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Approximation theory on the electromagnetic fields in metal cavity

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Abstract: A new method in perturbation theory of electromagnetic fields in metal cavity has been proposed in this paper. When a medium is introduced in metal cavity, the electromagnetic fields and oscillating frequency will be changed. The changed fields can be expanded as the fields of all modes in the original empty metal cavity. The expressions of the fields and the frequency in the various orders of approximation have been derived, that is important for such applications as the precise measurement of high permittivity of microwave materials.

INTRODUCTION

With the rapid development and popularity of the techniques about wireless transmission and Global position system and personal communication system, the microwave ceramics have been widely used in the range of telecommunication and satellite broadcast and military. Hence, the method of measurement and value on microwave ceramics becomes more and more important. The main methods are the dielectric resonator method, the Fabry–Perot open resonator method and the metal cavity perturbation method [1,2,3]. Based on the Maxwell's equations and boundary conditions, the theory basis of metal cavity perturbation has been founded, but in this theory, only the approximation formulas of the sample's permittivity and loss be given out, and the electromagnetic distribution variations from perturbation in cavity have not been considered [4,5,6]. This method cannot be used to measure the high permittivity materials yet.

For the regular ideal conductor cavity, the analytical solutions to electromagnetic fields and resonant frequency have been well known. But for the irregular sample, the analytical solutions cannot be derived, and only be substituted by the approximation and numerical method [7]. In this paper, a new perturbation theory of electromagnetic fields in metal cavity has been presented. When a medium is introduced in metal cavity, the electromagnetic fields and oscillating frequency may be changed. The changed fields can be expanded as the fields of all modes in the original empty metal cavity. If the medium is small enough or its permittivity and permeability are approximately equal to one, it can be taken as perturbation. We have derived the approximate expressions of the fields and the frequency in the various orders of approximation, and that is important for such applications as the precise measurement of high permittivity of microwave materials.

THEORY MODEL AND DISCUSSIONS

To give out Hamiltonian, we write the Helmholtz equation as

$$\frac{1}{\mu\varepsilon}\nabla^2\vec{E} = -\omega^2\vec{E}, \quad \frac{1}{\mu\varepsilon}\nabla^2\vec{H} = -\omega^2\vec{H} \tag{1}$$

Where the Hamiltonian $\hat{H}$ is $\frac{1}{\mu\varepsilon}\nabla^2$ and the eigenvalue is $-\omega^2$

The eigenvalues of Hamiltonian and the electromagnetic fields without perturbation are known, that is the frequencies and the fields of all modes in empty cavity. They satisfy Helmholtz equation

$$\begin{cases} \hat{H}_0\vec{E}_n^0 = -\omega_{0n}^2\vec{E}_n^0 \\ \hat{H}_0\vec{H}_n^0 = -\omega_{0n}^2\vec{H}_n^0 \end{cases} \tag{2}$$

This is the zeroth-order eigenvalue equation of electromagnetic field.
When a perturbation medium is putted into the cavity with relative permittivity ε_1 and permeability μ_1 and volume V_1, the frequency and the electromagnetic field will be changed. The changed field can be expanded by the original fields of all modes

$$\vec{E} = \sum_n a_n\vec{E}_n^0 \quad , \qquad \vec{H} = \sum_n b_n\vec{H}_n^0 \tag{3}$$

$$\hat{H} = \frac{1}{\mu\varepsilon}\nabla^2 = \frac{1}{\mu_0\varepsilon_0}\nabla^2 + (\frac{\mu_0\varepsilon_0}{\mu\varepsilon} - 1)\frac{1}{\mu_0\varepsilon_0}\nabla^2$$

$$= \hat{H}_0 + (\frac{\mu_0\varepsilon_0}{\mu\varepsilon} - 1)\hat{H}_0 = \hat{H}_0 + \hat{H}'$$

$$\hat{H}' = \begin{cases} 0 & V - V_1 \\ (\frac{\mu_0\varepsilon_0}{\mu_1\varepsilon_1} - 1)\hat{H}_0 & V_1 \end{cases} \tag{4}$$

