ANALYSIS OF THE PREWHITENED CONSTANT MODULUS COST FUNCTION

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ABSTRACT

We provide an analysis of the constant modulus (CM) cost function under the assumption of a white equalizer input. This can be achieved by means of an adaptive prewhitening all-pole filter and has been suggested in previous works as a means for both MSE improvement and DFE cold start-up. For white inputs, it is seen that CM-optimizing the spherical component of the equalizer parameter vector is equivalent to minimizing the fourth moment of its output, regardless of the value of the radial component. This leads to an eigenvector interpretation of prewhitened CM receivers, and to a blind initialization procedure from an eigenvector of the quadricovariance matrix of the whitened data. Connections with iterative eigenvector-based schemes are also explored.

1. INTRODUCTION

The Godard or constant modulus algorithm (CMA) [2, 11] is probably the most popular blind equalization scheme due to its computational simplicity and efectiveness [4]. However, due to the nonquadratic nature of the associated cost function, the analysis of CMA behavior is difficult and often only possible under some simplifying assumptions.

Recently it has been suggested [5] that the received signal be prewhitened by a recursive filter and then processed by the CMA transversal equalizer. Inclusion of the prewhitener is justified by the fact that the denominator of the transfer function of the minimum mean squared error (MMSE) equalizer under the only constraint of causality coincides with the minimum phase spectral factor of the received signal's power spectral density [8]. As shown in [8], IIR equalizers including this whitening filter outperform FIR structures in terms of MSE for the same number of coefficients. Also, [1, 5] discuss the possibility of using the parameters of the IIR configuration, once convergence is achieved, to blindly initialize a decision feedback equalizer (DFE), while [6] studies the impact of prewhitening in the convergence speed of CMA.

We present an analysis of the CM cost assuming that the received signal has been whitened. By parameterizing the equalizer in polar form, it is found that under such condition CM stationary points are eigenvectors of a certain matrix associated to its largest eigenvalue. This reveals certain eigenvector-based iterative blind equalization approaches such as that of Jelonnek and Kammeyer [3] or the power method [10] as seeking the extrema of the CM cost, and suggests an initialization for the equalizer parameter vector from an eigenvector of the quadricovariance matrix of the prewhitened data.

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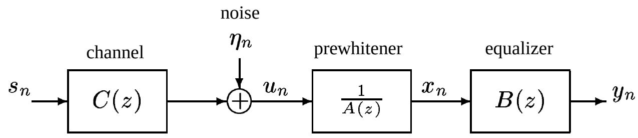

Figure 1: Discrete-time symbol rate channel-equalizer configuration.

2. MATHEMATICAL FRAMEWORK

We consider a single input single output channel-equalizer configuration as shown in Figure 1. The sequence $\{s_n\}$ is assumed to be sub-Gaussian, i.i.d. with variance $E[|s_n|]^2 = \sigma_s^2$, and circular (i.e. $E[s_n^2] = 0$) if complex. The noise $\{\eta_n\}$ is a zero mean white Gaussian process with variance σ_η^2 and independent of $\{s_n\}$. The channel transfer function $C(z)$ is FIR: $C(z) = \sum_{k=0}^{L} c_k^* z^{-k}$.

The received signal $u_n = C(z)s_n + \eta_n$ is processed by the all-pole filter $1/A(z)$, whose goal is to whiten its output $\{x_n\}$ [5]. Since $\{u_n\}$ is a moving average (MA) process of order L, perfect whitening of $\{x_n\}$ is achievable as long as the order of $A(z)$ is no less than L. Adaptation of $1/A(z)$ can be done with second-order statistics based blind algorithms [5, 7]. We assume that convergence of $1/A(z)$ has taken place so that $\{x_n\}$ is white, i.e. $E[x_n x_{n-k}^*] = \sigma_x^2$ for $k = 0$ and 0 otherwise.

