1(z)H_2(z) - S_2(z)H_1(z)$ be an unknown system, then the system identification problem can be stated. In this problem, since $H_1(z)$, $H_2(z)$, $S_1(z)$ and $S_2(z)$ are already given, an estimate of $H_{opt}(z)$ can be obtained as the coefficients of $\widehat{A}(z)$ which are updated by using adaptive algorithm, such as NLMS algorithm.

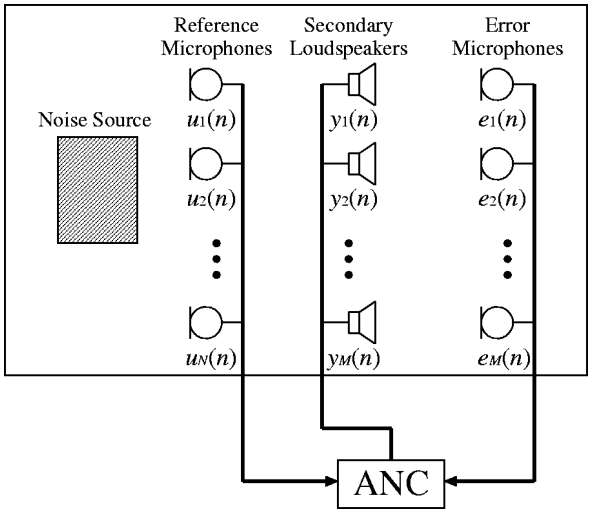


Fig. 3 Multiple channel ANC system.

2.2 Multiple Channel Case [7]

In the multiple-channel ANC system shown in Fig. 3, the numbers of reference microphones, secondary loudspeakers and error microphones are N, M and M, respectively. This case, called the $N-M-M$ system, satisfies conditions under which the simultaneous equations method can be applied, as described in Ref. [7]. In this section, we treat the $N-M-M$ case.

Let the Z transform of the input to the reference microphones be

$$\boldsymbol{U}(z) = [U_1(z), U_2(z), \cdots, U_N(z)]^T,$$

the Z transform of an input to an error microphone be

$$\boldsymbol{E}(z) = [E_1(z), E_2(z), \cdots, E_M(z)]^T,$$

and $A_{ij}(z)$, $H_{ij}(z)$ and $C_{ij}(z)$ be the transfer function from each reference microphone to each loudspeaker, the corresponding transfer function of the noise control filter and the transfer function of the secondary path from each loudspeaker to each error microphone. Then,

$$\begin{aligned}\boldsymbol{E}(z) &= \boldsymbol{C}(z)(\boldsymbol{A}(z) - \boldsymbol{H}(z))\boldsymbol{U}(z)\\ &= \boldsymbol{S}(z)\boldsymbol{U}(z)\end{aligned} \tag{8}$$

is held, where

$$\boldsymbol{A}(z) = \begin{bmatrix} A_{11}(z) & A_{12}(z) & \cdots & A_{1N}(z)\\ A_{21}(z) & A_{22}(z) & \cdots & A_{2N}(z)\\ \vdots & \vdots & & \vdots\\ A_{M1}(z) & A_{M2}(z) & \cdots & A_{MN}(z)\end{bmatrix}$$

$$\boldsymbol{H}(z) = \begin{bmatrix} H_{11}(z) & H_{12}(z) & \cdots & H_{1N}(z)\\ H_{21}(z) & H_{22}(z) & \cdots & H_{2N}(z)\\ \vdots & \vdots & & \vdots\\ H_{M1}(z) & H_{M2}(z) & \cdots & H_{MN}(z)\end{bmatrix}$$

$$\boldsymbol{C}(z) = \begin{bmatrix} C_{11}(z) & C_{12}(z) & \cdots & C_{1M}(z) \\ C_{21}(z) & C_{22}(z) & \cdots & C_{2M}(z) \\ \vdots & \vdots & & \vdots \\ C_{M1}(z) & C_{M2}(z) & \cdots & C_{MM}(z) \end{bmatrix}$$

and

$$\boldsymbol{S}(z) = \begin{bmatrix} S_{11}(z) & S_{12}(z) & \cdots & S_{1N}(z) \\ S_{21}(z) & S_{22}(z) & \cdots & S_{2N}(z) \\ \vdots & \vdots & & \vdots \\ S_{M1}(z) & S_{M2}(z) & \cdots & S_{MN}(z) \end{bmatrix}.$$

Therefore, it is assumed that

$$S_{ij}(z) = \sum_{k=1}^{m} C_{ik}(z)(A_{kj}(z) - H_{kj}(z)) \tag{9}$$

is valid for each element. Equations (8) and (9) are the principle for the multi-channel case.

If there is a correlation between inputs of reference microphones, (9) does not necessarily hold. In Ref. [7], no problem arises in applying this algorithm based on this equation, however it becomes the reason for the slow convergence of $S_{ij}(z)$ under correlated inputs.

If the degradation of noise reduction performance is allowed in (9) as in the case of the single channel, an arbitrary transfer function can be set for $H_{kj}(z)$. Therefore we consider the case where $H^1_{kj}(z)$ and $H^2_{kj}(z)$ are assigned to $H_{kj}(z)$. The following equations are obtained:

$$S^1_{ij}(z) = \sum_{k=1}^{m} C_{ik}(z)(A_{kj}(z) - H^1_{kj}(z)) \tag{10}$$

$$S^2_{ij}(z) = \sum_{k=1}^{m} C_{ik}(z)(A_{kj}(z) - H^2_{kj}(z)) \tag{11}$$

where $S^1_{ij}(z)$ and $S^2_{ij}(z)$ are the transfer functions of the auxiliary filters corresponding to $H^1_{kj}(z)$ and $H^2_{kj}(z)$, respectively.

