DOCUMENT RESUME

ED 302 415 SE 050 250

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|---|---|
| AUTHOR | Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed. |
| TITLE | Investigations in Mathematics Education, Volume 21, Number 2. |
| INSTITUTION | Ohio State Univ., Columbus, Ohio. Information Reference Center for Science, Mathematics, and Environmental Education. |
| PUB DATE | 88 |
| NOTE | 70p.; For previous edition, see ED 296 888. |
| AVAILABLE FROM | SMEAC Information Reference Center, 1200 Chambers Road, Room 310, Columbus, OH 43212 (U.S. Subscription $8.00; $2.75 single copy). |
| PUB TYPE | Collected Works - Serials (022) -- Reports - Research/Technical (143) -- Information Analyses - ERIC Information Analysis Products (071) |
| JOURNAL CIT | Investigations in Mathematics Education; v21 n2 Spr 88 |
| EDRS PRICE | MF01/PC03 Plus Postage. |
| DESCRIPTORS | Arithmetic; Computer Assisted Instruction; Computer Science; *Computer Uses in Education; Educational Research; *Experiential Learning; Longitudinal Studies; *Mathematics Achievement; *Mathematics Anxiety; Mathematics Education; Mathematics Instruction; *Problem Solving; Secondary Education; *Secondary School Mathematics |

ABSTRACT

This publication is an abstracted compilation of 11 investigations. The information includes purpose, rationale, research design and procedures, findings, interpretations, abstractor's comments, previous research, procedure and statistical methods, and further research. Contained within are: (1) "Gender-Based Differential Item Performance in Mathematics Achievement Items"; (2) "The Relationship between Mathematics Anxiety and Achievement Variables"; (3) "A Teaching Strategy for Elementary Algebraic Fractions"; (4) "Resequencing Skills and Concepts in Applied Calculus Using the Computer as a Tool"; (5) "Computer Programming and Logical Reasoning"; (6) "The Influence on Mathematics Test Scores, by Ethnicity and Sex, of Prior Achievement and High School Mathematics Courses"; (7) "Mathematics Achievement as a Function of Language"; (8) "Verbal Clarifying Behaviors, Mathematics Participation, Attitudes"; (9) "The Development of Informal and Formal Mathematical Thinking in Korean and U.S. Children"; (10) "Mathematics Education Research Studies in Journals as Indexed by Current Index to Journals in Education, October-December 1987"; and (11) "Mathematics Education Research Studies Reported in Resources in Education, October-December 1987." (CW)

# MATHEMATICS EDUCATION INFORMATION REPORT



INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 21, Number 2 - Spring 1988

THE ERIC SCIENCE, MATHEMATICS AND
ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
The Office of Educational Research and
Improvement, U.S. Department of Education
and
The SMEAC Information Reference Center,
College of Education, The Ohio State University

INVESTIGATIONS IN MATHEMATICS EDUCATION



Published quarterly by

SMEAC Information Reference Center
The Ohio State University
1200 Chambers Road, Room 310
Columbus, Ohio 43212

Telephone: (614) 292-6717

With the cooperation of the ERIC Clearinghouse for Science, Mathematics and Environmental Education

Volume 21, Number 2 - Spring 1988

Subscription Price: $8.00 per year. Single Copy Price: $2.75
$9.00 for Canadian mailings and $11.00 for foreign mailings.

Spring 1988

An editorial comment . . .

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You may have noticed that the arrival of IME has become somewhat erratic.

There are reasons for this.

The primary one lies with some of our reviewers.

Some reviewers send the request form back immediately, indicating "yes, I can review the article" or "no, I can't do a review this time." Their prompt replies are really appreciated.

Other reviewers never send the request form back -- despite a follow-up letter asking them which option they're taking. I'm left not knowing whether or not to request another reviewer to prepare the manuscript.

And then there are the reviewers who say "yes" to the request -- but never send a manuscript.

Even after a reminder -- or two -- nothing is heard from them again.

Last year I put a request for reviewers in the SIG/RME Newsletter, asking anyone interested in reviewing for IME to let me know. Upon receipt of their letters, I sent each a request for a review. Over half of them did not reply with either a "yes" or "no" -- which really puzzles me, since they had indicated that they wanted to review articles.

We have considered ending publication of IME, because of the difficulty in publishing it on schedule. As of now, it appears that it will survive another year -- a year when it continues to be on probation. Individual subscribers seldom complain when issues fail to arrive on time, but libraries do, and this creates real problems.

Approximately 3800 copies of IME are distributed each year (four issues per year plus back sales). In addition, it is abstracted for ERIC's Resources in Education (RIE), and is thus available from the ERIC Document Reproduction Service. User studies indicate that each issue is used by over 1000 people, primarily graduate students, researchers, and mathematics teachers.

I believe, as previous editors have, and as I believe many of you do, that IME serves a special, needed function in the realm of mathematics education research. It has done this ever since its origin as an in-house publication of the School Mathematics Study Group (SMSG). But my beliefs are not going to "save" IME without the help of reviewers.

Perhaps I have failed to communicate before this how vital your help is. I know that I have not had time for the past year or so to send out cards telling you your manuscript was received (there is no secretarial help except for typing the journal and, occasionally, sending out the requests). This is distressing to me as well as to you -- but the time available is spent on editing and proofing and requesting -- and requesting again, and reminding ...

So what can you do?

1) Of course , reply when you receive a request for a review -- and remember that a prompt reply is especially welcome!

2) When you agree to prepare a review, send it as close to the due date as you can -- please!

3) If you know of qualified people who would like to prepare reviews -- and would actually prepare them! -- let me know or encourage them to write to me.

IME really does need your help!

Marilyn N. Suydam

Doolittle, Allen E. and Cleary, T. Anne. GENDER-BASED DIFFERENTIAL ITEM PERFORMANCE IN MATHEMATICS ACHIEVEMENT ITEMS. Journal of Educational Measurement 24: 157-166; Summer 1987.

Abstract and comments prepared for I.M.E. by LINDA JENSEN SHEFFIELD, Northern Kentucky University.

## 1. Purpose

The investigation was designed to determine the relationships between characteristics of mathematics achievement and gender differences in performance. Differential item performance was studied for the areas of arithmetic and algebraic operations, arithmetic and algebraic reasoning, geometry, intermediate algebra, number and numeration concepts, and advanced topics.

## 2. Rationale

Research indicates that male high school students as a group perform better than female high school students on mathematics achievement tests. This is partially due to the fact that males typically receive more mathematics instruction than females in high school. In this study, an attempt was made to control for instructional background and then determine if certain categories of items were easier for males or females.

## 3. Research Design and Procedures

The data for this study were drawn from a sample of college-bound high school seniors who took the ACT Assessment Mathematics Usage Test (ACTM) in October 1985. Only those students who had had a course in geometry, advanced algebra, or algebra II, and either or both trigonometry and advanced mathematics (including precalculus) were included in the study. This reduced but did not eliminate differences in mathematics instruction between males and females. For example, 18 percent of the females and 23 percent of the males had studied calculus.

The ACTM is a 40-item, 50-minute measure of mathematical reasoning ability. In general, the test emphasizes the mathematical skills acquired in plane geometry and first- and second-year algebra. The test includes four items on arithmetic and algebraic operations, 14 word problems on arithmetic and algebraic reasoning, eight items on geometry, eight items on intermediate algebra, four items on number and numeration concepts, and two questions on advanced topics.

A modification of an index by Linn and Harnish was used to measure differential item performance. The index gives a relative measure of differential item performance which assumes that the total test score is unbiased. Differential item performance exists when the relative performance on an item for the two groups is not in line with the overall expectations.

A single-factor design with replicated experiments was used, with the item category considered a fixed effect and the test form a random effect. The six categories of items were crossed with the eight unique forms of the test which were given. The eight forms were used essentially as replications. Negative values of the index represented items which were relatively easier for males and positive items indicated items which were relatively easier for females. Analysis of variance was used to determine if there was a significant effect of item category on gender-based differential item performance.

## 4. Findings

The means averaged across all eight forms of the ACTM for the six categories are as follows (the standard deviations are in parentheses): geometry -.99 (1.18); arithmetic and algebraic reasoning -.55 (1.56); advanced topics -.47 (1.23); number and numeration concepts .10 (1.19); intermediate algebra .57 (1.10); and arithmetic and algebraic operations .60 (1.68). There are some stable patterns, in spite of a fairly wide range of means across the eight forms. Geometry and arithmetic and algebraic reasoning items have negative means on each of the eight forms and intermediate algebra and arithmetic and algebraic operations items have predominantly positive means on the forms.

Analysis of variance showed only the item category main effect to be significant. Using the Scheffe procedure to test for differences among the means suggested that geometry and arithmetic and algebraic reasoning were relatively more difficult for females and intermediate algebra and arithmetic and algebraic operations were relatively less difficult for females.

5. Interpretations

The results of the study suggest that some gender-based differential item performance exists in mathematics achievement which is not simply the result of differential instruction at the high school level. The cause of the differences cannot be determined by this study, however.

The authors suggested three possible reasons for the differences:

1. "The ACTM is "biased" in the sense of some items' unfairly measuring performance on extraneous content". The authors then stated, however, that it is not useful to think in terms of the test being biased because the items appear to be of a type typically encountered in high school courses.

2. Group differences in instruction or background may have been too well established before and during high school for balancing on the basis of the high school curriculum to be enough of a control.

Background needs to be broadly interpreted to include attitudes, extracurricular activities, and a wide array of sociocultural factors. Also, the advanced instruction in areas such as calculus received by some of the males may have helped.

3. "There may be real differences between males and females in the measured abilities." The relative ease of the geometry for the male group may suggest that the diagrams in many of the items may have been easier to use by the males because of certain spatial skills which they have developed more than the females. Males may have stronger mathematical reasoning skills while females have relatively stronger computational skills. The authors point to the relative performance on the arithmetic and algebraic operations and arithmetic and algebraic reasoning sections which require essentially the same knowledge of mathematical concepts with the primary difference being in the context.

"This study has identified item categories that relate to gender-based differential item performance among males and females with formal instruction at the level required by the test. Further investigation will be needed to clarify the differences in the dimensionality of the achievement construct for these two groups of students."

## Abstractor's Comments

The issue of male/female differences in performance on mathematics achievement tests is a very difficult one, and the authors are to be commended for attempting to minimize the effects of the differences in mathematics instruction. With over 1300 students in this study taking each form of the ACTM, the amount of data available is impressive.

