Robust train formation planning

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Abstract: Train formation planning models determine the routing and frequency of trains and assign the wagons to trains. In this article, a new robust mixed integer model for the train formation problems is proposed where the input data are subject to uncertainty. The optimal solution of the proposed model of this article is believed to be difficult to determine and a heuristic approach to find the near-optimal solution is presented. The implementation of the proposed model of this article is demonstrated for a real-world case study and the results are discussed.

Keywords: train formation problem, robust approach, perturbation, mixed integer programming, heuristic algorithm

1 INTRODUCTION

Freight train formation is one of the most important research areas in transportation planning. The train formation planning (TFP) models are often formulated as a mixed integer problem and the optimal solution is normally difficult to find. Practically, one may use trial and error to find a practical solution and this could increase the cost of transportation significantly. On the other hand, there are many practical cases where the exact value for the system's information is not accessible and, even if the optimal solution exists, a small change in data could make the optimal solution virtually infeasible. The primary objective of this article is to present a mathematical model to solve TFP by considering noise in the input data. The proposed model of the article is formulated as a mixed integer non-linear problem where the input data are subject to uncertainty. It has been explained that the resulting model involves a large number of binary variables and the optimal solution cannot be found in a reasonable amount of time. Therefore, a heuristic approach is presented in order to locate a near-optimal solution.

Based on the survey conducted by Cordeau *et al.* [**1**], the TFP, sometimes called the routing and makeup problem (RMP), is a subclass of network routing problems. The other subclasses are the blocking problem and compound routing and scheduling problems. The TFP can also be named as the RMP where the frequency of trains and the assignment of blocks to trains are determined. Furthermore, the blocking policy may either be determined endogenously or be given as an input (see, for example, Thomet [**2**]). Crainic *et al.* [**3**] proposed a model and a heuristic for tactical planning. The model is non-linear, mixed integer programming (MIP), which minimizes the operating and delay costs. Keaton [**4**] presented an MIP model and a heuristic method based on Lagrangian relaxation [**5**]. Lin [**6**] also presented an implicit enumeration algorithm with ε-optimality to solve the TFP model. Marin and Salmeron [**7**, **8**] proposed a local search heuristic for the tactical design of rail freight networks where the objective was to minimize the operating and time costs. Morlok and Peterson [**9**] introduced a linear MIP model and applied it to a very small instance where the optimal solutions were found using the branch and bound procedure. Huntley *et al.* [**10**] presented a non-linear MIP model with an adaptation of simulated annealing to locate a near-optimal solution. Gorman [**11**] offered an application of genetic and tabu searches for the same problem. Finally, Godwin *et al* [**12**] presented a heuristic approach for this problem.

This article is organized as follows. First, the necessary definitions are presented in section 2. Section 3 presents the mathematical formulation of the problem and the robust TFP is explained in section 4. Section 5 is devoted to a heuristic and a case study for an Iranian transportation problem. Finally, the concluding remarks are given at the end to summarize the contribution of this article.

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2 PROBLEM DEFINITION

A railway is considered as a directed graph $G(N, A)$, consisting of shunting yards as nodes $n \in N$ and paths, and the distances between shunting yards as arcs $a \in A$, which are shown by doublets $a = (n_1, n_2)$ where $n_1, n_2 \in N$. In the railway network, there are some compartments transported by trains from their origins to their destinations. Each compartment may be divided into some subsets of wagons so that each subset could be assigned to a certain train. The subsets are sometimes called blocks. The problem is to form the required kinds of trains to transport all wagons of compartments. The primary constraints are the maximum length and tonnage of trains in each path and the objective is to minimize the cost of transportation.

There are different objectives involved in our proposed model such as the costs related to train formation, wagon classification works in shunting yards, and idle time of wagons waiting for trains in yards, which are discussed in the next section.

In this article, two kinds of trains are considered: trains with one locomotive, called short trains, and trains with two locomotives, called long trains. Generally, forming trains with more than two locomotives is not recommended in many countries such as Iran [**13**] and is not considered in this article. The length of the trains is dependent on the hauling power of the locomotives and the topography of the paths. In other words, since the hauling power is always constant, the maximum wagons that can be hauled by a specific train are directly related to the steepest gradient of the path. In addition, every station has a limited length so that the length of the formed train cannot exceed this limit. In real-world applications, the mean weight of each wagon and also the number of wagons for each compartment are subject to disturbances, and as a result, in practice, it may be different from the amount considered in the planning stage. In this situation, the pre-organized plan may be neither infeasible nor optimum any more. A practical approach is to consider an empty space for each train. A more sophisticated approach for an optimal allocation of the spaces is explained.

3 THE MODEL

The following assumptions are considered through this article.

1. There is no limit for the classification works in shunting yards.
2. Long trains have the ability of moving twice the number of wagons moved by short trains. In other words, if a short train can move n wagons, then a long train can transport $2 \times n$ wagons.

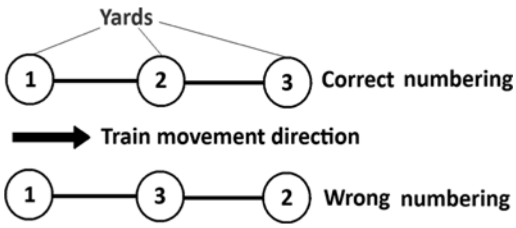

Fig. 1 Yards numbering

3. Yards are numbered such that the numbers increase in the train movement direction. Fig. 1 shows two different numberings.
4. Two different assumptions are considered in the case of arriving trains at stations. The simpler one is that there is a tight and accurate timetable for all trains that arrive at stations. The other one is that the arrival of freight trains to stations is unknown but is based on a given stochastic distribution.

3.1 Notations

Appendix 1 summarizes the necessary notations needed for our proposed model.

As explained, the proposed model of this article considers three objective functions that are discussed in the following subsections.

3.2 Minimizing the train formation cost

This cost consists of train personnel wages (including the locomotive driver, the locomotive driver's assistant, the repairman, and the train chief), the consumed oil and gasoline, and the amortization of the locomotive. Equation (1) indicates this objective function. Note that $(1 - t_i) = 1$ if train i is a short one.

$$D_1 = \sum_{i \in T} \sum_{j \in N} \sum_{k \in N} \left\{ cs_{jk} \times \left[a_{i,(j,k)} \times (1 - t_i) \right] + cl_{jk} \left(a_{i,(j,k)} \times t_i \right) \right\} \tag{1}$$

3.3 Minimizing wagon classifications works cost

Classification works in shunting yards contain separating and connecting wagons to/from trains and increase the cost of transportation which is dependent on the quantity of the classified wagons. In addition, there are some fixed costs associated with classification works which are as follows.

1. A fixed cost for trains that stop in yards for classification operations.
2. A fixed cost for the number of blocks assigned to each train.

Note that any compartment can be divided into various blocks with a different number of wagons and each block is assigned to an individual train. As the number of blocks assigned to a train increases, classification works in shunting yards become more complicated.

Classification works are divided into two categories: first, those works that are related to all compartments in their origins and destinations that equally appear in all feasible solutions and therefore can be ignored, and second, the classification works that are done in the middle yards (i.e. yards between the origin and the destination of compartments). Equation (2) shows the formulation of the second objective

$$D_2 = \sum_{i\in T}\sum_{p\in P}\sum_{k=\mathrm{or}_p+1}^{\mathrm{de}_p-1} cc_k \times \left|\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)}\right|$$
$$+ \sum_{i\in T}\sum_{k\in N} ct_k \times S_{ik}$$
$$+ \sum_{i\in T}\sum_{p\in P}\sum_{k=\mathrm{or}_p}^{\mathrm{de}_p-1}\sum_{h=j+1}^{\mathrm{de}_p}\left(cb_k \times b^p_{i(k,h)}\right) \qquad (2)$$

where $\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)}$ is the number of wagons of compartment p transported by train i to yard k and $\sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)}$ is the number of wagons of compartment p in yard k transported to the next yard by train i.

If in yard k, no wagons of compartment p are disconnected from train i

$$\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} = 0$$

else

$$\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} \neq 0$$

Therefore

$$\left|\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)}\right|$$

indicates the number of disconnecting and connecting works of wagons of compartment p to/from train i in yard k.