Let $\boldsymbol{S}^1(z) = [S^1_{ij}(z)]$, $\boldsymbol{S}^2(z) = [S^2_{ij}(z)]$, $\boldsymbol{H}^1(z) = [H^1_{ij}(z)]$ and $\boldsymbol{H}^2(z) = [H^2_{ij}(z)]$. Then, from (10) and (11), the following equation is held:

$$\widetilde{\boldsymbol{S}}(z) = \boldsymbol{C}(z)\widetilde{\boldsymbol{H}}(z) \tag{12}$$

where $\widetilde{\boldsymbol{S}}(z) = \boldsymbol{S}^2(z) - \boldsymbol{S}^1(z)$ and $\widetilde{\boldsymbol{H}}(z) = \boldsymbol{H}^1(z) - \boldsymbol{H}^2(z)$.

From (12), the following equation can be derived:

$$\boldsymbol{C}(z) = \widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T(\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)^{-1} \tag{13}$$

where the nonsingularity of $(\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)$ is assumed. Hence, $\boldsymbol{C}(z)$ can be obtained.

Also, from (8),

$$\boldsymbol{S}^1(z) = \boldsymbol{C}(z)(\boldsymbol{A}(z) - \boldsymbol{H}^1(z)) \tag{14}$$

can be derived. Since $\boldsymbol{C}(z)$ is a square matrix, it is assumed that $\boldsymbol{C}^{-1}(z)$ exists. Then,

$$\boldsymbol{A}(z) = \boldsymbol{C}^{-1}(z)\boldsymbol{S}^1(z) + \boldsymbol{H}^1(z) \tag{15}$$

is held and $\boldsymbol{A}(z)$ can be obtained.

Now, (13) is substituted into (15), the following is derived:

$$\begin{aligned}(det\,\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T)\boldsymbol{A}(z) &= (\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)(adj\,\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T)\boldsymbol{S}^1(z) \\ &\quad + (det\,\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T)\boldsymbol{H}^1(z)\end{aligned} \tag{16}$$

where $det\,\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T$ and $adj\,\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T$ mean the determinant and adjoint matrices of $\widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T$, respectively. Similarly to the case of the single channel, by using system identification to solve (16), $\boldsymbol{A}(z)$ can be obtained in terms of $S^0_{ij}(z)$, $S^1_{ij}(z)$, $H^0_{ij}(z)$ and $H^1_{ij}(z)$.

When $\widetilde{\boldsymbol{H}}(z)$ is a square matrix, (13) can be simplified as follows:

$$\begin{aligned}\boldsymbol{C}(z) &= \widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^T(\widetilde{\boldsymbol{H}}(z)^T)^{-1}\widetilde{\boldsymbol{H}}(z)^{-1} \\ &= \widetilde{\boldsymbol{S}}(z)\widetilde{\boldsymbol{H}}(z)^{-1}\end{aligned} \tag{17}$$

Then, (16) can also be rewritten as

$$\begin{aligned}(det\,\widetilde{\boldsymbol{S}}(z))\boldsymbol{A}(z) &= \widetilde{\boldsymbol{H}}(z)(adj\,\widetilde{\boldsymbol{S}}(z))\boldsymbol{S}^1(z) + (det\,\widetilde{\boldsymbol{S}}(z))\boldsymbol{H}^1(z).\end{aligned}$$

3. Proposed Algorithm

3.1 Single Channel Case

Figure 4 shows the block diagram of the simultaneous equations method in the case of $P(z) = C(z)A(z)$ where $A(z)$ means a transfer function from a reference microphone to a canceling loudspeaker. In this case, the same identification scheme can be also used [6].

Let the input signals which are used for obtaining $S_1(z)$ and $S_2(z)$ be $U_1(z)$ and $U_2(z)$, respectively, then

$$S_1(z) = \frac{E_1(z)}{U_1(z)} \tag{18}$$

and

$$S_2(z) = \frac{E_2(z)}{U_2(z)} \tag{19}$$

are held, where $E_1(z)$ and $E_2(z)$ mean the Z transform of outputs of error microphones for $U_1(z)$ and $U_2(z)$, respectively. To eliminate $S_1(z)$ and $S_2(z)$, (18) and (19) are substituted into (7). Then

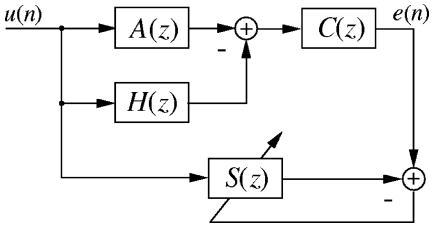


Fig. 4 Block diagram of the simultaneous equations method.

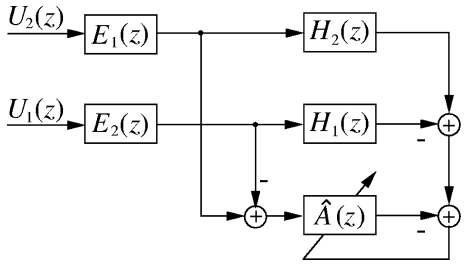


Fig. 5 System identification scheme for the proposed method.

$$\{E_1(z)U_2(z) - E_2(z)U_1(z)\}\widehat{A}(z) = E_1(z)U_2(z)H_2(z) - E_2(z)U_1(z)H_1(z) \quad (20)$$

is obtained.