There are some questions raised by the study which may suggest new areas for further research or alternate causes for the differential item performance by male and female students:

1. Is the modification of the index by Linn and Harnisch to determine differential item performance the best index to use for this study? The authors note that this index is a "small sample" alternative to other indices based on item response theory. With over 1300 students taking each form of the ACTM, a small sample index seems unnecessary. The Linn and Harnisch index assumes that the total test score is an unbiased measure of ability or achievement, but it is not at all clear that the total ACTM is unbiased. No means for males and females are given for a combination of the eight forms, but the means on individual forms range from 22.7 for females to 27.9 for males. Female means are lower than male means on every form, but the authors do not state if these differences are significant. It appears as though they would be significant since the differences range generally between two and three points with a standard deviation around seven and an N above 1300. It therefore appears that the ACTM is indeed biased in favor of males.

2. How does the number of items in each category affect the overall score on the ACTM? If males do better on arithmetic and algebraic reasoning and females do better on arithmetic and algebraic operations, it would certainly seem that having 14 items on reasoning and only four items on operations would bias the test in favor of the males. If males and females traditionally perform better in different areas, should we balance those areas to give females a better opportunity at an overall score which is equal to males? What is the reason for having eight items on geometry and 14 on arithmetic and algebraic reasoning, the two areas in which males performed better, and only four items on arithmetic and algebraic operations and eight items on intermediate algebra, the two areas in which females performed better? It is reasonable to expect that changing the balance of these items would change the overall performance of males and females on the test. Do we wish to do that? IQ tests also have subtests with different means for males and females but they appear to be balanced on their overall scores. Should we not attempt to balance the overall scores on the ACTM knowing the influence these scores have in the important areas of college acceptance, especially in traditionally male-dominated fields, and scholarships?

3. How did the differences in items within each category affect the overall category ratings? The area of advanced topics is a good example since there were only two questions in that category on each of the eight forms. The items covered such topics as "trigonometric functions, permutations and combinations, probability, statistics and logic." Means in this area ranged from a -1.7 to 1.1 across the eight forms. It would be interesting to note whether different topics were covered on Form H with a mean of -1.7 and Form C with a mean of 1.1. The same questions can be raised for other areas such as number and numeration concepts and arithmetic and algebraic operations which had a wide range of means and only four items testing each area.

4. The final question I would like to raise is in the area of bias. What is the meaning of bias? In this article, biased is defined as "unfairly measuring extraneous content" and the concept is then dismissed as irrelevant because questions which appear in the ACTM typically appear in high school mathematics courses. In his article, "Human Intelligence Testing: A Cultural-Ecological Perspective", Ogbu (1988) discussed the "cultural bias" in the IQ testing of Black Americans. At least three of the arguments used by Ogbu might also be used in the context of females taking a mathematics achievement test.

1. Because of a stratified opportunity structure, Ogbu (1988, p. 28) noted that Blacks have been "excluded from the more desirable technoeconomic, social and political roles which demand and promote White middle-class types of cognitive skills...which denied involuntary minorities the opportunity to develop the ways of speaking, conceptualizing and thinking of the Whites." Thus, Ogbu argued that the cognitive skills of the two groups may be different.

It may be that females develop different cognitive skills due to the encouragement of females to learn and follow rules without questioning the teacher. This influence is much stronger for girls than for boys throughout school and may account for some of the differences in arithmetic and algebraic operations as opposed to arithmetic and algebraic reasoning. Girls are also not encouraged to play with spatial structures as much as boys and this may account for some differences in geometric abilities. Doolittle and Cleary noted these as background differences in males and females, but do not consider them a cultural bias of the test as Ogbu does.

2. Ogbu noted that even though Blacks may possess the same mathematical concepts and other cognitive skills which make up the IQ items, it does not necessarily mean that their test scores will be as high. Part of the reason for this is that doing well scholastically did not historically bring the same rewards to Blacks as it did to Whites. Students therefore became ambivalent about taking IQ tests. In a like manner, females have not historically been rewarded for doing well on a test of mathematical ability. Careers which require mathematics ability have traditionally been male-dominated and females have not seen the need or the rewards for excelling in that area. They therefore may not put out the same effort as males to perform well on the tests. It is therefore possible that females have the same knowledge as males in certain areas which do not show up in test results.

3. Ogbu noted another reason for Blacks not performing well on IQ tests in spite of possessing the knowledge

which is equally applicable to females. That is, many Blacks look down upon other Blacks who do well in traditionally White-dominated areas. They put successful Blacks down for "acting White". In a like manner, girls are often put down for doing well in mathematics because it is "only for boys". They may feel they are losing their feminine identity if they outperform boys in a traditionally male-dominated area such as mathematics. They may therefore, consciously or unconsciously experience dissonance in both the preparation for and the taking of mathematics tests which may negatively affect their test performance and mask their true abilities.

The bias of the test is therefore a very complex issue and should not be lightly dismissed by the authors.

## Reference

Ogbu, John U. (1988). Human intelligence testing: a cultural-ecological perspcetive. National Forum, 48(2): 23-29.

Gliner, Gail S. THE RELATIONSHIP BETWEEN MATHEMATICS ANXIETY AND ACHIEVEMENT VARIABLES. School Science and Mathematics 87: 81-87; February 1987.

Abstract and comments prepared for I.M.E. by THOMAS O'SHEA, Simon Fraser University, Burnaby, B.C.

1. Purpose

The purpose was to study the relationship among several variables in order to "gather more definitive information involved in the development of mathematics anxiety and [its] relationship to achievement in mathematics."

2. Rationale

The relationship betwen attitude and achievement in mathematics is not clear. While higher levels of mathematics anxiety are related to lower mathematics achievement, there may not be sex differences in mathematics anxiety. Factors such as grade level, courses completed, spelling, vocabulary, language mechanics, and computational and applied mathematics achievement may be differentially related to mathematics anxiety and to each other.

3. Research Design and Procedures

The original sample consisted of 154 students in grades 9 through 12 at an urban high school of approximately 1200 students in Denver, Colorado. Of these, the parents of 95 students signed the necessary consent form, and the final sample consisted of 50 boys and 45 girls. Students were enrolled in courses having a wide range of mathematics content: General Math, Consumer Math, Algebra 1, Geometry 1X, Algebra 3, Computers, and Analytic Geometry. Courses were coded from 1 to 7, reflecting a progression of difficulty.

Two criterion variables were used: mathematics anxiety as measured by the Mathematics Anxiety Rating Scale-Form A (MARS-A), a modification of the original MARS adapted for high school students, and mathematics achievement as demonstrated by percentile rank for the total mathematics score on the California Test of Basic Skills.

The predictor variables consisted of current mathematics course, sex, age, grade level, grade point average, number of mathematics courses completed, previous score on the CTBS, and eight CTBS subtest scores in verbal and mathematical skills. The MARS-A test score was used also as a predictor of achievement.

Two separate multiple regression analyses were used to identify variables predicting mathematics anxiety and mathematics achievement. Two two-way analyses of variance using the MARS-A score as the dependent measure were performed, one for sex by grade and the second for sex by course.

4. Findings

For the anxiety criterion variable, students' grade point average entered the equation first, and accounted for 7 percent of the variance. The second variable to enter was course enrolled in, yielding an R2 value of 0.12. The final three variables were sex, spelling, and language-expression, each adding about .03 to the R2 value, for a final R2 of 0.21.

For the achievement criterion variable, the reading-comprehension subtest score accounted for 71 percent of the variance. Eight other variables, including sex, but not including MARS-A, entered the equation, bringing, the final R2 value to 0.85.

The two analyses of variance showed no reliable differences on the MARS-A score between sex, grade, or course.

5. Interpretation

The analyses confirm the findings of previous researchers: mathematics achievement is not a significant variable in predicting mathematics anxiety scores. However, verbal skills seem to be good predictors of overall mathematics achievement. Students who have developed good verbal skills may also have acquired the skills needed to read and solve mathematics problems. Teachers should stress careful reading and understanding of words and symbols in mathematics problems.

## Abstractor's Comments

Technically, the regression equations should have been included in the article. For example, although course enrolled in was included as a predictor of anxiety, albeit making a very small contribution, it was difficult to determine the direction of the contribution. The positive zero-order correlation of .02 could have become negative in the regression equation, given the relatively small sample size of 95 used to analyze the relationship among 16 variables.

I sympathize with the author when she strains to find something to say about her findings. The major variable of interest, mathematics anxiety, turned out to be a dud. The analyses of variance showed that mathematics anxiety was not related to sex, grade, or course, and the regression analysis showed it did not help to predict mathematics achievement. Furthermore, the five variables that entered the prediction equation for anxiety accounted for only 21 percent of the variance. Grade point average was the most important variable in that analysis, with a significant ($p=.01$) zero-order correlation of -.26. In other words, the better you are academically, the lower your mathematics anxiety.

Apart from the very limited discussion of the results, there were inconsistencies that indicated a lack of clarity as to the nature of the research. In the introduction the author claims that previous research has pointed to an inverse relationship between mathematics achievement and anxiety. In discussing her results, however, she says: "As predicted by previous researchers, math achievement was not a significant variable in predicting math anxiety scores." Perhaps this contradiction simply illustrates the continuing lack of consistency and difficulties in interpreting research in the area of achievement, attitudes, anxiety, and sex.

Gordon, John T. A TEACHING STRATEGY FOR ELEMENTARY ALGEBRAIC FRACTIONS. Focus on Learning Problems in Mathematics 10: 29-36; Fall 1988.

Abstract and comments prepared for I.M.E. by DAVID KIRSHNER, Louisiana State University.

1. Purpose

This article explores the effects of expository writing about algebraic symbol manipulation exercises upon subsequent errors on similar exercises.

2. Rationale

Gordon rehearses a litany of ills (fear, lack of comprehension, a tendency to respond "automatically and compulsively" (p. 29) to mathematics problems) which are the all-too-familiar accompaniments of school mathematics experience. He also notes that "writing is frequently used as an analytic tool in history and literature courses" (p. 30), and that theorists such as "Vygotsky (1962) and Odell (1980) see an important link between writing and learning" (p. 30). As Emig (1977) observes, "the act of writing gives students the opportunity to formulate, organize, internalize and evaluate ideas" (p. 30). Perhaps, then, as Johnson (1983) suggests, "writing may help students deal with their feelings toward mathematics, [and] organize and clarify their thoughts about mathematical ideas and processes" (p. 29).

