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Motion repulsion arises from stimulus statistics when analyzed with a clustering algorithm

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Abstract. Motion repulsion is the perceived enlargement of the angle between the directions of motion of two transparently moving patterns. An explanation of this illusion has long been sought for in the neural circuitry of the brain. We show that motion repulsion already arises from the statistical properties of the motion transparency problem when analyzed with a clustering algorithm.

1 Introduction

When viewing two superimposed, translating groups of dots each moving coherently in a different direction, humans overestimate the direction difference (Marshak and Sekuler 1979). Ever since the discovery of this so-called 'motion repulsion' phenomenon, it has been conjectured that this illusion is mediated by inhibitory interactions between directionally tuned motion detectors in the brain (Marshak and Sekuler 1979; Mather and Moulden 1980; Hiris and Blake 1996; Wilson and Kim 1994; Kim and Wilson 1996, 1997). Here, we investigate to what extent motion repulsion can be explained in terms of the statistical properties of the motion transparency problem and its algorithmic analysis, rather than any particular neural implementation.

2 Unsupervised classification of transparently moving random dots

We consider a transparent motion pattern, in which a number of identical dots are randomly distributed in the visual field (Fig. 1a). Half of the dots (group 1) move coherently in a certain direction, and the other half (group 2) move with the same speed but in a different direction. If the direction difference, δ, is larger than a certain amount, then the resulting pattern will be perceived by a human observer as two surfaces moving transparently (Mather and Moulden 1980; Braddick et al. 2002). How can an observer estimate the direction difference? Crucially for this estimation task, neither the group identity of the individual dots nor their exact direction of motion is available to the observer. Available to the observer are solely the 'noisy' measurements of individual dot directions (Fig. 1b). This noise can be the result of optical as well as neuronal limitations along the visual pathway. Since the group identities of dots are not known, the observer does not know which measurements belong to which group and therefore is not able to average the measurements belonging to the same group. Instead, 'unsupervised classification' algorithms (Duda et al. 2000; Mahani et al. 2004) have to be applied to the noisy measurements to derive single-trial estimates, m_1 and m_2, of the two directions.

In our simulations, two groups, each consisting of 50 coherently moving dots, were moving in two directions with a direction difference, δ, having values between 0° and 180°, at 4-deg intervals. For each trial, direction measurements were simulated by drawing 50 random numbers from a Gaussian distribution with zero mean and standard deviation $\sigma = 40°$ and another 50 from a Gaussian distribution with a mean of δ and the same standard deviation. Subsequently, the set of 100 measurements were mixed, i.e. unlabelled, and the iterative K-means clustering algorithm (Duda et al. 2000) was performed on them to find the single-trial direction estimates, m_1 and m_2. The algorithm has the following steps: (1) Initialize the estimates m_1 and m_2 as the mean of the measurements plus or minus one standard deviation, respectively. (2) Generate two clusters of measurements by assigning each measurement to the estimate to which it is closer. (3) Average the measurements in each cluster and use the two averages as the new estimates m_1 and m_2. (4) Repeat step two and three until the estimates do not change any more in subsequent iterations. The single-trial estimate of the direction difference is then $|m_1 - m_2|$. Here, the absolute value reflects mathematically the fact that the two groups of dots are indistinguishable except for the direction that defines their identity. Thus the group labels 1 and 2 are arbitrary and only the absolute value of the direction difference is meaningful. For each value of δ, 1000 trials were performed and the mean estimated direction difference, $\hat{\delta}$, is the average

