

Adjusting the Vibratory Response of a Micro Mirror via Position and Velocity Feedback

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Abstract

We changed the vibratory (dynamic) response of a micro mirror experimentally using a feedback circuit. For this purpose, we first measured the vibrational velocity of the mirror using a Laser Doppler Vibrometer and then multiplied it with a gain and fed the signal back to the mirror to change its effective damping. We also investigate the influence of velocity and position feedback on the dynamic response of the same mirror through numerical simulations. For this purpose, first, a transfer function of the mirror was obtained based on the experimental frequency sweep data of the first two vibration modes. Then, the dynamic response of the mirror was investigated for different feedback gains via simulations. The results of our study show that the vibration amplitude or settling time of the mirror can be adjusted by altering the velocity feedback gain and the coupled vibration modes of the mirror can be separated from each other by adjusting the position feedback gain.

Keywords

Dynamic properties of micro systems, feedback control

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1. Introduction

Many compact visual displays or scanning devices utilize micro mirrors rotating about one or two axes. Also, for several applications utilizing optical switches, projectors, head-up and head-worn displays, barcode readers, endoscopic cameras, the dynamical characteristics of micro mirrors play a crucial role in the output performance of these systems (Urey, 2003; Changho et al., 2006; Yalcinkaya et al., 2007). The dynamical response of a micro mirror is related to its energy storage (i.e. stiffness) and dissipation capacity (i.e. damping), which in turn may effect the scanning range and resolution, the transient response and signal to noise ratio.

In today's practice, a micro mirror is specifically designed and manufactured based on the requirements of an application. However, achieving an optimum design that satisfies both the space constraints and the dynamical requirements of the application is not trivial and still the subject of current research. Even if this is achieved, the same mirror may not be easily used in another application having different dynamical requirements.

In this study, a velocity and position feedback system is introduced in order to change the effective damping and the

stiffness characteristics of a micro mirror. The mirror used in this study is actuated by Lorentz's forces created by an external magnetic field and an alternating current (see Figure 1). The mirror is used as a scanner to deflect a modulated light beam on an image plane to create a display. The scanning angle, operation frequency and quality factor are among the design parameters of the micro scanners that should be highly taken into consideration.

If the micro mirror is modeled as a simple spring-mass-damper system, its equation of motion can be written as

$$\ddot{\theta} + b\dot{\theta} + k\theta = T_o \cos(\omega_R t) \quad (1)$$

$$\omega_n = \sqrt{\frac{k}{I}}$$

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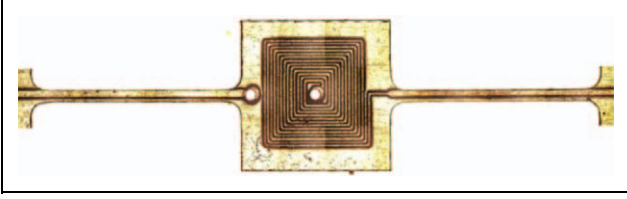


Figure 1. Electromagnetically actuated micro mirror.

where θ is the mechanical rotation angle of the mirror, I , b , k and Q are the effective moment of inertia, damping coefficient, and spring constant of the mirror for the n^{th} vibration mode, respectively. ω_n is the natural frequency at that mode, ω_R is the excitation frequency, and T_o is the amplitude of the external driving torque. The Q-factor of the micro mirror is defined as $Q \cong \frac{\omega_n}{\omega_2 - \omega_1} \cong \frac{I\omega_n}{b}$ where ω_1 and ω_2 are the corresponding frequencies at the half power points (Rao, 2004). The Q factor of a mirror is an indicator of its energy dissipation capacity and is inversely proportional to the damping present in the mirror. A mirror with high Q factor (low damping) dumps its energy slower, resulting in higher-amplitude steady-state oscillations and a sharp resonance curve. On the other hand, low Q factors are desired for improved transient response with shorter settling times (Maithripala et al., 2005). Changes in damping and stiffness of a mirror directly affect its Q-factor and the resonance frequency, respectively. In order to alter the dynamics of effectively, the modal characteristics of the mirror must be well understood. For that purpose, before implementing the velocity and position feedback control, we dynamically characterize the micro scanner using experimental modal analysis techniques and identify the natural frequencies and the mode shapes of the system (Ververi and Basdogan, 2007). Dynamic characterization technique of the micro systems and part of the experimental system used in this study was developed in an earlier study (Anac and Basdogan, 2008). In this paper, we investigate the influence of velocity and position feedback on the dynamic response of the mirror through experimental and numerical studies. For a velocity and position feedback signal, the equation of motion of the system can be written as

$$\begin{aligned}
 I\ddot{\theta} + b\dot{\theta} + k\theta &= T_o \cos(\omega_R t) + G\dot{\theta} + H\theta \\
 I\ddot{\theta} + (b - G)\dot{\theta} + (k - H)\theta &= T_o \cos(\omega_R t) \\
 k^* &= (k - H) \\
 \omega_n^* &= \sqrt{\frac{(k - H)}{m}} \\
 Q^* &= \frac{I\omega_n^*}{(b - G)}
 \end{aligned} \tag{2}$$

where G and H determine the amount of velocity and position feedback gain, respectively. Equation (2) shows that pure velocity feedback ($H = 0$) only modifies the Q factor of the system, whereas any position feedback leads to the

variation of the Q factor as well as the natural frequency, ω_n .

