Empirical Investigations of Beef Gain Roughage-Concentrate Substitution

Francis Epplin, Shashanka Bhide, and Earl O. Heady

The shape of the beef gain roughage-concentrate isoquant is examined. Data from two beef-feeding experiments designed to investigate response from alternative combinations of roughage and concentrate are utilized. Isoquants derived from estimated production functions are compared with directly estimated isoquants. The estimates indicate that the isoquant is not strictly convex to the origin. The nonconvex region is more pronounced in estimates obtained from data of steers fed diethylstilbestrol.

Key words: beef feeding, concentrate, instrumental variables, isoquant, roughage.

Several studies published during the last decade have suggested that the beef gain roughage-concentrate isoquant may not be strictly convex to the origin (Brokken et al.; Brokken 1977a, b; Byers, Matsushima, Johnson; Goodrich et al.; Perry and Beeson). In general, the studies suggest that a nonconvex region may exist over a range of rations extending from where the energy concentration is so low that gut fill restricts intake, to where roughage intake is so low (high energy concentration) that digestive problems result. In addition, Brokken's work suggests that successive isoquants tip away from the roughage axis such that the ability of roughage to compete economically with concentrate diminishes as animal weight increases. This tipping phenomenon, along with a nonconvex region in the isoquant, would explain the popularity of feeding high roughage rations to light animals, followed by a high concentrate ration at heavier weights.

Soilage Experiment

A study of soilage (fresh-chopped alfalfa-bromegrass) and corn grain substitution reported by Heady et al. shows isoquants which are convex to the origin. The isoquants were derived from a statistically estimated production function. Given the knowledge of the time, the convex isoquants were deemed acceptable. However, based on more recent findings, we might expect a concave region in the soilage-corn grain isoquant.

Hereford yearling steers weighing approximately 840 pounds were assigned randomly to one of two farms and one of six rations. Rations at one location included diethylstilbestrol (DES) (with DES), while those at the other location did not (without DES). The dry matter (DM) ratios of soilage to corn grain were approximately as follows: 100:0, 83:17, 70:30, 54:46, 41:59, and 32:68. The estimated net energy for gain (NE_g) in terms of megacalories per kilogram (Mcal./kg.) for the six rations was: 0.71, 0.84, 0.94, 1.06, 1.16, and 1.23 (National Academy of Sciences).

Based on Brokken's work, the energy concentration of the full soilage ration was so low that gut fill would restrict energy intake to something less than the physiological demand for energy. Thus, this ration would fall in the convex range of Brokken's proposed sigmoid isoquant (Brokken 1977a). However, the energy concentration of the other five rations was sufficiently high that the physiological demand for energy could be met prior to encountering the gut fill restriction. They would fall in the concave range of the proposed sigmoid isoquant. If the isoquant is sigmoid, a quadratic gain-production function estimate based on the six rations combined could give misleading results. The convex region might

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Journal Paper No. 9442 of the Iowa Agriculture and Home Economics Experiment Station, Project No. 2102.

The assistance of M. P. Hoffman and Allen Trenkle is gratefully acknowledged. The authors also wish to acknowledge the assistance of Wayne Fuller and Ray Brokken who suggested the use of polar coordinates.

distort a slightly concave region and result in an overall convex isoquant as Brokken et al. suggest. Thus, the data were reanalyzed.

Data indicated that the rates of gain were slower on the full soilage ration such that half of the pens receiving that ration did not gain 200 pounds during the pasture season. Therefore, the full soilage ration (100:0) was dropped from the data set, and estimates were obtained from the remaining five rations. The estimated NE_g for these five rations ranged from 0.84 to 1.23 Mcal./kg. Based on the work of Brokken et al., these five rations would be expected to fall in the concave region of the isoquant.

Corn Silage Experiment

The corn silage feeding trial was conducted over a three-year period. Yearling crossbred steers, primarily of Angus and Hereford breeding, were placed on feed in the fall and slaughtered in the following spring. A total of 278 cattle were fed: 96 in the first and third years and 86 in the second.