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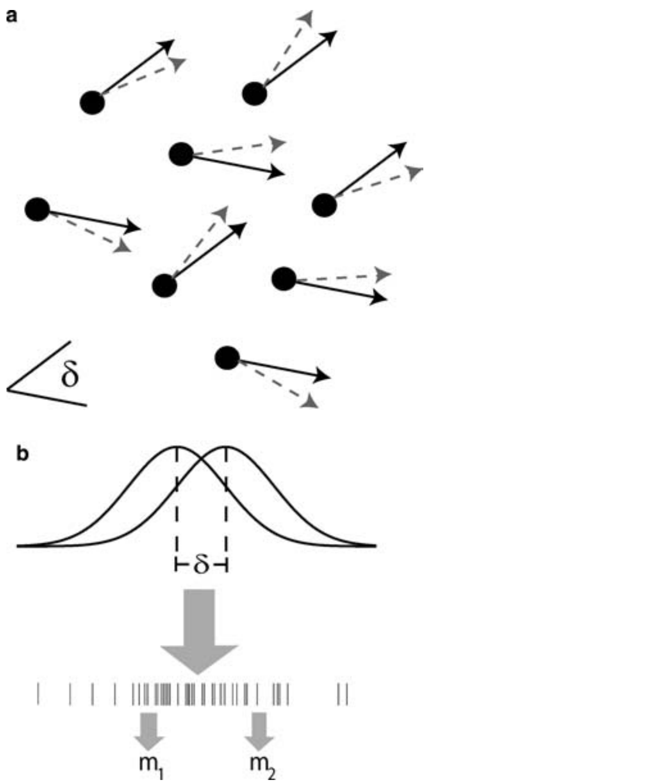

Fig. 1. Schematic of the motion transparency problem. **a** Two groups of randomly distributed and identical dots, with one group moving coherently in one direction and the other group moving coherently in a different direction (*black arrows*). The angle between the directions is δ. The *gray, dashed arrows* indicate a single-trial noisy measurement of the direction of motion for each dot. **b** From two Gaussian-distributed directions (*black curves, top*) of width, σ, and direction difference, δ, single-trial noisy measurements of individual dot directions (*gray vertical lines, bottom*) are generated in equal numbers. From these noisy measurements, a classification algorithm derives the single-trial estimates, m_1 and m_2, of the two directions

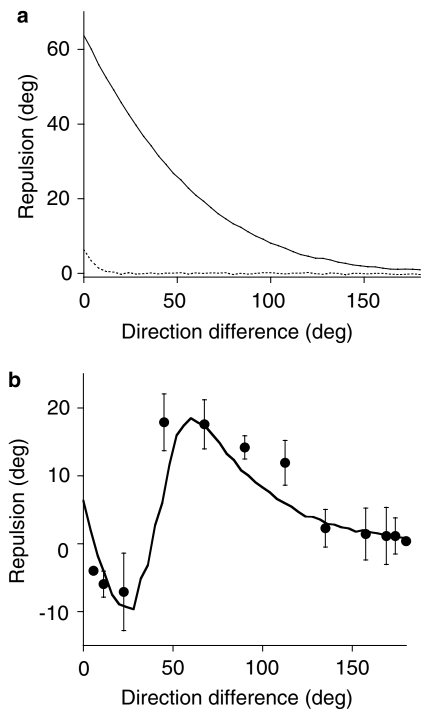

Fig. 2. Simulated motion repulsion. **a** Repulsion, $\hat{\delta} - \delta$, as a function of direction difference, δ, for the random-dot motion transparency stimulus for two different cases: (1) when direction measurements are noisy ($\sigma = 40°$) and the dot group identities are not known, using the K-means clustering algorithm (*solid curve*); (2) when direction measurements are noisy ($\sigma = 40°$) and *dots* are identified to belong to a certain direction group (*dotted curve*). **b** Same as **a**(i), but combined with a threshold of 72°, below which single trial estimates, $|m_1 - m_2|$, are set to zero (*thick curve*). Results from psychophysical experiments (Braddick et al. 2002) are shown for comparison (*solid circles*). Error bars are standard errors

of the single-trial estimates, i.e., $\hat{\delta} = \langle |m_1 - m_2| \rangle$, where the brackets denote average over many trials. We are interested in the behavior of $\hat{\delta}$ as a function of the actual direction difference, δ.

The simulation result reveals motion repulsion (Fig. 2a). The estimated direction difference is, on average, larger than the real direction difference, i.e., the repulsion, $\hat{\delta} - \delta$, is always positive. The repulsion increases with decreasing direction difference.

