Estimates of Heat Transfer and Pressure Drop Characteristics of the Fractal Heat Exchanger

Introduction

Any new heat exchanger analysis should include the heat transfer and pressure drop analysis. Estimates will be done on the fractal heat exchanger to find the approximate Nusselt number and friction factors. For established heat exchanger configurations there are many correlations for the Nusselt number and friction factors to choose from. Since the fractal heat exchanger is unique, the existing equations need to be modified and even then, it has to be kept in mind that the equations are only an estimate and additional data should supplement it. Once the correlations for heat transfer and pressure drop were found, the performance of the heat exchanger can be established.

In this chapter the fractal heat exchanger was analysed. To find the correct analysis method the compactness of the exchanger will be established first, after which the analysis method would be applied to the exchanger to find the relative increase in heat transfer compared to an ordinary heat exchanger.

Geometry of the Fractal Heat Exchanger

Figure 1 is a representation of the fractal heat exchanger after two applications of the fractal.

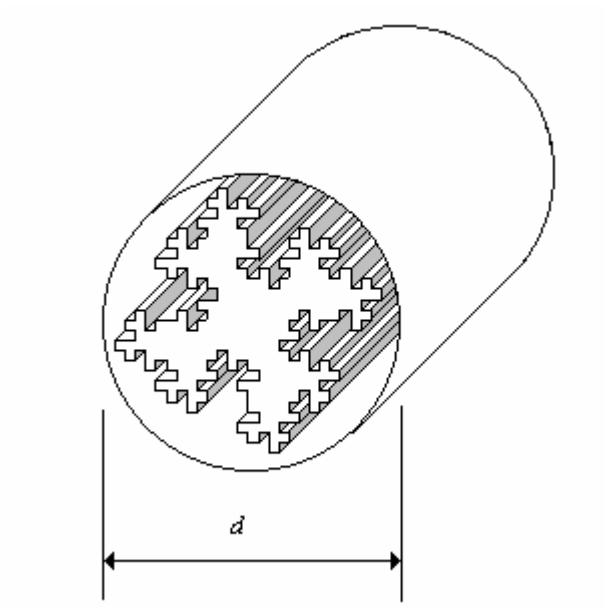


Figure 1: Fractal heat exchanger, $n = 2$

The fractal heat exchanger will be compared to the benchmark heat exchanger, which is a square tube in round tube heat exchanger as shown below in Figure 2.

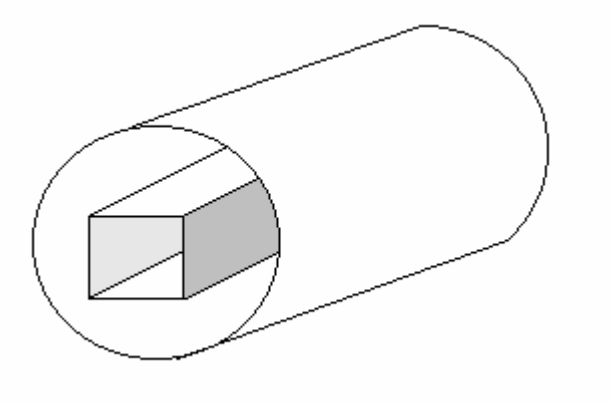


Figure 2: Square tube-in-tube exchanger

Consider the inner tube in Figure 3.

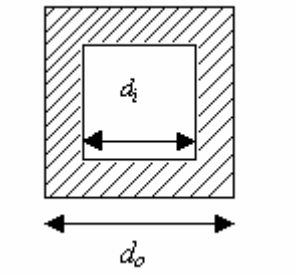


Figure 3: Inner tube dimensions

The inner tube has an outside side length of d_o and an inside length of d_i. Let *Dd* be the thickness of the inner tube and be defined as:

$$\Delta d = \frac{d_o - d_i}{2} \qquad [1]$$

The fractal will be applied to the inner tube of the heat exchanger.

With each application of the fractal to the inner tube, the inner tube becomes a little bit larger. Since the inner tube must fit in the outer tube, this must be taken into account when choosing a size for the outer tube.

To calculate the dimension of the outer tube, it must be first establish what the total length of the inner tube will be after the fractal has been applied several times. The length of the inner tube in the limit is given as equation [15] from the chapter, *Mathematical Analysis of the Quadratic Koch Island and the Koch Snowflake.*

$$X_o = \frac{5}{3} x_0 \qquad [2]$$

The tube inner diameter for the outer tube should be X_0 or larger and the outer diameter is X_0 + twice the thickness.