The equalizer output is

$$y_n = B(z)x_n = \sum_{k=0}^{N} b_k^* x_{n-k} = \mathbf{b}^H \mathbf{x}_n, \qquad (1)$$

where $\mathbf{b} = [\ b_0 \quad b_1 \quad \cdots \quad b_N\ ]^T$ is the equalizer parameter vector, and $\mathbf{x}_n = [\ x_n \quad x_{n-1} \quad \cdots \quad x_{n-N}\ ]^T$. It is instructive to parameterize $\mathbf{b}$ in polar form: $\mathbf{b} = r\bar{\mathbf{b}}$, where $\bar{\mathbf{b}}^H\bar{\mathbf{b}} = 1$ and $r = ||\mathbf{b}||$. The output of the unit-norm filter is denoted as $\bar{y}_n = \bar{\mathbf{b}}^H \mathbf{x}_n$, while its scaled version $y_n = r\bar{y}_n$ yields the equalizer output. Note that since $\{x_n\}$ is assumed white, one has $E[\mathbf{x}_n\mathbf{x}_n^H] = \sigma_x^2\mathbf{I}$.

3. THE CM COST FUNCTION

The CM criterion is based on the minimization of the following cost function w.r.t. $\mathbf{b}$:

$$F(\mathbf{b}) = E[(\gamma - |y_n|^2)^2], \qquad \gamma \triangleq E[|s_n|^4]/E[|s_n|^2].$$

Since $E[|y_n|^2] = \mathbf{b}^H E[\mathbf{x}_n \mathbf{x}_n^H]\mathbf{b} = r^2\sigma_x^2$, one readily finds that

$$F(\mathbf{b}) = \gamma^2 + r^4 E[|\bar{y}_n|^4] - 2\gamma r^2 \sigma_x^2. \quad (2)$$

From (2) it is seen that minimizing F in terms of the spherical component $\bar{\mathbf{b}}$ is equivalent to minimizing $E[|\bar{y}_n|^4]$, regardless of the value of the radial parameter r. In fact, r could be chosen by any other means (e.g. in order to set $E[|y_n|^2] = \sigma_s^2$) without affecting the location of the minima of F in terms of $\bar{\mathbf{b}}$. Thus we are left with the problem

$$\text{minimize } J(\mathbf{b}) \triangleq E[|\mathbf{b}^H \mathbf{x}_n|^4] \text{ subject to } ||\mathbf{b}|| = 1. \quad (3)$$

Observe that $J(\mathbf{b})$ can be written as $J(\mathbf{b}) = \mathbf{b}^H \mathbf{\Phi}(\mathbf{b})\mathbf{b}$, where the matrix $\mathbf{\Phi}(\mathbf{b})$ is given by

$$\mathbf{\Phi}(\mathbf{b}) \triangleq E[|\mathbf{b}^H \mathbf{x}_n|^2 \mathbf{x}_n \mathbf{x}_n^H]. \quad (4)$$

The Lagrangian for the problem (3) is given by $L(\mathbf{b}) = J(\mathbf{b}) + \lambda(1 - \mathbf{b}^H \mathbf{b})$, where λ is the Lagrange multiplier. Using the standard definition of the gradient operator

$$\nabla = \begin{cases} \frac{\partial}{\partial \mathbf{b}} & \text{(real case)} \\ \frac{1}{2}\left(\frac{\partial}{\partial\,\mathrm{Re}\{\mathbf{b}\}} + j\frac{\partial}{\partial\,\mathrm{Im}\{\mathbf{b}\}}\right) & \text{(complex case)} \end{cases}$$

one has $\nabla J(\mathbf{b}) = a\mathbf{\Phi}(\mathbf{b})\mathbf{b}$, where $a = 4$ (real case) or 2 (complex case). In both cases, setting $\nabla L(\mathbf{b}) = \mathbf{0}$ it is seen that any stationary point $\mathbf{b}_\star$ of the constrained problem (3) must satisfy $2\mathbf{\Phi}(\mathbf{b}_\star)\mathbf{b}_\star = \lambda_\star \mathbf{b}_\star$. By using $\mathbf{b}_\star^H \mathbf{b}_\star = 1$, the value of the multiplier is found to be $\lambda_\star = 2\mathbf{b}_\star \Phi(\mathbf{b}_\star)\mathbf{b}_\star = 2J(\mathbf{b}_\star)$. Thus the equilibria of (3) satisfy the eigenequation

$$\mathbf{\Phi}(\mathbf{b}_\star)\mathbf{b}_\star = J(\mathbf{b}_\star)\mathbf{b}_\star, \qquad ||\mathbf{b}_\star|| = 1, \quad (5)$$

i.e. $(J(\mathbf{b}_\star), \mathbf{b}_\star)$ is an eigenpair of the matrix $\mathbf{\Phi}(\mathbf{b}_\star)$, with $\mathbf{b}_\star$ having unit norm.