3. Research Design and Procedures

A fairly standard quasi-experimental methodology seemed to be employed in the study. Six intact non-credit, developmental study elementary algebra classes of 15-26 students each at Georgia State University were assigned to various treatment and control conditions. On the first day of class, students in three of the sections (call these the writing sections) were requested to write, anonymously and briefly, about their feelings about being in a developmental mathematics class. Their answers were read and discussed in class.

In the third week of class, just after studying equations and inequalities in one variable, these same students "were asked to use one, two or three sentences" (p. 31) to answer the following questions:

(a) In what ways is the work different when solving equations and when solving inequalities? and

(b) In what ways is the work alike when solving equations and when solving inequalities? (p. 31)

Sometime later in the course students in all sections were given the following three exercises to perform individually:

1. Solve for y:

$$\frac{4}{y-2} + 2 = \frac{8}{y^2-2y}$$

2. Perform the indicated operations. Simplify if possible.

$$\frac{4}{y-2} + 2 - \frac{8}{y^2-2y}$$

3. Perform the indicated operations. Simplify if possible.

$$\frac{4}{y-2} + 2 + \frac{8}{y^2-2y} \quad \text{(p. 32)}$$

Each student in the writing sections was given a total of 25 minutes to compare and contrast (in writing) the work done for each of the three pairs of problems, (1,2), (1,3), (2,3). The professors immediately worked the three problems on the board. The students' written responses were not discussed.

A slightly different tack was taken in one of the non-writing sections. The professor in this section led a class discussion on the topics about which writing section students had written. The problems were worked by the students in the course of the discussion. (It was reported that the discussion was quite exciting for the students and motivating for the instructor.)

In the remaining two non-writing sections, (call these the extra-problem sections), students were given three problems similar in form to the original problems to work. They were "encouraged to work together" (p. 33). The professor worked the original problems on the board and then walked around the class helping students with the new problems.

About three days after these sessions, each student was given a quiz consisting of three problems similar in form to the original set. The students' work was marked and errors were tallied and grouped according to the mathematical concept involved in the error. In the case of several errors within a single problem, only the first error encountered was included.

## 4. Findings

For each of the three problems, the overall ratio of errors to students did not vary notably between sections; however, for the first problem there were some marked between-section differences in kind-of-error. For the extra-problem sections, 17 percent and 26 percent, respectively, of the errors "involved signs and the combining of like terms" (p. 34), whereas less than 6 percent of the errors in the other sections were of this sort. Students from these two sections also appeared to be sloppier in their work habits on question three than other students. They more frequently dropped an uninvolved term at one stage of solution, only to bring it back again when needed further on. Additional description of the kinds of errors made is reported in the article.

5. Interpretations

Early in the report the author disclaims the scientific intent of this "teaching experiment" (p. 30): "Hypotheses were not being tested" (p. 31). The bulk of his discussion section is a recapitulation of the findings, and an acknowledgement that "from this limited study one cannot conclude that the differences in treatment caused the differences in error patterns" (p. 35). The report concludes with suggestions for algebra topics for which similar studies could be undertaken.

## Abstractor's Comments

The effects of expository writing about mathematics on students' learning and affect is an important and provocative topic which has received considerable recent attention. Several authors specifically have analyzed students' written explanations about their own solution processes (Schmidt, 1985; McMillen, 1986; Ackerman, 1987). Other researchers have considered other kinds of written exercises (King, 1982, had students express their questions about their mathematics in writing; Pallmann, 1982, asked for descriptions of order of operation rules, rules for adding signed numbers, etc.; Bell & Bell, 1985, studied effects of writing about word-problem solving; Selfe, Peterson & Nahrgang, 1986, studied effects of journal writing activities; and Abel, 1987, had students imagine themselves to be a mathematical symbol or a geometric figure and write about their experiences).

This is only a sampling from a burgeoning literature, with which the author appears to be unfamiliar. (Look for Rose (in preparation) for a more complete review.) Needless to say, the precise framing of a question, the selection of research design and instrumentation, etc., must build upon the contributions of previous investigators in order for systematic progress to occur.

The new wrinkle which this study introduces is the use of error patterns in routine manipulative work as a dependent variable. Generally speaking, other researchers have chosen more obviously conceptually-linked domains in which to seek effects of verbalization. The nature of the psychological processes involved in manipulative work are still very much in dispute (see, for example, Gagné, 1983a; a reaction by Steffe and Blake, 1983; and Gagné's, 1983b, rebuttal), perhaps involving "visually-moderated sequences" (Davis, 1979, p. 26) hence relatively impermeable to verbalization inputs. Nevertheless, hunting for such effects is certainly a reasonable undertaking which could reflect importantly upon the nature of these mental processes; theory can build upon evidence, as well as evidence upon theory.

That said, the study does not seem to be optimally designed to bring home the bacon. Quasi-experimental design is problematic at the best of times, with non-random assignment to treatment groups introducing a possible source of uncontrolled variance. Even so, useful results can be gleaned from well-designed studies in which sources of variance are controlled to the greatest extent possible.

There was no consistent effort in this study to eliminate extraneous sources of variance. For example, introducing a discussion of feelings about being in a developmental mathematics class on the first day of classes may have stimulated class dynamics and patterns of interaction which would reverberate on through the semester, and the discussion of feelings is not even germane to the principal independent variable, writing about mathematics. Attempting to understand the study entirely as an exploratory e rcise does not succeed either, since the richest data source, the actual content of students' written work, is not reported. The study brings us tantalizingly close to a perhaps profound relationship between verbalization experiences and avoidance of error-types involving "signs and the combining of like terms" (p. 34), only to warn us off at the last moment for the dangers of overinterpretation.

## References


Abel, J. P. (1987). Using writing to teach mathematics. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Anaheim, CA.

Ackerman, A. (1987). Writing across the curriculum: hints for the mathematics classroom. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Anaheim, CA.

Bell, E. S., & Bell, R. N. (1985). Writing and mathematical problem solving: arguments in favor of synthesis. School Science and Mathematics, 85: 210-221.

Davis, R. B., & McKnight, C. C. (1979). The conceptualization of mathematics learning as a foundation of improved measurement (Development report number 4. Final report). Washington, DC: University of Illinois, Urbana/Champaign. (ERIC Document Reproduction Service No. ED 180 786).

Emig, J. (1977). Writing as a mode of learning. College Composition and Communication, 28: 122-128.

Gagne, R. M. (1983a). Some issues in the psychology of mathematics instruction. Journal for Research in Mathematics Education, 14: 7-18.

Gagne, R. M. (1983b). A reply to critiques of some issues in the psychology of mathematics instruction. Journal for Research in Mathematics Education, 14: 214-216.

Johnson, M. L. (1983). Writing in mathematics classes: a valuable tool for learning. Mathematics Teacher, 76: 117-119.

King, B. (1982). Using writing in the mathematics class: theory and practice. In Griffin, C. W. (Ed.), New Directions for Teaching and Learning: Teaching Writing in All Disciplines, San Francisco: Jossey-Bass, 39-44.

McMillen, L. (1986). Science and math professors are assigning writing drills to focus students' thinking. Chronicle of Higher Education, 19-21.

Odell, L. (1980). The process of writing and the process of learning. College Composition and Communication, 31.

Pallmann, M. (1982). Verbal language processes in support of learning mathematics. Mathematics in College, 49-55.

Rose, J. R. (in preparation). Using expressive writing to learn mathematics. Doctoral Dissertation, The University of Rochester, (Borasi, R., Director).

Schmidt, D. (1985). Writing in math class. In Gere, A. R. (Ed.) Roots in the Sawdust, Urbana, IL: National Council of Teachers of English, 104-116.

Selfe, C. L., Peterson, B. T., & Nahrgang, C. (1986). Journal writing in mathematics. In Young, A. & Fulwiler, T. (Eds.) Writing across the Disciplines, Upper Montchair, NJ: Boynton/Cook Publishers, Inc., 192-207.

Steffe, L. P., & Blake, R. N. (1983). Seeking meaning in mathematics instruction: a response to Gagne. Journal for Research in Mathematics Education, 14: 210-213.

Vygotsky, L. (1962). Thought and Language. Cambridge, MA: MIT Press.

Heid, M. Kathleen. RESEQUENCING SKILLS AND CONCEPTS IN APPLIED CALCULUS USING THE COMPUTER AS A TOOL. Journal for Research in Mathematics Education 19: 3-25; January 1988.

Abstract and comments prepared for I.M.E. by KENNETH A. RETZER, Illinois State University.

1. Purpose

This study explored the effects of a modified sequence of skills and concepts in an applied calculus course upon student understandings - given that the computer was used to perform computational skills and support concept development.

2. Rationale

With the availability and capability of computers to do computational tasks with speed and accuracy, the traditional goal of mastery of computational skills in secondary and early college mathematics courses is questioned. Doing time-consuming computational procedures, students often lose sight of interrelationships among the concepts involved. Computers can both (1) deliver the results of algorithmic procedures and (2) provide easy access to a wide range of exemplars of those concepts. Using the computer as a tool, a decrease in emphasis on hand computations and hand-generated concept exemplars can be coupled with an increased emphasis on concepts, including a wider than usual range of computer-generated exemplars of them. Because of current dissatisfaction with the outcomes of this introductory calculus course, it is taken as an ideal course in which to test the effects of using the computer in concept development.

## 3. Research Design and Procedures

The study addressed two questions (p. 4):

> "1. Can the concepts of calculus be learned without the concurrent or previous mastery of the usual algorithmic skills of computing derivatives, computing integrals or sketching curves?"
>
> "2. How does student understanding of course concepts and skills attained on a concepts-first course differ from that attained by students in a traditional version of the course?"

### Subjects

Subjects were business, architecture, or life science majors in a large public university enrolled in two sections of a first-semester applied [as contrasted to a traditional] calculus course. Professor Heid (Instructor A) taught both sections of this experimental course completed by 39 (87%) of the 45 students enrolled -- meeting the classes for three 50-minute periods each week. These students also completed assignments requiring the use of an Apple II Plus computer system reserved for them in a computer room and available at least 34 hours a week at times of high student demand.

Comparison data were gathered from 100 (82%) of the 122 students enrolled in a traditional large lecture section of the same course -- taught by a mathematics professor (Instructor B) and meeting for two 75-minute periods weekly. Each student in the large lecture section was scheduled for one of six small group 50-minute recitation sections taught by graduate teaching assistants. No data are given on the regularity of attendance of the students involved. Students could have selected the experimental sections because of the times they met or because they were the only two small sections of the course offered that semester, but they reported no prior information about the intended course content.