Note that the trains and compartments can pass any arc based on the third assumption (i.e. the beginning node must have a lower number than the end node). As a result of the above-mentioned fact, further to railway lines, railway networks can also be studied under the condition of the third assumption. In other words, to study a railway network, one must assign numbers to shunting yards such that the beginning node of each arc has a smaller number than the end node.

3.4 Minimizing the cost of idle time of wagons in shunting yards waiting for trains

The RMP class problems are not involved with scheduling trains and wagons. Moreover, as stated before, two different assumptions regarding the arrival time of trains at stations are considered. The first one is that freight trains are moved on the basis of an accurate timetable generated after determining the train routings. In this case, there is no idle time and this objective function can be disregarded. There is also the other case where there is no timetable for the freight trains entering into each station and the entrance is considered to have a Poisson distribution. In order to understand each component of the objective function, each is explained separately. The number of wagons that must wait in yard k for the arrival of already formed trains to ship to the next yards is computed on the basis of the following

$$\sum_{i\in T}\sum_{p\in P}\left\{\max\left[\left(\sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} - \sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)}\right), 0\right] \times S_{ik}\right\}$$

Proposition 1

The expected entrance time of kth train to yard j during a one-day operation is $k/\sum_{i\in T} S_{ij}$, which means that it takes place at $k/\sum_{i\in T} S_{ij} \times 24$ o'clock.

Proof

Since the arrival time schedule has Poisson distribution, the interval time is also considered to have an exponential distribution. The sum of k exponentially distributed arrival times of trains to yard j has a gamma distribution with a scale parameter $n = \sum_{i\in T} S_{ij}$ and a shape parameter k. Therefore, the expected entrance time of kth train to yard j is equal to the mean of the gamma distribution, where n is the number of preformed trains arriving at yard j during the one-day operation.

Proposition 2

Let n be the number of trains arriving to yard j during a one-day operation. The expected summation of the entrance time of n trains is $(n + 1/2)$ day, $n > 0$.

Proof

Since $\sum_{i\in T} S_{ij} = n$ for yard j, the summation of arrival times of n trains, each of which is based on gamma distribution, is $(1 + 2 + \ldots + n/n)$. Since $1 + 2 + \ldots + n = (n(n + 1)/2)$, then $n(n + 1)/2n = (n + 1/2)$ days or $12 \times (n + 1)$ hours.

Equation (3) summarizes the third objective function

$$D_3 = \sum_{k \in N} \begin{pmatrix} 12 \times \dfrac{\sum_{i \in T} S_{ik} + 1}{\sum_{i \in T} S_{ik}} \times \sum_{i \in T} \sum_{p \in P} \left\{ \max \left[\left(\sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} - \sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} \right), 0 \right] \times ci^p \times S_{ik} \right\}, & \text{if } \sum_{i \in T} S_{ik} > 0 \\ 0, & \text{otherwise} \end{pmatrix} \quad (3)$$

The model P1 is described as follows

$$\min D_1 + D_2 + D_3 \quad (4)$$

subject to

$$\sum_{i \in T} \sum_{j=\mathrm{or}_p+1}^{\mathrm{de}_p} y^p_{i(\mathrm{or}_p,j)} = r^p, \quad \forall p \in P \quad (5)$$

$$\sum_{i \in T} \sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{i \in T} \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} = 0, \quad \forall p \in P, k \in N \quad (6)$$

$$\sum_{i \in T} \sum_{j=\mathrm{or}_p}^{\mathrm{de}_p-1} y^p_{i(j,\mathrm{de}_p)} = r^p, \quad \forall p \in P \quad (7)$$

$$\sum_{p \in P} \left(l^p \times y^p_{i(j,k)} \right) <= v_{jk}, \quad \forall (j,k) \in A, k > j, \quad j \text{ and } k \in N, \forall i \in T \quad (8)$$

$$\sum_{p \in P} \left(w^p \times y^p_{i(j,k)} \right) <= m_{jk} \times (1 + t_i), \quad \forall (j,k) \in A, k > j, j \text{ and } k \in N, \forall i \in T \quad (9)$$

$$\sum_{p \in P} \left(y^p_{i(j,k)} \right) <= M \times \left(S_{ij} + O_{ij} \right), \quad \forall (j,k) \in A, k > j, j \text{ and } k \in N, \forall i \in T \quad (10)$$

$$\sum_{p \in P} \left(y^p_{i(j,k)} \right) <= M \times a_{i(j,k)}, \quad \forall (j,k) \in A, k > j, j \text{ and } k \in N, \forall i \in T \quad (11)$$

$$\sum_{j \in N} O_{ij} \leqslant 1, \quad \forall i \in T \quad (12)$$

$$S_{ik} \leqslant \sum_{h=1}^{k-1} O_{ih}, \quad \forall i \in T, \forall k \in N \quad (13)$$

$$a_{i(j,k)} \leqslant \frac{1}{2} \times (O_{ij} + S_{ij} + S_{ik}) - \sum_{h=j+1}^{k-1} a_{i(j,h)}, \quad \forall (j,k) \in A, k > j, j \text{ and } k \in N, \forall i \in T \quad (14)$$

$$a_{i(j,k)} \geqslant (O_{ij} + S_{ij} + S_{ik} - 1) - \sum_{h=j+1}^{k-1} a_{i(j,h)}, \quad \forall (j,k) \in A, k > j, j \text{ and } k \in N, \forall i \in T \quad (15)$$

$$b^p_{i(j,k)} \geqslant \frac{1}{M} y^p_{i(j,k)}, \quad \forall (j,k) \in A, k > j, \quad j \text{ and } k \in N, \forall i \in T \quad (16)$$

where M is a big positive number with $M \geqslant \sum_{p \in P} r^p$. The objective function (4) is the summation of the three defined single objectives shown by equations (1) to (3). Constraints (5), (6), and (7) ensure that all compartments leave their origins, pass middle yards, and reach their destinations one after another. Constraints (8) and (9) prevent assigning wagons more than the maximum allowable to trains from length and weight points of view, respectively. Constraints (10) and (11) state that train i is allowed to transport wagons using path (j, k) if and only if this path is allocated to the set of paths passed by train i and also nodes j and k are assigned to the node sets that are met by train i. Constraint (12) specifies that each train has its own origin. Constraint (13) shows that train i can dwell in yard j if and only if the origin of train i is set before this yard. Constraint (14) indicates that path (j, k) can be passed by train i whenever yards j and k are met by train i. Constraint (15) ensures that arc (j, k) is passed by train i not only if train i stops in yards j and k, but also if no yards between j and k are met by the train. Constraint (16) indicates that a block of compartment p is assigned to train i if at least one of its wagons is transported by train i.

As it is explained, the proposed model of this article is formulated as a non-linear one because of the first, second, and third objective functions. The proposed model can be simplified into an ordinary MIP. The details are explained in Appendix 2.

4 APPLYING ROBUST APPROACH TO THE MODEL

Consider a typical optimization problem where there are different input parameters. When there are uncertainty with one or limited number of parameters, one can use the traditional sensitivity analysis to analyse the optimal solutions. However, when almost all input parameters are contaminated with noise, a method is needed to ensure that a small change in input data will not violate constraints. There are two principal methods that address data uncertainty: stochastic programming and robust optimization. The former case suffers from two issues: the exact distribution of the data needs to be known and the resulting model is often formulated in the form of a highly non-linear optimization problem that makes it difficult to solve. The second approach called robust optimization is computationally tractable and also does not require any information about the uncertain data distribution. The robust method has recently become popular among practitioners. Generally, robust optimization is defined as the method for guaranteeing the feasibility and optimality of the solution for the worst cases of the parameters. In other words, when the robust optimization method is used, one is willing to accept a suboptimal solution for the nominal values of the data in order to ensure that the solution remains feasible and near optimal when the data change. Ben-Tal and Nemirovski [**14**] show that a small perturbation on the input data for some benchmark problems may result in solutions that are not only optimal but also may be infeasible. In robust optimization, it is assumed that the bounds of the uncertain parameters are known, and the aim is to find the solutions that are guaranteed to remain feasible for all parameters values. Figure 2 depicts the effect of the robust approach graphically.