4. EIGENVALUE BEHAVIOR OF $\mathbf{\Phi}$

Note that (5) applies to all stationary points of (3) regardless of whether they are minima, maxima or saddle points. Given the resemblance of the constrained problem (3) with a standard Rayleigh quotient, one could expect to classify these equilibria according to whether $J(\mathbf{b}_\star)$ corresponds to the largest, smallest or an intermediate eigenvalue of $\mathbf{\Phi}(\mathbf{b}_\star)$. This is not the case, however, due to the dependence of the matrix $\mathbf{\Phi}$ in (4) with $\mathbf{b}$.

Lemma 1 *Let $\mathbf{b}_\star$ satisfy (5). Then $J(\mathbf{b}_\star)$ is the largest eigenvalue of $\mathbf{\Phi}(\mathbf{b}_\star)$ and satisfies $J(\mathbf{b}_\star) \geq \sigma_x^4$. The remaining eigenvalues satisfy $\lambda \leq \sigma_x^4$. $J(\mathbf{b}_\star)$ has multiplicity one unless the source has a constant modulus distribution and $\mathbf{b}_\star$ perfectly equalizes the channel.*

Proof: Introduce the function

$$K(\mathbf{b}) \triangleq \mathbf{b}^H \mathbf{\Phi}(\mathbf{b}_\star)\mathbf{b}.$$

(Note that $K(\mathbf{b}_\star) = J(\mathbf{b}_\star)$). By defining the processes $\alpha_n = \mathbf{b}_\star^H \mathbf{x}_n$, $\beta_n = \mathbf{b}^H \mathbf{x}_n$, one has $K(\mathbf{b}) = E[|\alpha_n|^2 |\beta_n|^2]$. The signals α_n, β_n are generated as shown in Fig. 2, where $B_\star(z)$, $B(z)$ are the filters with coefficients $\mathbf{b}_\star$, $\mathbf{b}$ respectively, and

$$\varsigma \triangleq \frac{\sigma_\eta^2}{\sigma_s^2}, \qquad s_n^{(0)} \triangleq s_n, \qquad s_n^{(1)} \triangleq \frac{1}{\sqrt{\varsigma}}\eta_n.$$

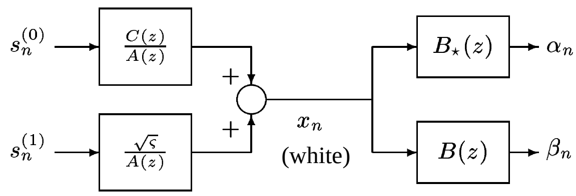

Figure 2: Generation of the signals α_n, β_n.

We can write

$$\alpha_n = \sum_{i=0}^{1} G_i(z) s_n^{(i)}, \qquad \beta_n = \sum_{i=0}^{1} H_i(z) s_n^{(i)},$$

where

$$G_0(z) = \frac{C(z)B_\star(z)}{A(z)} = \sum_{k=0}^{\infty} g_k^{(0)} z^{-k},$$
$$G_1(z) = \frac{\sqrt{\varsigma}B_\star(z)}{A(z)} = \sum_{k=0}^{\infty} g_k^{(1)} z^{-k}$$

and $H_i(z)$ analogously defined. Assume now that the vector $\mathbf{b}$ is orthogonal to $\mathbf{b}_\star$. In that case, $E[\alpha_n \beta_n^*] = \sigma_x^2 \mathbf{b}_\star^H \mathbf{b} = 0$, i.e.

$$\sum_{k=0}^{\infty} [g_k^{(0)} h_k^{(0)*} + g_k^{(1)} h_k^{(1)*}] = 0. \quad (6)$$

Using (6), one can show that

$$\begin{aligned} \frac{K(\mathbf{b})}{\sigma_s^4} = & (\kappa_s^{(0)} - \kappa_g) \sum_{k=0}^{\infty} |g_k^{(0)}|^2 |h_k^{(0)}|^2 \\ & + (\kappa_s^{(1)} - \kappa_g) \sum_{k=0}^{\infty} |g_k^{(1)}|^2 |h_k^{(1)}|^2 \\ & + \left(\sum_{i=0}^{1}\sum_{k=0}^{\infty} |g_k^{(i)}|^2\right)\left(\sum_{i=0}^{1}\sum_{k=0}^{\infty} |h_k^{(i)}|^2\right), \end{aligned} \quad (7)$$