While methods for controlling the effective damping and stiffness of macro systems have been developed during the last decade, it has been recently applied to micro/nano systems such as micro gyroscopes, micro resonators and scanning atomic force microscope probes (Pannu et al., 2000; Gunev et al., 2007). The necessity to alter the system parameters in micro mirror may arise from several reasons, one of which is the fabrication imperfections (Park and Horowitz, 2004). One of the novel methods to handle variations in resonance frequency due to fabrication imperfections in a micro resonator is to use an inbuilt sensor and tuner, which are thin films deposited on the hinges (Kobayashi, et al., 2007). The piezoelectric sensor placed at the hinges of the resonator detects small shifts in resonance frequency. The variations in the resonant frequency are compensated by the tuner, which changes the effective spring constants of the hinges through inverse piezoelectric effect. Since the compensation through the tuner is limited only small changes are possible. Another study demonstrated that velocity feedback can dramatically improve the dynamic response and external disturbance rejection of a magnetic micro resonator (Nguyen and Howe, 1992). The motional current output of the micromechanical resonator is electronically sensed by a sense electrode, converted to a voltage, and then added to or subtracted from the driving input signal to change the effective damping of the resonator. Improving the transient response is another reason for using velocity feedback. For example, in electrophotographic processes, widely used in laser printers where an array of surface micro-machined cantilever beams are generally required, velocity feedback is also employed. This approach improves the settling time of the microbeams and reduces the image banding, a type of image artifact due to variations in the velocities of mirrors (Cheng et al., 2001). Another example of velocity feedback usage is in electrostatically actuated parallel plate capacitors where low damping may result in long settling times or undesired electrode contact (Maithripala et al., 2005).

Most of the earlier studies discussed above have implemented either position or velocity feedback to change the resonance frequency or the damping characteristics of a micro resonator. Our study has focused on changing both the frequency and/or damping by investigating the relation between the feedback gains and the dynamical characteristics of a mirror through experiments and numerical simulations. Moreover, in majority of the earlier studies, vibration amplitude or the angular rotation of the mirror is measured. Then, the measured signal is scaled and shifted in phase to obtain the velocity signal. In our study, the true velocity of the mirror is directly measured using a Laser Doppler Vibrometer (LDV) unlike many of the earlier studies (Lawrence et al., 2002). The velocity signal is then amplified by an adjustable gain through an analog circuit and fed

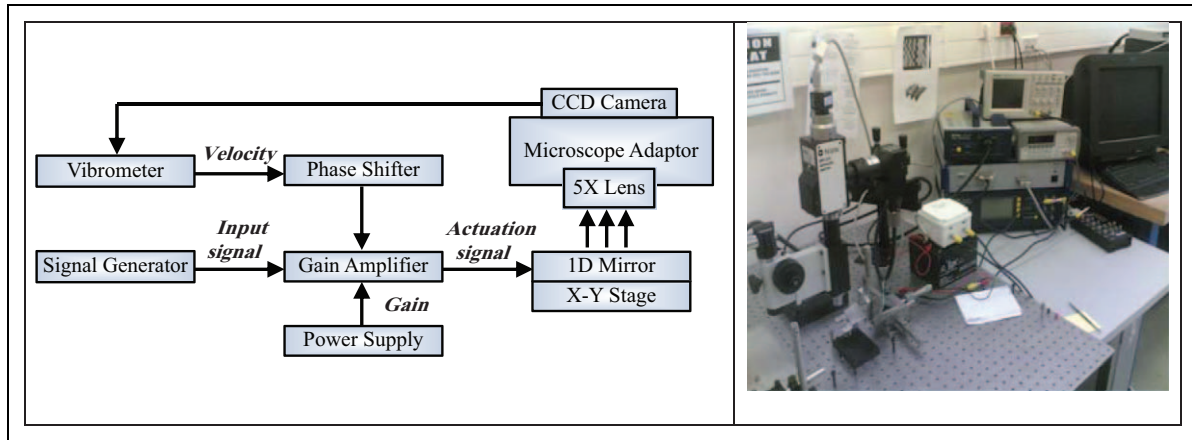


Figure 2. (a)The schematic of the experimental set-up. (b) A photograph of the experimental set-up.

back to the system to change the effective damping of the mirror. This approach is more robust especially if the non-linear effects due to friction or external forces significantly influence the output response of the mirror. Under these circumstances, phase shifting and simply scaling the position signal to obtain the velocity signal is not a reliable approach.

Finally, only a few studies in the literature investigated the dynamic response a micro mirror under simple position and velocity feedback through numerical simulations supported by experiments (Zhu et al., 2007). Most of the earlier studies have focused on the development of a control law to achieve the desired response. To better understand the influence of velocity and position feedback gains on the vibratory response of the mirror, we performed numerical simulations in SIMULINK®. A transfer function of the mirror is developed based on the experimental frequency sweep data of the first two resonant frequencies. Then, the effect of changing the feedback gains on the dynamic response of the mirror is investigated. The numerical simulations enabled us to investigate the system behavior under ideal conditions (free of environmental disturbances) and also determine the range of gain values that are used in the actual experiments.

In the next section, the details of the experimental set-up, feedback mechanism, and the results of our experiments are presented. Development of the mirror model and the results of the numerical simulations for different feedback gains are given in Section 3. Finally, we discuss the results of both experiments and simulations in Section 4.

2. Experimental Set-up

The experimental set-up consists of a LDV (Polytec GmbH, Germany) is used for measuring the vertical vibrations of the mirror, an optical microscope, a charge coupled device (CCD) camera, and a X-Y stage are used for exact positioning of the laser beam (see Figure 2). A data

acquisition system and an analog circuit are used for velocity feedback. The LDV having a bandwidth of 1.5 GHz measures the out of plane velocity of a point on the probe by collecting and processing the backscattered laser light. It is composed of a controller (OFV-5000) and a fiber interferometer (OFV-551). The sensor head of OFV-551 delivers a laser beam to a measurement point on the mirror and collects the reflected light. Using the optical microscope, the laser beam is focused. The controller OFV-5000 processes the data collected from OFV-551 using a wide bandwidth velocity decoder (VD-02) having a resolution of 0.15 $\mu\text{m/s}$. The sinusoidal input signal (generated by a signal generator) and the velocity signal acquired by LDV are sent to the computer for further analysis via the acquisition system.