Among all parameters defined in the proposed model, parameter r^p, the number of wagons that belong to compartment p, and parameter w^p, the mean weight of each wagon of compartment p, are subject to perturbation in real-world applications. In this case, in order to have a robust routing in which facing with some alterations in the estimation of data related to demands (i.e. transporting wagons, makes no changes on the optimum solution, the proposed model is developed by applying the robust approach exhibited by Bertsimas and Sim [**15**]).

Although the idea of robust optimization has recently gained much attention, its origin goes back to Soyster [**16**] where his approach admits the highest protection. Therefore, the objective function is determined in the worst-case condition. There have been two more popular robust approaches since then, which address a more conservative solution. Ben-Tal and Nemirovski [**17**, **18**], Ben-Tal *et al.* [**19**], El Ghaoui and Lebret [**20**], and El Ghaoui *et al.* [**21**] have applied robust optimization to linear programming problems, which results in conic quadratic programmes. Bertsimas and Sim introduced a new different robust approach in which the robust counterpart is of the same class and size as the nominal problem. In this case, if the nominal problem is a linear/mixed integer, the final robust model will remain a linear/mixed integer. This approach has the advantage of having the ability to control the degree of conservatism for every constraint and guarantees feasibility for the robust optimization problem.

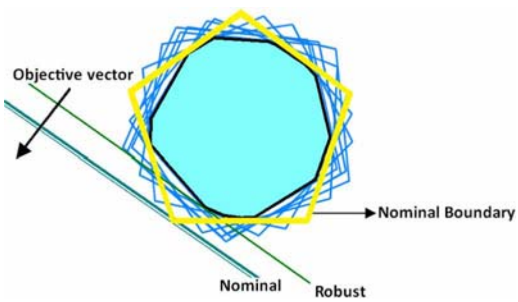

Fig. 2 The effect of the robust approach to the classical optimization problem

Taking into account the advantages of the Bertsimas and Sim method, this idea has been used for the proposed model. A brief description of making a robust optimization problem presented by Bertsimas and Sim is worth looking at for further transparency of the issue.

Consider the following nominal linear optimization problem

$$\begin{aligned} &\max\ C'X \\ &\text{subject to}:\ \mathbf{A}X \leqslant B \\ &\qquad\qquad L \leqslant X \leqslant U \end{aligned}$$

First, note that one can assume, without any loss of generality, that the data uncertainty affects only the elements of the left-hand-side matrix coefficients, **A**, for the following reasons:

(a) the objective function can be transformed to a constraint;
(b) when the right-hand-side constant b_i is subject to uncertainty, one can introduce a new variable x_{n+1} with a fixed value of 1. In other words, the constraint $\sum_{j=1}^{n} a_{ij}x_j \leqslant \tilde{b}_i$ can be replaced by $\sum_{j=1}^{n} a_{ij}x_j - \tilde{b}_i x_{n+1} \leqslant 0$, where $1 \leqslant x_{n+1} \leqslant 1$.

Consider row i of matrix **A** and let J_i represent the set of coefficients in row i, which are subject to perturbation $(\tilde{a}_{ij},\ j \in J_i)$. In other words, the constraint $\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i$ is expanded to $\sum_{j \notin J_i} a_{ij}x_j + \sum_{j \in J_i} \tilde{a}_{ij}x_j \leqslant b_i$. It is assumed that each uncertain coefficient $\tilde{a}_{ij},\ j \in J_i$ independently takes values according to a symmetric distribution with a mean that is equal to the nominal value a_{ij} and of half length $\hat{a}_{ij}$. In other words, $\tilde{a}_{ij}$ belongs to the interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$.

It is unlikely to suppose that all uncertain parameters are equal to their worst-case bound; as a result, parameter Γ_i, $0 \leqslant \Gamma_i \leqslant |J_i|$ is used to adjust the conservatism level of the final solution. It means that at last only $\lfloor \Gamma_i \rfloor$ of the parameters subjected to perturbation are allowed to change in constraint i, and one coefficient a_{ie_i} changes by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{ie_i}$. Bertsimas and Sim developed an approach in which if, in practice, only a subset of $\lfloor\Gamma_i\rfloor$ coefficients changes, then the robust solution will be feasible, and even if more than $\lfloor\Gamma_i\rfloor$ changes, then the robust solution will be feasible with very high probability. In the Bertsimas and Sim approach, the robust formulation of the linear programme is formulated as the following model

$$\begin{aligned}
&\max \quad C'X \\
&\text{subject to} \sum_j a_{ij}x_j + \max_{\{F_i\cup\{e_i\}|F_i\subseteq J_i,|F_i|=\lfloor\Gamma_i\rfloor,e_i\in J_i,e_i\notin F_i\}} \\
&\qquad \times \left\{\sum_{j\in F_i}\hat{a}_{ij}\left|x_j\right| + (\Gamma_i - \lfloor\Gamma_i\rfloor)\hat{a}_{ie_i}\left|x_{e_i}\right|\right\} \leqslant b_i\,\forall\, i \\
&\qquad L \leqslant X \leqslant U
\end{aligned} \tag{17}$$

where $F_i \subseteq J_i$ indicates a subset that has $\lfloor\Gamma_i\rfloor$ members of noise affected parameters in the ith constraint. The added statement to the left-hand side of constraint i in model (17) is called the protection function. The role of parameter Γ_i is to adjust the robustness of the proposed method against the level of conservatism of the solution. If $\Gamma_i = |J_i|$ is considered, the Bertsimas method leads to the solution with the maximum conservatism; on the other hand, if $\Gamma_i = 0$, the approach is inactive. The model (17) is non-linear. In order to reformulate it as a linear optimization model, Bertsimas and Sim have proved that the protection function of model (17) equals the objective function of the following linear optimization problem [**15**]

$$\begin{aligned}
&\max \quad \sum_{j\in J_i}\hat{a}_{ij}|x_j^*|z_{ij} \\
&\text{subject to:} \sum_{j\in J_i} z_{ij} \leqslant \Gamma_i \\
&\qquad 0 \leqslant z_{ij} \leqslant 1 \quad \forall j \in J_i
\end{aligned} \tag{18}$$

Next, they showed that model (17) has an equivalent linear formulation, model (19), which appears by developing the dual of model (18) and substituting it to model (17). The dual of model (18) is shown as follows

$$\begin{aligned}
&\min \quad \sum_{j\in J_i} p_{ij} + \Gamma_i z_i' \\
&\text{subject to } z_i' + g_{ij} \geqslant \hat{a}_{ij}|x_j^*|, \quad \forall i, j \in J_i \\
&\qquad g_{ij} \geqslant 0 \quad \forall j \in J_i \\
&\qquad z_i' \geqslant 0 \quad \forall i
\end{aligned}$$

where z_i' and g_{ij} are the dual variables of the first and second constraints of model (18), respectively. The linear formulation of model (17) is as follows

$$\begin{aligned}
&\max \quad C'X \\
&\text{subject to} \sum_j a_{ij}x_j + z_i'\Gamma_i + \sum_{j\in J_i} g_{ij} \leqslant b_i \quad \forall i \\
&\qquad z_i' + g_{ij} \geqslant \hat{a}_{ij}y_j \quad \forall j \in J_i \\
&\qquad -y_j \leqslant x_j \leqslant y_j \quad \forall j \\
&\qquad l_j \leqslant x_j \leqslant u_j \quad \forall j \\
&\qquad g_{ij} \geqslant 0 \quad \forall j \in J_i \\
&\qquad y_j \geqslant 0 \quad \forall j \\
&\qquad z_i' \geqslant 0 \quad \forall i
\end{aligned} \tag{19}$$

For more details, refer to reference [**15**].