To calculate the Nusselt number and friction factors, the Reynolds number for the inner tube and annulus is needed. First the hydraulic diameters will be derived after which the Reynolds equations will be found.

The Reynolds number for the inner tube is calculated as follows:

$$\mathrm{Re}_i = \frac{\rho v_i d_{h,i}}{\mu} = \frac{\rho v_i d_i}{2^n \mu} \qquad [3]$$

with $d_{h,i}$ the hydraulic diameter for the inner tube and is calculated as

$$d_{h,i} = \frac{4A_i}{p_{w,i}} = \frac{4d_i^2}{4 \cdot 2^n d_i} = \frac{d_i}{2^n} \qquad [4]$$

The Reynolds number for the annulus is:

$$\mathrm{Re}_o = \frac{\rho v_o d_{h,o}}{\mu} = \frac{\rho v_o}{\mu}\left(\frac{\pi d^2 - 4d_o^2}{\pi d + 4 \cdot 2^n d_o}\right) \qquad [5]$$

with $d_{h,o}$ the hydraulic diameter for the outer tube and is calculated as

$$d_{h,o} = \frac{4A_o}{p_{w,o}} = \frac{4(\frac{1}{4}\pi d^2 - d_o^2)}{\pi d + 4 \cdot 2^n d_o} = \frac{\pi d^2 - 4d_o^2}{\pi d + 4 \cdot 2^n d_o} \qquad [6]$$

The basic equations for the fractal heat exchanger have been established and the heat transfer and pressure drop can now be estimated.

Heat Transfer

Compactness of the Fractal Heat Exchanger

A heat exchanger's surface compactness determines if it is classified as a compact heat exchanger or a non-compact heat exchanger. Compact heat exchangers have a large heat transfer area per unit exchanger volume and the advantages include decrease in space, weight, power requirements and costs. The flow arrangement is typical single-pass crossflow or counterflow and multipass cross-counterflow. Before the surface compactness is discussed, a couple of definitions of compact heat exchangers are given.

Gas-to-fluid compact heat exchanger: A gas-to-fluid exchanger with a heat transfer surface area density greater than 700 m^2/m^3 or with a hydraulic diameter smaller than 6 mm for the gas side and a surface area density greater than 400 m^2/m^3 for the liquid side.
Laminar flow heat exchanger (meso heat exchanger): Exchanger with a surface area density greater than 3 000 m^2/m^3 or a hydraulic diameter between 100 µm and 1 mm.
Micro heat exchanger: Exchanger with a surface area density greater than 15 000 m^2/m^3 or a hydraulic diameter between 1 µm and 100 µm.
Liquid/two phase compact heat exchanger: The surface area density on any fluid side is greater than 400 m^2/m^3.

The surface area density (α) calculations are shown below (Shah, 2000).

- For tube-fin and shell-and-tube exchangers:

$$a = \frac{A_h}{V_{total}} = \frac{A_c}{V_{total}} \qquad [7]$$

V_c and V_h are the volumes whose surface is taken up by the hot or cold fluid heat transfer surface area.

The above surface area densities can be used in the specific heat exchanger performance equation that is expressed below:

$$\frac{q}{\Delta T_m} = UA = UaV \qquad [8]$$

A high value of a reduces the exchanger volume for a specified heat exchanger performance.

For a tube-in-tube heat exchanger, equation number [7] will be used for a tube-in-tube arrangement. The outer diameter of the exchanger is 88.9 mm, the segment length of the exchanger is 10 mm and the length of the exchanger is 3.91 m.

V_{total} = *outer tube area* x *length of heat exchanger* [9]
$V_{total} = ¼ \times \pi \times 0.0889^2 \times 3.91$
$V_{total} = 0.02427\ m^3$

$A_c = A_h$ = *heat transfer circumference*[1] x *length of exchanger* [10]
$A_c = A_h = 4(0.01 \times 2^n) \times 3.91$
$A_c = A_h = 0.1564 \times 2^n\ m^2$

$$a = \frac{0.14 \cdot 2^n}{0.02173} = 6.444 \cdot 2^n \qquad [11]$$

For an exchanger to be compact the value of a should be higher than $400 m^2/m^3$:
$a_0 = 6.444\ m^2/m^3$
$a_1 = 12.888\ m^2/m^3$
. . .
$a_5 = 206.208\ m^2/m^3$
$a_6 = 412.416\ m^2/m^3$

Thus a fractal heat exchanger with six or more iterations is a compact heat exchanger and the e – NTU method should be used to analyse it. A fractal heat exchanger with five or less iterations is non-compact and the log-mean temperature difference method will be used to analyse it.