The computer room office was staffed by Professors A, B, or a student aide during computer room hours. No information is given about the individual help given students of the experimental sections. Professor B's students met with him during his office hours but almost never used the computers.

Treatment

The treatment in the experimental classes incorporated materials developed in a semester-long pilot study. Using Polya's problem-solving model language, it may be said that for 12 weeks these class sessions used the microcomputer for the carry out the plan stage of most of the algorithms on which the comparison section spent most of their time. Instructor A demonstrated the use of the computer where appropriate, and focused the time saved on the understanding ["...use the meaning of concepts to analyze problem situations."], plan making ["...executive decision making..."] and looking back ["...concentrated on further analysis of the computer generated results." (p. 6)] problem-solving stages. A major difference between the experimental and comparison classes is that the teaching of concepts had priority over and preceded the teaching of skills. This emphasis was reflected consistently in class discussions, assignments, and test items. Explicit information is not given on the higher level questions used in discussions and test items which contrasted to those in the comparison class. Instructor A demonstrated basic examples of the algorithms the computer was processing in one experimental class but not in the other.

During the last 3 weeks of the 15-week semester, Instructor A covered traditional algorithms and procedures with little attention to problem solving or applications. She emphasized skills in demonstrations, assignments, quizzes, and examinations. This was the kind of emphasis on skills that Instructor B did all 15 weeks in a traditional manner.

## Curriculum

To illustrate how concepts were developed in greater depth and with a greater variety of exemplars than done traditionally in the central course topics [graphs, derivatives, modeling, the fundamental theorem of calculus, Riemann sums and partial derivatives], the experimental and traditional treatments of graphs and derivatives are contrasted in some detail in the report.

For example, the traditional approach is described as mastering the methods of using derivatives to locate maximum and minimum points in order to graph polynomial functions of, primarily, second and third degree. Function values are used to identify behavior of rational functions near points of discontinuity of those which may have linear or vertical asymptotes. Traditional students rarely are asked to draw conclusions from these graphs or questioned about relationships between graphs.

But in the experimental sections during the first 12 weeks, students saw a wide variety of computer-generated graphs and were asked for conclusions and comparisons. They examined graphs of linear and other polynomial functions up to degree five plus a variety of rational functions. They sketched their own graphs using computer-generated data about derivative values. They deduced graphical properties from information about inverse functions, intercepts, asymptotes, intervals of concavity, and slopes. They interpreted graphs to analyze applications of the mathematics. Only in the last three weeks did they construct graphs by hand.

Another major difference of the experimental approach is seen in the traditional way of posing a problem based on nonsymbolic formulations (e.g., tables, graphs, applications), quickly translating the problem in terms of algebraic symbols or formulas to be solved, and retranslating into the original context. Decision-making and quantitative reasoning is done within the context of algebraic manipulation. By contrast, the experimental approach freely used these nonsymbolic representations and asked students to reason with them. The computer was used to superimpose graphs of families of functions generated by changing parameters. Reasoning about graphical and other nonalgebraic representations characterized concept development in the experimental classes. Similar contrasts for the traditional and experimental experiences with derivatives are also described.

Computers and Software

Programs used in the experimental classes include Mu Math, Graph Functions, Fit Functions to Data, Table of Values, and other demonstration programs such as those providing graphical demonstrations of Reimann sum estimates for the area under curves and for volumes of solids of revolution. Three major functions of the computers in the experimental sections are described (pp. 10, 11):

"1. Computers decreased the time and attention usually directed toward mastery of computational skills."

"2. Computers provided concrete data for the discussion of calculus ideas. They were used to provide data that students could examine in their search for patterns, to generate initial representations on which students could base their reasoning and to display examples and counterexamples with which students could corroborate or disprove their conjectures."

"3. Computers lent flexibility to the analysis of problem situations. Their easy display of concepts in a large range of representations made feasible the consideration of more difficult problems, opened avenues for exploring several methods of solution for a single problem, and created an environment amenable to convenient exploration of changing parameters."

Data

Data on which the effects of the experimental method upon understanding was studied were collected in the form of 69 audiotapes, copies of student assignments, quiz and examination papers, notes taken by several students and voluntarily submitted, field notes from experimenter observation in 15 comparison classes and student contacts outside of class, questionaire responses and interviews. From these data sources only three were used to describe student understandings of concepts and skills: interview transcripts, conceptual comparison question results, and final examination results.

Monetary compensation, an unnamed amount, was offered to encourage students to volunteer for interviews. From the volunteers a stratified random sample was chosen for interviews from four categories: those who had ranked either (a) high or (b) low on a questionnaire given at the beginning of the course designed to determine (1) their need for algorithms and (2) their need for creative work. Fifteen were interviewed from the experimental classes and five from the comparison class; most were interviewed for 4 or 5 audiotaped one-hour sessions using a fixed set of questions to assess their understanding of course concepts, among other things of interest to the researcher.

Conceptual comparison questions, written by Instructors A and B, were chosen jointly by both instructors and were distributed among the quizzes and tests of both experimental and comparison classes. These questions tested concepts and applications of elementary calculus without the necessity of performing algorithms; by contrast, the common final examination, constructed by Instructor B, measured execution of standard algorithms developed in the textbook.

## 4. Findings

Since other details of differences in teaching and learning patterns with this experimental treatment are reported elsewhere, only information on student understanding of concepts is reported in this article.

### Interviews

The experimenter states that students in the experimental classes showed more evidence of conceptual understanding than those in the comparison classes. Support cited indicates that interviewees in the experimental classes (1) used a broader array of associations with the concept of derivative; (2) caught and corrected their own errors about concepts and reconstructed their statements about such things as concavity, Riemann sums, and applications using basic concepts of derivative and area; (3) used their own wording in speaking about concepts; (4) sometimes constructed a different form of the Riemann sum concept than used in either class or text; and (5) verbalized connections between concepts such as the concept of the derivative and its mathematical definition.

On 14 of the 16 parts of the conceptual comparison questions, both experimental sections got higher scores than the comparison section. This indicated that they were better able to do such things as (1) draw conclusions about slopes, (2) identify graphic representations of quantitative statements, (3) use mathematical statements to draw conclusions about applied situations, (4) interpret facts about derivatives, and (5) match exponential and logarithmic functions with their graphs. For the 200-point final examination the respective means for both experimental classes and the comparison class were 105, 115, and 117, with standard deviations of 43.6, 40.9, and 36.7 respectively. These were not seen as substantially different. There were no indications that statistically significant differences on scores on the conceptual comparison questions or the final exam were sought.

5. Interpretation

The experimenter concludes, "Students from the experimental classes spoke about the concepts of calculus in more detail, with greater clarity, and with more flexibility than did students of the comparison group. They applied calculus more appropriately and freely" (p. 21). They were better able to translate a mathematical concept from one representation to another. They performed almost as well on the final examination, suggesting that increased attention to concepts coupled with concentrated attention to skill development was not necessarily harmful to skills as measured on that exam.

Reflections

In discussing her study, the experimenter reflects that this concepts-first, computer-assisted, calculus curriculum challenged popular beliefs that students could not adequately understand concepts without prior or concommitant mastery of basic skills. Even though they are reported elsewhere, she felt that it may be instructive to share some student feelings about this curriculum. Some felt that the computer (1) relieved them of some of the manipulative work in calculus, (2) gave them time to understand concepts and plan solutions to algorithm without worrying about getting wrong answers, and (3) helped them focus on more global aspects of problem solving.

A noted limitation of this study is that it is not an experiment with tight controls. Differences in class sizes and the fact that the same person was an instructor, the interviewer, and the investigator could have affected the observations and subjective conclusions.

## Abstractor's Comments

A key to understanding the report of this study is an awareness that an overabundance of data was collected in order to search for emerging patterns of understandings, skill acquisition, attitudes, and beliefs about learning calculus using the computer. It is clear that for the purpose of this research report Professor Heid intended to focus on the effects of these curriculum experiences on student understandings of concepts and skills, but the data on this were collected within such a broad context that it is hard for a reader to sort out of the report what collection procedures and data are truly relevant to the criterion variables. Even after relevant data are identified, findings come not from statistical processing of data gathered to fit a traditional research design, but from subjective judgments of the researcher, supported with samples of data that may encourage the reader to form similar judgments and/or use them in curricular modification.

The course was an applied calculus course, historically called "Calculus for Business," rather than a calculus course for mathematics and/or physical science majors.

While these understandings may be helpful to a mathematics teacher reading this report, they are not a criticisms. The investigator is aware of and has well expressed the characteristics and limitations of the study. The power of the computer as a tool, the availability of relevant software, and the continual search for a proper balance among concepts, generalizations, and skills in each mathematics course makes this type of study a helpful prelude to controlled curricular experiments. It is a timely and interesting study.

More Information?

Even so, there are several points, noted above, where more information would have been helpful. It is amazing that one might accomplish any roughly equivalent educational tasks in 3 weeks compared to a traditional 15. Yet professors with experience teaching large lecture mathematics classes know that class attendance is a common problem unless an attendance policy is in place and enforced. If spotty attendance, and hence limited opportunity to learn, was not uncommon in the comparison class, it would be a bit less surprising that 3 weeks of concentrated work on calculus skills produced on the final exam nearly the same competency as 15 weeks. However, using the same opportunity-to-learn criterion, it is not surprising that the experimental classes did better on the concept comparison test items if 12 weeks of work with concepts is compared to 0 weeks.

It would be interesting to know if and how the individual help outside of class sessions affected the interviews with respect to concepts. Quality of interview content would seem to depend on both the student understandings and verbalization ability. It would be interesting to know whether individual help available in the computer room office for students in the experimental classes included conversations about concepts which were sufficiently similar to interview material as to give them practice with feedback on verbalization skills. Details of the nature and content of the higher-level questions used in the experimental classes in discussion and test items might also give some additional insight on ability to respond well to interview questions.

Skill Performance Adequate?

The final exam results are interesting. Class means ranged from 105 to 117 points out of 200, distressingly near the 50% level. If it would have been reasonable to expect students in this study to succeed on the final exam at the same percentage rate appropriate for the letter grade they are given, both the experimental and comparison classes failed miserably to demonstrate skill mastery. In that case, immediate remediation, rather than experimentation, would be called for in both experimental and comparison classes.

In contrast to the final exam, the concept comparison test items were frequently answered correctly over the 80% level. But if percentage level of responses on the skill-oriented final examination scores in this study did not trigger an alarm for the experimenter, it may be presumed that the concepts developed with computer assistance are not coupled with a perceived failure of both experimental and comparison groups at mathematical skill development.