Six rations were fed. Rations were composed of whole plant corn silage, whole shelled corn grain, dehydrated alfalfa pellets, and a soybean meal base supplement fortified with vitamins and minerals. Corn grain and alfalfa pellets were fed in a constant ratio of two parts corn grain to one part alfalfa pellets on an as-fed basis. This fixed proportion mixture is referred to as concentrate. The DM ratios of corn silage to concentrate, ignoring the supplement, were approximately: 100:0, 82:18, 63:37, 44:56, 23:77, and 0:100. The 100:0 ration was composed entirely of corn silage and supplement. Corn grain, alfalfa pellets, and supplement made up the 0:100 ration. All of the animals were implanted with 30 milligrams of DES immediately prior to the feeding trial. The supplement was fed at a constant rate of about 0.8 kilograms per animal per day.

In each year, twelve pens of cattle were fed. Six pens were fed a constant quantity of metabolizable energy per day. Hence, their intake was restricted. The other six pens were fed *ad libitum*.

The energy concentration, as measured by NE_g, ranged from 0.99 Mcal./kg. for the full corn silage ration, to 1.22 Mcal./kg. for the full concentrate ration (National Academy of Sciences). Based solely on the criterion of energy concentration, these rations would be expected to fall in the concave region of the proposed sigmoid isoquant. However, the National Research Council suggests that a minimal amount of roughage generally is necessary to prevent rumen dysfunction. Morrison also recognized the need for some roughage in the diet to prevent animals from being thrown off feed. Thus, the full concentrate ration might fall outside the proposed concave region.

Estimation Procedures

Traditionally, gain isoquants have been derived from production functions estimated by regressing cumulative weight gain on cumulative feed intake. For example, see Heady and Dillon. Several statistical problems are generally encountered. One potential problem is autocorrelated residuals.

Autocorrelation

Because successive measurements are made on the same experimental units over time, ordinary least squares (OLS) residuals associated with these measurements may be correlated. Several methods have been devised to correct for autocorrelation. If the number of experimental units per trial is relatively large, the full covariance matrix of the disturbance terms can be estimated. For example, see Dahm, Heady, and Sonka. Unfortunately, the number of experimental units, pens, per trial was limited to eight in the soilage study and six in the corn silage study.

A more common method of correcting for autocorrelation is to make an assumption about the autoregressive nature of the residual terms. Animals with low (high) rates of gain in one time period generally compensate with high (low) rates of gain in the next period. The effects of these compensatory gains generally are expected to dissipate in ten to fourteen days. Because the measurement periods for the experiments analyzed ranged from twenty-one to twenty-eight days, a first-order autoregressive scheme seems to be a reasonable assumption (Heady et al.). Transformation matrices were estimated for each trial and the data were transformed for production function estimation (Johnston, p. 260).

Stochastic Regressors

An additional statistical problem may be encountered in production function estimation.

When animals are fed *ad libitum,* they, and not the research workers, control the rate of feed intake. Thus, the recorded feed intake quantities may contain a stochastic component which may not be independent of the equation error. Fuller (1977) has developed a statistical test to determine if OLS estimates will be nearly unbiased. The F-test was applied to the data of each trial. The null hypothesis that OLS estimators would be nearly unbiased was not rejected at the 0.05 level of probability for any of the data sets. Thus, we could proceed under the assumption that OLS is appropriate.

Alternatively, if the results of the F-test were not satisfactory, and if variables which can be used as instruments are available, the instrumental variables (IV) procedure can be used to estimate the production function (Fuller 1976).

The IV procedure can be used to obtain consistent estimates of the regression coefficients for both situations involving stochastic regressors as well as situations involving errors in variables. However, the stochastic nature of feed consumption by the animals is probably of greater importance for the present analysis than any "measurement error" associated with the regressors. Hence, the gain production function model is essentially a regression model with stochastic independent variables which may be correlated with the equation error.

Direct Estimation of Isoquants

Sonka, Heady, and Dahm used the IV procedure to estimate directly swine gain isoquants. They used ration protein percentage as the instrument for estimating the quantity of supplement consumed. The isoquant was estimated by regressing recorded values of corn consumption on predicted values of supplement consumption. Burt suggested that the isoquant estimate was not needed because the isoquant could be determined from the initial estimate. This would hold if a theoretical relationship exists between the random independent variables and the instruments and if the relationship is properly specified by the functional form. This requirement is not necessary for the IV procedure.