Heat Transfer Analysis

The heat transfer analysis is dependent on the type of heat flow. Examples of different flow arrangements are parallel flow, counterflow and crossflow. The arrangement

[1] Equation [7] from the chapter *Mathematical Analysis of the Quadratic Koch Island and the Koch Snowflake*

with the best heat transfer is counterflow. Below is the set up for counterflow where the hot water flows in the inner tube and the cold water in the outer tube.

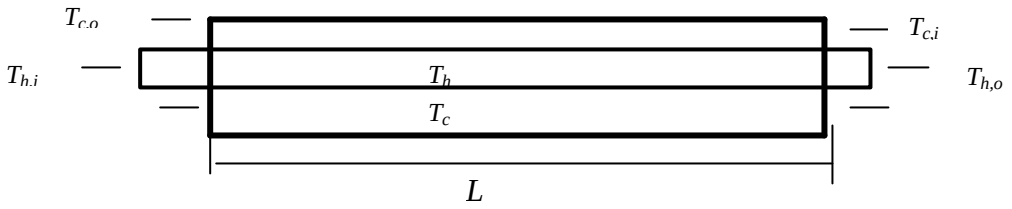


Figure 4: Counterflow configuration

The heat transfer analysis for direct transfer type exchangers will be presented in this section. To do a heat transfer analysis, the energy balance need to be derived for a simple system.

The first law of thermodynamics can be applied to the above system. The first law is simplified by assuming that the flow is steady state and that the system is overall adiabatic.

$$dq = q''dA = -(\dot{m}c_p)_h\, dT_h = -(\dot{m}c_p)_c\, dT_c \qquad [12]$$

$$dq = q''dA = -C_h dT_h = -C_c dT_c \qquad [13]$$

There is a negative sign because the temperatures increase as the area decreases. To find the heat transfer, the above equation can be integrated over the whole area:

$$q = \int_A q''dA = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}) \qquad [14]$$

Another expression for the heat transfer can be found by considering the first law on a local basis:

$$q'' = \frac{dq}{dA} = U(T_h - T_c)_{local} = U\Delta T \qquad [15]$$

From the above the heat transfer can be expressed in terms of mean quantities:

$$q = U_m A \Delta T_m \qquad [16]$$

where ΔT_m is the true mean temperature difference (TMTD).

To find an expression for U the different resistances of heat flow need to be considered:

- Hot-side film-convection resistance (convective thermal resistance between the flowing fluid and the heat transfer surface)

$$R_h = \frac{1}{(h_o hA)_h} \qquad [17]$$

- Hot-side scale resistance (resistance due to fouling)

$$R_{s,h} = \frac{1}{(h_o h_s A)_h} \quad [18]$$

- Wall thermal resistance

$$R_w = \frac{\Delta d}{k_w A} \quad [19]$$

- Cold-side scale resistance

$$R_{s,c} = \frac{1}{(h_o h_s A)_c} \quad [20]$$

- Cold-side film-convection resistance

$$R_c = \frac{1}{(h_o h A)_c} \quad [21]$$

h_o is the total surface temperature effectiveness of an extended (fin) surface. For prime surfaces $h_o = 1$.

Equation [16] can be rewritten as

$$q = UA(T_h - T_c) \quad [22]$$

where $$\frac{1}{UA} = \frac{1}{(h_o h A)_h} + \frac{1}{(h_o h_s A)_h} + \frac{\Delta d}{p_w A} + \frac{1}{(h_o h_s A)_c} + \frac{1}{(h_o h A)_c} \quad [23]$$

Dimensional analysis will be used to complete the heat transfer analysis because it is difficult to work with so many variables, it will reduce design time and the results can be presented graphical. The dimensional groups are the effectiveness-number of transfer units method (e-NTU or P-NTU) and the log-mean temperature difference method (*LMTD*). To use dimensional analysis the following idealisations are made:

- The flow is steady state.
- The entrance velocity and temperature are uniform.
- Uniform flow rate.
- Uniform temperature over every cross section.
- Specific heat capacity of each fluid is constant throughout the exchanger.
- Overall heat transfer coefficient between fluids is constant.
- Uniformly distributed heat transfer area.
- No phase change (or the change must occur under certain conditions) – this research will assume no phase change.
- Longitudinal heat conduction in fluid and wall is negligible.
- Heat losses to surroundings are negligible.
- No thermal energy sources in heat exchanger.