The mirror is actuated using a velocity feedback circuit as shown in Figure 2. The main components of the feedback circuit are the phase shifter and the gain amplifier. The analog phase shifter is integrated into the signal processing circuit to adjust the phase deviations in velocity measurements at different frequencies due to the intrinsic time delay in the LDV. Although the delay observed in LDV is constant and small at the operating frequency, it increases proportionally with increasing frequency, resulting in an undesired positional signal in the feedback loop.

This shift can be tuned by adjusting the capacitance or the resistance in the phase shifter circuit (see Figure 3). To observe the behavior of this circuit for varying resistance and capacitance values, a pSpice model of the phase shifter is developed. As shown in Figure 4, increasing the resistance or the capacitance by the same amount has the same effect on the phase shift (e.g. if the resistance is increased and the capacitance is decreased by a factor of ten simultaneously, then there is no change in the phase). Hence, after selecting a value for the capacitance, the resistance is fine tuned in its allowable range to obtain a desired shift in phase. A single adjustment is adequate when the system is only fed back with velocity gain. In position

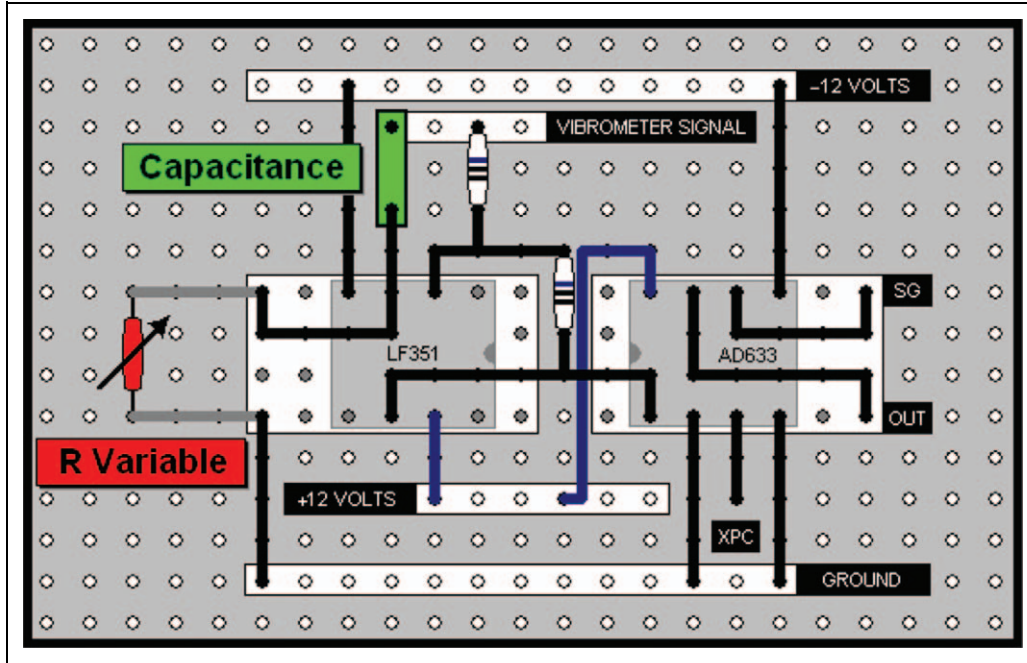


Figure 3. The components of the velocity feedback circuit.

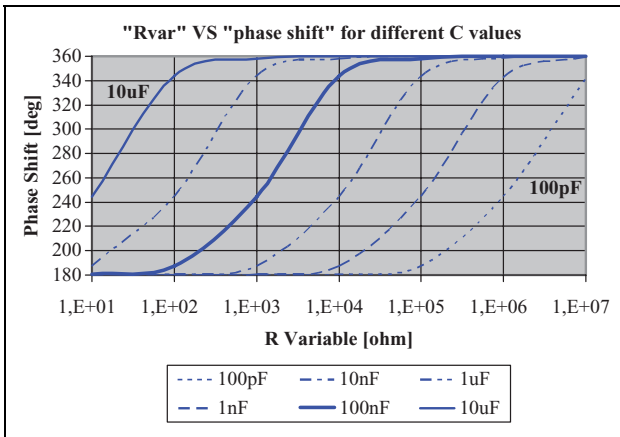


Figure 4. Phase shift in degrees for various values of resistance and capacitance.

feedback, adjusting the resistance for the new resonant frequency is necessary.

The gain amplifier (AD633JN, Analog Devices, Norwood, MA) is used to multiply the phase shifter output signal by a predefined constant. The amplified signal is then added to the input signal to generate the final actuation signal. The frequency sweep curves for three different gain values are shown in Figure 5. The quality factors corresponding to the curves are $Q = 62.7, 96.1$ and 169.6 for the velocity feedback gains of $G = 0, 0.01$ and 0.02 , respectively.

It is observed that the vibration amplitude does not increase for gain values greater than $G = 0.02$. To better

understand the limitations of the velocity feedback approach, the feedback gain is set to zero and the mirror velocity is measured at increasing voltages (see Figure 6). Up to a certain input voltage (~ 140 mV), the velocity of the mirror is linearly proportional with the voltage. After that point, the increase in velocity is small and the output amplitude tends to saturate at a limiting value of 150 mm/s ($6V \times 25$ mm/s).

Similarly, to see the behavior under increasing gain, the actuation signal amplitude around the resonant frequency is plotted in Figure 7. For $G = 0.02$, the amplitude of the actuation signal increases with the velocity feedback near the resonant frequency as expected, but the curve for $G = 0.03$ flattens when the magnitude of the actuation signal is reached to approximately 140 mV. As we have shown in Figure 6, the changes in mirror velocity are small after this value. Hence, the gains values greater than $G = 0.02$ are not very effective. Even with the addition of an extra operational amplifier to the circuit, we could not amplify the gain any further. This result may occur due to the very low resistance of the scanner ($1-10$ ohms) and suggests that rate feedback is applicable up to a certain saturation point, which may vary from mirror to mirror depending on the resistance of the mirror.