Significance for Curriculum

Curricular decisions, including balancing concepts, generalizations, and skills within a mathematics course, are affected by (1) individual values, (2) shared experiences, and (3) controlled experiments. The balance among concepts (knowledge of mathematical objects), generalizations (knowledge that certain things are always true) and skills (knowledge how to perform a task) has traditionally been weighted in favor of skills. Skills, including those calculus skills needed by business, architecture, or life science majors for applications of mathematics to their fields, seem to comprise an indispensible core of the basics of mathematics. While generalizations (axioms, theorems) are not explicitly mentioned in this report (although they might be subsumed under the term "concepts"), it seems to be a value of the experimenter that concepts are worth greater emphasis in that balance.

That such a value prompted this study will reinforce mathematics educators at all levels who hold similar values. To the extent shared experiences can change our values and our curricular practices, this study shows applied calculus [traditionally, "calculus for business"] professors a way, namely, using computers, to possibly enhance conceptual attainment without harming the present level of skill attainment in this course. The curricular issues in calculus are sufficiently different from those of applied calculus that caution should be exercised in generalizing these outcomes to a first course in calculus, in particular, or to other mathematics courses, in general. Since it is not a controlled experiment, ones not already predisposed to the increasing the attention given to concepts in the curriculum will not be convinced that a replication will produce the same outcomes in significant amounts.

However, this study may stimulate a replication of the treatment in the same course under controlled conditions which would determine if similar outcomes are found and, if so, whether they are statistically significant. It may stimulate similar experimentation in other mathematics courses which can add to our shared experiences. In particular, the prior college algebra for business course seems a ripe candidate for such a study because computer assistance in handling matrices, including the simplex method, would seem to be a welcomed change. Both controlled experiments and further similar experimentation would be desirable.

Whether or not the balance between concepts and skills or which is prerequisite for the other is at issue, the use of computers for what they do best, carry out plans, freeing teachers and students to understand, plan, and look back at the results is such a desirable step forward that it should be considered a basic consideration of computer use in any mathematics course.

If this study convinces those teaching the applied calculus course that a reasonable level of skill development is possible in a much shorter time than is traditionally devoted, then one who is interested in that skill development alone might seriously consider breaking up the traditional large lecture 15-week applied calculus classes into 5 three-week small concentrated classes.

That possibility creates an interesting twist, provided that this skill attainment is a reproducible phenomena. The experimenter asserts that the results of this study challenged popular beliefs that students could not adequately understand concepts without prior or concomitant mastery of basic skills. If, however, content from the 12-week portion of the experimental classes were needed to prepare them to achieve as much in a comparable three-week skills concentration, the question of which is the prerequisite might need to be reversed. One might explore whether the concepts, computer experience, or both are among those necessary prerequisites for skills needed.

Given that such a course should remain 15-week course, thought might be given to whether the concepts taught in the treatment are the best possible set of concepts for this course. One might review the specific concepts involved to see if mathematicians and representatives from the other content fields would feel that these concepts would contribute most to their understanding of mathematics and/or their fields - somewhat akin to determining what the "critical mass" of those concepts should be. Whether or not computers and software are available to help teach those concepts would be another consideration in curricular modification in this course.

To justify moving the balance of content of any mathematics away from the present amount of skills emphasis for some mathematics educators would require an emerging pattern from many statistically sound, controlled experiments. The admitted limitations of this study indicate that it will not contribute to curricular change in this way. It can serve to surface issues for further study. This study is an important contribution to mathematics education.

Jansson, Lars C.; Williams, Harvey D.; and Collens, Robert J. COMPUTER PROGRAMMING AND LOGICAL REASONING. School Science and Mathematics 87: 371-379; May 1987.

Abstract and comments prepared for I.M.E. by WILLIAM H. KRAUS, Wittenberg University.

1. Purpose

Three studies are described in the report. Each study was designed to test the hypothesis that experience in computer programming will enhance performance on conditional reasoning tasks.

2. Rationale

Some educators (e.g., Moursund, 1975; Anderson, Klassen, and Johnson, 1981) have argued that computer programming experience is not a necessary element of the computer literacy experiences that we provide to students. Others (e.g., Luehrmann, 1981) have argued that computer programming experience is essential for all students.

One of the purported benefits of computer programming is that it will improve skills in problem solving and reasoning (Linn, 1985; Roberts and Moore, 1984). However, there is currently only limited research to support this claim.

3. Research Design and Procedures

Since IF...THEN reasoning is frequently used in programming, the authors decided to study the effects of experience in computer programming on performance on conditional reasoning tasks. Three parallel studies were designed.

Subjects in Study 1 were junior high school students. The experimental group were students in a class taking a six-week unit on computers using BASIC programming. The control group were students in a class that would take the computer unit later in the year.

Subjects in Study 2 were teacher education students. The experimental group were students in an elective class in computer applications in education. Logo was the primary language used in the course, the same BASIC was taught. The control group were students in an elective course that apparently did not involve any computer experiences.

Subjects in Study 3 were arts and sciences students. The experimental group were students enrolled in a first course in computer science. Programming was done in Pascal. The control group were students enrolled in a Computers and Society course in which no programming was done.

Although data were gathered from all students, only the data from students in each study who had had no prior experiences in programming were used in the analyses.

Two pencil-and-paper tests, designed by one of the authors (Jansson, 1977) were used in each study. Each test consisted of 32 items, 8 each of each of the four basic principles of conditional reasoning: detachment, inversion, conversion, and contraposition. One form of the test, the Concrete Familiar, used premises for which the truth value was empirically neutral. In the other form, the Suggestive, at least one part of one premise in each item was contrary to observable fact. The test form was randomly assigned to students; each student was given the same test as both pretest and posttest.

## 4. Findings

Analysis of variance was used for the statistical analyses in each study. There were no significant differences due to treatment. No mean scores were reported for pre- or posttests.

5. Interpretations

The authors conclude that "the results of the present investigation appear to place the burden of proof on the shoulders of the advocates of computer programming as a means of developing logical thinking skills" (p. 378). They recommend further investigation of the relationships between experiences in computer programming and problem solving and reasoning.

## Abstractor's Comments

The report of these studies does little to shed light on the important question of the relationship between experiences in computer programming and problem solving and reasoning. The rationale for the studies is well-developed in the report, but design limitations and incomplete reporting of the studies detract significantly from the impact of the studies.

Intact classes were used in all of the studies, and in two of the studies these were apparently elective classes. Thus it is possible that there was substantial bias in the samples. The treatments are not clearly described; we know only what computer languages were used in the experimental groups. In the experimental groups, we do not know how much . what kind of programming experiences were used nor do we know if any attempt was made to relate those experiences to general reasoning skills. In the control groups, we do not know if there were any experiences that might affect the reasoning skills of the students. No scores are reported for the pre- or posttests, leaving unanswered questions about students' performance on the tests (e.g., was there a "ceiling effect" in any of the studies?).

There is considerable room for further study in the areas the authors have identified. Studies such as those conducted by the authors, but more carefully controlled, could be of value. Since transfer from specific skills to more general skills seldom happens without some direct effort to facilitate the transfer, studies where a direct effort is made in the treatment to help students make connections between programming and problem solving or reasoning would seem to be especially worthwhile.

## References

Anderson, R. E., Klassen, D. L., & Johnson, D. L. (1981). In defense of a comprehensive view of computer literacy--a reply to Luehrmann. The Mathematics Teacher, 74: 687-690.

Jansson, L. C. (1977). Conditional reasoning in adolescents. Alberta Journal of Educational Research, 23: 118-127.

Linn, M. C. (1985). The cognitive consequences of programming instruction in classrooms. Educational Researcher, 14(5): 14-16, 25-29.

Luehrmann, A. (1981). Computer literacy--what should it be? The Mathematics Teacher, 74: 682-686.

Moursund, D. (1975). What Is Computer Literacy? Eugene, Oregon: Oregon Council for Computer Education.

Roberts, R. S., & Moore, M. L. (1984). Programming to learn problem solving. In M. J. Zweng (Ed.), Computers in Mathematics Education, 1984 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 162-170.

Jones, Lyle V. THE INFLUENCE ON MATHEMATICS TEST SCORES, BY ETHNICITY AND SEX, OF PRIOR ACHIEVEMENT AND HIGH SCHOOL MATHEMATICS COURSES. Journal for Research in Mathematics Education 18: 180-186; May 1987.

Abstract and comments prepared for I.M.E. by JOANNE ROSSI BECKER, San Jose State University.

1. Purpose

This study of the 1980 sophomore cohort from High School and Beyond (HS&B) compared the influence of mathematics course taking on test scores among four subgroups: Black males, Black females, white males and white females.

2. Rationale

Previous studies of data from the National Assessment of Educational Progress and the HS&B project have found that mathematics test scores are strongly related to the number of high school mathematics courses taken. A study of 1982 seniors from HS&B (Jones, Davenport, Bryson, Bekhuis, and Zwick, 1986) found that taking advanced mathematics affected senior year mathematics test scores, even after adjusting for three critical variables: socioeconomic status (SES), verbal test scores, and scores on the same mathematics test taken two years earlier. This study was designed to determine whether these earlier findings hold equally for subgroups classified by ethnicity and gender.

3. Research Design and Procedures

Data are available for nearly 7500 members of the 1980 sophomore cohort from HS&B. These data include: 1982 mathematics test scores (computation, arithmetic reasoning, graph reading, elementary algebra, and geometry); high school transcripts; scores on the mathematics and verbal tests given in 1980; and, questionnaires from both 1980 and

1982. For this study, the linear combination of SES, 1980 verbal test score, and 1980 mathematics test score was used to predict 1982 mathematics test score. Level of course taking ranged from 0 to 5, the number of transcript credits in mathematics (algebra 1 or above). The sample size was 538 Black males, 616 Black females, 3030 white males, and 3277 white females.

The regression coefficients from Jones et al. (1986) were used to form the linear combination (X) of the three background variables with maximum linear correlation with the 1982 mathematics test score (Y) for each level of course taking. The intercept term was omitted from the regression equation because the intercept values were similar for the six levels of course taking. Regression was used in a descriptive way; no statistical tests of inference were reported.

To compare the relation between $\overline{Y}$ (the observed mean test score and $\overline{X}$ (the mean linear composite of predictors) for the four student groups, Jones plotted $\overline{Y}$ against $\overline{X}$ for all students and for each subgroup of students separately. An additional analysis was done for students with three credits of mathematics, separating those who had taken a precalculus/analysis or calculus course from those who had not.