Now the adjustment of the robust approach to the introduced TFP model is shown. As stated before, two parameters r^p and w^p of the main model are subject to disturbances. First focus on parameter r^p and specifically constraint (8), and the robustness against the perturbation affected by parameter w^p in constraint (9) is discussed in Appendix 3. The number of wagons of compartments r^p is known to belong to an interval centred at its nominal value $\bar{r}^p$ and of half-length $\hat{r}^p$ (i.e. $r^p \in [\bar{r}^p - \hat{r}^p, \bar{r}^p + \hat{r}^p]$, but its exact value is unknown). Consider train i, which is planned to transport $r'^1, r'^2, \ldots, r'^p$, wagons of compartments $1, 2, \ldots, p$, respectively, in path (j, k). Since the number of wagons of each compartment is not known exactly, in practice, the total number of wagons planned to be hauled by train i, $\sum_{h\in P} r'^h$ can be different compared with the planned quantity. The difference, especially when $\sum_{h\in P} r'^h$ is near to the ultimate capacity of train i in path (j, k), v_{jk}, may lead to void planning, which itself causes much unwillingness.

To avoid such malfunctions, the mathematical formulation has been enriched using the explained robust procedure introduced by Bertsimas and Sim to find a robust plan for train routing and makeup. To make the proposed MIP model robust against explained disturbances only, constraints (8) and (9), which prevent assigning more than the maximum allowable wagons to trains, from length and weight points of view, must be studied. At first, the robust approach is applied to inequality (8). The results are extended to inequality (9) in Appendix 3.

Since inequality (8) does not contain unknown parameters r^p, to apply the robust approach, one can replace variable $y^p_{i(j,k)}$ with $r^p \times x^p_{i(j,k)}$ where $x^p_{i(j,k)}$ indicates the proportion of wagons of compartment p pass path (j, k) by train i to all wagons of compartment p. Therefore inequality (8) can be re-written as inequality (20). It is worth specifying that this replacement holds theoretically but is not applicable when tried to be solved by computer programs, owing to rounding problems

$$\sum_{p\in P}(l^p \times \tilde{r}^p \times x^p_{i(j,k)}) <= v_{jk}, \quad \forall i,j,k \tag{20}$$

Inequality (20) can be protected against disturbances by adding the protection function shown by statement (21) into the left-hand side of inequality (20) and replacing the unknown parameter $\tilde{r}^p$ with its nominal value $\bar{r}^p$.

$$\max_{\{F_{i(j,k)}\cup\{e_{i(j,k)}\}|F_{i(j,k)}\subseteq J_{i(j,k)},|F_{i(j,k)}|=\lfloor\Gamma_{i(j,k)}\rfloor,e_{i(j,k)}\in J_{i(j,k)},e_{i(j,k)}\notin F_{i(j,k)}\}} \times \left\{\left[\sum_{j\in F_{i(j,k)}}(l^p\hat{r}^p x^p_{i(j,k)})\right] + (\Gamma_{i(j,k)} - \lfloor\Gamma_{i(j,k)}\rfloor) \times l^{e_{i(j,k)}}\hat{r}^{e_{i(j,k)}} x^{e_{i(j,k)}}_{i(j,k)}\right\} \tag{21}$$

Note that since $x^p_{i(j,k)} \geqslant 0$, there is no need to consider its absolute value in statement (21). Moreover, $J_{i(j,k)}$ is interpreted as the set of compartments under disturbances in which at least one of the related wagons is transported from yard j to yard k by train i. Furthermore, at most, only $\lfloor\Gamma_{i(j,k)}\rfloor$ number of the compartments that are subject to perturbation (i.e. the parameters that belong to subset $F_{i(j,k)} \subseteq J_{i(j,k)}$ are allowed to change and one compartment, $e_{i(j,k)}$, changes by a portion of $\Gamma_{i(j,k)} - \lfloor\Gamma_{i(j,k)}\rfloor$). Statement (21) equals the objective function of the linear optimization problem (22)

$$\begin{aligned} &\max \sum_{j\in J_i} l^p\hat{r}^p x^{*p}_{i(j,k)} z^p_{i(j,k)} \\ &\text{subject to} \sum_{p\in J_{i(j,k)}} z^p_{i(j,k)} \leqslant \Gamma_{i(j,k)} \\ &\qquad 0 \leqslant z^p_{i(j,k)} \leqslant 1 \quad \forall p \in J_{i(j,k)} \end{aligned} \tag{22}$$

The dual of model (22) is equal to

$$\begin{aligned} &\min \Gamma_{i(j,k)} z'_{i(j,k)} + \sum_{p\in J_{i(j,k)}} g^p_{i(j,k)} \\ &\text{subject to } z'_{i(j,k)} + g^p_{i(j,k)} \geqslant l^p\hat{r}^p x^{*p}_{i(j,k)} \quad \forall p \\ &\qquad g^p_{i(j,k)} \geqslant 0 \quad \forall p \\ &\qquad z'_{i(j,k)} \geqslant 0 \end{aligned} \tag{23}$$

where $z'_{i(j,k)}$ and $g^p_{i(j,k)}$ are the dual variables of the first and second constraints of model (22). Finally, the linear robust counterpart of inequality (20) is emerged from the embedding model (23) in inequality (20). Inequalities (24) to (27) indicate the linear robust representation of inequality (20) altogether

$$\sum_{p\in J_{i(j,k)}} l^p y^p_{i(j,k)} + \Gamma_{i(j,k)} z'_{i(j,k)} + \sum_{p\in J_{i(j,k)}} g^p_{i(j,k)} \leqslant v_{jk}, \quad \forall p,i,j,k \tag{24}$$

$$z'_{i(j,k)} + g^p_{i(j,k)} \geqslant \frac{\hat{r}^p}{r^p} l^p y^p_{i(j,k)}, \quad \forall p,i,j,k \tag{25}$$

$$g^p_{i(j,k)} \geqslant 0, \quad \forall p,i,j,k \tag{26}$$

$$z'_{i(j,k)} \geqslant 0, \quad \forall i,j,k \tag{27}$$

The protection parameter $\Gamma_{i(j,k)}$ can be defined as a fixed value or based on the following equation

$$\Gamma_{i(j,k)} = \alpha \times \sum_{p\in P} b^p_{i(j,k)} \tag{28}$$

where α is the conservatism factor and is defined in the range of [0, 1]. If $\alpha = 0$, then the robust approach would be inactive. Also, if $\alpha = 1$, the most conservative robust solution that considers the worst case appears.

In the remaining of this section, to illustrate the effects of applying the robust approach to the presented mathematical formulation, some examples are discussed.

Example 1

Consider a railway line that contains four shunting yards. Only those compartments that are supposed to be transported in the same direction are concentrated on, e.g. south to north. There are four different compartments as illustrated in Table 1. The maximum number of wagons and the maximum tonnage that can be hauled by trains in each path are shown in Table 2.

In Table 1, the perturbation tolerances are shown in parentheses. Table 2 specifies that the distance between yards 3 and 4 is more restrictive in the manner of the maximum number and weight of wagons that could be hauled by a certain train.

The goal is to find the origin and the destination of the formed trains as well as the assignment of the wagons to trains such that the related costs shown in Fig. 3 are minimized.

In order to study the changes on the robust objective function against the crisp one, the crisp and robust

Table 1 Characteristics of the compartments

	Compartment 1	Compartment 2	Compartment 3	Compartment 4
Origin	1	1	2	3
Destination	4	3	4	4
Length of wagons	14	14	14	14
Number of wagons	92(±9)	42(±4)	28(±3)	15(±2)
Mean weight	60(±6)	70(±7)	70(±7)	60(±6)

Table 2 Characteristics of the railway network

	Path (1, 2)	Path (1, 3)	Path (1, 4)	Path (2, 3)	Path (2, 4)	Path (3, 4)
Maximum length of train	1120	1120	700	1120	700	700
Maximum weight of train	3000	3000	2000	3000	2000	2000

Table 3 Noise-affected trains and paths

Train	Path	Total weight	Weight limit
1	(3, 4)	1980	2000
2	(1, 3)	2950	3000
3	(1, 3)	2970	3000
5	(2, 4)	1960	2000

$$cs_{jk} = \begin{bmatrix} 0 & 8000 & 20000 & 28000 \\ 0 & 0 & 12000 & 20000 \\ 0 & 0 & 0 & 8000 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad cl_{jk} = \begin{bmatrix} 0 & 16000 & 40000 & 56000 \\ 0 & 0 & 24000 & 40000 \\ 0 & 0 & 0 & 16000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$cc_j = \begin{bmatrix} 15 & 15 & 15 & 15 \end{bmatrix} \quad cb_j = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$ct_j = \begin{bmatrix} 100 & 100 & 100 & 100 \end{bmatrix} \quad ci^p = \begin{bmatrix} 5 & 5 & 5 & 5 \end{bmatrix}$$

Fig. 3 Costs related to example 1

problems have been solved using different values for the protection parameters defined as $\Gamma^{v}_{i(j,k)} = \Gamma^{w}_{i(j,k)} = \gamma, \forall i \in T, (j,k) \in A$. First the crisp problem is considered, where $\gamma = 0$. Figure 4 depicts the result. Trains and yards are shown by lines and circles, respectively. The quantity of wagons of each compartment is shown in parenthesis. Note that all trains are long ones.