3. The SIMULINK Model

A SIMULINK® model of the mirror and the feedback loop are developed to justify the experimental measurements and to further investigate the effect of velocity and position feedback on the dynamical behavior of the mirror. First, we

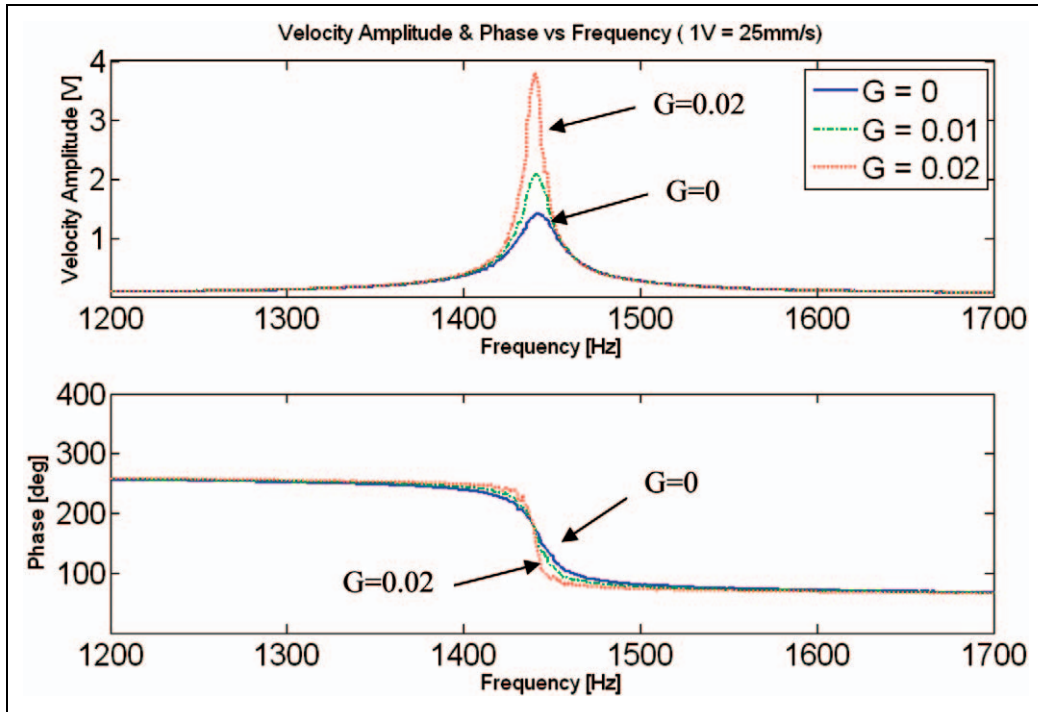


Figure 5. Experimental frequency sweep curves for different velocity gains.

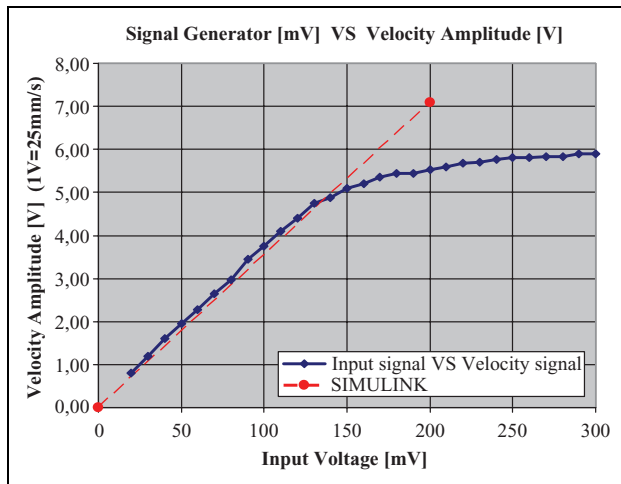


Figure 6. Amplitude of mirror velocity for increasing input voltages.

obtained the transfer function of the mirror using the amplitude and phase data collected experimentally over a range of frequencies. This is achieved using the “invfreqs” function in MATLAB. A transfer function, between the input voltage signal and output velocity of the mirror, having 3rd and 6th order polynomials in the numerator and denominator respectively yields satisfactory results, as shown in Figure 8. However, in order to analyze the dynamical characteristics of the mirror more easily, we prefer to use a second order polynomial for each resonance frequency of the

mirror and a delay function. For this purpose, the resonance frequencies of the mirror ($\omega_1 = 939 \text{ Hz}$ and $\omega_2 = 1442 \text{ Hz}$) are determined from the amplitude plot first. Then, the corresponding quality factors are calculated using the half-power method around the resonant peaks. Finally, these two transfer functions are combined and connected in series to form the new transfer function, which also gives satisfactory results as shown in Figure 9.

After obtaining the transfer function of the mirror, the next step is to change the velocity gain, G in the SIMULINK[®] model to test if the numerical simulations are consistent with the experimental measurements. As shown in Figure 10, the response of the numerical model is very close to experimental data for different gain values.

As a further step in simulation, position feedback is introduced into the feedback loop in the SIMULINK[®] model to investigate its effect on the dynamical response of the mirror (see Figure 11). By applying negative or positive position gains (H), the resonance frequency of the mirror is shifted towards to the right or left, respectively (see Figure 12). Observe that two resonance curves are approaching to each other as the gain H is increased due to the coupling of the resonance modes. For the positive H gains, the curves are superimposed to each other, resulting in an increase in the peak amplitude. Note that the opposite effect (i.e. decrease in peak amplitude) is also observed for the negative H gains where the peaks are separated from each other. As the gain H decreases further, the peak amplitude of the second peak converges to a certain