The repulsion results from the noise in the direction measurements. In the case when the measurement noise is reduced to zero, the estimated direction difference becomes identical to the real direction difference, and the repulsion vanishes completely. The measurement noise, σ, is a free parameter in our model. Since we did not adopt a population-coding scheme such as that occurring in the cortex in modeling the measurement process, it is hard to choose a 'realistic' range for σ. This is especially true due to lack of accurate estimates of the number of neurons encoding the direction of motion per location, plus the complicating effect of the observed correlation in the firing rate of cortical neurons (Zohary et al. 1994).

Repulsion is further enhanced by the lack of group assignment for each dot. In contrast, when dots are identified to belong to a certain direction group, measurements belonging to the same group can be averaged and the repulsion is drastically reduced (Fig. 2a). It has, indeed, been shown in psychophysical experiments (Braddick et al. 2002; Rauber and Treue 1999) that repulsion is significantly reduced when the paradigm allows for group identification. This situation is realized when two moving patterns are spatially segregated (Braddick et al. 2002) or when the direction of only one moving group is evaluated against a known reference direction (Rauber and Treue 1999), the so-called 'reference repulsion'.

On the other hand, the simulated repulsion is independent of the number of dots, as the latter affects only the standard deviation of the repulsion. This observation is consistent with results from psychophysical experiments where the density of dots was varied and the observed repulsion remained constant (Braddick et al. 2002; Rauber and Treue 1999).

3 Negative repulsion

Psychophysical experiments have shown that for small direction differences repulsion can turn into attraction, i.e., negative repulsion (Braddick et al. 2002). At these small direction differences, subjects start to perceive the two groups of dots moving in two directions as one group of coherently moving dots (Mather and Moulden 1980; Braddick et al. 2002). Adding a threshold to the above procedure reproduces quantitatively the observed transition from repulsion to attraction (Fig. 2b). For each trial, we either set $|m_1 - m_2|$ to zero (corresponding to coherent motion), when its value is below a certain threshold, or otherwise keep its value. As the direction difference becomes smaller, it is more likely for single-trial estimates, $|m_1 - m_2|$, to be smaller than the threshold. As a result, the repulsion, $\hat{\delta} - \delta$, which is the average over many trials, changes gradually with decreasing direction difference and displays a minimum. Depending on the value of the threshold, the minimum may occur at negative values of repulsion, thus reproducing attraction (Fig. 3). Therefore, the experimentally observed attraction (Braddick et al. 2002) follows from the fact that subjects tend to perceive coherent motion for small direction differences. The value for the threshold can approximately be determined by the angle at which this perceptual transition from transparent to coherent motion happens ($\sim$10 deg). Our choice of threshold should cause $\hat{\delta}$ to be zero for δ's smaller than this angle. The threshold of 72° used to produce Fig. 2b does so.

4 Other unsupervised classification methods

Several other methods are suggested in the statistical literature for solving 'unsupervised classification' or 'mixture density estimation' problems, namely the method of moments (Pearson 1894; McLachlan and Peel 2000) and the maximum likelihood estimation using Expectation-Maximization (Dempster et al. 1977; McLachlan and Krishnan 1997). Applied to the motion transparency problem, both these methods exhibit mathematically undesirable behaviors. The method of moments can produce imaginary outputs for $|m_1 - m_2|$ when δ is small, while the expectation–maximization method shows increasingly slow convergence in the same regime. We therefore chose not to include these two methods in our simulations. A seemingly straightforward method, i.e. finding the modes of the histogram of measurements, suffers from a similar shortcoming: If the separation, δ, of the two Gaussians from which measurements are sampled is smaller than their spread, σ, then the modes of individual Gaussians merge and the resulting histogram will, on average, have a single flattened peak rather than two separate peaks.

We did test however, a non-iterative classification algorithm, the Naïve estimation method (Mahani et al. 2004). In this method, initially the global average of all measurements is calculated. Next, each measurement is assigned a group identity based on whether it is larger or smaller than this global average. Finally, the average of each group is calculated to find m_1 and m_2. Variations of the Naïve estimation method, such as replacing the global and/or group averages by the global and/or group medians, were tested as well.