where $\kappa_s^{(i)} = E[|s_n^{(i)}|^4]/E^2[|s_n^{(i)}|^2]$ are the normalized kurtoses of $\{s_n^{(i)}\}$, and κ_g is the normalized kurtosis of a Gaussian process ($\kappa_g = 3$ in the real case and 2 in the complex case). Since the source is sub-Gaussian, $\kappa_s^{(0)} < \kappa_g$ so that the first term in (7) is negative. For Gaussian noise, $\kappa_s^{(1)} = \kappa_g$ and the second term vanishes. The third term is just $E[|\alpha_n|^2]E[|\beta_n|^2]/\sigma_s^4$. Therefore,

$$K(\mathbf{b}) \leq E[|\alpha_n|^2]E[|\beta_n|^2] = \sigma_x^4(\mathbf{b}^H\mathbf{b})(\mathbf{b}_\star^H\mathbf{b}_\star) \quad (8)$$

for all $\mathbf{b}$ orthogonal to $\mathbf{b}_\star$. Now let $\mathbf{b}$ be a unit-norm eigenvector of the Hermitian matrix $\mathbf{\Phi}(\mathbf{b}_\star)$ associated to an eigenvalue $\lambda \neq J(\mathbf{b}_\star)$. Then $\mathbf{b}_\star^H \mathbf{b} = 0$ and (8) holds, showing that $\lambda \leq \sigma_x^4$. On the other hand, applying the Cauchy-Schwartz inequality one has

$$J(\mathbf{b}_\star) = E[|\alpha_n|^4] \geq E^2[|\alpha_n|^2] = \sigma_x^4. \quad (9)$$

Thus $J(\mathbf{b}_\star)$ is the largest eigenvalue. Suppose that its multiplicity is greater than one; then there exist a unit-norm $\mathbf{b}$ such that

$\mathbf{\Phi}(\mathbf{b}_\star)\mathbf{b} = J(\mathbf{b}_\star)\mathbf{b}$ and $\mathbf{b}_\star^H\mathbf{b} = 0$, so that $K(\mathbf{b}) = J(\mathbf{b}_\star) \leq \sigma_x^4$ in view of (8). Then from (9), $J(\mathbf{b}_\star) = \sigma_x^4$, or $E[|\alpha_n|^4] = E^2[|\alpha_n|^2]$. This means that $\{\alpha_n\}$ has a constant modulus distribution, which is possible only if $\{s_n\}$ does, $\sigma_\eta^2 = 0$ and $C(z)B_\star(z)/A(z)$ reduces to a delay. ∎

Eq. (5) is a first-order necessary condition for $\mathbf{b}_\star$ to be a stationary point of (3). If $\mathbf{b}_\star$ is a local minimum, then it must also satisfy the second-order necessary condition

$$\mathbf{b}^T[\nabla^2 J(\mathbf{b}_\star) + \lambda_\star \nabla^2 h(\mathbf{b}_\star)]\mathbf{b} \geq 0 \qquad \forall \mathbf{b} \text{ s.t. } \mathbf{b}^T\nabla h(\mathbf{b}_\star) = 0, \tag{10}$$

where $h(\mathbf{b}) = 1 - \mathbf{b}^T\mathbf{b}$ and $\lambda_\star = 2J(\mathbf{b}_\star)$ are the constraint function and Lagrange multiplier respectively, and we have assumed real signals and channels. The complex case requires taking partial derivatives w.r.t. the real and imaginary parts of $\mathbf{b}$. From (10), the following result is derived.

Lemma 2 *If $\mathbf{b}_\star$ is a local minimum of the constrained problem (3), then the eigenvalues λ of $\mathbf{\Phi}(\mathbf{b}_\star)$ satisfy, for both the real- and complex-valued cases,*

$$\frac{J(\mathbf{b}_\star)}{3} \leq \lambda \leq J(\mathbf{b}_\star).$$

In the real-valued case, if $\mathbf{b}_\star$ is a local maximum of (3), then the eigenvalues $\lambda \neq J(\mathbf{b}_\star)$ of $\mathbf{\Phi}(\mathbf{b}_\star)$ satisfy

$$\lambda \leq \frac{J(\mathbf{b}_\star)}{3}.$$

Note that for all unit norm vectors $\mathbf{b}$, the largest eigenvalue of $\mathbf{\Phi}(\mathbf{b})$ satisfies