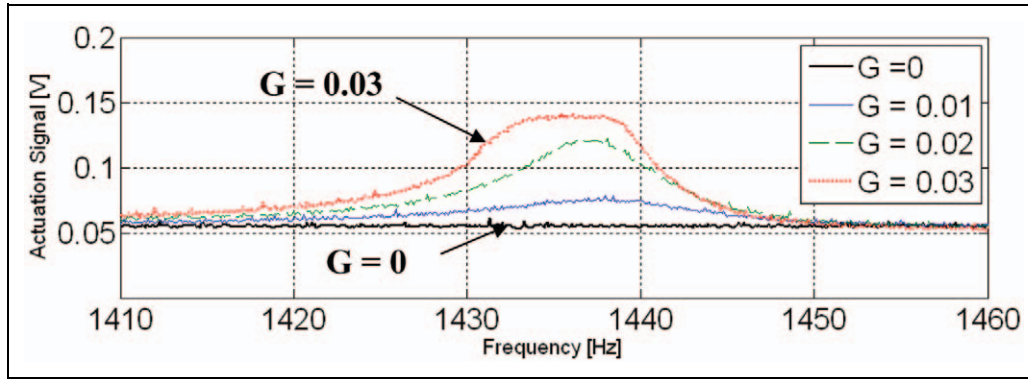


Figure 7. Actuation signal around the resonant frequency for different velocity gains.

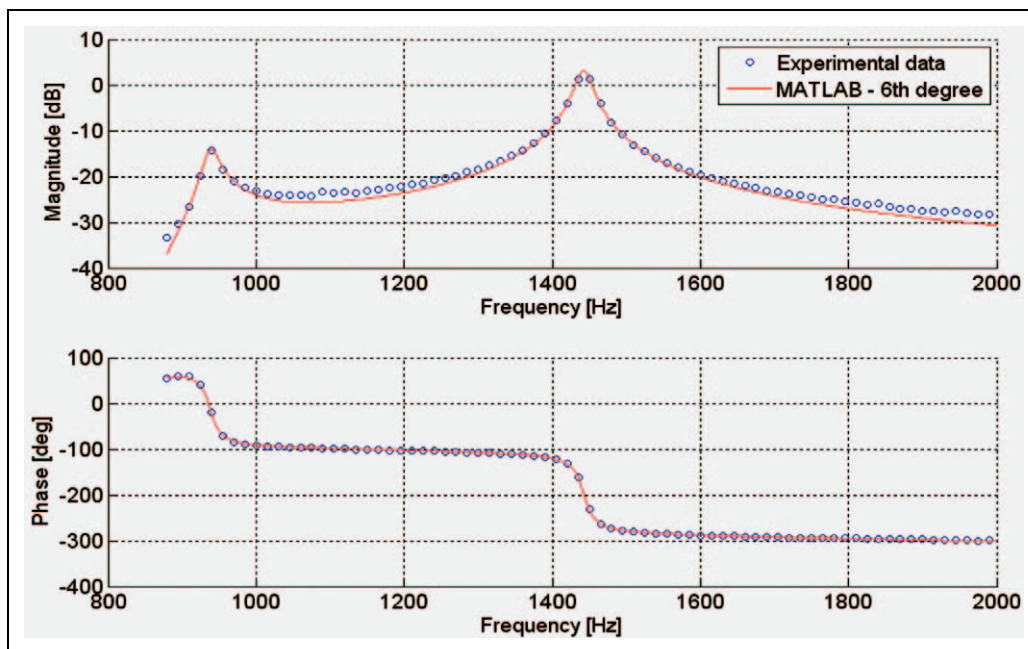


Figure 8. Curve fitting to the frequency response data using 3rd and 6th order polynomials in the numerator and denominator.

value since the effect of the first mode becomes insignificant.

By adjusting both the position and velocity gains, the resonance frequency and the damping characteristics of the system can be set. In Figure 13, the effect of changing velocity gain to adjust the effective damping of the system after shifting the resonance frequency to the right is shown. Once the resonance frequency is set, the effective damping of the system can be simply altered independently by adjusting the gain G . Table 1 reports the percent changes in damping and time constant of the system as the gains G and H are altered. One can quickly notice from Table 1 that changing the gain G has no influence on the resonant frequency as expected. However, due to the coupling effect of vibration modes, the alterations in the gain H have an influence on the peak amplitudes as well as the resonant frequency. With decreasing H , the

resonant frequency increases and the peak amplitude decreases. For negative values of G , the Q -factor and the time constant have a general trend of increasing with decreasing H , however an opposite tendency is observed for positive G gains.

To further analyze the transient response of the system at resonant frequency, the effect of altering G and H gains on the time constant of the mirror are plotted in Figure 14(a). In electromechanical systems, a small time constant is preferred for shorter settling times. In our system, we observed that the gain G has a major influence on the time constant. For example, the time constant (τ) of the mirror can be reduced by 50% when the velocity gain is set to $G = -0.03$. Although decreasing the position gain H reduces the time constant as well, its effect is not significant at negative values of gain G . Also, we observed that even though the time constant appears to be decreasing with decreasing