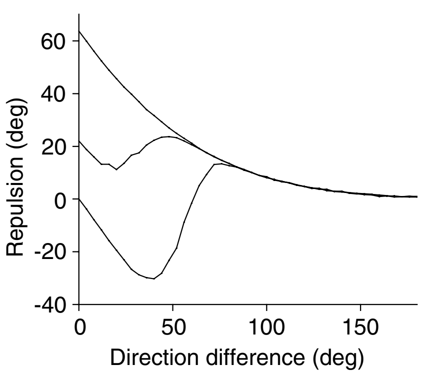

Fig. 3. The effect of the threshold value on the shape of the repulsion function. Threshold values are 40, 66, and 78° for the top, middle, and bottom curves, respectively. $\sigma = 40°$. K-means clustering is used for classification

5 Discussion

5.1 Generality of results

As described in Sects. 2 and 4, we examined several unsupervised classification algorithms. The results reported in Figs. 2 and 3 are obtained by the K-means clustering method (Duda et al. 2000). For those algorithms that had acceptable behavior in the small-δ regime, in which we are mainly interested, the repulsion persists and the magnitude of repulsion is much larger than that seen for the labeled case. The Naïve estimation algorithm, in particular, produced results very close to those of the K-means clustering algorithm. Notably, for those algorithms with unacceptable behavior in the small-δ regime, the simulated magnitudes of repulsion for larger δ's are smaller than the corresponding values found using other algorithms such as K-means clustering (data not shown here). It thus appears that large repulsion is the price being paid to have the ability to perceive motion transparency at small δ's. For example, if we were to use the modes of measurement histogram to find m_1 and m_2, and subsequently $\hat{\delta}$, we could only do so for δ's larger than σ, which is 40° in our simulations. This is while, using K-means clustering, we can distinguish δ's as small as 10° with reasonable accuracy. Choosing the ability to distinguish transparently moving surfaces with small direction differences over less bias in estimating each direction, can be understood from a behavioral standpoint: Two transparently moving surfaces may correspond to a prey or predator whose image is moving relative to the rest of the visual field. Detecting such stimuli are vital to the animal's survival.

5.2 Attraction at small δ's

The experimentally observed attraction (Braddick et al. 2002), i.e. negative motion repulsion, follows from the fact that subjects tend to perceive two transparently moving groups of dots as coherent motion for small values of δ (Fig. 2b). The perceptual transition from transparent to coherent motion happens as δ becomes smaller than approximately 10 deg (Braddick et al. 2002; Mather and Moulden 1980), which is exactly the region where repulsion turns into attraction. On the other hand, attraction was not observed in different experimental paradigms (Rauber and Treue 1999; Marshak and Sekuler 1979) where subjects were instructed to report the direction of motion of one group of dots with respect to a reference group of dots moving in a known direction. For decreasing δ motion repulsion gradually vanished without ever going negative. It is possible that the asymmetric treatment of the two directions (Rauber and Treue 1999; Marshak and Sekuler 1979) biased the subjects towards perceiving two transparently moving groups of dots even at small values of δ, at which the symmetric treatment of the two directions (Braddick et al. 2002) would lead to the perception of coherent motion.

5.3 Interpretation of the single-trial results

If we interpret the single-trial estimates of the direction difference as perception, then we should expect our subjects to either perceive coherent motion, or transparent motion at angles larger than the threshold, which is 72° according to our fit. This is unlikely to be true, as subjects frequently report angles much smaller.

Alternatively, we can consider an average over a few individual trials as our perception of the direction difference. For example, if the random dots move for 500 ms, we can assume that the measurements of direction of motion for individual dots are updated every 100 ms, leading to five individual trials. Since in each update a new random set of measurements are used to estimate $|m_1 - m_2|$, in some of these trials this estimate will be larger than threshold while in others it will be smaller. Correspondingly, in some trials our estimate of the direction difference is estimated to be zero and in others it will be larger than threshold (72°). The average over these trials can now have values between zero and the threshold.

5.4 Conclusion

In conclusion, this study demonstrates that the statistics of the motion transparency problem, namely the noisy measurements of direction and the lack of individual dot group identity, when analyzed with a clustering algorithm, lead to the motion repulsion illusion. It will be interesting to see whether a rationale similar to the one presented here can account for illusory repulsions observed in other domains such as orientation perception (Blakemore et al. 1970) newpage and stereoscopic depth perception (Stevenson et al. 1991).

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