$$\lambda_{\max}[\mathbf{\Phi}(\mathbf{b})] = \max_{||\mathbf{p}||=1} \mathbf{p}^H\mathbf{\Phi}(\mathbf{b})\mathbf{p} \geq \mathbf{b}^H\mathbf{\Phi}(\mathbf{b})\mathbf{b} = J(\mathbf{b}),$$

with equality holding at the stationary points of (3). Thus, in order to minimize $J(\mathbf{b})$ (subject to $\mathbf{b}^H\mathbf{b} = 1$), it makes sense to select $\mathbf{b}$ so as to make $\lambda_{\max}[\mathbf{\Phi}(\mathbf{b})]$ small. Noting also that

$$\lambda_{\max}[\mathbf{\Phi}(\mathbf{b})] \leq \operatorname{trace}\mathbf{\Phi}(\mathbf{b}) = \mathbf{b}^H E[\mathbf{x}_n\mathbf{x}_n^H\mathbf{x}_n\mathbf{x}_n^H]\mathbf{b},$$

one finds that a possible initial estimate for the equalizer vector $\mathbf{b}$ could be the unit-norm eigenvector associated to the smallest eigenvalue of the quadricovariance matrix

$$\mathbf{\Psi} \triangleq E[\mathbf{x}_n\mathbf{x}_n^H\mathbf{x}_n\mathbf{x}_n^H]. \tag{11}$$

This choice minimizes $\operatorname{trace}\mathbf{\Phi}(\mathbf{b})$ and it can be computed from the received data alone, since $\mathbf{\Psi}$ does not depend on $\mathbf{b}$.

5. CONNECTIONS WITH ITERATIVE SCHEMES

The previous section has revealed that the search for a local minimum of the CM cost in the prewhitened case can be linked to a search for the eigenvector associated to the largest eigenvalue of certain matrix. This problem, however, is difficulted by the fact that the matrix in question depends on the equalizer parameters. Nevertheless, the eigenvector formulation lends itself to the application of iterative methods. We comment on several of these next.

1. A natural iterative procedure can be formulated as follows:

   choose $\mathbf{b}_{k+1}$ as the unit-norm eigenvector of $\mathbf{\Phi}(\mathbf{b}_k)$ associated to its largest eigenvalue.

   This method seems to converge to a local *maximum* of $J(\mathbf{b})$ (subject to $||\mathbf{b}|| = 1$). Therefore it is not well suited to problems with sub-Gaussian sources. It could be useful, however, in settings in which the source is super-Gaussian since in those cases the CM cost should be maximized w.r.t. the spherical component $\bar{\mathbf{b}}$ of the equalizer vector in order to reduce intersymbol interference [9].

2. Given the extremal eigenvalue character of the problem, the power method [10] can also be used, for example:

   $$\tilde{\mathbf{b}} = \mathbf{\Phi}(\mathbf{b}_k)\mathbf{b}_k, \qquad \mathbf{b}_{k+1} = \tilde{\mathbf{b}}/||\tilde{\mathbf{b}}||.$$

   Again, this procedure seems to converge to a local maximum of J. It can be reversed in order to have it converge to a local minimum:

   $$\mathbf{\Phi}(\mathbf{b}_k)\tilde{\mathbf{b}} = \mathbf{b}_k, \qquad \mathbf{b}_{k+1} = \tilde{\mathbf{b}}/||\tilde{\mathbf{b}}||.$$

   With this modification, a linear equation set must be solved at each step.

3. In [3], Jelonnek and Kammeyer proposed a method based on the maximization of the absolute value of the fourth-order cross-cumulant of the outputs of the equalizers at steps k and $k+1$. Specifically, if we let $\alpha_n = \mathbf{b}_k^H\mathbf{x}_n$, $\beta_n = \mathbf{b}_{k+1}^H\mathbf{x}_n$, then

   $$\begin{aligned} c_4^{\alpha\beta} \;\triangleq\; & E[|\alpha_n|^2|\beta_n|^2] - |E[\alpha_n\beta_n]|^2 \\ & - |E[\alpha_n^*\beta_n]|^2 - E[|\alpha_n|^2]E[|\beta_n|^2], \end{aligned}$$

   and the iteration maximizes $|c_4^{\alpha\beta}|$ w.r.t. $\mathbf{b}_{k+1}$ subject to $E[|\beta_n|^2] =$ constant. When $\{x_n\}$ is white, one finds that