The above equation means that $E_1(z)U_2(z)$ and $E_2(z)U_1(z)$ are substituted into $S_1(z)$ and $S_2(z)$ for (7). Therefore, to solve this equation, the system identification in Fig. 2 whose $S_1(z)$ and $S_2(z)$ are replaced with $E_1(z)U_2(z)$ and $E_2(z)U_1(z)$, can be used.

However, $E_1(z)U_2(z)$ and $E_2(z)U_1(z)$ have very large number of taps, since they are convolution of the input signal and the output signal. To avoid this problem, (20) is rewritten as

$$\widehat{A}(z)\{E_1(z)U_2(z) - E_2(z)U_1(z)\} = H_2(z)E_1(z)U_2(z) - H_1(z)E_2(z)U_1(z). \quad (21)$$

Then this system identification problem can be considered as the one with two inputs, namely $S_1(z)$ and $S_2(z)$ are replaced with $E_1(z)$ and $E_2(z)$, respectively, and $U_1(z)$ and $U_2(z)$ are used as the inputs for the system identification. This system identification scheme is shown in Fig. 5.

The tap lengths of $E_1(z)$ and $E_2(z)$ may be longer than that of $S_1(z)$ and $S_2(z)$, however they are shorter than that of $E_1(z)U_2(z)$ and $E_2(z)U_1(z)$. If the statistical characteristics of $U_1(z)$ and $U_2(z)$ can be assumed to be almost the white noise, the performance for the estimation of $\widehat{A}(z)$ is not deteriorated compared to the method in Ref. [6]. In colored case, the performance is somewhat inferior and this point is discussed in Sect. 4.

The noise reduction procedure of the proposed algorithm is summarized in the following and shown in Fig. 6. In this procedure, $\boldsymbol{h}_i$, $\boldsymbol{u}_i$ and $\boldsymbol{e}_i (i = 1, 2)$ mean impulse response vectors of $H_i(z)$, $U_i(z)$ and $E_i(z)$, respectively.

1. As coefficients of the noise control filter, $\boldsymbol{h}_1 = \boldsymbol{0}$ is given.
2. Record the inputs of the reference and error microphone, $\boldsymbol{u}_1$ and $\boldsymbol{e}_1$ whose lengths are N_s.
3. $\boldsymbol{h}_2 = [a, 0, \cdots, 0] (a \neq 0)$ is given to the coefficients of the noise control filter.
4. Record the inputs of the reference and error microphones, $\boldsymbol{u}_2$ and $\boldsymbol{e}_2$
5. By using the system identification structure shown in Fig. 5, calculate $\hat{\boldsymbol{a}}$ which is the estimation of the noise control filter.
6. Set $\hat{\boldsymbol{a}}$ to the coefficient of the noise control filter and record the input of the reference and error microphones as the new $\boldsymbol{u}_2$ and $\boldsymbol{e}_2$. Let the old $\boldsymbol{u}_2$ and $\boldsymbol{e}_2$ be $\boldsymbol{u}_1$ and $\boldsymbol{e}_1$, then obtain new $\hat{\boldsymbol{a}}$ by the structure shown in Fig. 5.
7. Repeat Step 6.

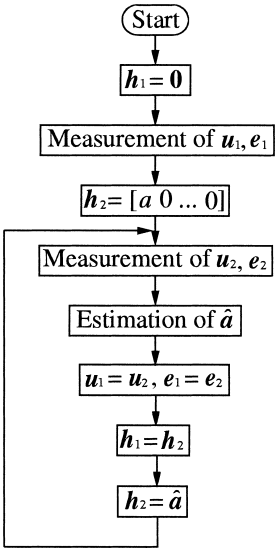


Fig. 6 Update procedure of proposed method in single channel case.

3.2 Multiple Channel Case

According to the single channel case, $\boldsymbol{H}(z) = [H_{ij}(z)]$ in (8) is set to $\boldsymbol{H}^1(z) = [H^1_{ij}(z)]$. After N_p times measurements of the inputs to the reference and error microphones, let $\boldsymbol{S}(z) = [S_{ij}(z)]$ be $\boldsymbol{S}^1(z) = [S^1_{ij}(z)]$, the following relation can be obtained:

$$\widetilde{\boldsymbol{E}}^1(z) = \boldsymbol{S}^1(z)\widetilde{\boldsymbol{U}}^1(z) \quad (22)$$

where

$$\widetilde{\boldsymbol{U}}^1(z) = [\widetilde{\boldsymbol{U}}^1_1(z), \widetilde{\boldsymbol{U}}^1_2(z), \cdots, \widetilde{\boldsymbol{U}}^1_{N_p}(z)] = \begin{bmatrix} U^1_{1,1}(z) & U^1_{1,2}(z) & \cdots & U^1_{1,N_p}(z) \\ U^1_{2,1}(z) & U^1_{2,2}(z) & \cdots & U^1_{2,N_p}(z) \\ \vdots & \vdots & & \vdots \\ U^1_{N,1}(z) & U^1_{N,2}(z) & \cdots & U^1_{N,N_p}(z) \end{bmatrix}$$