Descriptive statistics presented 1982 test score means broken down by level of course taking, gender, and ethnicity, and percentage distribution of each subgroup of students in each level of course taking.

## 4. Findings

The mean test score for Black students as a whole was 41% correct; for white students, 60%. The mean for all females was 56%; for males, 60%. As the level of course taking increased, the mean test score also increased, from 37% for zero credits to 82% for five credits.

The distribution of students by level of course taking varied considerably by ethnicity, somewhat less so by gender within ethnicity.

From the multiple regressions, it was found that, at all levels of course taking, the contribution of the earlier mathematics test was great (beta ranging from 0.48 to 0.66), the contribution of the verbal test score was more modest (beta from 0.15 to 0.20), and the contribution of SES was small (beta from 0.04 to 0.11). $R^2$ ranged from a low of 0.38 for zero courses to a high of 0.62 for five courses.

Inspection of the plot of $\overline{Y}$ versus $\overline{X}$ suggested one line for 0, 1, or 2 courses, and a parallel line with greater $\overline{Y}$ intercept for 4 or 5 courses. The points for 3 courses fell between these two lines. The pattern was the same, regardless of ethnicity or gender, except for 3 courses of credit. However, both $\overline{Y}$ and $\overline{X}$ were higher for white males than for white females, and for Black males than for Black females. Also, white students had higher $\overline{Y}$ and $\overline{X}$ values than Black students for each level of course taking.

For students with 3 credits of mathematics, those who had received credit for calculus, precalculus, or analysis had both $\overline{Y}$ and $\overline{X}$ higher than students who did not have such a credit. This result held for all four subgroups of students.

## 5. Interpretations

The author concluded that four or five credits in mathematics, or three credits, one of which is calculus or precalculus/analysis, contribute a sizable amount to mathematics test scores beyond the values predicted from SES and earlier performance. Thus the content of earlier mathematics instruction is better understood by students who have taken more advanced courses. This is true for all students, white or Black, male or female. Jones states that mean test score differences among these groups in the senior year are fully explained by subgroup differences present in the sophomore year, and by disproportionate representation in advanced mathematics courses.

## Abstractor's Comments

The main contribution of this study is to add to the paucity of achievement data which are broken down by both gender and ethnicity, although the latter is limited to Black and white students. However, the paper fails to present any theoretical background for hypothesizing either gender or ethnic differences in the importance of advanced mathematics courses for senior year test scores. I have yet to read of any mathematics educator suggesting that advanced mathematics courses are less critical for females or Black students. Is there reason to believe they are more critical? Still, it is reassuring to confirm the salutory effects of continued study of mathematics on achievement, however limited the measure of achievement used.

I was a bit perplexed by the approach the author used to analyze the data. In the earlier study by Jones et al. (1986), analysis of covariance was used to determine the effect of advanced mathematics courses on test scores, adjusting for SES, 1980 verbal score, and 1980 mathematics score. I wonder why this technique was not used here. Instead, Jones determined the contribution of SES, verbal score, and mathematics score for each level of course taking for the total sample. Then, he plotted the observed mean 1982 test score against the mean predicted score (minus the intercept term) for each subgroup of students in each of the six levels of course taking. At this point, as far as I can tell, Jones visually drew in two parallel lines which seemed to fit the plotted points, one line for 0, 1, or 2 credits in mathematics, and the second, with higher $\overline{Y}$ intercept, for 4 or 5 credits. I wonder about this informal approach to the data. What techniques were used, and assumptions made, to arrive at these two lines?

I also was confused by the case of three credits of mathematics. I cannot figure out how students received only three credits of mathematics, algebra 1 and above, yet still had a course in calculus, or even precalculus/analysis. I think a typical sequence of course work would be algebra 1, geometry, algebra 2-trigonometry, analysis/precalculus, calculus. Is it possible course work in junior high school was not counted for some students? I think this issue merited some comment from the author, since he uses the analysis of the three-credit case to conclude that precalculus/analysis or calculus was especially important for student achievement.

A caution should be made about the small sample sizes for Black students with five credits in mathematics; I calculated that 8 Black males and 16 Black females were in this category.

The author needed to explain more fully his conclusion that differences in senior-year scores among Black females, Black males, white females, and white males were "fully explained" jointly by subgroup differences evident in sophomore year and disproportionate representation in advanced mathematics courses. Jones writes that about half of the variance of 1982 mathematics test scores was explained by numbers of credits in mathematics. Was the other half accounted for by SES, verbal score, and 1980 mathematics score?

A comment on the printing of the paper. A great deal of interpretation is made by the author from Figure 1, the plot of $\overline{Y}$ versus $\overline{X}$. I found the graph very difficult to read because it tried to distinguish six data points for each of five subgroups with different symbols. A larger figure is needed if the figure is to be the sole evidence for major conclusions in a paper.

Finally, Jones suggested that we need policies that would lessen the average performance differences between white and Black, female and male students prior to high school. I could not agree more. I would also suggest that we need research that identifies how we can lessen those performance differences which persist today.

## Reference

Jones, L. V., Davenport, E. C., Bryson, A., Bekhuis, T., & Zwick, R. (1986). Mathematics and science test scores as related to courses taken in high school and other factors. Journal of Educational Measurement, 23(3): 197-208.

Miura, Irene T. MATHEMATICS ACHIEVEMENT AS A FUNCTION OF LANGUAGE. Journal of Educational Psychology 79: 79-82; March 1987.

Abstract and comments prepared for I.M.E. by ROBERT B. KANE, Purdue University.

1. Purpose

To ascertain how Japanese- and English-speaking children represent numbers.

2. Rationale

The author notes that explanations for the superior mathematics achievement of Asian American students have ranged from parental support and cultural emphasis on educational achievement to superior innate ability. An additional alternative is suggested--national language characteristics. Students from Far Eastern countries share a common numerical language, rooted in ancient Chinese, in which the number names relate closely to base ten numeration. For example, in Asian languages, 11 is spoken as ten-one, 15 as ten-five, 30 as three-tens, 46 as four-tens-six, and so forth. Thus number names such as fourteen and forty, which are confusingly similar in English, are entirely different in Japanese or Chinese, namely ten-four and four-tens. Asian numerical language may facilitate arithmatic learning because it reflects base ten numeration so faithfully. The introduction of manipulatives such as base ten blocks and the pictorial and expanded notation forms prior to standard numeration are common in American schools and considered unnecessary in Japan. Furthermore, Asian students who said that "English is not my best language" scored higher on the 1981 administration of the SAT-Math than Asian students who said that "English is my best language." Similar results were observed in the 1979-80 California assessment program data for students in grades 3 and 6.

## 3. Research Design and Procedures

Subjects for this study were 21 six- and seven-year-olds for whom Japanese was the dominant language and 20 six- and seven-year-olds for whom English was their only language. The first experimental task (after appropriate modeling) was to read a number aloud and show the number by using tens and units blocks. Each child had 100 unit blocks and 20 ten blocks for his use. The numerals 11, 13, 28, 30, and 42 were presented in random order. After completing the first task, the child was shown his constructions and asked if he could show each number another way with his supply of blocks. A response was scored as correct if the blocks summed to the stimulus number. Correct responses were partitioned into three classes: constructions using only unit blocks, canonical base ten representations, and noncanonical base ten constructions (i.e., a construction which used some ten blocks but also used more than nine unit blocks).

## 4. Findings

Japanese speakers constructed more canonical representations on task 1 than did English speakers. On task 2 there were no such differences. English speakers used more one-to-one constructions on task 1; Japanese speakers used more of these on task 2. English speakers used no noncanonical constructions on task 1. Over both tasks the mean number of noncanonical constructions for English speakers was 0.65; for Japanese speakers, 1.38. Correlations between standardized mathematics achievement test scores and experimental data were developed for the English-speaking subjects. The number of canonical base ten constructions was positively correlated with test scores. The number of items for which two correct constructions were made also correlated positively with test scores.

## 5. Interpretations

The cognitive representation of numbers differs for Japanese- and English-speaking children and this difference may affect achievement in arithmetic computation. Since Japanese-speaking subjects were more likely to produce canonical base ten constructions on task 1, numbers are organized as structures of tens and ones for speakers of Asian languages and place value appears to be an integral part of the representation. Conversely, English speakers were less likely to use base ten structures to represent numbers. The findings suggest more extensive investigation of the reason for the Japanese speakers' greater use of base ten representations of numbers.

## Abstractor's Comments

Miura is right to call for more evidence on the role of differential correspondences to base ten numeration among natural languages. This study, while useful as an alert to the possibility of a salient language factor, is far too limited in scope to cause one to feel comfortable with suggesting such a conclusion. I look forward to seeing additional studies along Miura's line of inquiry. For example, do the differences displayed in her data persist for a year or longer? What differences exist between English speakers and speakers of European languages where the degrees of correspondence are more similar between natural language number names and canonical base ten constructions? We should encourage such inquiry and then evaluate the larger corpus of evidence. For now the social factors which distinguish Japanese from American schooling and family life remain the strongest candidates for factors which explain the observable differences.

Smith, Lyle R. VERBAL CLARIFYING BEHAVIORS, MATHEMATICS PARTICIPATION, ATTITUDES. School Science and Mathematics 87: 40-49; January 1987.

Abstract and comments prepared for I.M.E. by WALTER SZETELA, University of British Columbia.

1. Purpose

The purpose of this investigation was to examine the combined instructional effects of student participation and use of realistic levels of vagueness terms (uncertainty and bluffing terms) on student achievement and on student attitudes.

2. Rationale

Previous research concerning teacher vagueness terms and student participation in mathematics classes has focused mainly upon effects on student achievement, rather than on students' attitudes and perceptions. Much of such research has been based on lessons with unusually high levels of vagueness terms. In the present study, the additional focus upon student attitudes broadened the scope of effects of using vagueness terms while investigating possible effects using realistic levels of vagueness terms. Whereas previous studies had used approximately 7 to 9 vagueness terms per minute, in typical classrooms Smith reports that teachers use an average of 2.2 vagueness terms.

3. Research Design and Procedures

A total of 96 sixth-graders, predominantly Caucasian and middle class, participated in the study. There were two conditions each for uncertainty, bluffing, and student participation. Students were randomly assigned to one of the eight groups (n = 12 in each group) in the 2 X 2 X 2 factorial design.