Our objective is to estimate directly beef gain roughage-concentrate isoquants. The equation to be estimated is

$$(1) \qquad C = f(S,\Psi) + U,$$

where C and S are quantities of the two feeds, f is the functional form, Ψ a set of parameters, and U the residual error term. Neither C nor S satisfy the classical regression model assumptions with respect to independent variables. Thus, OLS estimates may be seriously biased as a result of the correlation between U and S. The IV procedure defined by Fuller (1976, p. 220) can be used to obtain consistent estimates if information is available on fixed variables which are correlated with the "independent variables" and not correlated with the random error term of the model. For both experiments, treatments were fed along fixed ration lines. In other words, the percentage of roughage (and concentrate) fed to each pen was fixed experimentally. This variable, call it N, can be used to build instrumental variables to estimate equation (1). In addition, the vector of "ones," along with the dummy variables, call them M, can also be used as instruments.

Powers of S can be used in equation (1) to permit a nonlinear isoquant. Thus, we could estimate

$$(2) \qquad S^*, S^{2*} = G(N,M),$$

and use the predicted values of S^* and S^{2*} in equation (1). However, we arbitrarily selected the following alternative procedure. First we estimate

$$(3) \qquad \hat{S} = G(N,M),$$

and use the predicted values and powers of predicted values from this estimate, call them Z, in "second round" estimates.

$$(4) \qquad S^*, S^{2*} = h(Z,M).$$

Predicted values from these estimates are used to estimate directly the isoquant, equation (1).

We note that the residuals from equations (2), (3), and (4) "may be a sum of random and fixed components" (Fuller 1976, p. 221), in which case it would be inappropriate to solve them for the isoquant. The appropriate residuals for the isoquant estimate, equation (1), should be based on the recorded values of the "independent" variable. Thus, they were directly computed and used to calculate the estimated variance and t-values.

Soilage (with DES) Estimates

The soilage data were interpolated to a weight gain level of 200 pounds. The IV procedure was used to provide estimates of soilage consumption that are independent of the estimates of the quantity of corn consumed such that isoquants could be estimated directly. Linear, quadratic, and cubic functional forms were used to fit the isoquants. The quadratic estimate resulted in the lowest mean square error (MSE). It is reported in table 1. The negative coefficient on the square term for equation (1.1) is statistically significant at the 0.01 level of probability. The isoquant is graphed in figure 1. It is concave to the origin.

Gain production functions also were estimated from the five rations of the soilage experiment which were expected to fall within the concave range of the overall isoquant. Quadratic and square root functional forms were used. The square root estimate resulted in the smallest MSE. It is reported in table 2.[1] The 200-pound isoquant derived from equation (2.1) is graphed in figure 1. It is concave to the origin.

Because the square root term on S in equation (2.1) is of questionable significance, it was dropped and equation (2.2) was estimated. The 200-pound isoquant derived from equation (2.2) is graphed in figure 2. It is also concave to the origin.

As noted previously, if we do not "trust" the results of the F-test regarding OLS bias, and if instruments are available, the full production function can be estimated with the IV procedure. Thus, because isoquants derived from the production functions, equation (2.1) and (2.2), are concave, and because this is the

[1] In the original analysis, a quadratic function was fit. It did not contain an intercept term or dummy variables for the three trials. However, a temperature variable was included (Heady et al.). Unfortunately, the temperature data were not located.

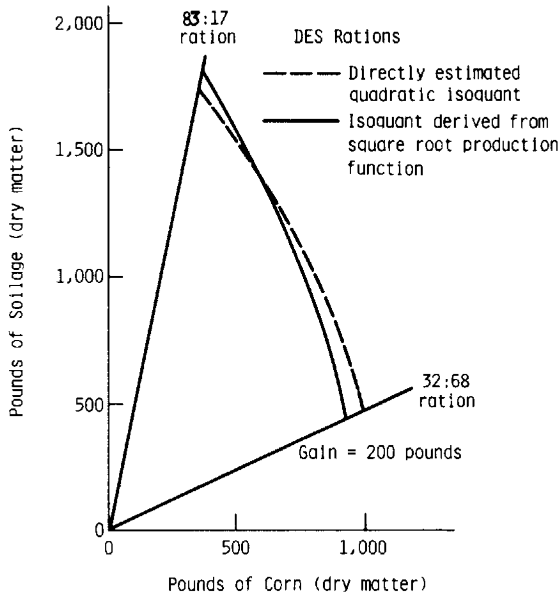

Figure 1. Soilage with DES isoquant—direct estimate versus production function estimate

crucial issue, we utilized the IV procedure to estimate the full production function for the soilage with DES data set.