The log-mean temperature difference represents the maximum temperature potential for heat transfer that can only be obtained in a counterflow heat exchanger. To find an expression for the heat transfer, the *LMTD* (log-mean temperature difference) method

will be used (Kakaç *et al.*, 1983), because only the non compact iterations will be studied in this research.

Definition of log-mean temperature difference is given below:

$$LMTD = \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2} \quad [24]$$

For a counterflow heat exchanger

$$DT_1 = T_{h,i} - T_{c,o} \quad [25]$$

$$DT_2 = T_{h,o} - T_{c,i} \quad [26]$$

For any other flow arrangement (except parallel flow which has different equations for DT_1 and DT_2) the heat exchanger is hypothetically considered as a counterflow arrangement with the same terminal temperatures. *LMTD* is thus calculated using equations [24], [25] and [26].

For a counterflow arrangement the arithmetic mean and log-mean temperature values are the same:

$$DT_m = DT_{lm} \quad [27]$$

The heat transfer from the inner tube to the outer tube is given as

$$q = UADT_{lm} \quad [28]$$

where U is the overall heat transfer coefficient and is shown below. Equation [23] was used, but the scale resistance terms were ignored.

$$U_i A_i = \frac{1}{\frac{1}{h_i A_i} + \frac{\Delta d}{4k2^n d_i L} + \frac{1}{h_o A_o}} \quad [29]$$

where

$$A_i = 4d_i 2^n L \quad [30]$$

$$A_o = 4d_o 2^n L \quad [31]$$

Substituting equations [30] and [31] into [29] and simplifying, the following is found:

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{\Delta d}{k} + \frac{d_i}{h_o d_o}} \quad [32]$$

Thus the heat transfer (equation [28]) becomes

$$q_i = U_i A_i DT_{lm}$$
$$= U_i 4d_i 2^n L DT_{lm}$$
$$= 2^n [U_i 4d_i L DT_{lm}]$$
$$= 2^n [q_i¢] \quad [33]$$

where $q_i¢$ is the heat transfer without the fractal application.

From equation [33] it follows that an increase of 100% in heat transfer can be expected for a fractal heat exchanger with each iteration.

In the heat transfer analysis it was assumed for simplicity purposes that there was no dependency of the convection coefficient (h) on the amount of iterations (n). The correlation between the Nusselt number and the convection coefficient was used to establish how this assumption should be modified and whether experimental data was needed. For the following correlations it was assumed that the flow was fully developed and turbulent.

To calculate the Nusselt number the Dittus-Boelter correlation for a circular duct was used (Holman, 1992) in the absence of a more appropriate correlation.

$$Nu_i = 0.023 \cdot \mathrm{Re}_i^{0.8} \mathrm{Pr}_i^{0.3} \quad [34]$$

$$Nu_o = 0.023 \cdot \mathrm{Re}_o^{0.8} \mathrm{Pr}_o^{0.4} \quad [35]$$

Replacing the above Reynolds numbers with the values of equations [3] and [5], the following values for the Nusselt numbers were found:

$$Nu_i = 0.023 \cdot \left(\frac{r v_i d_i}{2^n m} \right)^{0.8} \mathrm{Pr}^{0.3} \quad [36]$$

$$Nu_o = 0.023 \cdot \left(\frac{r v_o}{m} \cdot \frac{p d^2 - 4 d_o^2}{p d + 4 \cdot 2^n d_o} \right)^{0.8} \mathrm{Pr}^{0.4} \quad [37]$$