   $$c_4^{\alpha\beta} = \mathbf{b}_{k+1}^H\mathbf{\Upsilon}(\mathbf{b}_k)\mathbf{b}_{k+1}, \qquad E[|\beta_n|^2] = \sigma_x^2||\mathbf{b}_{k+1}||^2,$$

   where

   $$\mathbf{\Upsilon}(\mathbf{b}) \triangleq \mathbf{\Phi}(\mathbf{b}) - (\kappa_g - 1)\sigma_x^4\mathbf{b}\mathbf{b}^H - ||\mathbf{b}||^2\sigma_x^4\mathbf{I}.$$

   For sub-Gaussian (super-Gaussian) sources, $c_4^{\alpha\beta}$ is negative (positive). Therefore, $\mathbf{b}_{k+1}$ is given by the unit-norm eigenvector associated to the minimum eigenvalue of $\mathbf{\Phi}(\mathbf{b}_k) - (\kappa_g - 1)\sigma_x^4\mathbf{b}_k\mathbf{b}_k^H$ in the sub-Gaussian source case, and to the maximum eigenvalue in the super-Gaussian case. In both cases, at any fixed point of the iteration, the condition (5) must hold, so that these coincide with the stationary points of the constrained problem (3). The iterative process seems to converge to a local minima of (3), but as will be shown in the next section, there may exist local minima which are not convergent.

6. NUMERICAL EXAMPLES

In order to illustrate the points of the previous sections, we consider a transversal equalizer of order $N = 2$. In the real-valued case, the spherical component $\bar{\mathbf{b}}$ can be parameterized as

$$\bar{\mathbf{b}} = [\;\cos\theta_1\cos\theta_2 \quad \sin\theta_1\cos\theta_2 \quad \sin\theta_2\;]^T,$$

where $|\theta_i| \leq \pi/2$. Any unit-norm $\bar{\mathbf{b}}$ can be parameterized by θ_1, θ_2 in this range, up to a sign change which is inherent to the blind equalization problem.

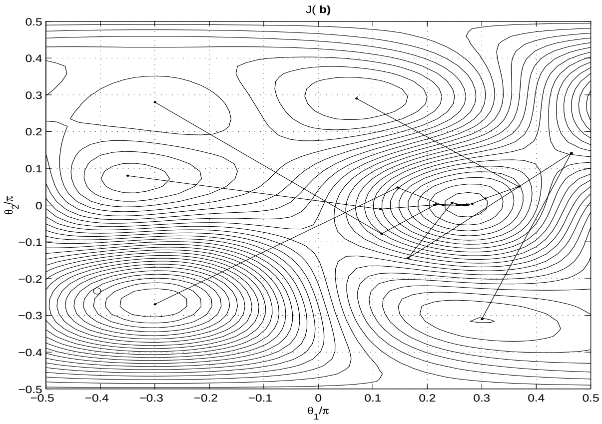

Figure 3: Contour plot of $J(\bar{\mathbf{b}})$ and trajectories of the JK iteration. The circle marks the initial estimate from the eigenvector of $\mathbf{\Psi}$.

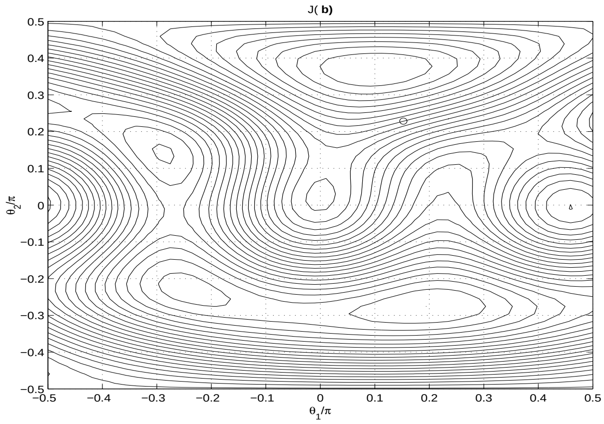

Figure 4: Contour plot of $J(\bar{\mathbf{b}})$. The circle marks the initial estimate from the eigenvector of $\mathbf{\Psi}$.

The channel-prewhitener combination $C(z)/A(z)$ is taken as a second-order all-pass system

$$\frac{C(z)}{A(z)} = \frac{a_2 + a_1 z^{-1} + z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}},$$

and it is assumed that the noise is absent: $\sigma_\eta^2 = 0$.