The procedure is similar to that used in the direct estimation of isoquants. Following Fuller (1976, p. 224), we used time on feed, proportion of roughage in the ration, and the interaction of time with ration as instruments on which to regress C and S. Predicted values from these regressions along with their powers were used in second round regressions to prepare "independent variables" for the production function estimate. The resulting estimate is reported as equation (2.3) in table 2.

The 200-pound isoquant derived from equa-

Table 1. Soilage-Corn Grain Directly Estimated Isoquants with Corn Grain as the Dependent Variable (840 to 1,040 Pounds)

	Explanatory Variables							
Equation	Intercept	S^{*a}	S^{2*}	$D1$	$D2$	R^2	MSE	df
With DES								
1.1	1,013.3	0.0346	−0.00025	334.1	−249.3	.95	4,277.3	14
	(12.14)[b]	(0.24)	(4.07)	(8.47)	(7.08)			
Without DES								
1.2	1,070.3	−0.2687	−0.000019	373.5	−109.6	.74	18,020.8	15
	(11.10)	(2.73)	(0.71)	(4.36)	(1.53)			
1.3	1,109.7	−0.3286		368.1	−109.9	.75	16,082.8	16
	(14.94)	(6.94)		(4.57)	(1.62)			

[a] The IV procedure was used to obtain estimates of S and S^2 which are independent of corn grain consumption (S is pounds dry matter of soilage; $D1$ and $D2$ are dummy variables for years and equal 1 for years 1 and 2, respectively, and zero otherwise).
[b] Numbers in parentheses are t-values (absolute value).

Table 2. Soilage-Corn Grain Production Function Estimates with Weight Gain from 840 Pounds as the Dependent Variable

	Explanatory Variables										
Equation	Intercept	C^a	S	$C^{½}$	$S^{½}$	$(CS)^{½}$	$D1$	$D2$	R^2	MSE	df
With DES											
2.1	12.6	0.1562	0.0779	1.6882	0.7390	−0.0938	−31.8	36.2	0.95	225.07	113
	(0.83)[b]	(6.69)	(3.11)	(1.52)	(0.59)	(2.40)	(5.41)	(6.48)			
2.2	18.8	0.1535	0.0900	2.0102		−0.0968	−31.2	36.7	0.95	224.43	114
	(1.71)	(6.65)	(7.00)	(2.09)		(2.54)	(5.36)	(6.62)			
IV Estimate											
2.3[c]	17.2	0.1368	0.0789	1.6412		−0.0558	−30.4	37.9	0.95	229.64	114
	(1.20)	(5.71)	(5.21)	(1.25)		(1.20)	(5.08)	(6.72)			
Without DES											
2.4	44.3	0.0888	0.0357	1.7560	0.2980	−0.0050	−39.6	12.3	0.96	190.06	113
	(3.54)	(5.34)	(2.61)	(2.14)	(0.38)	(0.21)	(9.50)	(3.81)			

[a] C and S refer to quantities (dry matter) of corn and soilage consumption; $D1$ and $D2$ are dummy variables for years and equal 1 for years 1 and 2, respectively, and zero otherwise.
[b] Numbers in parentheses are t-values (absolute value).
[c] The IV procedure was used to estimate this production function.

tion (2.3) is graphed in figure 2. It is concave to the origin over the data range. Thus, the directly estimated isoquant, equation (1.1), the "traditional" production function estimates, equations (2.1) and (2.2), and the IV production function estimate, equation (2.3), all yield isoquants that are slightly concave to the origin over the data range. We feel that we can conclude that the beef gain soilage-corn grain isoquant derived from the soilage with