Each of the eight groups was presented with a 20-minute audiotaped mathematics lesson on traversibility of curves and Euler's formula concerning networks in the plane (F + V - 1 = E). In all eight groups, students observed overhead projections and demonstrations on the chalkboard presented by the same person. The audiotaped lessons ensured control for extraneous variables and desired levels of uncertainty, bluffing, and participation. One half of the lessons were constructed so that 2.46 uncertainty terms per minute of teacher talk were included. Student participation consisted of filling out results for two networks in a handout. Following is an excerpt from a lesson containing both uncertainty terms and bluffing terms. Five minutes of preliminary work had taken place in the lesson prior to the excerpt. The uncertainty terms are *italicized*, and the bluffing terms are CAPITALIZED.

> OBVIOUSLY, there is an equation that *generally* tells a relationship between the number of faces, edges, and vertices in any network. The equation is F + V - 1 = E, where F is the number of faces, V is the number of vertices, and E is the number of edges. (Overhead projector is used to show the equation.) Looking at the table we have constructed on the blackboard, for the first network we drew we have 3 + 6 - 1 = 8. For the second network we drew, we have 4 + 10 - 1 = 13. Our equation *pretty much* works for all networks we can draw, SO TO SPEAK. *Maybe* we can use the equation to solve problems about networks. For example, if a network has two faces and four vertices, we can *somehow* find out how many edges the network has, YOU KNOW.

Immediately after the lesson, a 20-item test of students comprehension of the lesson was completed (split-half reliability was .84). Immediately after the comprehension test a 12-item lesson evaluation was completed to determine students' attitudes and perceptions. Sample items of the test are shown in Table 1.

Table 1

Lesson Evaluation

| | Score | | | |
|---|---|---|---|---|
| Item | Definite no | no | yes | Definite yes |
| 1. The teacher was confident. | 1 | 2 | 3 | 4 |
| ... | | | | |
| 5. The teacher's explanations were clear to me. | 1 | 2 | 3 | 4 |
| ... | | | | |
| 8. The speech pattern of the teacher irritated me. | 4 | 3 | 2 | 1 |
| ... | | | | |
| 12. The teacher appeared lazy. | 1 | 2 | 3 | 4 |

## 4. Findings

There were no significant effects on achievement scores. Group means ranged from 9.6 to 11.7 with a mean of 10.8 out of 20. The 12 items of the Lesson Evaluation were analyzed separately based upon earlier research which showed that some of the items such as item 1 (teacher confident) and item 11 (teacher prepared) were shown to be quite independent of each other. For item 5 (clear explanations) and item 11 (teacher prepared), teacher uncertainty caused a significant decrease in ratings. Teacher bluffing significantly reduced ratings on item 6 (stayed on main subject). Student participation significantly increased ratings on item 1 (teacher confident) and item 5 (clear explanation). However, on item 12 (teacher lazy), student participation also significantly increased students' perception of teacher as lazy. The combination of no uncertainty terms and no bluffing terms resulted in higher ratings for items 1 and 5 (teacher confident and clear explanations). Among three significant three-way interactions, item 5 (irritating speech) had highest ratings under the combinations of no uncertainty, no bluff, no participation and no uncertainty, bluff, participation. Lowest scores on this item resulted from the combinations of uncertainty, bluff, no participation and no uncertainty, bluff, no participation. For item 8 (irritating speech) the no uncertainty, no bluff, no participation combination produced the highest rating while the two combinations of no uncertainty, no bluff, participation and no uncertainty, bluff, no participation gave the lowest ratings. On item 12 (teacher lazy) students rated teachers lazier except in combination with no uncertainty and no bluffing.

5. Interpretations

The investigator cautions that in this study participation was confined to the completion of tables on worksheets by students. Also, the recorded lesson produced "a degree of artificiality in the teaching-learning act." The author concludes that even with relatively low levels of teacher uncertainty and bluffing students' perceptions of the lesson can be significantly influenced. The requirement of student participation resulted in higher ratings of the teacher on clarity and on confidence. Further research is suggested to explain the surprising result of perceptions of teachers as lazy under the participation condition.

## Abstractor's Comments

The two main focal points of this study, teacher vagueness and student participation, certainly do merit the attention of researchers. The investigator's aim to determine what effects these variables have on students' perceptions of the teacher and of the lesson as well as on achievement is commendable. When compared with other educational studies, this investigation rates very highly on aspects of control. It is unflawed by such common weaknesses in classroom studies as differences in teacher personality, background, enthusiasm, and implementation of and attention to the numerous instructional details of such studies. Here we have eight groups of 12 students each, randomly assigned to all possible combinations of the conditions of uncertainty, bluffing, and participation. Furthermore, audiotapes ensure the homogeneity of the lesson content and presentation except for the intended variables. The study has a pristine quality with clear direction and impeccable research design.

At the same time such control and purity exacts a price, one which the investigator recognizes himself. The artificiality of the audiotaped lesson is far removed from typical classroom situations. In each of the four participation groups, participation in the audiotaped lesson was limited to students filling out tables on worksheets with no student-teacher interactions. Furthermore, each lesson spanned only 20 minutes. However, the investigator did attempt to simulate more realistic teacher behaviors by using only a moderate, more typical number of vagueness terms.

The investigator's decision to analyze the lesson evaluation item by item rather than in clusters of related items on the basis that some of the items have been shown to be independent is questionable. Item 4 (frustration) and item 9 (irritation) appear to be closely related as the results indicate. Similarly, item 1 (teacher confidence) and item 11 (teacher preparation) appear to belong to a natural grouping. There were eleven significant effects out of 84 possible effects. There may have been proportionately more significant effects if items had been analyzed by cluster instead of individually.

The investigator's report and analysis focuses mainly on students' evaluation of the lesson rather than on achievement. The lesson evaluation was completed immediately after the achievement test. One must consider how the achievement test might have influenced the responses to the lesson evaluation items, especially after a lesson of only 20 minutes duration. Despite some of the concerns raised here, the fact remains that some significant effects were achieved in a very brief lesson without severely loading the treatment variables. Most of these effects were in the direction that one would expect, which gives further credence to the study. What emerges is that use of vagueness terms by teachers does negatively influence students' perceptions of the teacher and the lesson. Student participation, limited though it was in this study, also increased perceptions of teacher's confidence and clarity of teacher's explanation. These may be already commonly held beliefs, but when reinforced by results of research, they provide clearer directions for teachers and teacher educators to more consciously avoid vagueness in instruction and to increase student participation. The finding that students perceive the teacher as more lazy under the participation condition seems to be anomalous. The investigator's call for more research to explain this phenomenon seems reasonable. Perhaps it might be embedded in further research which might attempt to retain some of the very commendable attributes of this study while strengthening some of its weaknesses.

Song, Myung-Ja and Ginsburg, Herbert P. THE DEVELOPMENT OF INFORMAL AND FORMAL MATHEMATICAL THINKING IN KOREAN AND U.S. CHILDREN. Child Development 58: 1286-1296; October 1987.

Abstract and comments prepared for I.M.E. by YOUNGSHIM KWON and KAREN C. FUSON, Northwestern University.

1. Purpose

This study was conducted to investigate whether Korean children exhibit superior levels of performance in school mathematics compared to U.S. children, whether any such superiority is related to an early advantage in informal mathematical thinking by the Korean children, and whether any superior achievement is qualitatively different from U.S. children's achievement either in rote activity or in deeper understanding.

2. Rationale

Recent research has shown that before they are taught written mathematics in school, young children acquire proficiency in informal mathematics. This study attempted to relate children's early school success to their informal mathematical thinking at the preschool level.

Cross-national studies have shown that the mathematcis achievement of school-age children in the United States lags behind that of children in Asian countries. However, little is known about the nature of this difference and in particular whether it appears only in rote calculation tasks or also in tasks assessing understanding. This study attempted to extend the previous research by examining cognitive factors: these attributes of performance.

3. Research Design and Procedures

The subjects were 315 Korean children and 538 U.S. children at age levels 4, 5, 6, 7, and 8. The U.S. subjects at all age levels and

Korean subjects at ages 4 and 5 represented a reasonable social class spread. At ages 6, 7, and 8 the Korean sample was generally lower-middle-class by Korean standards.

The TEMA (Test of Early Mathematical Ability), which was derived from the previous research of Ginsburg and his colleagues, was used to measure informal and formal mathematical thinking. Twenty-three informal mathematics items focused on three kinds of informal subskills: concept of relative magnitude, counting, and calculation (e.g., mental addition and subtraction). Twenty-seven formal mathematics items involved four kinds of formal subskills: knowledge of convention, number facts, calculation (written addition and subtraction and some multiplication), and base-10 concepts. For Korean children, test items were translated into that language by the first author, a native speaker of Korean.

Subjects were individually tested in a structured and standardized interview strictly following the procedure described in the TEMA manual. Items were given in order of increasing difficulty. If a child failed five items in succession, testing was stopped on the assumption that there was little chance of further success.

Children received one point for every item answered correctly: therefore, 50 was the maximum possible total score. Three-way ANOVAs by culture, age, and sex for unbalanced data were performed on various TEMA scores. T-tests were used for the analyses of differences between cultures within age groups, and Duncan's Multiple-Range Tests were used for post-hoc analyses between age groups. Cultural differences in performance on individual items were analyzed by chi-square tests. Correlations were computed for cultural comparisons of the rank order of difficulty of items.

4. Findings

Korean children's performance in informal mathematics was inferior to that of U.S. children at age 4, 5, and 6, and there was no significant difference in formal mathematics between the two cultures at these ages. However, Korean children exhibited superior performance in both formal and informal mathematical thinking at age 7 and 8. Korean children at age 7 and 8 were superior in every sub-area of formal mathematics but they were especially strong on principle and process items. A high and significant correlation was obtained between the average ranks of Korean and U.S. children on all TEMA items.

5. Interpretations

Korean children do not begin with a "head start" in informal mathematical thinking. Thus, Korean children's superior performance in school arithmetic cannot be explained by a head start in informal mathematics. The discouragement of preschool children's intellectual work at home, the two Korean counting systems, and deficiencies in Korean preschool education may contribute to Korean preschool children's poor performance in informal mathematics.

Korean children are not superior only because they can learn in a rote fashion. Their school mathematics achievement does not appear to be qualitatively different from that of U.S. children. Korean children's relative success and U.S. children's relative failure in early school mathematics seem to stem from such environmental-cultural factors as classroom practices, teacher attitudes and skills, expectancies, and parental demands, values, and assistance.