$$\widetilde{\boldsymbol{E}}^1(z) = [\widetilde{\boldsymbol{E}}^1_1(z), \widetilde{\boldsymbol{E}}^1_2(z), \cdots, \widetilde{\boldsymbol{E}}^1_{N_p}(z)] = \begin{bmatrix} E^1_{1,1}(z) & E^1_{1,2}(z) & \cdots & E^1_{1,N_p}(z) \\ E^1_{2,1}(z) & E^1_{2,2}(z) & \cdots & E^1_{2,N_p}(z) \\ \vdots & \vdots & & \vdots \\ E^1_{M,1}(z) & E^1_{M,2}(z) & \cdots & E^1_{M,N_p}(z) \end{bmatrix}.$$

and $U^k_{i,l}(z)$ and $E^k_{i,l}(z)$ mean the Z tansform of an input to i-th reference and error microphones for $\boldsymbol{H}^k(z)$ at l-th iteration, respectively. This relation is substituted into (8), then

$$\boldsymbol{S}^1(z) = \widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1} = \boldsymbol{C}(z)\boldsymbol{A}(z) - \boldsymbol{C}(z)\boldsymbol{H}^1(z) \quad (23)$$

is held, where $N = N_p$ and the existence of $\widetilde{\boldsymbol{U}}^1(z)^{-1}$ are assumed. Next, $\boldsymbol{H}^2(z) = [H_{ij}^2(z)]$ is substituted into $\boldsymbol{H}(z) = [H_{ij}(z)]$ and let $\boldsymbol{S}(z) = [S_{ij}(z)]$ be $\boldsymbol{S}^2(z) = [\boldsymbol{S}_{ij}^2(z)]$ in this time. Similarly,

$$\begin{aligned}\boldsymbol{S}^2(z) &= \widetilde{\boldsymbol{E}}^2(z)\widetilde{\boldsymbol{U}}^2(z)^{-1} \\ &= \boldsymbol{C}(z)\boldsymbol{A}(z) - \boldsymbol{C}(z)\boldsymbol{H}^2(z) \end{aligned} \tag{24}$$

is held, where

$$\widetilde{\boldsymbol{U}}^2(z) = [\widetilde{\boldsymbol{U}}_1^2(z), \widetilde{\boldsymbol{U}}_2^2(z), \cdots, \widetilde{\boldsymbol{U}}_{N_s}^2(z)]$$
$$\widetilde{\boldsymbol{E}}^2(z) = [\widetilde{\boldsymbol{E}}_1^2(z), \widetilde{\boldsymbol{E}}_2^2(z), \cdots, \widetilde{\boldsymbol{E}}_{N_s}^2(z)].$$

From (23) and (24),

$$\widetilde{\boldsymbol{F}}(z) = \boldsymbol{C}(z)\widetilde{\boldsymbol{H}}(z) \tag{25}$$

is obtained, where

$$\widetilde{\boldsymbol{F}}(z) = \widetilde{\boldsymbol{E}}^2(z)\widetilde{\boldsymbol{U}}^2(z)^{-1} - \widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1}$$

and $\widetilde{\boldsymbol{H}}(z) = \boldsymbol{H}^1(z) - \boldsymbol{H}^2(z)$.

If $(\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)^{-1}$ exists, from (25),

$$\boldsymbol{C}(z) = \widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T(\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)^{-1} \tag{26}$$

is held.

Equation (23) can be rewritten as

$$\boldsymbol{C}(z)\boldsymbol{A}(z) = \widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1} + \boldsymbol{C}(z)\boldsymbol{H}^1(z) \tag{27}$$

and $\boldsymbol{A}(z)$ can be obtained in the following:

$$\boldsymbol{A}(z) = \boldsymbol{C}^{-1}(z)\widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1} + \boldsymbol{H}^1(z). \tag{28}$$

Substituting (26) into (28), then

$$\begin{aligned}&(det\widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T)\boldsymbol{A}(z) \\ &\quad= (\widetilde{\boldsymbol{H}}(z)\widetilde{\boldsymbol{H}}(z)^T)(adj\widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T)\widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1} \\ &\qquad+ (det\widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T)\boldsymbol{H}_1(z)\end{aligned} \tag{29}$$

is given, where $det\ \widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T$ and $adj\ \widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T$ mean the determinant and adjoint matrices of $\widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T$, respectively. By system identification, $\boldsymbol{A}(z)$ can be also obtained by a similar method of the single channel case.

If $\widetilde{\boldsymbol{H}}(z)$ is a square matrix, (26) can be simplified as

$$\begin{aligned}\boldsymbol{C}(z) &= \widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^T(\widetilde{\boldsymbol{H}}(z)^T)^{-1}\widetilde{\boldsymbol{H}}(z)^{-1} \\ &= \widetilde{\boldsymbol{F}}(z)\widetilde{\boldsymbol{H}}(z)^{-1}. \end{aligned} \tag{30}$$

In this case, (29) can be rewritten as

$$\begin{aligned}&(det\widetilde{\boldsymbol{F}}(z))\boldsymbol{A}(z) \\ &\quad= \widetilde{\boldsymbol{H}}(z)(adj\widetilde{\boldsymbol{F}}(z))\widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1} \\ &\qquad+ (det\widetilde{\boldsymbol{F}}(z))\boldsymbol{H}^1(z).\end{aligned} \tag{31}$$

In the above discussion, the existence of the several inverse matrices is assumed. These matrices may become singular, because of ill conditions of inputs or the location of microphones. Theoretical discussion for singularity of these matrices is difficult. However, the system expressed by (29) can not converge in such a case. Therefore this can be avoided by no exchange of the coefficients of the noise control filters until the stable solution is given, or rearrangement of the microphones.