Thus

$$Nu_{i,o} \propto 2^{-0.8n} \quad [38]$$

The correlation between the convective coefficient and the number of iterations are shown below:

$$Nu_i = \frac{h_i d_i}{2^n k} \propto 2^{-n} h_i(n) \quad [39]$$

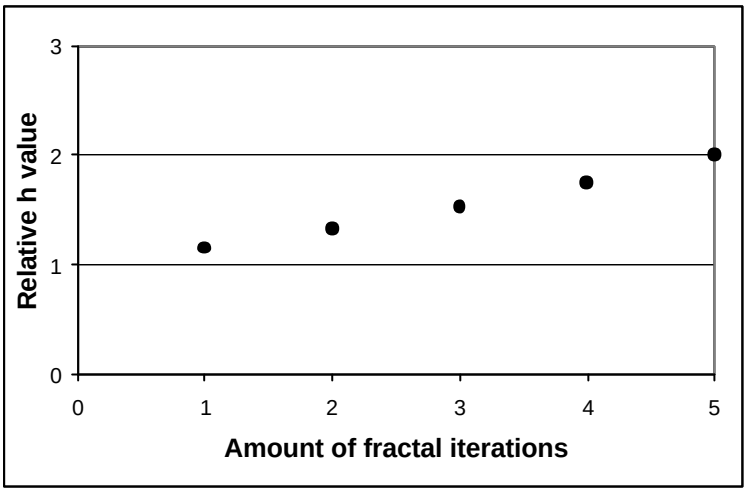
$$Nu_o = \frac{h_o}{k} \left(\frac{p d^2 - 4 d_o^2}{p d + 4 \cdot 2^n d_o} \right) \propto 2^{-n} h_o(n) \quad [40]$$

Comparing equation [38] with [39] and [40] it was concluded that there seemed to be a dependency of the convective coefficient on the number of iterations, thus

$$Nu_{i,o} \propto 2^{-n} \cdot h(n) \propto 2^{-0.8n} \quad [41]$$

$$h_{i,o} \propto 2^{0.2n} \quad [42]$$

Therefore the heat transfer coefficient will increase exponentially by 0.2 with every fractal iteration as shown in Graph 1.



Graph 1: Increase of the heat transfer coefficient

Pressure Drop Analysis

Friction Factor Correlations

The amount of effort needed to pump the fluid through the exchanger is dependent on the pressure drop of the heat exchange system. To establish the pressure drop, correlations are needed for the friction factor. There are many friction factor correlations depending on the type of flow and for this analysis it will be assumed that the flow is fully developed. The analysis will be done for laminar and turbulent flow.

Laminar

For laminar flow, the friction factor relationship of a smooth tube is used in the absence of a more appropriate relationship.

$$f_l = \frac{64}{\mathrm{Re}} \qquad [43]$$

By substituting the Reynolds equations [3] and [5] the friction factor is found. For the inner tube the friction factor is expressed as:

$$f_{l,i} = \frac{64 \cdot 2^n m}{r v_i d_i} \qquad [44]$$

The friction factor expression for the outer tube is:

$$f_{l,o} = \frac{64m}{r v_o}\left(\frac{pd + 4 \cdot 2^n d_o}{pd^2 - 4d_o^2}\right) \qquad [45]$$

For both the inner and outer tubes, the friction factor is dependent on the amount of iterations and doubles with every fractal iteration:

$$f_l \propto 2^n \qquad [46]$$

Turbulent

There are several correlations for the Nusselt number and friction factors in the literature. The rectangular configuration shown below in Figure 5 will be used to derive correlations for the friction factor and Nusselt number.

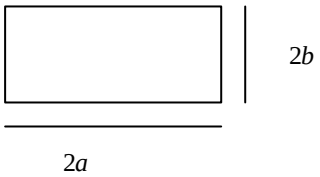


Figure 5: Rectangular configuration

The hydraulic diameter and aspect ratio are defined below:

Hydraulic diameter:

$$d_h = \frac{4ab}{a+b} \qquad [47]$$

Aspect ratio:

$$a^* = \frac{2b}{2a} \qquad [48]$$

$$\frac{d_l}{d_h} = \frac{2}{3} + \frac{11}{24} a^* (2 - a^*) \qquad [49]$$

The following equations are recommended by Shah and Bhatti (Kakaç *et al.*, 1988):

To calculate the friction factor:

- Use the above calculated d_l in the circular duct equation:

  $$f = A + \frac{B}{\mathrm{Re}^{\frac{1}{m}}} \qquad [50]$$

  Where the values for *A*, *B* and *m* are shown in Table 1.