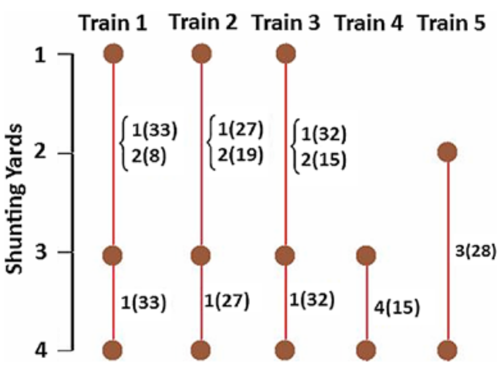

Fig. 4 Results of the crisp type of example 1

Table 3 shows the trains and paths that are near to its final limit and therefore a small noise can affect the solution so that it would not be feasible any more.

Note that since all formed trains are short, therefore no trains are near to the length limits. The results of Table 3 show that to protect the solution against the bad effects of noise, applying the robust approach is vital. Figure 5 shows the different objective function value for a different amount of γ. The problem is solved by the assumption of forming at most 3, 4, and 5 trains individually. The CPU time, which is achieved by a Pentium IV Laptop with an Intel Core 2 Duo processor running at 2.53 GHz and 2 GB of RAM memory, is reported in the figure.

Moreover, it has been fixed that $\Gamma^{v}_{i(j,k)} = \Gamma^{w}_{i(j,k)} = 1$, $\forall i \in T, (j,k) \in A$, and example 1 has been solved considering a different perturbation tolerance. Table 4 summarizes the final optimal solutions where $\tau = \hat{r}^p / r^p, \upsilon = \hat{w}^p / w^p$, and $\forall p \in P$. Note that the third objective functions for all the instances are found to be equal to zero. The CPU time is reported in the last column of the table.

As it could be expected, as the protection function values increase, the total objective function value is increased too. The case of $\tau = 0$ and $\upsilon = 0$ shows the crisp solution, which is obtained without the robust approach. Moreover, the first objective function has a more important effect on the total objective function value than the second and third objective functions.

The robust approach aims to find solutions that are robust against the perturbations, in the case

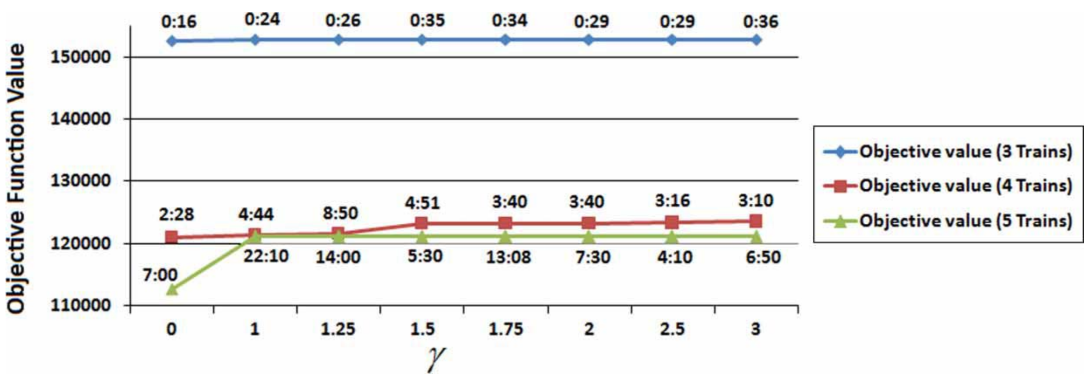

Fig. 5 Results of example 1

Table 4 Results of example 1

Protection parameter	Objective function value				Protection parameter	Objective function value			
	First	Second	Total	CPU time		First	Second	Total	CPU time
$\tau = 0, \upsilon = 0$	112 000	800	112 800	4:48	$\tau = 0, \upsilon = 0.5$	140 000	1135	141 135	6:18
$\tau = 0.2, \upsilon = 0$	112 000	800	112 800	1:51	$\tau = 0.2, \upsilon = 0.5$	140 000	1410	141 410	5:00
$\tau = 0.5, \upsilon = 0$	112 000	800	112 800	7:40	$\tau = 0.5, \upsilon = 0.5$	148 000	2010	150 010	12:22
$\tau = 0.8, \upsilon = 0$	112 000	1230	113 230	5:10	$\tau = 0.8, \upsilon = 0.5$	156 000	1775	157 775	6:00
$\tau = 0, \upsilon = 0.2$	120 000	1390	121 390	10:10	$\tau = 0, \upsilon = 0.8$	156 000	1730	157 730	6:40
$\tau = 0.2, \upsilon = 0.2$	120 000	1390	121 390	9:52	$\tau = 0.2, \upsilon = 0.8$	156 000	1730	157 730	5:04
$\tau = 0.5, \upsilon = 0.2$	120 000	1390	121 390	9:20	$\tau = 0.5, \upsilon = 0.8$	170 000	1560	171 560	2:36
$\tau = 0.8, \upsilon = 0.2$	120 000	1390	121 390	20:36	$\tau = 0.8, \upsilon = 0.8$	176 000	1960	177 960	7:00

of negative perturbations (i.e. wagons cancellations in the compartment), the feasibility of the solutions will not be jeopardized. In fact, in these cases, the trains will take fewer quantity of trains, which itself leads to an increase in the speed of trains in routes.

5 HEURISTIC METHOD

As explained, there are many binary variables involved in a real-world problem formulation. Therefore, one may need to look for a near-optimal solution as an alternative to using a heuristic approach.

Before explaining the proposed heuristic method, it is worth stating that the solution of model P1 can be broken down into three parts: the routes of compartments are determined first, allocating the necessary trains for shipping the compartments is the second part, and the exact allocation of each wagon to trains is determined in the last step. The proposed heuristic of this article consists of two phases.

Phase 1

The first phase addresses the first and second parts of the solution. Model P2 is introduced to find the routes of compartments and to allocate the necessary trains such that the first objective function, which is the most important among all the other objective functions, is minimized.

Phase 2

The output of the first phase is now considered as the input of the second phase. In phase 2, the model P1 is used under the condition that the first and second parts of the solution are known. Therefore, the output of phase 2 is the exact assignment of the wagons to each train where the second and third objective functions are minimized.

The model P2 is given as follows

$$\text{(P2)} \quad \min D_1 = \sum_{i\in T}\sum_{j\in N}\sum_{k\in N}\{cs_{jk} \times [a_{i,(j,k)} \times (1-t_i)] + cl_{jk}(a_{i,(j,k)} \times t_i)\} \tag{29}$$

$$\sum_{k=\text{or}_p+1}^{\text{de}_p} q^p_{\text{or}_p,k} = 1, \quad \forall p \in P,\ \forall k \in N \tag{30}$$

$$\sum_{j=\text{or}_p}^{k-1} q^p_{j,k} - \sum_{h=k+1}^{\text{de}_p} q^p_{k,h} = 0, \quad \forall p \in P,\ \forall k \in N \tag{31}$$

$$\sum_{j=\text{or}_p}^{\text{de}_p-1} q^p_{j,\text{de}_p} = 1, \quad \forall p \in P,\ \forall j \in N \tag{32}$$

$$\sum_{p\in P}(l^p \times r^p \times q^p_{j,k}) <= \upsilon_{jk} \times \sum_{i\in T} a_{i(j,k)}, \quad \forall (j,k) \in A \tag{33}$$

$$\sum_{p\in P}(w^p \times r^p \times q^p_{j,k}) <= m_{jk} \times \sum_{i\in T}[a_{i(j,k)} \times (1+t_i)], \quad \forall (j,k) \in A \tag{34}$$

where $q^p_{j,k}$ is a binary variable, which is 1 if compartment p passes path (j,k) and 0 otherwise.