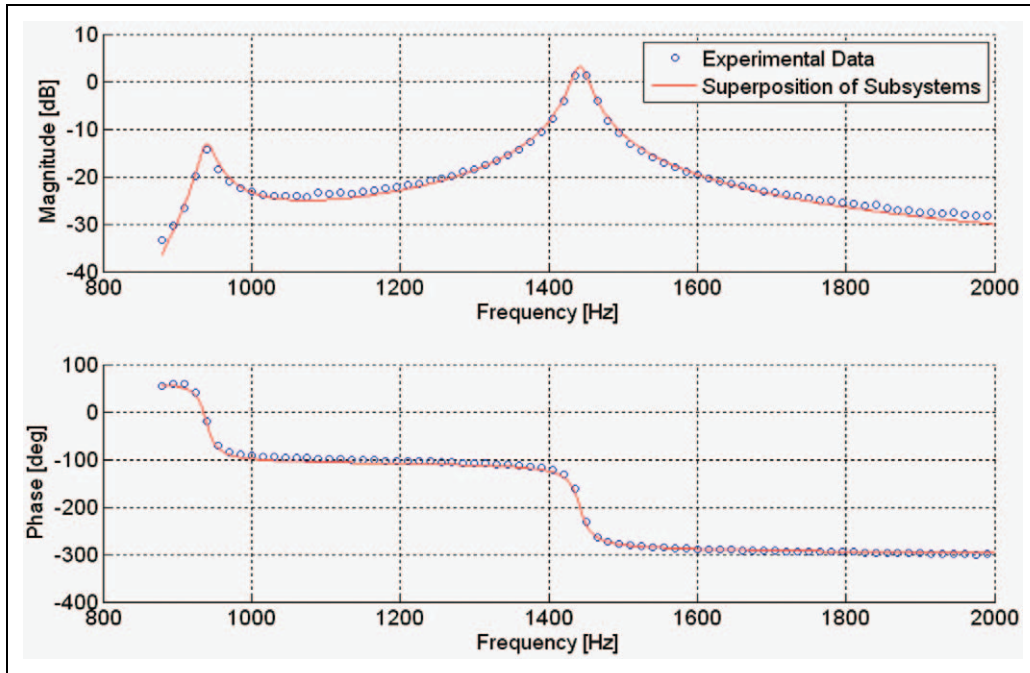


Figure 9. Curve fitting to the frequency response data using “two” 2nd order models.

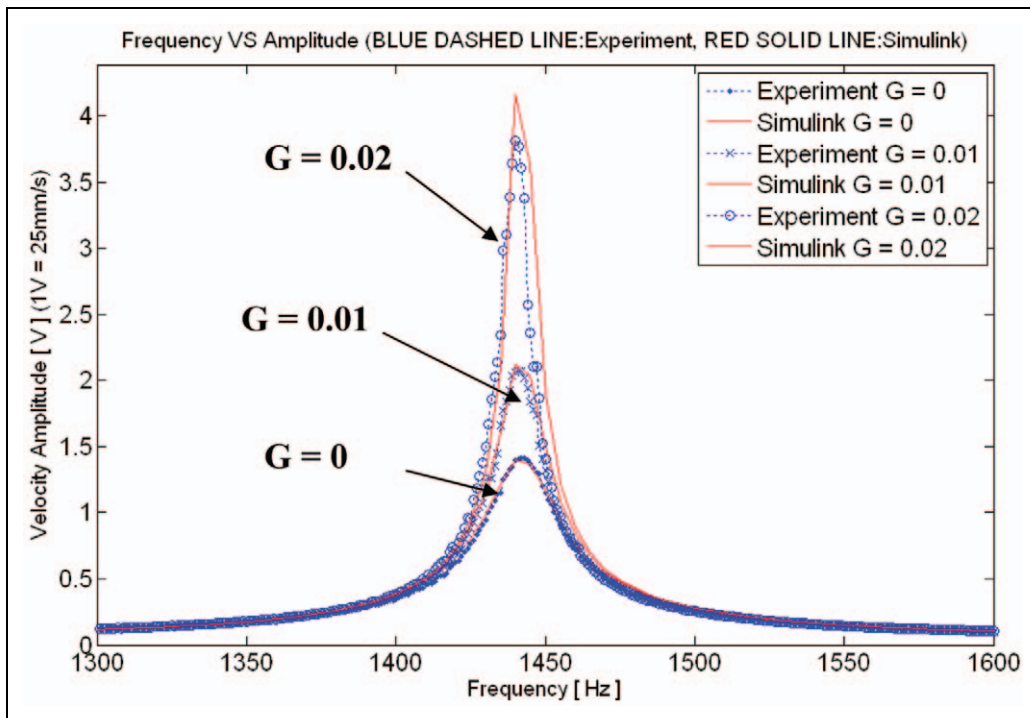


Figure 10. Simulink vs experimental results for different velocity gains.

H for positive values of G, there is also a simultaneous decrease in the peak amplitudes.

Most of these behaviors are again the result of the coupling effect of the first two vibration modes. Recall that we consider both resonance frequencies in our model of the

mirror for better representation of the mirror dynamics. To investigate the isolated behavior of the mirror without the influence of the first resonance mode, only the transfer function corresponding to the second resonance frequency is considered and the percent change in time constant of the mirror

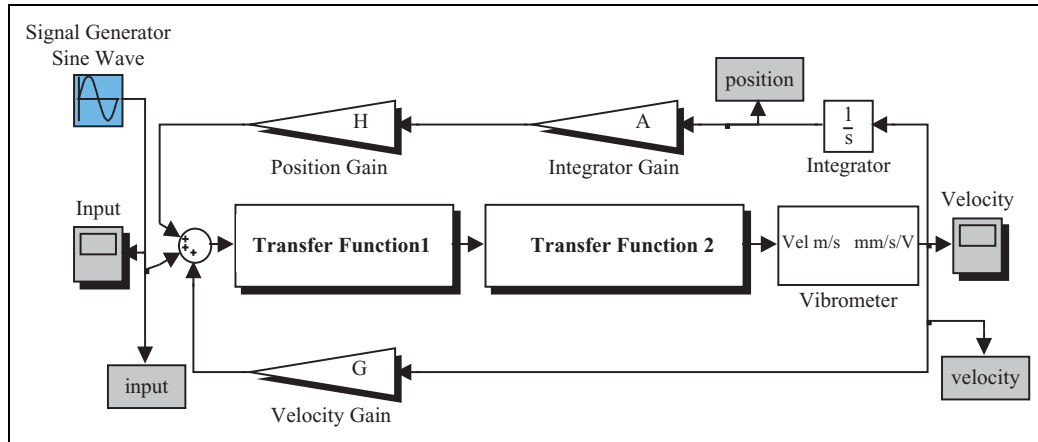


Figure 11. The Simulink model of the mirror with the feedback circuit.