Where V is the volume of cavity. From this formula we can derive the frequency value and the expression of electromagnetic field in the various orders of approximation

$$\begin{cases} \omega_k^2 = \omega_{0k}^2 - H'^{EE}_{kk} - \sum_{k\neq n}\frac{H'^{EE}_{kn}H'^{EE}_{nk}}{\omega_{0k}^2 - \omega_{0n}^2} + \cdots \\ \omega_k^2 = \omega_{0k}^2 - H'^{HH}_{kk} - \sum_{k\neq n}\frac{H'^{HH}_{kn}H'^{HH}_{nk}}{\omega_{0k}^2 - \omega_{0n}^2} + \cdots \end{cases} \tag{5}$$

$$\begin{cases} \vec{E}_k = \vec{E}_k^0 + \sum_{m\neq k} \frac{H'^{EE}_{mk}}{\omega_{0m}^2 - \omega_{0k}^2} \vec{E}_m^0 + \\ \sum_{m\neq k} \left[-\frac{H'^{EE}_{mk} H'^{EE}_{kk}}{(\omega_{0m}^2 - \omega_{0k}^2)^2} + \sum_{n\neq k} \frac{H'^{EE}_{mn} H'^{EE}_{nk}}{(\omega_{0n}^2 - \omega_{0k}^2)(\omega_{0m}^2 - \omega_{0k}^2)} \right] \vec{E}_m^0 + \cdots \\ \vec{H}_k = \vec{H}_k^0 + \sum_{m\neq k} \frac{H'^{HH}_{mk}}{\omega_{0m}^2 - \omega_{0k}^2} \vec{H}_m^0 + \\ \sum_{m\neq k} \left[-\frac{H'^{HH}_{mk} H'^{HH}_{kk}}{(\omega_{0m}^2 - \omega_{0k}^2)^2} + \sum_{n\neq k} \frac{H'^{HH}_{mn} H'^{HH}_{nk}}{(\omega_{0n}^2 - \omega_{0k}^2)(\omega_{0m}^2 - \omega_{0k}^2)} \right] \vec{H}_m^0 + \cdots \end{cases} \tag{6}$$

Where $\begin{cases} \boldsymbol{H'^{EE}_{nk}} = (1 - \frac{1}{\mu_{r1}\varepsilon_{r1}})\omega_{0k}^2 \int_{V1} \vec{\boldsymbol{E}}_n^{0*} \cdot \vec{\boldsymbol{E}}_k^0 \boldsymbol{dV} \Big/ \left[\int_V \left|\vec{\boldsymbol{E}}_n^0\right|^2 \boldsymbol{dV} \int_V \left|\vec{\boldsymbol{E}}_k^0\right|^2 \right]^{1/2} \\ \boldsymbol{H'^{HH}_{nk}} = (1 - \frac{1}{\mu_{r1}\varepsilon_{r1}})\omega_{0k}^2 \int_{V1} \vec{\boldsymbol{H}}_n^{0*} \cdot \vec{\boldsymbol{H}}_k^0 \boldsymbol{dV} \Big/ \left[\int_V \left|\vec{\boldsymbol{H}}_n^0\right|^2 \boldsymbol{dV} \int_V \left|\vec{\boldsymbol{H}}_k^0\right|^2 \right]^{1/2} \end{cases}$

If we considered the first-order approximation only, using $\Delta\omega_k = \omega_k - \omega_{0k}$, $\omega_k + \omega_{0k} \approx 2\omega_{0k}$ and $\mu_{r1} = 1$ results in

$$\Delta\omega_k / \omega_{0k} = -(\varepsilon_{r1} - 1) \int_{V1} \frac{1}{\varepsilon_{r1}} \vec{E}_k^{0*} \cdot \vec{E}_k^0 dV \Big/ 2\int_V \vec{E}_k^{0*} \cdot \vec{E}_k^0 dV$$

The above formula is identical to that of references [4], [5], [6].

There are only TE and TM modes exist in the regular ideal metal cavity. After the perturbation sample has been introduced in metal cavity, normally it can be seen from the formulas of (5) (6), that the first-order correction and the second-order correction to the component E_z of TE mode will not be zero when the coefficient of one TM mode's zeroth electromagnetic fields is not equal to zero, due to the TE mode's expansion with all zeroth electromagnetic fields of TE and TM modes. In this case TE mode is changed into EH mode. For the same reason TM mode is changed into HE mode. It can be easily understood that if a rigorous solution to electromagnetic fields of the metal cavity with perturbation dielectric sample can be obtained, on account of the boundary conditions of conductor's surface is still homogeneous, but that of dielectric surface is no longer homogeneous, then the mode characteristics equation is heterogeneous. Hence the electromagnetic fields are HE and EH modes.

SUMMARY

The perturbation theory for metal cavity electromagnetic fields can be used to measure the permittivity or permeability of microwave materials, its first-order approximation is the same as that of the method being presently used. But the first-order approximation is unsuitable for the situations of high permittivity dielectrics, as the error is serious. Hence, the second-order correction we derived from perturbation theory is important for such applications as the precise measurement of high permittivity of microwave materials, and then we can even get the third-order, the fourth-order…approximation corrections.

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