4. Simulation

4.1 Single Channel Case

In this simulation, the following conditions were applied:

1. The primary noise and disturbance to the error microphone were white Gaussian and the power ratio of them was 40 dB.
2. The number of taps of $A(z)$ and $C(z)$ were 128, respectively.
3. The number of stored samples of $U(z)$ and $E(z)$ were $N_s = 2048$ or 1024.
4. The coefficients of $A(z)$ and $C(z)$ were given by white noise with mean 0 and variance 1.
5. The NLMS algorithm with the step-size parameter $\mu = 1.0$ was used for the estimation of $\widehat{A}(z)$ in the proposed algorithm.
6. The number of updating for the coefficients of $S(z)$ and $\widehat{A}(z)$ were $M_1 = 2048$ or 1024, and $M_2 = 16,384$ times, respectively.

To evaluate the noise reduction performance, the mean of 10 times trials for the following estimation error of noise control filters was used:

$$EstimationError\,[\mathrm{dB}] = 10\log_{10}\frac{\|\widehat{\boldsymbol{a}} - \boldsymbol{a}\|^2}{\|\boldsymbol{a}\|^2} \tag{32}$$

where $\boldsymbol{a}$ and $\widehat{\boldsymbol{a}}$ mean the impulse responses of $A(z)$ and $\widehat{A}(z)$, respectively.

The behaviors of the estimation error of the noise control filter for 12 iterations are shown in Figs. 7 ($N_s = 2048$) and 8 ($N_s = 1024$), respectively. In these figures, each block means N_S samples. About -40 dB estimation error was achieved in each case. In 5th block of Fig. 7, the impulse responses of $A(z)$ and $C(z)$ were changed. The estimation error was increased in 5th and 6th blocks, however the estimation error was decreased after that and almost the same estimation error could be obtained again.

For comparison, the behaviors of the estimation error

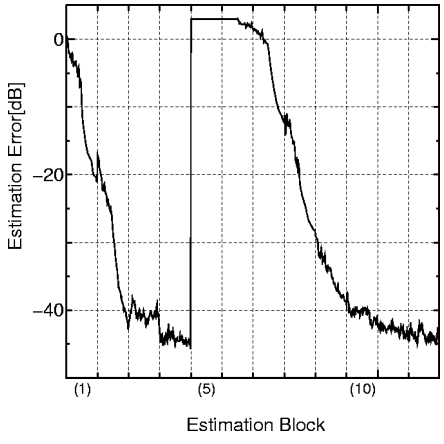


Fig. 7 Estimation error of the proposed algorithm ($N_s = 2048$).

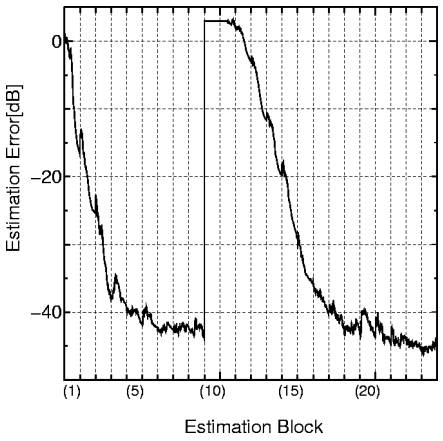


Fig. 8 Estimation error of the proposed algorithm (N_s = 1024).

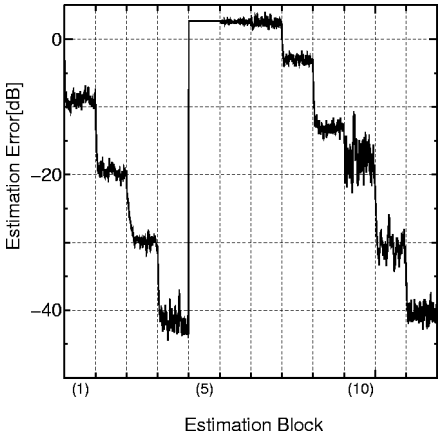


Fig. 9 Estimation error of Ref. [6] (M_1 = 2048).

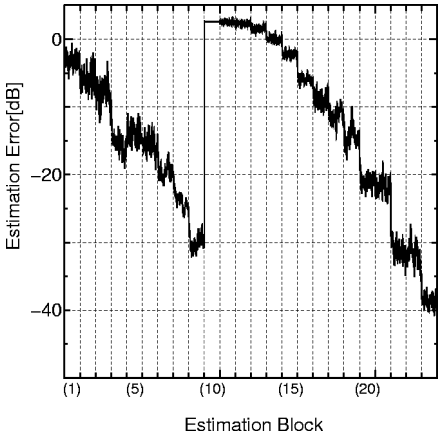


Fig. 10 Estimation error of Ref. [6] (M_1 = 1024).

of the noise control filter by the method proposed in Ref. [6] are shown in Figs. 9(M_1 = 2048) and 10(M_1 = 1024), respectively. In these cases, the number of samples in each block is M_1 for the estimation of $S(z)$. From these results, the proposed algorithm shows higher noise reduction performance by smaller number of samples than the algorithm in Ref. [6].