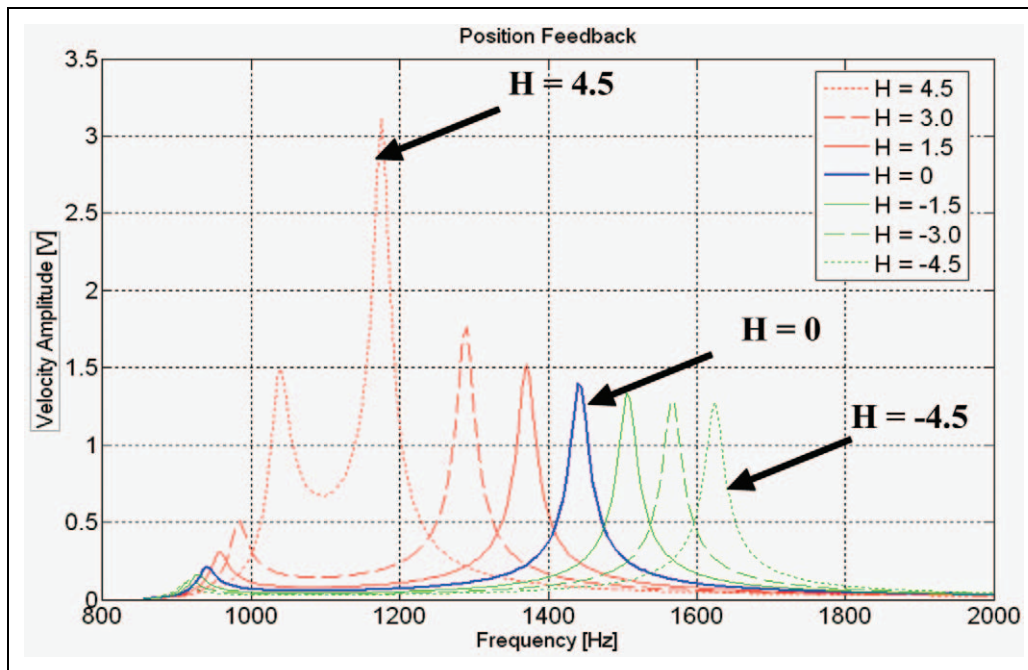


Figure 12. The frequency response of the mirror for different position gains.

is plotted in Figure 14(b). As shown in the Figure, the time constant decreases as gain G is reduced, but the value is independent of the position gain H as expected. These results clearly show that the dynamics of the mirror is influenced by the coupling effect of the vibration modes. To isolate these modes, position feedback can be used (see Figure 12).

4. Conclusions

We showed how the effective damping of a micro mirror can be adjusted experimentally using a feedback circuit. In our implementation, the dynamical response of the mirror can be altered in real-time, which is beneficial in mirror applications requiring quick changes in scan angles or amplitudes.

In our approach, the velocity of the mirror is measured directly using a LDV. Obtaining the velocity signal from the measured position signal through differentiation is not preferred since the noise in the position signal is amplified through differentiation. Typically, the position signal is scaled and phase shifted to obtain the velocity signal. However, this approach could also be problematic if there are time-varying disturbance forces or frictional forces affecting the dynamics of the mirror as it operates. Under these circumstances, the time-shifted position signal will not be the “true” velocity of the mirror and the velocity feedback approach may lead to errors.

Using a SIMULINK® model, we investigated the influence of both position and velocity feedback on the effective

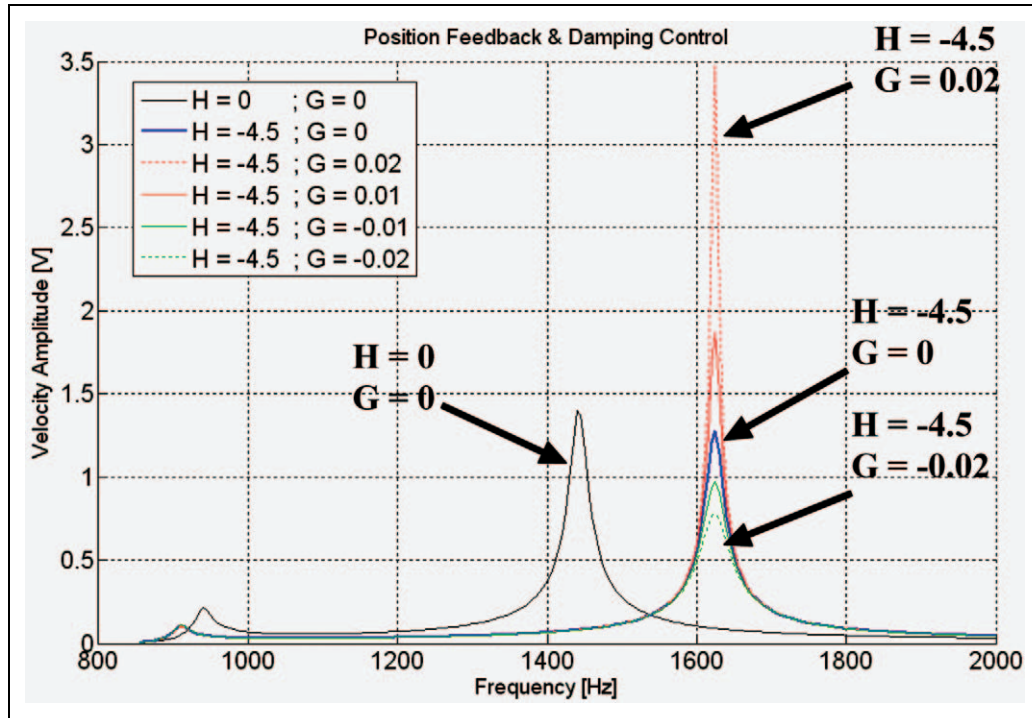


Figure 13. Adjustment of resonance frequency and damping by changing position and velocity gains incrementally.

Table I. Percent changes in peak amplitudes, damping ratio, and time constant for positive and negative values of G & H gains.