The robust form of P2 is similar to P1 and is not repeated here.

Note that any solution from this model does not necessarily have to be feasible for P1; therefore, some modification is needed in order to adjust the results. Algorithm 1 shows a step-by-step procedure, first to achieve a feasible solution and second to validate it among neighbourhood solutions. The following notations shown in Appendix 1 are used in our heuristic algorithms.

Algorithm 1

The heuristic approach.

Step 1: let $h \leftarrow 0$ and $\text{exv}^h_{(j,k)} \leftarrow 0$ and $\text{exw}^h_{(j,k)} \leftarrow 0$, $\forall\ (j,k) \in A$. The two last ones that are called expanding variables are used to expand the solution space of model P2. The expanding variables are the main tools for achieving neighbourhood solutions.

Step 2: if the terminating criterion is satisfied, stop; else for all $(j, k) \in A$, add $\mathrm{exv}^h_{(j,k)}$ and $\mathrm{exw}^h_{(j,k)}$ to the left-hand side of constraints (33) and (34), respectively, to expand the solution space. Run model P2.

Step 3: given the results of model P2, run model P1.

Case 1: if no feasible solution is found, find the infeasible paths that cause the infeasibility using algorithm 2 and go to step 4.

Case 2: if a feasible solution is found, modify the best solution $ofv_{\mathrm{best}} = ofv_{\mathrm{h}}$ if necessary. Select one of the paths in which the total capacity of all the trains is tightly shared among the trains (i.e. the path that is near to the infeasibility condition using algorithm 2). Go to step 4.

Step 4: let $\lambda^h_{(j,k)} = \alpha \times \lambda^{h-1}_{(j,k)}$, $\mu^h_{(j,k)} = \beta \times \mu^{h-1}_{(j,k)}$, $\mathrm{exv}^h_{(j,k)} = \lambda^h_{(j,k)} \times \sum_{i\in T} a_{i(j,k)}$, and $\mathrm{exw}^h_{(j,k)} = \mu^h_{(j,k)} \times \sum_{i\in T} a_{i(j,k)}$, $\forall (j, k) \in A$, where $\alpha, \beta > 1$. Let $h \leftarrow h + 1$. Go to step 2.

5.1 Termination criterion

Algorithm 1 is stopped when a certain number of iterations are elapsed or the objective function value of model P2 exceeds the total objective functions value of the best found solution.

The following algorithm finds the infeasible or near-to-infeasible paths.

Algorithm 2

Step 1: let $j = 1$ and $k = 1$. Empty infeasible path list and go to step 2.

Step 2: if $j = |N| - 1$ and $k = |N|$, then go to step 5; else if $k < |N|$, then $k = k + 1$, or $j = j + 1$ and $k = j + 1$. Go to step 3.

Step 3: if at least one train passes path (j, k), go to step 4, else go to step 2.

Step 4: based on the results of model P2, consider the trains that pass path (j, k) and also the compartments that are assigned to the path. Use model P1 to find a feasible assignment of wagons to trains in path (j, k). If no feasible solution is obtained, add path (j, k) to the infeasible path list. Otherwise, compute $\mathrm{Slv}_{(j,k)}$ and $\mathrm{Slw}_{(j,k)}$. Go to step 2.

Step 5: if the infeasible path list is not empty, return the list; otherwise return the path that has the minimum $\mathrm{Slv}_{(j,k)}$ and $\mathrm{Slw}_{(j,k)}$. Terminate the algorithm.

In the remaining part of this section, an example of the Iranian railway line is represented.

5.2 Example 2: the case study

The railroad network of Iran is shown in Fig. 6(a). This network is divided into nine regions. The case that is studied contains the south, Lorestan, Arak, and some parts of Tehran regions depicted in Fig. 6(b).

There are 61 stations in the studied line, and 11 shunting yards among them can service shunting operations. The topography of the line and maximum length of stations changes in different parts. The distances among all shunting yards are shown in Fig. 7. Each shunting yard is addressed by a number.

We intend to form the necessary trains to transport 12 different compartments and assign the wagons of compartments to the trains. The characteristics of the compartments are specified in Table 5. The destinations of compartments numbered 4 and 12 are out of the considered line; therefore, in these cases, Tehran station is considered as their destination.

In Table 5, NI, I, and VI stand for non-important, important, and very important compartments, respectively. On the basis of topography alterations, the studied line can be divided into two parts. If the Andimeshk station is considered as the centre of the studied line, the southern part is almost flat and the stations located in this part have long internal tracks that can embed long trains, while the northern part is mountainous and the length of the internal track yards is limited. The characteristics of the studied railway line are shown in Table 6.

The cost of classification works for 1 h is assessed as 1.8 times the cost of one-hour short train running in the main line, which is estimated to be near 1000 units. The cost for the long train formation is 1.9 times greater than that of the short one. Furthermore, the fixed cost of trains and blocks that stop in the yards for classification works are appraised around 150 and 50 units per hour, respectively. The average speed of freight trains is about 40 km/h; hence, the cost of train movement in the main line is $1000/40 = 25$ per kilometre. One-hour classification works, performed by shunting locomotive and necessary operators, give services to 40 wagons. As a result, the classification cost of each wagon is $1800/40 = 45$ units per wagon. The cost related to the idle time of different wagons in yards is experimentally assessed to be 2, 5, and 10 units per hour for each wagon belonging in non-important, important, and very important compartments, respectively. It is supposed that at most only one of the compartments of each train is disturbed practically. Moreover, it is also assumed that at last nine short trains and nine long trains can be formed in the studied line.

Given the distances among all yards depicted in Fig. 7, the implementation of the proposed heuristic is described in the following section.

Fig. 6 (a) Railroad network of Iran and (b) the studied route

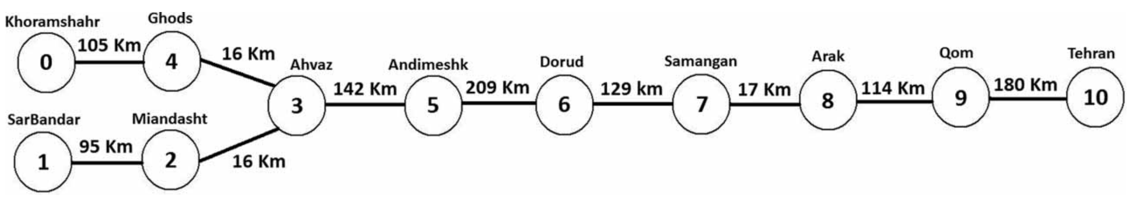

Fig. 7 Characteristics of the studied railway network

Table 5 Characteristics of compartments concerning the case study

Compartment	1	2	3	4	5	6	7	8	9	10	11	12
Origin	1	1	1	1	2	3	5	7	7	7	8	8
Destination	5	8	9	10(Tabriz)	4	10	10	8	9	10	9	10(Karaj)
Number of wagons	20	30	48	45	41	10	50	10	30	30	25	12
Maximum perturbation (number of wagons)	2	3	6	5	5	1	5	1	3	3	3	2
Mean weight	80	70	60	80	60	80	50	60	80	70	60	65
Maximum perturbation (mean weight)	8	7	6	8	6	8	5	6	8	7	6	6.5
Length	20	20	14	20	20	20	14	14	20	14	14	14
Importance factor	NI	NI	VI	NI	NI	NI	I	I	NI	NI	I	NI

Table 6 Maximum allowable capacity of trains in different parts of the network

From	To	Maximum length	Maximum tonnage
South part	Andimeshk	935	5000
Andimeshk	Tehran	595	1500

Table 7 Iterations of solving the case study

K	OF-1	OF-2	OF-3	CPU time
1	80 950	Infeasible {path (8, 9): 3 long +1 short trains}		2:15
2	81 100	Infeasible {path (8, 9): 2 long+3 short trains}		2:52
3	82 450	Infeasible {path (7, 8): 2 long+3 short trains}		8:48
4	82 450	21 965	264	11:29+30:00

5.3 Results of the case study

The model P1 is unable to even find a feasible solution in a reasonable amount of time. As a result, the proposed heuristic algorithm is applied to the case study. Table 7 shows the abstract of the iterations to solve the case study where OF stands for the objective function value.