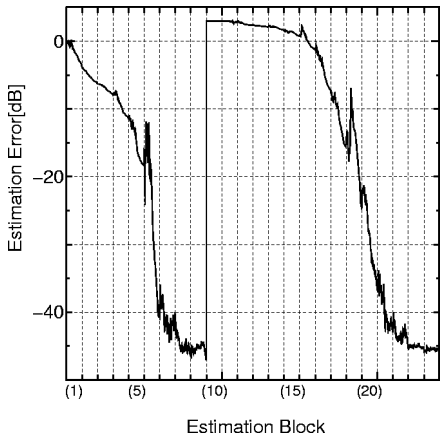


Fig. 11 Estimation error of the proposed algorithm with auto-correlated inputs (N_s = 2048).

Next, we consider the correlated inputs case. In this case, the input signal $u(n)$ was defined by $u(n) = 0.8u(n) + 0.2w(n)$ where $w(n)$ meant the white Gaussian noise whose mean and variance were 0 and 1. The disturbance to error microphone was white Gaussian and its power ratio with the input was 45 dB. Any other conditions were same as that in the previous one. The behavior of the estimation error of the noise control filter is shown in Fig. 11 (N_s = 2048). The number of iteration was 24 times and the impulse responses of $A(z)$ and $C(z)$ were changed in 9th iteration. Compared to the white noise case, it takes longer time to converge, however almost the same estimation error can be obtained.

4.2 Multiple Channel Case

Next, the results for multiple channel system for the 2-2-2 case are shown. The following conditions were applied:

1. The primary noise and disturbance to the error microphones were white Gaussian and the power ratio of them was 40 dB.
2. The number of taps of $A_{ij}(z)$ and $C_{ij}(z)$ were 128, respectively.
3. The number of stored samples for $U^k_{i,l}(z)$ and $E^k_{i,l}(z)$ were $2N_s - 3 - 512$. To avoid the increase of estimation errors, the first and last 256 samples, which have less correlation with each signal, were omitted from total number of samples, $2N_s - 3$ in each term of each microphone.
4. The coefficients of $A(z)$ and $C(z)$ were given by white noise with mean 0 and variance 1.
5. The NLMS algorithm with the step-size parameter μ = 0.4 was used in the proposed algorithm.

For N_s = 1024, the behaviors of the estimation error of the noise control filter are shown in Fig. 12. In 19th block of Fig. 12, the impulse responses of the system were changed. For comparison, the behavior of the estimation error of the noise control filter by the method proposed in Ref. [7] are shown in Fig. 13. In this case, the number of samples to be

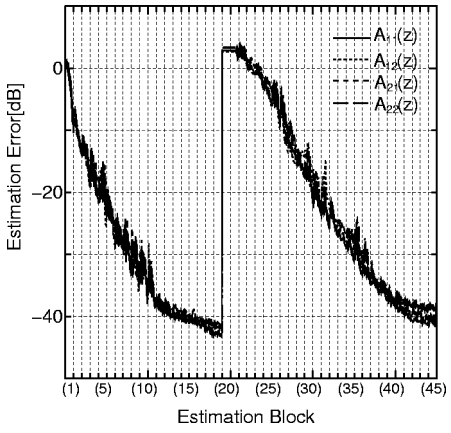


Fig. 12 Estimation error of the proposed algorithm.

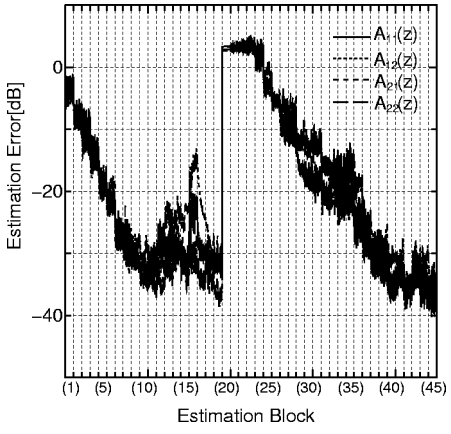


Fig. 13 Estimation error of Ref. [7].

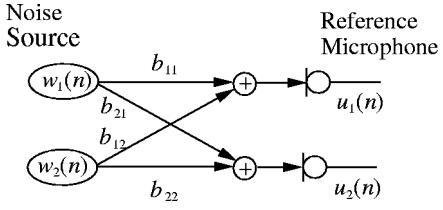


Fig. 14 Input model.

used for the estimation of $S_{ij}(z)$ in the ordinary method was selected as 2048 according to the same reason with the single channel case. In these figures, the number of samples in each block is 2048 samples. From these results, the proposed algorithm shows higher noise reduction performance by smaller number of samples than the algorithm in Ref. [7].

We also simulated for the correlated inputs case. Figure 14 shows an input model with cross-correlation to reference microphones. In this model, $w_1(n)$ and $w_2(n)$ were independent white noise sources and the parameters were $b_{11} = 1.0$, $b_{12} = 0.9$, $b_{21} = 0.7$, $b_{22} = 1.2$.