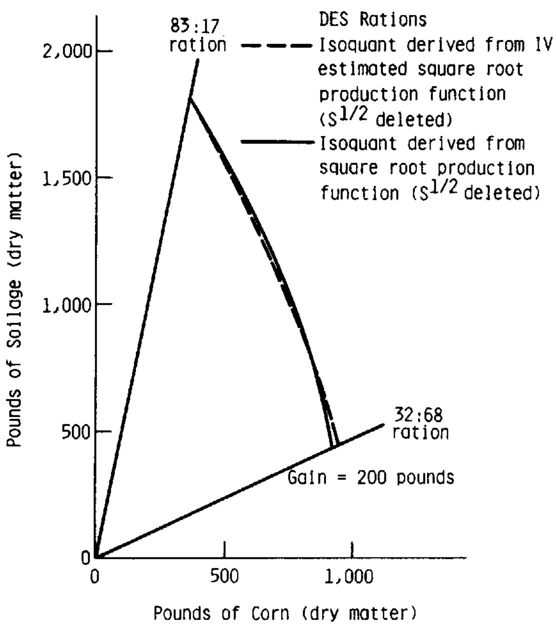

Figure 2. Soilage with DES isoquant—production function estimate versus instrumental variables-production function estimate

DES experiment has a nonconvex range. Further, the full soilage ration, which very likely restricted energy intake, masked the nonconvex range in the previous analysis of the data.

Soilage (without DES) Estimates

Data from the soilage without DES experiment also were reanalyzed. The directly estimated 200-pound gain quadratic isoquant is reported in table 1. The graph of equation (1.2) is slightly concave to the origin. But, the coefficient of S^2 is of questionable significance. Thus, the square term was dropped. Equation (1.3), the linear fit, resulted in a lower MSE than the quadratic fit. It is graphed in figure 3.

The production function for the soilage without DES experiment is reported in table 2. All estimated coefficients for equation (2.4) are statistically significantly different from zero at the 0.05 level of probability except for the square root of S and the interaction term. The 200-pound isoquant derived from equation (2.4) is graphed in figure 3. It is very slightly convex to the origin.

The isoquant estimates obtained from the data of the pens which did not receive DES are more nearly linear than those estimated from the DES data. A linear isoquant has the same ration and cost implications as a concave isoquant. Isoquants derived from the nonDES data lie above those derived from the DES data for all rations. This finding is consistent with the popularity of using the hormone addi-

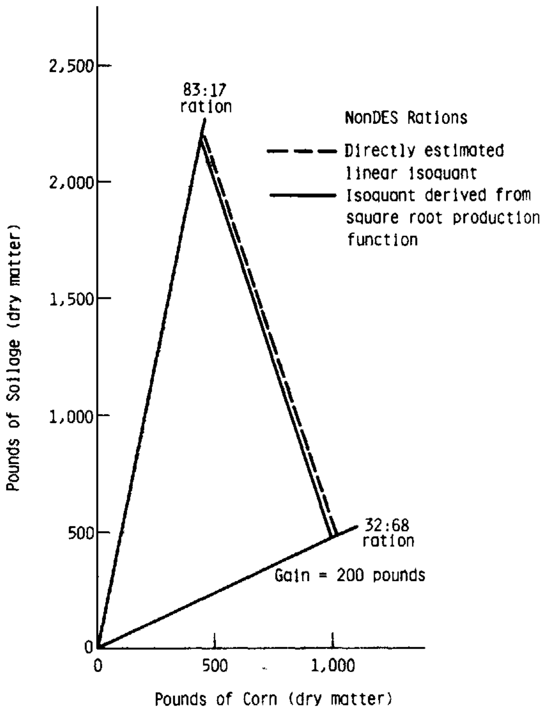

Figure 3. Soilage without DES isoquant—direct estimate versus production function estimate

tive. Also, the two isoquants are more divergent for the higher roughage rations. Perhaps, the impact of DES varies across rations. Because the data were collected on two different farms, soilage quality and weather could not be fixed across farms. Thus, the difference in the estimates may be due to factors other than the DES.

The soilage experiment was limited in time to the length of the growing season. Hence, not all of the steers, especially those receiving the full soilage ration, had equal opportunity to gain equivalent amounts. The steers of the corn silage experiment were permitted to gain equivalent weights, and, consequently, did not face this problem. Because one of the rations of the corn silage experiment did not contain any roughage, we might expect a sigmoid isoquant with a convex tail on the high concentrate end.

Functional Forms

Conventional functional forms for production function estimation, such as Cobb-Douglas, quadratic, and square root, do not permit sigmoid isoquants. The grafted polynomial approach developed by Fuller (1969) and applied to the beef gain problem by Melton et al. can be used to permit sigmoid isoquants. However, the grafting technique requires some method for estimating a join line. Production functions estimated in polar coordinates provide a more general approach in that functional forms may be chosen which place fewer restrictions on derived quantities. They will permit sigmoid isoquants as well as tipping. Thus, the corn silage production functions were estimated in polar coordinates.