The plan of train formation and the assignment of wagons to trains are depicted in Fig. 8.

In Fig. 8, the routes that are passed by trains are shown by lines. Moreover, the yards that are visited by trains for classification works are represented by circles. As can be observed, 13 trains must be formed to transport all the 12 compartments. The last four trains are long trains with two locomotives.

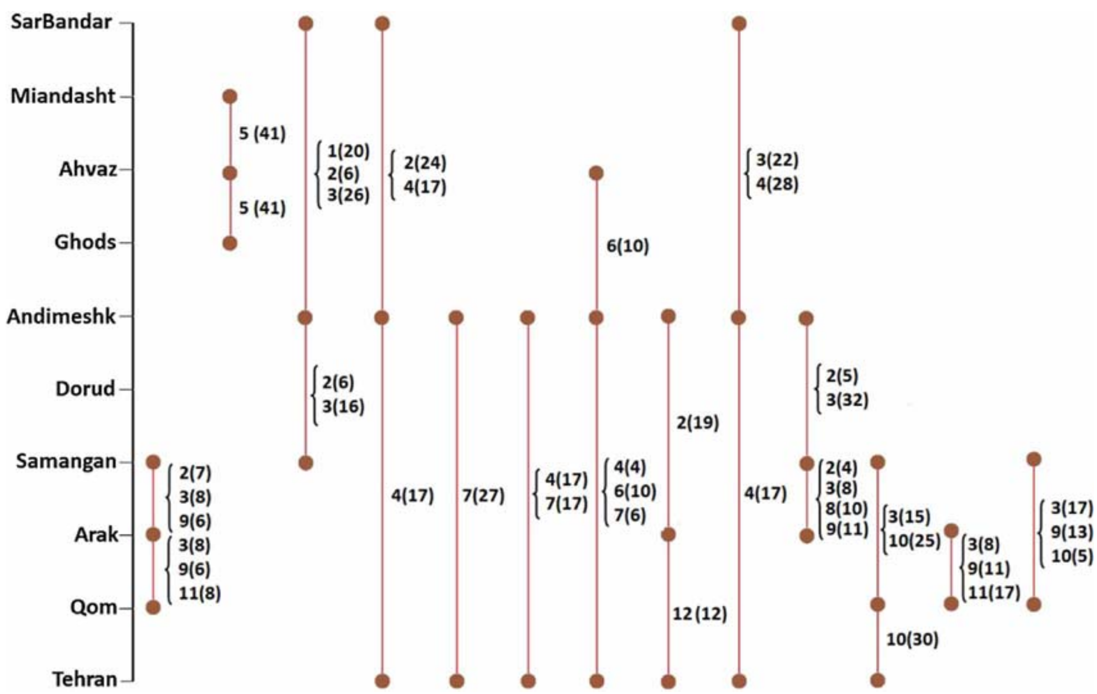

Fig. 8 TFP of the case study

6 CONCLUSIONS

In this article, train RMP, where the number and weight of wagons that belong to compartments were subject to disturbances, has been studied. A mathematical model for the train formation problem has been presented as a mixed integer problem. The proposed model of this article is able to solve not only the line railroads, but also the network railroads. It has been explained that any real-world applications deal with uncertain input data. Therefore, a robust approach has been applied, which not only effectively absorbs the noises in data but also has the ability of controlling the conservative level. A heuristic algorithm has been introduced to find some near-optimal solutions in a reasonable amount of time. The implementation of the proposed method of this article has been applied for the real-world case study in the Iranian railroad industry.

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APPENDIX 1

Notation

$a_{i(j,k)}$	a binary variable, 1 if train i pass path (j, k), and 0, otherwise
$at_{i(j,k)}$	a binary variable, 1 if train i is a long formed train that passes arc (j, k)
A	set of arcs (paths)
$b^p_{i(j,k)}$	a binary variable which is 1, if at least one of the wagons of compartment p is transported by train i in path (j, k), and 0, otherwise
cb_j	constant classification cost in yard j for a block, a subset of compartment
cc_j	classification cost in yard j for a wagon
ci^p	one-hour idle time cost of each wagon that belongs in compartment p
cl_{jk}	long train formation cost in arc (j, k)
ct_j	constant classification cost in yard j for a train
cs_{jk}	short train formation cost in arc (j, k)
de_p	destination of compartment p

$\mathrm{exv}^h_{(j,k)}$ train length expanding variable for path (j,k) in hth iteration
$\mathrm{exw}^h_{(j,k)}$ train weight expanding variable for path (j,k) in hth iteration
l^p length of wagons that belong in compartment p
m_{jk} maximum allowable tonnage that can be hauled by a short train in arc (j,k)
N set of nodes (yards)
O_{ij} a binary variable, 1 if the origin of train i is yard j, and 0, otherwise
$\mathrm{ofv}_{\mathrm{best}}$ best found objective function value
ofv_h objective function value in hth iteration
or_p origin of compartment p
P set of compartments
r^p number of wagons that belong in compartment p
S_{ij} a binary variable, 1 if train i dwells in yard j for wagon separating and connecting works, and 0, otherwise
$\mathrm{Slv}_{(j,k)}$ mean of slack values related to the train's length which pass path (j,k) concerning constraint 8
$\mathrm{Slw}_{(j,k)}$ mean of slack values related to the train's weight which pass path (j,k) concerning constraint 9
$SSyS_{ti}$ an integer variable that shows the number of wagons that leave yard i by an existed formed train
$Sy^p_{i(k)}$ an integer variable that represents the quantity of wagons of compartment p that are disconnected from train i in yard k and wait for arrival of some already existing formed trains
SyS_i cost related to the idle time of wagons that wait in yard i for the arrival of an existing formed trains/the number of existing formed trains
t_i a binary variable, 1 if train i is a long one, and 0, otherwise
T set of trains
v_{jk} maximum length of a train in arc (j,k)
w^p mean weight of wagons that belong in compartment p
$y^p_{i,(j,k)}$ an integer variable that indicates the number of wagons of compartment p that are transported by train i for passing path (j,k)
$y'^p_{i(k)}$ an integer variable that represents the quantity of wagons of compartment p that are disconnected from train i in yard k and must be shipped to the next destination by other trains
$yy^p_{i(k)}$ an integer variable that represents the quantity of wagons of compartment p that are connected or disconnected to/from train i in yard k
$\lambda^h_{(j,k)}$ train length expanding coefficient for path (j,k) in hth iteration
$\mu^h_{(j,k)}$ train weight expanding coefficient for path (j,k) in hth iteration

APPENDIX 2

Generating MIP model

As stated in section 3, the objective functions are non-linear. How the functions can be linearized is shown here.

First objective function

Let rewrite equation (1) as follows

$$D_1 = \sum_{i\in T}\sum_{j\in N}\sum_{k\in N} cs_{jk}\times a_{i,(j,k)} + \sum_{j\in N}\sum_{k\in N}\left((cl_{jk}-cs_{jk})\sum_{i\in T} a_{i,(j,k)}\times t_i\right)$$

Note that $a_{i,(j,k)}$ and t_i are both binary variables. One can replace $a_{i,(j,k)}\times t_i$ by a new binary variable $at_{i(j,k)}$ such that the following if-condition is satisfied.

If $t_i = 1$ and $a_{i,(j,k)} = 1$, then $\mathrm{at}_{i,(j,k)} = 1$, else $\mathrm{at}_{i,(j,k)} = 0$.