The behaviors of the estimation error of the noise control filter by the proposed method are shown in Fig. 15. The simulation conditions are same without the input of the reference microphones. Figure 16 shows the behaviors of the

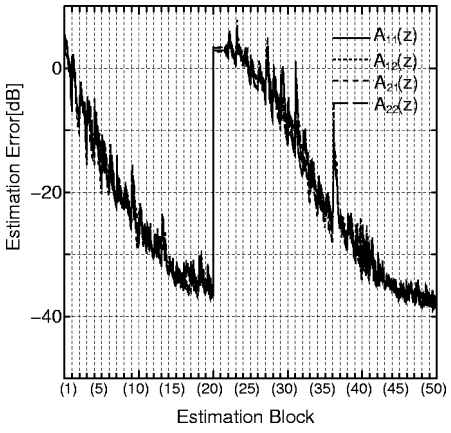


Fig. 15 Estimation error of the proposed algorithm with correlation inputs.

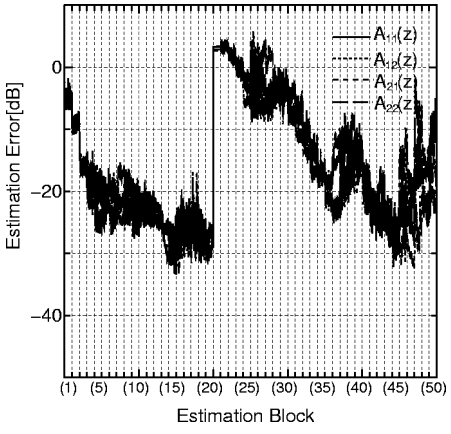


Fig. 16 Estimation error of Ref. [7] with correlation inputs.

estimation error of the noise control filter by the ordinary method. In this case, we set the limit of updating the adaptive filter for the estimation of $S_{ij}(z)$ to 22528 ($= 2048 \times 11$) times. In Figs. 15 and 16, the numbers of samples in each block are 2048 and 22528, respectively.

In the above example, good noise reduction performance can be obtained by the proposed algorithm and the required numbers of samples are less than that in the ordinary method. Especially, for the correlated inputs, the proposed method has better performance than that of the ordinary method.

5. Conclusion

This paper has been proposed an ANC system based on simultaneous equations method without the identification of auxiliary filters. To elimination of auxiliary filters, the error signals and input signals within a given term are stored and used for the identification of the noise control filter. We can achieve same performance of noise reduction with more reduced number of samples and the undesirable assumption in the multiple channel case can be discarded. The simulation result has shown that its noise reduction performance has

been almost same as that of the ordinary algorithm.

References

[1] P.A. Nelson and S.J. Elliott, Active control of sound, Academic Press, London, 1992.
[2] S. Kuo and D.R. Morgan, Active noise control systems, John Wiley & Sons, New York 1996.
[3] B. Widrow and S.D. Stearns, Adaptive signal processing, Prentice-Hall, New Jersey, 1985.
[4] N. Saito, T. Sone, T. Ise, and A. Akiho, "Optimal on-line modeling of primary and secondary paths in active noise control systems," J. Acoust. Soc. Jpn. (E), vol.17, no.6, pp.275–282, Nov. 1996.
[5] Y. Kajikawa and Y. Nomura, "Frequency domain active noise control system without a secondary path model via perturbation method," IEICE Trans. Fundamentals, vol.E-84A, no.12, pp.3090–3098, Dec. 2001.
[6] K. Fujii, M. Muneyasu, and J. Ohga, "Simultaneous equations method not requiring secondary path filter," Proc. Active'99, Fort Lauderdale, pp.941–948, Dec. 1999.
[7] M. Muneyasu, T. Asai, K. Fujii, and T. Hinamoto, "Multiple channel active noise control system based on simultaneous equations methods," Proc. 2002 IEEE ISCAS, vol.2, pp.II-169–II-172, Phoenix, Arizona, 2002.

Appendix: Symbols

A.1 Single Channel Case

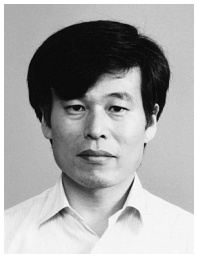
$P(z)$:Transfer function (TF) of a primary path from a reference microphone to an error microphone.
$C(z)$:TF of a secondary path from a canceling loudspeaker to an error microphone.
$S(z)$:TF of identified primary path with a noise cotrol filter.
$H(z)$:TF of the noise control filters.
$H_i(z)$:TF of the noise control filters with different coefficients
$S_i(z)$:TF of the identified primary path with $H_i(z)$.
$U_i(z)$:Z transform of the input signal to the reference microphone in case of $H_i(z)$.
$E_i(z)$:Z transform of the input signal to the error microphone in case of $H_i(z)$.
$H_{opt}(z)$:TF of the optimal noise control filter.
$\hat{A}(z)$:TF of the estimated optimal noise control filter.