Any point in a rectangular coordinate system can be described as being some distance, d, from the origin. Also, a line from the point to the origin forms an angle, θ, with the horizontal axis. The points of a polar coordinate system are described in this manner. Because the corn silage experiment can be described with only two variable inputs, the data can be transformed conveniently into polar coordinates in the following manner. Let

$$(5) \qquad W = f(d,\theta),$$

where W is kilograms of weight gain per steer for the period; d is $(A^2 + B^2)^{\frac{1}{2}}$; θ is $tan^{-1}(A/B)$; A is kilograms DM of concentrate; and B is kilograms DM of corn silage.

Starting Weight Adjustments

The starting weights of the steers in the corn silage study differed across trials. Thus, adjustments were necessary to account for trial-to-trial differences. Dummy variables were used to accommodate different starting weights for the directly estimated isoquants. The actual starting weights were 320 kg., 370 kg., and 395 kg. for trials 1, 2, and 3, respectively. If we let the 160 kg. gain isoquant for trial 1 be

$$(6) \qquad A = a_0 + a_1B + a_2B^2;$$

and the 110 kg. gain isoquant for trial 2 be

$$(7) \qquad A = b_0 + b_1B + b_2B^2,$$

then, by restricting a_1 to equal b_1 and a_2 to equal b_2, trial 2 can be represented as

$$(A - A_0) = b_0 + a_1(B - B_0) + a_2(B - B_0)^2,$$

where A_0 and B_0 are the unknown quantities of concentrate and corn silage utilized by the steers of trial 1 to achieve the initial 50 kilograms of gain. This equation can be written as

$$(8) \quad A = a_0 + a_1B + a_2B^2 + DO + D1B,$$

where $DO = b_0 - a_0 + A_0 - a_1B_0 + a_2B_0^2$ and $D1 = -2a_2B_0$.

A similar procedure can be followed to adjust for trial 3 differences. The regression equation used to estimate the quadratic gain isoquants for the three trials pooled is

$$(9) \quad \hat{A} = a_0 + a_1B + a_2B^2 + a_3D1 + a_4D1B + a_5D2 + a_6D2B,$$

where A and B are interpolated quantities of concentrate and corn silage required for the specified weight gain, $D1 = 1$ for trial 2, 0 otherwise, and $D2 = 1$ for trial 3, 0 otherwise (Dahm, Heady, Sonka).

For production functions estimated in polar coordinates, dummy "distance" variables can be used to account for differences among trials. Because rations, in terms of proportions of corn silage and concentrate in the feed, were fixed for all trials, the measure of "angles" does not vary among trials. The following example illustrates the type of dummy variables used and the form of a second degree function:

$$(10) \quad W = (a_0 + a_1\theta + a_2\theta^2)d + (a_3 + a_4\theta + a_5\theta^2)d^2 + (a_6D1 + a_7D2)d + U,$$

where $D1 = 1$ for trial 2, 0 otherwise, and $D2 = 1$ for trial 3, 0 otherwise. The variable θ is not used without interaction with d since this would imply a variable intercept term.

Corn Silage (*Ad Libitum* Intake) Estimates

The dummy variable techniques were used to adjust starting weights to 320 kilograms. Cubic, quadratic, and linear functional forms were fit. The cubic functional form reported in table 3 resulted in the lowest MSE. It reflects a gain of 160 kilograms from a starting weight of 320 kilograms.

Perhaps as a result of the limited number of observations, most of the estimated coefficients of equation (3.1) are not statistically significant. It is graphed in figure 4. The isoquant is concave with a convex tail near the concentrate axis.