	2 100 < Re < 4 000	4 000 < Re < 10^7
A	0.0054	1.28E-3
B	2.3E-8	0.1143
m	-⅔	3.2154

Table 1: Values for the constants *A*, *B*, and *m*

If the Reynolds number is larger than 4 000, the values of *A*, *B* and *m* are:
A = 1.28E-3
B = 0.1143

$m = 3.2154, \Rightarrow \frac{1}{m} = 0.311$

To use the friction equation [50] for a square configuration, d_h must be substituted with the d_l calculated below:

$$\frac{d_l}{d_h} = \frac{2}{3} + \frac{11}{24} a^* (2 - a^*) \qquad [51]$$

Another parameter that needs to be taken in account is the aspect ratio. The aspect ratio, a^* is the ratio of one side length to another. Since the shape of the inner tube is a square the aspect ratio is equal to one. Therefor equation [51] reduces to

$$\frac{d_l}{d_h} = \frac{19}{12} \qquad [52]$$

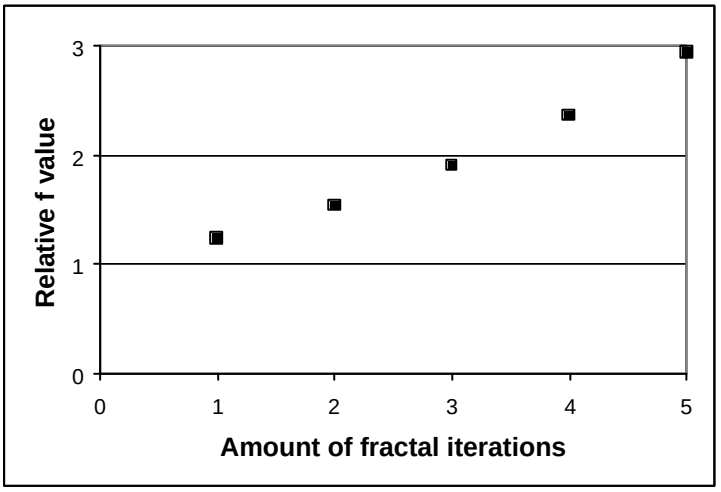
The friction factors for the inner and outer tubes are:

$$f_{t,i} = 1.28E-3 + 0.1143 \left(\frac{r v_i d_i {}^{19}\!/\!_{12}}{2^n m} \right)^{-0.311} \qquad [53]$$

$$f_{t,o} = 1.28E-3 + 0.1143 \left(\frac{r v_o {}^{19}\!/\!_{12}}{m} \left(\frac{pd^2 - 4d_o^2}{pd + 4 \cdot 2^n d_o} \right) \right)^{-0.311} \qquad [54]$$

$$f_t \propto \left(\frac{1}{2^n} \right)^{-0.311} = \left(2^{-n} \right)^{-0.311} = 2^{0.311n} \qquad [55]$$

From equation [55] it can be seen that the friction factor will increase exponentially by 0.311 with an increase in the amount of iterations. This tendency is shown in Graph 2.



Graph 2: Increase of the friction factors

Pumping Power

Since any advantage that is obtained with the heat transfer should be weighed against any rise in the pressure drop, an analysis of a heat exchanger is not complete without a pressure drop analysis. There are mainly two reasons for determining the pressure drop of a system:

1. The fluids need to be pumped through the exchanger, thus pumping power is required which is directly proportional to the pressure drop.
2. The heat transfer rate can be influenced by the saturation temperature change for a condensing/evaporating fluid for a large pressure drop.

The fluid pumping power is the product of the mass flow rate and pressure drop divided by the fluid density (White, 1986):

$$P = \frac{\dot{m}\Delta p}{\rho} \qquad [56]$$

The pumping power needed to pump the fluid through the exchanger is also an important design parameter. The improvement of the design in terms of the increase in heat transfer can be counter acted by the increase in pumping power. The amount of pumping power increases with an increase in the pressure drop. The pressure drop inside the exchanger is dependent on the amount of fractal iterations. The pressure drop can be calculated (Holman, 1992) using the following equation:

$$\Delta p = f \frac{L}{d_h} \rho \frac{v^2}{2} \qquad [57]$$

The pressure drop for the inner and outer tube can then be calculated by substituting equations [4], [6], [44] and [45] into [57]. The resulting equations are as follows:

The pressure drop for laminar flow for the inner and outer tube respectively is:

$$\Delta p_{l,i} = \frac{32 \cdot 2^{2n} \mu L v_i}{d_i^2} \propto 2^{2n} \qquad [58]$$

$$\Delta p_{l,o} = 32 \mu L v_o \frac{(\pi d + 4 \cdot 2^n d_o)^2}{(\pi d^2 - 4 d_o^2)^2} \propto 2^{2n} \qquad [59]$$

For laminar flow the pressure drop increases four times with every increase in the amount of iterations.