A.2 Multiple Channel Case

$A_{ij}(z)$:TF from i-th reference microphone to j-th canceling loudspeaker.
$H_{ij}(z)$:TF of the noise control filters from i-th reference microphone to j-th canceling loudspeaker.
$C_{ij}(z)$:TF of a secondary path from i-th loudspeaker to j-th error microphone.
$S_{ij}(z)$:TF of identified primary path from i-th reference microphone to j-th canceling loudspeaker.
$\boldsymbol{A}(z) = [A_{ij}(z)]$, $\boldsymbol{H}(z) = [H_{ij}(z)]$, $\boldsymbol{C}(z) = [C_{ij}(z)]$, $\boldsymbol{S}(z) = [S_{ij}(z)]$
$U_i(z)$:Z transform of input signal to i-th reference microphone
$E_i(z)$:Z transform of input signal to i-th error microphone
$\boldsymbol{U}(z) = [U_1(z), U_2(z), \cdots, U_N(z)]^T$
$\boldsymbol{E}(z) = [E_1(z), E_2(z), \cdots, E_M(z)]^T$
$H_{ij}^k(z)$:TF of the noise control filters from i-th reference microphone to j-th canceling loudspeaker with different coefficients.
$\boldsymbol{H}^k(z) = [H_{ij}^k(z)]$
$S_{ij}^k(z)$:TF of identified primary path from i-th reference microphone to j-th canceling loudspeaker with $\boldsymbol{H}^k(z)$.
$\boldsymbol{S}^k(z) = [S_{ij}^k(z)]$
$\widetilde{\boldsymbol{S}}(z) = \boldsymbol{S}^2(z) - \boldsymbol{S}^1(z)$
$\widetilde{\boldsymbol{H}}(z) = \boldsymbol{H}^1(z) - \boldsymbol{H}^2(z)$
$U_{i,l}^k(z)$:Z transform of input signal to i-th reference microphone for $\boldsymbol{H}^k(z)$ at l-th iteration.
$E_{i,l}^k(z)$:Z transform of input signal to i-th error microphone in case of $\boldsymbol{H}^k(z)$ at l-th iteration
$\widetilde{\boldsymbol{U}}_l^k(z) = [U_{1,l}^k(z), U_{2,l}^k(z), \cdots, U_{N,l}^k(z)]^T$
$\widetilde{\boldsymbol{E}}_l^k(z) = [E_{1,l}^k(z), E_{2,l}^k(z), \cdots, E_{M,l}^k(z)]^T$
$\boldsymbol{U}^k(z) = [U_{i,l}^k(z)]$, $\boldsymbol{E}^k(z) = [E_{i,l}^k(z)]$
$\widetilde{\boldsymbol{F}}(z) = \widetilde{\boldsymbol{E}}^2(z)\widetilde{\boldsymbol{U}}^2(z)^{-1} - \widetilde{\boldsymbol{E}}^1(z)\widetilde{\boldsymbol{U}}^1(z)^{-1}$

Mitsuji Muneyasu received B.E. and M.E. degrees in system engineering from Kobe University, in 1982 and 1984, respectively and Dr. E. degree from Hiroshima University, Japan in 1993. In 1984, he joined Oki Electric Industry Co., Ltd., in Tokyo, Japan. From 1990 to 1991, he was a Research Assistant at the Faculty of Engineering, Tottori University, Tottori, Japan. From 1991 to 2001, he was a Research Assistant and an Associate Professor at the Faculty of Engineering, Hiroshima University, Higashi-Hiroshima, Japan. Since 2001 he joined the Faculty of Engineering, Kansai University, Osaka, Japan, where he is currently a Professor. His research interests include image processing theory and nonlinear digital signal processing. He is a member of IEEE and IPSJ.

Osamu Hisayasu received B.E. and M.E. degrees in system engineering from Hiroshima University in 2000 and 2002, respectively. In 2002, He joined Mitsubishi Engineering Co. Ltd. in Hiroshima, Japan. His research interest includes digital signal processing.



Kensaku Fujii received B.E., M.E. and Dr. E. degrees in electrical engineering from Osaka Prefecture University in 1970, 1972 and 1992, respectively. He joined FUJITSU LTD., Kawasaki, Japan, in 1972 where was engaged in the design of electric telephone exchanger. From 1989 to 2001, he researched on the application of digital signal processing to echo canceller and active noise control systems. Since 2001, he has been with Himeji Institute of Technology, Himeji, Japan, where he is currently a professor of the Graduate School of Electrical Engineering. His research interests are in the area of digital signal processing and its application. He received the 1993 Best Paper Award from the IEICE. From 1994 to 1998, he was an Associate Editor of Trans. on Fundamentals of IEICE. He is a member of IEEE and ASJ.

Takao Hinamoto received the B.E. degree in electrical engineering from Okayama University, Okayama, Japan, in 1969, the M.E. degree in electrical engineering from Kobe University, Kobe, Japan, in 1971, and the Dr. Eng. degree in electrical engineering from Osaka University, Osaka, Japan, in 1977. From 1972 to 1988, he was with the Faculty of Engineering, Kobe University, Kobe, Japan. From September 1979 to March 1981, he was on leave from Kobe University as a Visiting Member of Staff in the Department of Electrical Engineering, Queen's University, Kingston, ONT, Canada. During 1988–1991, he was Professor of Electronic Circuits in the Faculty of Engineering, Tottori University, Tottori, Japan. Since January 1992, he has been Professor of Electronic Control in the Department of Electrical Engineering, Hiroshima University, Higashi-Hiroshima, Japan. His teaching and research interests include digital signal processing, system theory, and control systems engineering. He has coedited and coauthored the book "Two-Dimensional Signal and Image Processing (Tokyo, Japan: SICE, 1996). Prof. Hinamoto was an Associate Editor of the IEEE Transactions on Circuits and System II during 1993-1995 and served as the Chair of 12th Digital Signal Processing (DSP) Symposium held in Hiroshima in November 1997, which is the 12th of a series of annual symposia sponsored by the DSP Technical Committee of the IEICE. He also served as the Guest Editor of the special section on DSP in the August 1998 issue of the IEICE Transactions on Fundamentals.