The above if-condition can be replaced with the following constraints

$$\mathrm{at}_{i,(j,k)} \geqslant a_{i,(j,k)} + t_i - 1$$
$$\mathrm{at}_{i,(j,k)} \leqslant \frac{a_{i,(j,k)} + t_i}{2}$$

Second objective function

One can easily replace

$$\left|\sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)}\right|$$

by a new integer variable $yy^p_{i(k)}$ such that

$$-yy^p_{i(k)} \leqslant \sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)} - \sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} \leqslant yy^p_{i(k)}$$

Third objective function

First replace

$$\max\left[\left(\sum_{h=k+1}^{\mathrm{de}_p} y^p_{i(k,h)} - \sum_{j=\mathrm{or}_p}^{k-1} y^p_{i(j,k)}\right), 0\right]$$

with $y'^{p}_{i(k)}$. Therefore

$$y'^{p}_{i(k)} \geqslant \left(\sum_{h=k+1}^{\mathrm{de}_p} y^{p}_{i(k,h)} - \sum_{j=\mathrm{or}_p}^{k-1} y^{p}_{i(j,k)} \right)$$
$$y'^{p}_{i(k)} \geqslant 0$$

Second, consider the multiplication of the integer variable $y'^{p}_{i(k)}$ by the binary variable S_{ik}. A new integer variable $Sy^{p}_{i(k)}$ is defined such that the following if-condition is fulfilled.

If $S_{ik} = 1$ then $Sy^{p}_{i(k)} = y'^{p}_{i(k)}$, else $Sy^{p}_{i(k)} = 0$.

The above if-condition can be replaced with the following constraints

$$Sy^{p}_{i(k)} \leqslant M' \times S_{ik}$$
$$Sy^{p}_{i(k)} \leqslant y'^{p}_{i(k)}$$
$$Sy^{p}_{i(k)} \geqslant y'^{p}_{i(k)} - M'(1 - S_{ik})$$
$$Sy^{p}_{i(k)} \geqslant 0$$

where M' is a relatively big positive number with $M' \geqslant \max\{r^p; p \in P\}$. Now, consider the integer variable $Sy^{p}_{i(k)}$, which is divided by a summation of binary variables $\sum_{i\in T} S_{ik}$. In this case, a new real variable SyS_k is introduced, which is replaced with

$$\frac{\sum_{i\in T}\sum_{p\in P}(ci^p \times Sy^{p}_{i(k)})}{\sum_{i\in T} S_{ik}}$$

To that end, the following constraint must be satisfied

$$\sum_{i\in T}\sum_{p\in P}(ci^p \times Sy^{p}_{i(k)}) = \sum_{i\in T} S_{ik} \times SyS_k$$

The right-hand side of the above constraint is a multiplication of the real variable SyS_k by a summation of binary variables $\sum_{i\in T} S_{ik}$.

To make the constraint linear, one can replace $\sum_{i\in T} S_{ik} \times SyS_k$ by $\sum_{i\in T} SSyS_{ik}$ such that $SSyS_{ik}$ is a real variable such that the following if-condition is satisfied

If $S_{ik} = 1$, then $SSyS_{ik} = SyS_k$, else $SSyS_{ik} = 0$.

The above if-condition can be replaced with the following constraints

$$SSyS_{ik} \leqslant M \times S_{ik}$$
$$SSyS_{ik} \leqslant SyS_k$$
$$SSyS_{ik} \geqslant SyS_k - M(1 - S_{ik})$$
$$SSyS_{ik} \geqslant 0$$

where M is a big positive number with $M \geqslant \sum_{p\in P} r^p$.

Moreover

$$12 \times \frac{\sum_{i\in T} S_{ik} + 1}{\sum_{i\in T} S_{ik}} \times \sum_{i\in T}\sum_{p\in P}\left\{\max\left[\left(\sum_{h=k+1}^{\mathrm{de}_p} y^{p}_{i(k,h)} - \sum_{j=\mathrm{or}_p}^{k-1} y^{p}_{i(j,k)}\right), 0\right] \times S_{ik}\right\}$$

must be equal to 0, if $\sum_{i\in T} S_{ik} = 0$; therefore, the following constraint must hold

$$SyS_k \leqslant M \times \sum_{i\in T} S_{ik}$$

APPENDIX 3

Robust counterpart of constraint (9)

The robust counterpart of inequality (8) is represented in section 5. Noise in w^p in addition to r^p is considered. Trains can haul a limited tonnage in each path based on the topography of the path and also the hauling power of trains. In practice, the weight of trains may differ from the amount which is considered in the planning stage. As a result, if the total tonnage that must be transported in a certain path with a particular train exceeds the allowable level, the movement of trains will be affected by the disturbances and many other relative problems will occur.

To avoid such malfunctions, inequality (9) has been enriched using the robust procedure introduced by Bertsimas and Sim explained in section 5. Similar to constraint (8), one can re-write constraint (9) as inequality (35)

$$\sum_{p\in P}(w^p \times r^p \times x^{p}_{i(j,k)}) <= m_{jk} \times (1 + t_i) \qquad (35)$$

Inequality (9) can be protected against disturbances by adding the protection function shown by statement (36) into the left-hand side of inequality (9)

$$\max_{\{F'_{i(j,k)} \cup \{e'_{i(j,k)}\} | F'_{i(j,k)} \subseteq J'_{i(j,k)}, |F'_{i(j,k)}| = \lfloor \Gamma'_{i(j,k)} \rfloor, e'_{i(j,k)} \in J'_{i(j,k)}, e'_{i(j,k)} \notin F'_{i(j,k)}\}} \times \left\{\left(\sum_{j\in F'_{i(j,k)}} \hat{w}^p \times \hat{r}^p \times x^{p}_{i(j,k)}\right) + (\Gamma'_{i(j,k)} - \lfloor \Gamma'_{i(j,k)} \rfloor) \times \hat{w}^{e'_{i(j,k)}} \times \hat{r}^{e'_{i(j,k)}} \times x^{e'_{i(j,k)}}_{i(j,k)}\right\} \qquad (36)$$

Note that since $x^{p}_{i(j,k)} \geqslant 0$, there is no need to consider its absolute value in statement (36). The protection function (36) equals the objective function of the following linear optimization problem

$$\begin{aligned} &\max \quad \sum_{j\in J_i} \hat{w}^p \hat{r}^p x^{*p}_{i(j,k)} z''^p_{i(j,k)} \\ &\text{subject to} \quad \sum_{p\in J_{i(j,k)}} z''^p_{i(j,k)} \leqslant \Gamma_{i(j,k)} \\ &\qquad 0 \leqslant z''^p_{i(j,k)} \leqslant 1 \quad \forall p \in J_{i(j,k)} \end{aligned} \tag{37}$$

The dual of model (37) is equal to

$$\begin{aligned} &\min \Gamma'_{i(j,k)} z'''_{i(j,k)} + \sum_{p\in J_{i(j,k)}} g'^p_{i(j,k)} \\ &\text{subject to} \quad z'''_{i(j,k)} + g'^p_{i(j,k)} \geqslant \hat{w}^p \hat{r}^p x^{*p}_{i(j,k)}, \quad \forall p \\ &\qquad g'^p_{i(j,k)} \geqslant 0, \quad \forall p \\ &\qquad z'''_{i(j,k)} \geqslant 0, \quad \forall i, j, k \end{aligned} \tag{38}$$

Finally, the robust counterpart of inequality (9) is obtained from the inducing model (38) in inequality (9). Therefore, inequalities (39) to (42) make the robust counterpart of inequality (9)

$$\sum_{p\in J_{i(j,k)}} \bar{w}^p y^p_{i(j,k)} + \Gamma'_{i(j,k)} z'''_{i(j,k)} + \sum_{p\in J_{i(j,k)}} g'^p_{i(j,k)} \leqslant m_{jk} \times (1 + t_i), \quad \forall p, i, j, k \tag{39}$$

$$z'''_{i(j,k)} + g'^p_{i(j,k)} \geqslant \hat{w}^p \times \frac{\hat{r}^p}{r^p} \times y^p_{i(j,k)}, \quad \forall p, i, j, k \tag{40}$$

$$g'^p_{i(j,k)} \geqslant 0, \quad \forall p, i, j, k \tag{41}$$

$$z'''_{i(j,k)} \geqslant 0, \quad \forall i, j, k \tag{42}$$

Equation (43) shows the proposed formula for the protection parameter

$$\Gamma'_{i(j,k)} = \beta \times \sum_{p\in P} b^p_{i(j,k)} \tag{43}$$

where β is called the conservatism factor and has a similar role as that of α in equation (28).