*Key Engineering Materials* *Online: 2007-03-15*
*ISSN: 1662-9795, Vols. 334-335, pp 241-244*
*doi:10.4028/www.scientific.net/KEM.334-335.241*


# A proposal of Calculation Method for Equivalent Property of Composite Materials

Hiroaki NAKAI[1], Hiromasa TOMIOKA[2],
Tetsusei KURASHIKI[3] and Masaru ZAKO[4]

[1, 3, 4]Graduate school of Engineering, Osaka University
,2-1 Yamadaoka, Suita, Osaka, 565-0871, Japan

[2]Graduate Student of Osaka University, Japan

[1]nakai@mapse.eng.osaka-u.ac.jp, [2]h-tomioka@mit.eng.osaka-u.ac.jp,
[3]kurasiki@mit.eng.osaka-u.ac.jp, [4]zako@mit.eng.osaka-u.ac.jp

**Keywords:** FEM, mesh superposition method, equivalent mechanical properties, FRP.

**Abstract.** To predict the mechanical properties of composite materials by using computer is complicated, because it is difficult to model directly by ordinary FEM. A calculation method by using the mesh superposition method and periodic boundary condition has been proposed in order to obtain the equivalent mechanical properties of composite materials easily. The numerical results by proposed method have shown good agreement with ones by the traditional procedure. The proposed method is efficient for the materials with complicated structure like woven fabric composites etc.

## Introduction

Fiber Reinforced Plastics (FRP) have been widely applied for many structures, because of its superior properties. It is not efficient and not economical procedures to obtain the properties of FRP to design from experiments. Though the finite element method (FEM) is used to analyze and evaluate the mechanical behavior of structures or materials, we have a limitation for total number of elements. The homogenization method can not be applied to the complicated composite materials, because of difficulty in making finite element mesh. But it is effective to obtain the equivalent properties. In order to analyze composite materials easily, we must solve two problems. First is to make the finite element mesh for even woven fabric composites which have complicated structure such as plain, satin, twill weave. Second is to need the much memory in computer to carry out FEM.

In this study, a new calculation method of equivalent properties has been proposed. In the proposed method, a composite material has been modeled as meshes of matrix and inclusion individually, and the equivalent property of the composite material can be calculated by using two or more FE meshes.

## Modeling of composite materials

**Mesh superposition method.** In order to get the modeling of composite materials easy, we use a numerical technique concerned with FEM. The mesh superposition method [1] is one of the multi-meshes FEM. In this method, we can define two or more FE meshes, and the relation between these meshes is considered in making stiffness matrix as shown in Eq.1.

$$\begin{bmatrix} [\boldsymbol{K}^G] & [\boldsymbol{K}^{GL}] \\ [\boldsymbol{K}^{GL}]^T & [\boldsymbol{K}^L] \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{d}^G\} \\ \{\boldsymbol{d}^L\} \end{Bmatrix} = \begin{Bmatrix} \{\boldsymbol{F}_s^G\} \\ \{\boldsymbol{0}\} \end{Bmatrix}. \tag{1}$$

Where, $[K^G]$ and $[K^L]$ are the stiffness matrices of the global mesh and local mesh, respectively. $[K^{GL}]$ is the macro and micro correlation. Traditionally, it has been proposed that the size of local mesh is important to guarantee the precision of analysis and the boundary of local model must be expanded to uniform strain field in global mesh. In this paper, we have broken the traditional applicability and have proposed a new calculation method of equivalent mechanical properties. The local mesh only consists of the inclusion material and the global mesh only consists of the matrix.