Production function estimates were obtained in polar coordinates. Polynomial functions of the second, third, and fourth degree in angle θ were estimated. The fourth degree function did not reduce MSE. The second and third degree functions are reported in table 4. All estimated coefficients of equation (4.1), except for those on θd^2 and $\theta^2 d^2$, are significant at the 0.10 level of probability. However, only the coefficients on d, d^2, and the dummies are significant at the 0.10 level for equation (4.2). The 80-kg., 120-kg., and 160-kg. isoquants de-

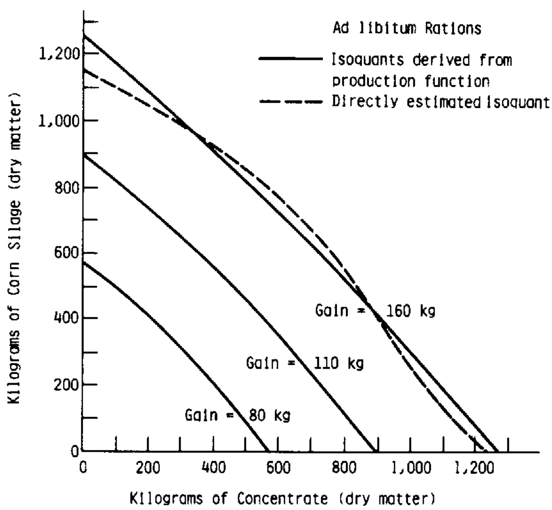

Figure 4. Corn silage *ad libitum* intake isoquants—direct estimate and production function estimates

Table 3. Corn Silage-Concentrate Directly Estimated Isoquants with Concentrate as the Dependent Variable (320 to 480 kg.)

Equation	Intercept	B^{*a}	B^{2*}	B^3	$D1$	$D1B^*$	$D1B^{2*}$	$D2$	$D2B^*$	$D2B^{2*}$	R^2	MSE	df
	Explanatory Variables												
Ad Libitum Intake													
3.1	1,228.7	−1.0736	0.0010	−0.0000009	−482.34	−0.2509	0.0004	−420.00	−0.3128	0.0001	0.99	3,272.0	8
	(18.37)[b]	(1.57)	(0.74)	(1.21)	(5.96)	(0.54)	(0.80)	(4.92)	(0.53)	(0.23)			
Restricted Intake													
3.2	1,226.9	−1.4659	0.0012	−0.0000008	−490.91	0.1307	0.0001	−324.22	−0.5056	0.0006	0.99	3,917.0	8
	(16.99)	(2.26)	(0.92)	(0.93)	(5.34)	(0.25)	(0.14)	(3.55)	(0.95)	(1.20)			

[a] The IV procedure was used to obtain estimates of B, B^2, and B^3 which are independent of concentrate consumption (B is corn silage consumption).

[b] Numbers in parentheses are t-values (absolute value).

Table 4. Corn Silage-Concentrate Production Functions Estimated in Polar Coordinates with Weight Gain from 320 kg. as the Dependent Variable

	Explanatory Variables												
Equation	d	θd	$\theta^2 d$	$\theta^3 d$	d^2	θd^2	$\theta^2 d^2$	$\theta^3 d^2$	$D1d$	$D2d$	R^2	MSE	df
Ad Libitum Intake													
4.1	0.1517	0.1101	−0.0697		−0.000020	−0.000015	0.000009		−0.0090	−0.0257	0.82	20.17	94
	(13.40)[a]	(2.93)	(2.97)		(2.70)	(0.51)	(0.52)		(1.76)	(5.30)			
4.2	0.1580	0.0335	0.0630	−0.0565	−0.000023	0.000011	−0.000035	0.000019	−0.0092	−0.0259	0.83	20.00	92
	(13.01)	(0.40)	(0.47)	(1.01)	(2.88)	(0.17)	(0.34)	(0.45)	(1.80)	(5.35)			
Restricted Intake													
4.3	0.1549	0.1492	−0.0850		−0.000016	−0.000037	0.000015		−0.0154	−0.0387	0.76	26.59	94
	(10.97)	(2.96)	(2.68)		(1.69)	(0.85)	(0.54)		(2.29)	(6.05)			

[a] Numbers in parentheses are t-values (absolute value).

rived from equation (4.1) are graphed in figure 4. They are concave over a wide range of rations. They are not progressively tipped away from the corn silage axis. Thus, for this particular study, in which the range of rations in terms of energy concentration is relatively small, and the range of gain considered is also relatively small, tipping is not evident. Of course, tipping would be easier to detect if both the range of gain and the range of energy concentration were increased.