Figure 3 shows the contours of J as a function of θ_1, θ_2 for $a_1 = 0.7$, $a_2 = 0$ and a source kurtosis $\kappa_s = 1.8$. The global minimum is located at $(\theta_1, \theta_2) \approx (-0.3\pi, -0.28\pi)$; in addition there are two local minima at $(\theta_1, \theta_2) \approx (0.27\pi, 0)$ and $(-0.3\pi, 0.28\pi)$. Also shown are the trajectories of the Jelonnek-Kammeyer iteration with different initializations. It is seen that only one of the local minima is convergent, while the global minimum is not.

The initial estimate obtained via the eigenvector of $\mathbf{\Psi}$ as described in section 4 is also marked in Figure 3; it is seen to lie in a basin of attraction of the global minimum. This is also the case in all the examples we have tested. For example, let $a_1 = 0.54$, $a_2 = 0.8$, $\kappa_s = 1$. The contour plot of J and the estimate from the eigenvector of $\mathbf{\Psi}$ are shown in Figure 4. The global minimum in this case is located at $(\theta_1, \theta_2) \approx (0.1\pi, 0.36\pi)$. Two local minima appear at $(0, 0.03\pi)$ and $(0.45\pi, 0)$.

7. CONCLUSIONS

A preliminary analysis of the CM cost with prewhitened input signals has been presented. It is seen that the CM-optimum value of the spherical component of the equalizer parameter vector corresponds to the minimization of the fourth moment of its output, independently of the value of the radial component. Stationary points are found to be eigenvectors of a certain matrix, corresponding to its largest eigenvalue. This suggests an initialization of the parameter vector from an eigenvector of the quadricovariance matrix of the received data. Future work should address the robustness of this approach in terms of finite-length data records and numerical stability issues.

8. REFERENCES

[1] R. A. Casas, C. Richard Johnson, Jr., J. Harp and S. Caffee, 'On initialization strategies for blind adaptive DFEs', *Proc. IEEE Wireless Comm. and Networking Conf.*, pp. 792-796, Sep. 1999.

[2] D. N. Godard, 'Self-recovering equalization and carrier tracking in two-dimensional data communication systems', *IEEE Trans. Comm.*, vol. 28, pp. 1867-1875, Nov. 1980.

[3] B. Jelonnek and K. D. Kammeyer, 'A closed-form solution to blind equalization', *Signal Processing*, vol. 36, pp. 251-259, Apr. 1994.

[4] C. R. Johnson Jr., P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown and R. A. Casas, 'Blind equalization using the Constant Modulus criterion: A review', *Proc. IEEE*, vol. 86, pp. 1927-1950, Oct. 1998.

[5] J. Labat, O. Macchi and C. Laot, 'Adaptive decision feedback equalization: can you skip the training period?', *IEEE Trans. Communications*, vol. 46, pp. 921-930, Jul. 1998.

[6] J. P. LeBlanc and I. Fijalkow, 'Blind adapted, pre-whitened constant modulus algorithm', *Proc. IEEE Int. Conf. Communications*, vol. 8, pp. 2438-2442, Jun. 2001.

[7] R. López-Valcarce and F. Pérez-González, 'On blind adaptive algorithms for IIR equalizers', *Proc. 9th DSP Workshop*, Oct. 2000. Online proceedings in http://spib.ece.rice.edu/SPTM/DSP2000/program.html .

[8] B. Mulgrew and C. F. N. Cowan, 'An adaptive Kalman equalizer: structure and performance', *IEEE Trans. ASSP*, vol. 35, pp. 1727-1735, Dec. 1987.

[9] P. A. Regalia, 'On the equivalence between the Godard and Shalvi-Weinstein schemes of blind equalization', *Signal Processing*, vol. 73, pp. 185-190, Feb. 1999.

[10] P. A. Regalia and E. Kofidis, 'The higher-order power method revisited: convergence proofs and effective initialization', *Proc. IEEE ICASSP*, vol. 5 pp. 2709-2712, Istanbul, Turkey, Jun. 2000.

[11] J. R. Treichler and B. G. Agee, 'A new approach to multipath correction of constant modulus signals', *IEEE Trans. ASSP*, vol. 31, pp. 459-472, Apr. 1983.