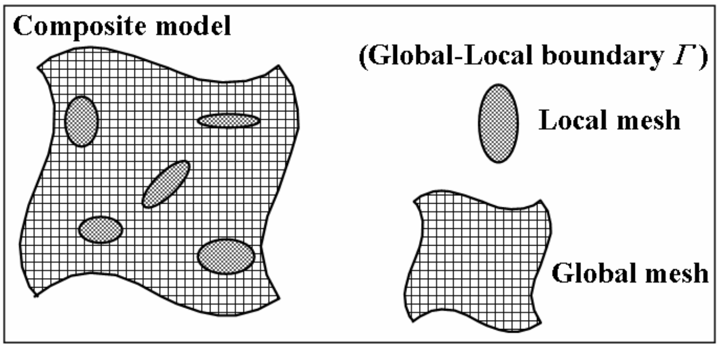
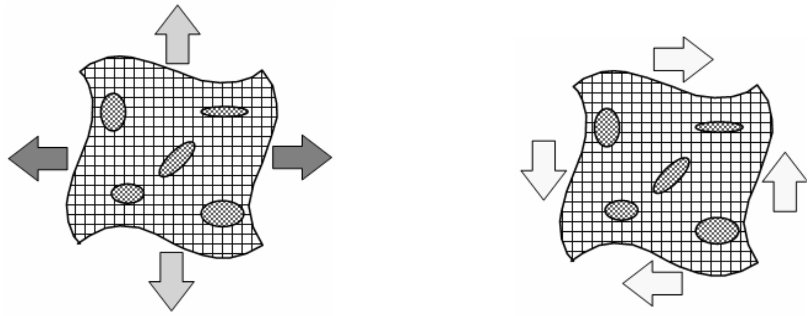



Fig.1 Modeling of composite materials with the mesh superposition method

**Equivalent mechanical property.** In general, some of purposes of FE analysis with unit cell are to find the macroscopic properties of materials and to detect the microscopic behavior in unit cell. The unit cell is defined as the representative volume and it is composite model in this paper as shown in Fig.1. Then, the boundary of the unit cell, which means the boundary of the global mesh, is applied periodic boundary condition. Some papers discussed about equivalent mechanical properties with periodic boundary condition [2, 3, 4]. Those are calculated by the average stress and the average strain in the unit cell with periodicity. In this paper, the equivalent properties of composite model are calculated from FE analysis in applying normal or shear strain as shown in Fig.2. The elastic moduli are obtained from the applied strain and reaction force in global mesh with periodicity. The Poison's ratios are obtained from transverse strain of global mesh.

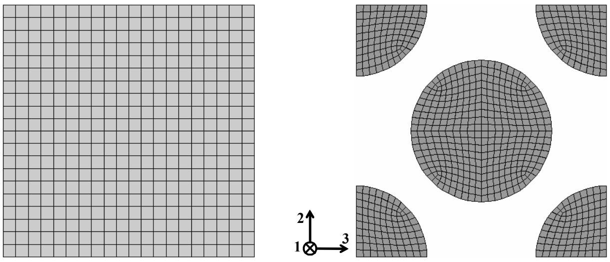


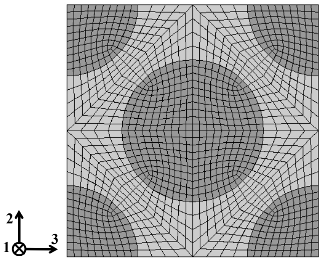
(a) Normal strain in x or y direction (c) Shear strain

Fig.2 Applying unit strain to periodic boundary condition

**Application to composite materials**

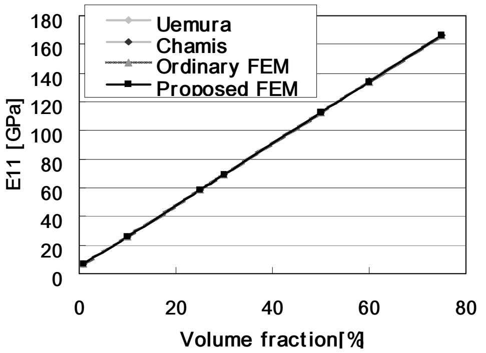
**Fibre-Matrix model.** We have modeled FRP as resin and unidirectional fibres as shown in Fig.3 and calculated the equivalent mechanical properties. The results are shown in Fig.5. In this figure, the subscript 1 means the fibre direction, 2 and 3 mean transverse direction, respectively. As a verification of the proposed method, the results by ordinary mesh in Fig.4 and empirical formulae of Uemura [5] and Chamis [6] are shown in the same figure. All results show good agreements except Poison's ratio($\nu23$). The reason why the difference occurs is that the empirical formulae of Uemura and Chamis have assume the relation $\nu23 = \nu23^{(f)}V_f + \nu^{(m)}(1\text{-}V_f)$ and $\nu23 = 2(E22/G23)\text{-}1$, respectively like as isotropic materials.