The pressure drop expression for the turbulent flow is calculated by substituting equations [4], [6], [53] [54] into [57]. The following is found for the inner and outer tube respectively:

$$\Delta p_{t,i} = (1.28E-3+0.1143\left(\frac{\rho v_i d_i {}^{19}\!/\!_{12}}{2^n \mu}\right)^{-0.311}) \cdot \frac{L \rho v_i^2}{2} \cdot \frac{2^n}{d_i} \propto 2^{1.311n} \qquad [60]$$

$$\Delta p_{t,o} = (1.28E-3+0.1143\left(\frac{\rho v_o {}^{19}\!/\!_{12}}{\mu}\left(\frac{\pi d^2 - 4 d_o^2}{\pi d + 4 \cdot 2^n d_o}\right)\right)^{-0.311}) \cdot \frac{L \rho v_o^2}{2} \cdot \frac{\pi d + 4 \cdot 2^n d_o}{\pi d^2 - 4 d_o^2} \propto 2^{1.311n} \qquad [61]$$

The pressure drop increases with an increase in the amount of iterations for turbulent flow as well.

Substituting equations [60] or [61] into the power equation [56], noting that $\frac{\dot{m}}{r}$ is independent of the fractal iteration, n (assuming that the velocity stays the same), the following was found:

$$P_t \propto 2^{1.311n} = 2^{1.311n} \qquad [62]$$

The above equations shows that pressure drop increases exponentially by 1.311 with an increase in the amount of iterations.

Conclusions

- The fractal heat exchanger is non-compact for $n = 0$ to $n = 5$ and compact for all values of n greater than 5.
- The *LMTD* method was used to analyse the non-compact fractal heat exchanger. The method only gave an estimate of the expected results.
- The heat transfer analysis showed than an increase of 100% can be expected over the benchmark heat exchanger for each iteration.
- The convective coefficient increases slightly with an increase in the amount of iterations.
- The pressure drop increases with the amount of iterations, n.

An analytical heat transfer analysis was done to a quadratic fractal heat exchanger. It was found that the heat transfer and the heat transfer coefficient increase independently of each other with every application of the fractal. The heat transfer at least doubled with every fractal iteration, while the pumping power increased at almost one and a half the rate.

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Nomenclature

a, A	Constant
A	Area [m^2]
b, B	Constant
c_p	Specific heat at constant pressure [kJ/kgK]
C	Heat capacity rate [W/K]
d	Inner width of outer tube [m]
d_i	Inner width of inner tube [m]
d_o	Outer width of inner tube [m]
Dd	Thickness on inner tube [m]
d_h	Hydraulic diameter [m]
d_l	Rectangular diameter [m]
f	Friction factor [ ]
h	Convective coefficient [W/m^2K]
k	Thermal conductivity [W/m^2K]
L	Length of heat exchanger [m]
$LMTD$	Log-mean temperature difference
m	Constant
$\dot{m}$	Mass flow rate [kg/s]
n	Amount of times fractal is applied
N_t	Number of tubes
NTU	Number of transfer units
Nu	Nusselt number [ ]
Dp	Pressure drop [Pa]
P	Power [W]
Pr	Prandtl number [ ]
p_w	Perimeter evaluated at wall conditions [m]
q	Heat transfer [W]
$q"$	Heat flux (heat transfer rate per unit surface area) [W/m^2]

R	Resistance [K/W]
Re	Reynolds number
T	Temperature [K]
DT_m	True mean temperature difference [K]
DT_{lm}	Log-mean temperature difference [K]
U	Overall heat transfer coefficient [W/m^{2}K]
v	Velocity [m/s]
V	Volume [m^3]
x_o	Length [m]
X_o	Total length of side [m]

Greek Letters

a^*	Aspect ratio
a	Compactness [m^2/m^3]
e	Effectiveness
m	Dynamic viscosity [Pa·s]
r	Density [kg/m^3]
h_0	Total surface temperature effectiveness of an extended surface

Subscripts

c	Cold
h	Hot
i	Inlet
i	Inner
l	Laminar
o	Outer
o	Outlet
s	Scale
t	Turbulent
w	Wall