Corn Silage (Restricted Intake) Estimates

The directly estimated 160-kilogram-restricted intake gain isoquant is reported as equation (3.2) in table 3. It is graphed in figure 5. This

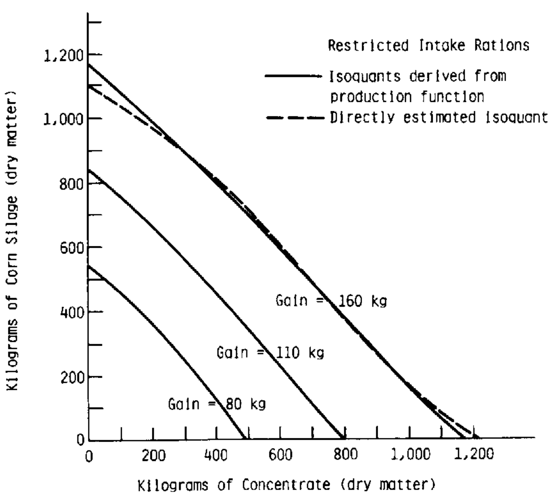

Figure 5. Corn silage restricted intake isoquants—direct estimate and production function estimates

estimated isoquant also has a nonconvex range.

The second degree polynomial production function estimate, equation (4.3), resulted in a lower MSE than the third and fourth degree functional forms. As with equation (4.1), the estimated regression coefficients on θd^2 and $\theta^2 d^2$ are not statistically significantly different from zero at the 0.10 level of probability. However, the other coefficients are significant at this level. Three isoquants derived from equation (4.3) are graphed in figure 5. They are also concave over a wide range of rations.

Equations (3.2) and (4.3) both exhibit sigmoid isoquants. Unlike Brokken's sigmoid isoquants, the corn silage isoquants show a convex range in the high concentrate region. The ration which is devoid of hay or silage is probably not a good ration for physiological reasons.

Even though the restricted intake scenario might be expected to increase the time on feed and, thus, the energy required for maintenance and, thus, the total feed required, the restricted intake isoquants lie below the corresponding *ad libitum* isoquants across all rations. This suggests that if time is ignored, *ad libitum* feeding is inefficient.[2]

Implications and Conclusions

The isoquants and production functions estimated from the data of the two experiments suggest that we cannot reject the hypothesis that the overall beef gain roughage-concentrate isoquant may have a linear or

[2] Because the marginal product of feed is zero beyond stomach capacity, Dillon (p. 99) has suggested that *ad libitum* feeding may not be optimal.

concave region. If time and grade are ignored, rations within a concave region would not be optimal. In general, for *ad libitum* feeding, as the energy concentration of the rations increases, the time required to achieve a given weight level declines. Thus, depending on the relative prices of roughage, concentrate, and time, either a high concentrate ration which contains a minimal amount of roughage to prevent rumen dysfunction, or a ration high in roughage, such as full corn silage, would be optimal.[3]

Roughage is bulky and relatively more difficult to transport, store, and feed than grains. Thus, producers who purchase feedstuffs might be expected to feed high concentrate rations. On the other hand, a corn producer who has silage production and storage facilities may feed a full corn silage ration. A full corn silage ration would yield more pounds of beef per acre. However, corn silage reduces soil cover, removes more nutrients, and increases the potential for erosion losses relative to corn grain.

Additional research is needed in several areas. For example, the NE_g of the soilage study (ignoring the full soilage ration) ranged from 0.84 to 1.23 Mcal./kg. and for the corn silage study from 0.99 to 1.24 Mcal./kg.[4] Experiments extending over a wider range of energy concentration could provide more information on the turning points between concave or linear and convex regions. Because of the nature of the function, six rations may be insufficient to detect accurately turning points. Work also is needed to determine the impact of nonnutrient additives and implants across rations. Finally, the impact of restricting intake should be explored.

We cannot conclude that a concave or linear region exists in the roughage-concentrate isoquant for all types of roughage and concentrates. Also, it is not entirely certain why the derived isoquants have the shape shown. These and other issues remain to be settled.

[*Received March 1979; revision accepted November 1979.*]

[3] Carcass quality grades and yield grades were not statistically significantly different among the corn silage rations. Steers from all pens consistently graded low choice to high good across years and rations. Thus, the output can be assumed to be homogenous across rations for the corn silage experiment.

[4] Many producers feed rations that exceed the energy concentration of those fed in either experiment. For example, an 85% corn grain-15% corn silage ration has an estimated NE_g = 1.41 Mcal./kg.

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