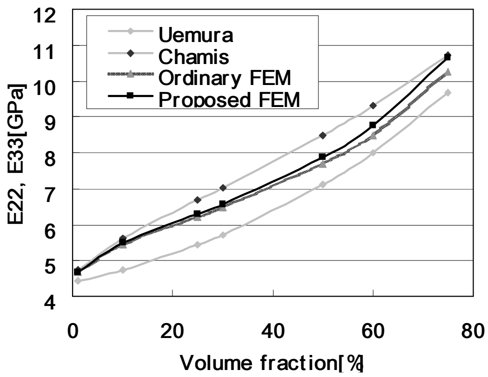




(a) Resin (Global mesh) (b) Fibre (Local mesh) Fig.4 Ordinary mesh

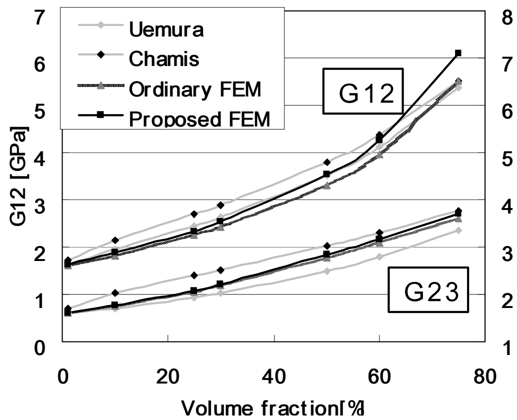
Fig.3 FE meshes of Fibre-Matrix model ($V_f$ = 50%)