		H = 3 (1288 Hz)	H = 1.5 (1371 Hz)	H = 0 (1442 Hz)	H = -1.5 (1507 Hz)	H = -3 (1567 Hz)	H = -4.5 (1624 Hz)
G = 0.02	Vmax [V] :	10.80	6.48	4.86	4.17	3.79	3.59
	ξ :	0.103	0.197	0.235	0.266	0.278	0.265
	$\Delta\tau$	790.1	338.0	249.2	194.9	171.5	174.8
G = 0.01	Vmax [V] :	3.19	2.47	2.19	2.04	1.94	1.88
	ξ :	0.483	0.511	0.520	0.514	0.511	0.496
	$\Delta\tau$	90.0	68.8	57.5	52.5	47.7	46.9
G = 0	Vmax [V] :	1.77	1.53	1.42	1.35	1.31	1.28
	ξ :	0.862	0.832	0.820	0.776	0.760	0.735
	$\Delta\tau$	6.5	3.6	0.0	-0.2	-0.7	-1.0
G = -0.01	Vmax [V] :	1.23	1.11	1.05	1.01	0.99	0.97
	ξ :	1.235	1.147	1.094	1.035	0.990	0.960
	$\Delta\tau$	-25.7	-24.8	-25.1	-24.2	-23.8	-24.2
G = -0.02	Vmax [V] :	0.94	0.87	0.83	0.81	0.79	0.78
	ξ :	1.613	1.462	1.370	1.295	1.232	1.199
	$\Delta\tau$	-43.1	-41.0	-40.2	-39.5	-38.8	-39.3
G = -0.03	Vmax [V] :	0.76	0.71	0.69	0.67	0.66	0.65
	ξ :	2.008	1.786	1.645	1.553	1.488	1.429
	$\Delta\tau$	-54.3	-51.7	-50.2	-49.5	-49.3	-49.1

damping, resonance frequency, and the settling time of the mirror. In experimental settings, it is difficult to repeat an experiment under the same environmental conditions due to variations in humidity, electrical noise, ground vibrations, temperature, etc. Moreover, even if the experimental conditions are fixed, the material and surface properties of the mirror (e.g. friction at the hinges) may change in time,

affecting the results of experiments adversely. All these factors make it difficult to quantitatively compare the results of the similar experiments under different settings.

The nonlinearities must be also taken into consideration when building more comprehensive models of the micro systems. For example, the actuation force of the micro system was modeled as a nonlinear function of angular

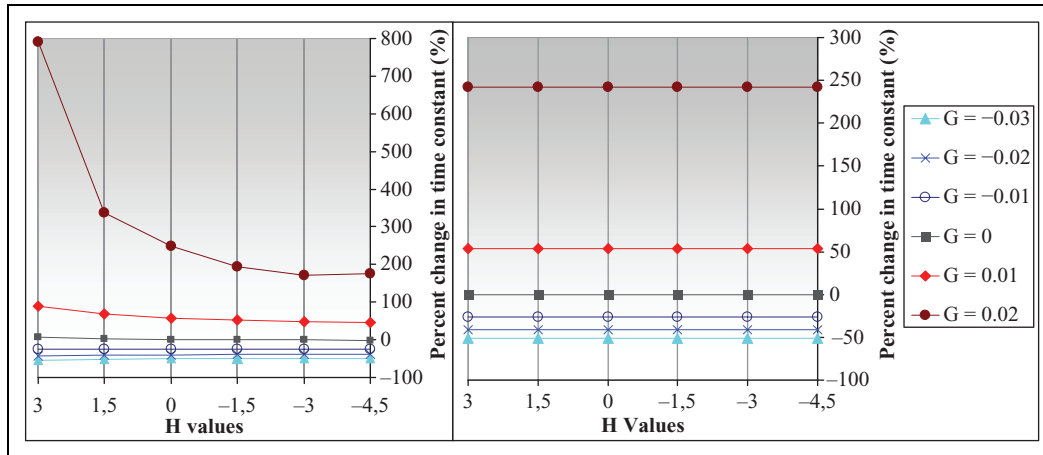


Figure 14. Percent change in time constant of the mirror as the feedback gains H and G are altered: (a) when the first two resonance modes of the mirror are considered in the model. (b) when only a single resonance mode is considered in the model.

displacement and time, in earlier studies (Juneau et al., 2003; Ataman and Urey, 2004; Zhu et al., 2006). As a result, the governing differential equation in the simulation model becomes nonlinear with time-varying coefficients. Our model does not include these nonlinearities. However, we believe that the linear assumption for our micro system is valid since it shows a linear behavior in the operating range that we currently use, as it can be observed from Figure 6. However, the effect of the nonlinearities for similar systems can be addressed in a future study.

We easily altered the resonance frequency and the effective damping of the mirror to investigate its dynamical response of the micro mirror using numerical simulations. In our approach, the transfer function of the mirror was developed based on the experimental data. We considered the first two vibration modes of the mirror to construct its transfer function. For each mode, we used a second order transfer function, which helped us to understand and analyze the influence of each vibration mode on the dynamical response of the mirror more effectively. Our study shows that coupling effects can significantly influence the response of the mirror as the feedback gain H is increased. We also investigated the effect of feedback gains G and H on damping and time constant of the system. In general, the velocity gain G has more influence on the time constant of the system. There is a trade-off between the Q -factor and the settling time. The negative values of G reduce the time constant of the system, which is desirable, but it also reduces the Q -factor and hence the amplitude of oscillations, which is not desirable. At positive values of G , the time constant of the system can be also reduced by increasing the position gain H . However, we observed that the Q -factor of the system changes more rapidly than its resonant frequency as the position feedback gain H is altered. Changing the position gain H has less influence on the time constant, however, it can be used to shift the resonance frequency of the system to reduce the coupling effect of the vibration modes. For

example, in our system, shifting the resonance frequency to the right reduces the coupling effect of the vibration modes, leading to more separated resonance frequencies. Hence, the resonance frequency of the mirror can be adjusted prior to adjusting the effective damping.

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