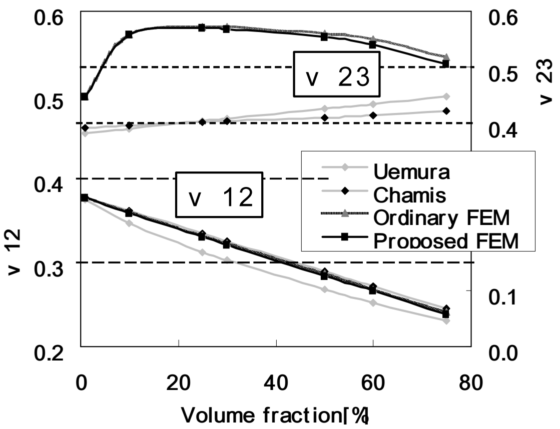

(a) Elastic modulus E11




(b) Elastic modulus E22, E33




(c)Shear modulus

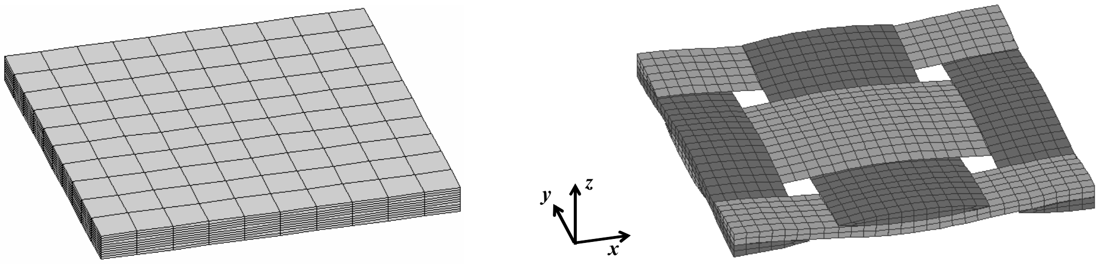



(d) Poison's ratio

Fig.5 Comparison of the equivalent properties of Fibre-Matrix model

Fibres:carbon $E1^{(f)}$=220GPa, $E2^{(f)}$=$E3^{(f)}$=13.8GPa, $G12^{(f)}$=$G31^{(f)}$=9.0GPa, $G12^{(f)}$=4.8GPa, $\nu12^{(f)}$=0.20, $\nu23^{(f)}$=0.47, $\nu31^{(f)}$=0.013, Resin:epoxy $E^{(m)}$=4.4GPa, $G^{(m)}$=1.6GPa, $\nu^{(m)}$=0.38

**Woven fabric composites.** The equivalent mechanical properties of woven fabric composite are calculated by the proposed method. The global mesh only consists of resin and the local mesh consists of fibre bundles which modeled as unidirectional FRP. These are shown in Fig.6. Otherwise, the ordinary mesh of woven fabric composite, which needs much time and techniques to produce, is shown in Fig.7. As shown in Table1, the results of two calculations are acceptable. In addition, total number of elements in proposed method is less than ordinary one. From these things, it is recognized that the complex geometry like textile composite can be simplified and be saved form the trouble of meshing and modeling.



(a) Resin (Global mesh) (b) Fibre bundles (Local mesh)

Fig.6 Finite element meshes of plain woven fabric composite

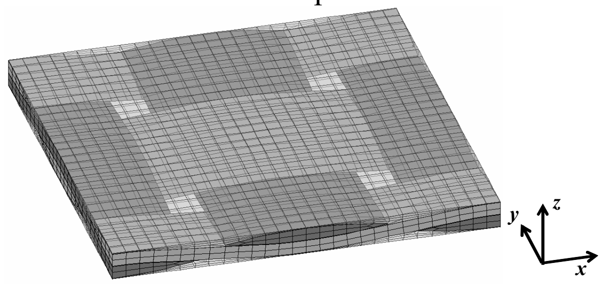


Fig.7 Ordinary mesh of plain woven fabric composite

Table1 Comparison of the equivalent properties of woven fabric composite

| Equivalent property | Ordinary FEM | Proposed method (error) | |
|---|---|---|---|
| Ex | 4030 MPa | 4060 MPa | (+0.7 %) |
| Ey | 4030 MPa | 4060 MPa | (+0.7 %) |
| Ez | 4855 MPa | 4920 MPa | (+1.4 %) |
| Gyz | 1517 MPa | 1640 MPa | (+8.1 %) |
| Gzx | 1517 MPa | 1640 MPa | (+8.1 %) |
| Gxy | 2185 MPa | 2236 MPa | (+2.3 %) |
| ν yz | 0.431 | 0.430 | (-0.2 %) |
| ν zx | 0.052 | 0.052 | (+0.4 %) |
| ν xy | 0.108 | 0.099 | (-7.9 %) |

## Conclusions

In order to get modeling of composite materials easy and to obtain the equivalent mechanical properties, a calculation method by the mesh superposition method and the periodic boundary condition has been proposed. The results have showed a good agreement with ordinary FEM and empirical formulae. It is convenient technique and efficient to obtain the equivalent property and macroscopic behavior of composite materials.

## References


[1] J. Fish, S. Markolefas, et al. J. App. Numer. Math. 14 (1994), p. 135-164.

[2] J. Whitcomb, C. Chapman etc.: Journal of Composite Materials, 34 (2000), p. 724-747

[3] V. Carvelli and A. Poggi, Compsites part A 32 (2001), p. 1425-1432.

[4] J.L. Lions, Some Methods in the Mathematical Analysis of Systems and Their Control, Science Press (1981)

[5] M. Uemura and N. Yamada, J. Society Mater. Sci., 24 (1975), p. 156-163.

[6] C.C. Chamis, J. Compos. Technol. Res. 11 (1989), p. 3-14.