## Abstractors' Comments

This study is one of several cross-cultural studies that have investigated why American children show inferior levels of performance in school mathematics. The focus of this study on early mathematical knowledge as a possible source of this difference and the attempt to assess both rote and conceptual abilities are strengths of this study. However, some questions might be raised about the interpretations of the study and about the measures of conceptual understanding.

These investigators had data with which to examine more closely three of their suggested sources of the relatively slow start Korean children make in informal mathematics learning. First, their sample of 4- and 5-year-olds was split between lower-class children attending a poorly furnished preschool and upper-middle-class children attending a preschool with a good educational environment. A comparison of the informal mathematical abilities of these two parts of the sample would have permitted some assessment of the relative strength of two of their suggested sources. No difference would have supported the argument that Korean parents and preschools do not encourage intellectual work, and a difference in favor of the better equipped school would have suggested that better furnished preschool environments do support informal mathematics or urban upper-middle-class parents provide this support (or both). A third possible source could have been examined using the items given on counting. The counting performance of the Korean 4- and 5-year-olds could have been analyzed for evidence of confusion between the two counting systems used in Korea (e.g., words intruded from one system into the other). In spite of the attention paid to this dual counting system in the text, we are not told in which counting system Korean children were tested, nor does there seem to have been an effort to gather data about their ability to count in both systems. Both of these kinds of data would have been informative with respect to the authors' suggestion of possible difficulties stemming from this dual counting system.

The very large reversal in mathematics performance at ages 7 and 8 strongly implicates the elementary schools in both countries as sources of this difference. We are told that more time is devoted to mathematics instruction in Korea; some estimate of the amount of time so spent in Korea would have been useful in understanding the strength of this factor. Three other factors not discussed in the text may also be playing important roles in this school difference; two of these concern the formal Korean counting sequence. First, children in

Japan, Mainland China, and Taiwan are taught a particular method for solving single-digit sums and differences between 10 and 18 (Fuson, Stigler, & Bartsch, in press). In this over-ten method one splits one addend into the amount that will make ten with the other number and the left-over amount; this left-over amount then is easily added to ten to give the sum (e.g., 8 + 5 = 8 + 2 + 3 = 10 + 3 = 13). This method is greatly facilitated by the regular number-word sequences used in these countries (and in Korea): an English translation of this procedure would be "eight plus five is eight plus two plus three is ten three". In Korean one merely has to find the left-over amount and then precede this by ten to say the answer. It is likely that many Korean children were using this approach because they were superior to the American children on TEMA items 25 and 34, which assess sums to ten and differences from ten, subskills for the over-ten method. The only other addition or subtraction facts on which Korean children were superior is on sums between 10 and 18 (differences of this size were not assessed). Because credit was given only if a child gave an answer within 3 seconds, the over-ten method (very rapid to accomplish in Korean) would have been credited while the slower counting strategies frequently used by American children would not have been.

Second, addition and subtraction topics are begun and finished earlier in Japan, Mainland China, and Taiwan than in the United States (Fuson et al., in press). Such an acceleration of topics in Korea would be a major contributor to the superior achievement of Korean 7 and 8-year-olds. This is particularly true because each item number of the TEMA in fact consisted of two or three items; one point was given only if all items were correct. Thus, a high level of skill and knowledge was required for each "item" listed in the study.

Third, the Korean language facilitates performance on the four base-ten items on which the Korean children were superior. The first two items (tens in 100, hundreds in 1000) are facilitated by the regular pattern of naming of tens up to 100 (counting by tens in

Korean would be "one ten, two ten, three ten, ..., eight ten, nine ten"); this plus the similar regular pattern for hundreds means that the regular ten-for-one trades are supported for Korean children by their counting words in every place. The next two items involved adding or subtracting tens (ten dollar bills) to or from one- or two-digit number words. These tasks are also much easier to do in the formal Korean counting sequence than in English. Three examples are: four plus three tens (given as four dollars plus three ten dollar bills) is "thirty four" in English but "three ten four" in Korean; 37 + one ten is "thirty seven plus one ten is forty seven" in English but "three ten seven plus one ten is four ten seven" in Korean; 35 - two tens is "thirty-five minus two tens is fifteen" in English but "three ten five minus two tens is one ten five" in Korean. Thus, these tasks are obviously much easier in Korean. It also is not clear whether in Korean these are really conceptual items because a more rote approach to such tasks seems possible.

Most of the tasks included in the principle and process items do not, in our opinion, qualify for these labels. Five of the seven items involve only multidigit adding or subtracting. Four of these five item clusters do contain only problems with trading (regrouping, carrying or borrowing), but these items were procedural rather than conceptual. Children were asked to carry out the calculation, and the results were assessed for accuracy. No differentiation seems to have been made betwee trading errors or fact errors, and children were not asked to justify or explain their procedure. These items are clearly more difficult than the items grouped within the rote items, but evidence is overwhelming that children in the U.S. carry out the multidigit addition and subtraction procedures in a rote, meaningless fashion. This study does not permit us to decide whether this is also true in Korea; i.e., whether the superior multidigit calculating performance of Korean children is accompanied by understanding of the procedure. Even the other two items seem open to a rote interpretation: children could learn to align uneven problems on the right through some mnemonic or other rote device. Again, the children do not seem to have been asked why alignment on the right was correct, which would have permitted a decision about whether this ability was conceptually based.

In conclusion, this study has contributed to our understanding of cross-cultural differences in mathematics performance in several ways. Korea now definitely joins other Asian countries with respect to a demonstrated achievement in addition and subtraction that is superior to the U.S. at age 7 and 8. This superiority clearly does not begin before school starts. Korean 7- and 8-year-olds are better at addition and subtraction tasks ranging across rapid single-digit, conventional two- and three-digit, and nonconventional addition and subtraction of tens to one- and two-place number words. The study did fall short on its goal of assessing conceptual understanding, so we do not know whether Korean children carry out these procedures with understanding. Assessing this and the difficult questions of social/cultural sources of the documented performance differences must await future research.

## Reference

Fuson, K. C., Stigler, J. W. & Bartsch, K. (in press). Grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan, and the United States. Journal for Research in Mathematics Education.

# MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED by CURRENT INDEX TO JOURNALS IN EDUCATION October - December 1987

EJ 355 468 Good, Thomas L.; Grouws, Douglas A. Increasing Teachers' Understanding of Mathematical Ideas through Inservice Training. Phi Delta Kappan, v68 n10, 778-83, June 1987.

EJ 355 492 Howell, Richard; And Others. The Effects of Computer Use on the Acquisition of Multiplication Facts by a Student with Learning Disabilities. Journal of Learning Disabilities, v20 n6, 336-41, June-July 1987.

EJ 355 884 Carraher, Terezinha Nunes. From Drawings to Buildings; Working with Mathematical Scales. International Journal of Behavioral Development, v9 n4, 527-44, December 1986.

EJ 355 887 Acredolo, Curt; Horobin, Karen. Development of Relational Reasoning and Avoidance of Premature Closure. Developmental Psychology, v23 n1, 13-21, January 1987.

EJ 355 888 Overton, Willis F.; And Others. Form and Content in the Development of Deductive Reasoning. Developmental Psychology, v23 n1, 22-30, January 1987.

EJ 356 042 Kelly, Gwendolyn N.; And Others. Working with Logo: Do 5th and 6th Graders Develop a Basic Understanding of Angles and Distances? Journal of Computers in Mathematics and Science Teaching, v6 n2, 23-27, Winter 1986-87.

EJ 356 073 Gabel, Dorothy L.; Enochs, Larry G. Different Approaches for Teaching Volume and Students' Visualization Ability. Science Education, v71 n4, 591-97, July 1987.

EJ 356 084 Veenman, Simon; And Others. Classroom Time and Achievement in Mixed Age Classes. Educational Studies, v13 n1, 75-89, 1987.

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EJ 356 272 Helmke, Andreas; Schrader, Friedrich-Wilhelm. Interactional Effects of Instructional Quality and Teacher Judgement Accuracy on Achievement. Teaching and Teacher Education, v3 n2, 91-98, 1987.

EJ 356 277 Carter, Kathy; And Others. Processing and Using Information About Students: A Study of Expert, Novice, and Postulant Teachers. Teaching and Teacher Education, v3 n2, 147-57, 1987.

EJ 356 317 Powell, Jack V.; And Others. Effects of Intergenerational Tutoring and Related Variables on Reading and Mathematics Achievement of Low Socioeconomic Children. Journal of Experimental Education, v55 n4, 206-11, Summer 1987.

EJ 356 341 Marjoribanks, Kevin. Ability and Attitude Correlates of Academic Achievement: Family-Group Differences. Journal of Educational Psychology, v79 n2, 171-78, June 1987.

EJ 356 342 Hess, Robert D.; And Others. Cultural Variations in Family Beliefs About Children's Performance in Mathematics: Comparisons Among People's Republic of China, Chinese-American, and Caucasian-American Families. Journal of Educational Psychology, v79 n2, 179-88, June 1987.

EJ 356 496 Wehr, Sara H.; Kaufman, Melvin E. The Effects of Assertive Training on Performance in Highly Anxious Adolescents. Adolescence, v22 n85, 195-205, Spring 1987.

EJ 357 061 Liddle, Ian; Wilkinson, J. Eric. The Emergence of Order and Class Aspects of Number in Children: Some Findings from a Longitudinal Study. British Journal of Educational Psychology, v57 pt2, 237-43, June 1987.

EJ 357 137 Collis, Betty. Research Windows. Computing Teacher, v14 n9, 8-9, June 1987.

EJ 357 303 Cobb, Paul. An investigation of Young Children's Academic Contexts. Educational Studies in Mathematics, v18 n2, 109-24, May 1987.

EJ 357 304 Treffers, A. Integrated Column Arithmetic According to Progressive Schematization. Educational Studies in Mathematics, v18 n2, 125-45, May 1987.

EJ 357 305 Isaacs, Ian. Using Problem Solving Teaching Styles to Prepare Candidates for the CXC Basic Proficiency Examinations. Educational Studies in Mathematics, v18 n2, 177-90, May 1987.

EJ 357 334 Suydam, Marilyn N. Research on Mathematics Education Reported in 1986. Journal for Research in Mathematics Education, v18 n4, 243-324, July 1987.

EJ 357 592 Stevenson, Harold W. The Asian Advantage: The Case of Mathematics. American Educator: The Professional Journal of the American Federation of Teachers, v11 n2, 26-31, 47, Summer 1987.

EJ 358 371 Hess, Robert D.; McGarvey, Linda J. School-Relevant Effects of Educational Uses of Microcomputers in Kindergarten Classrooms and Homes. Journal of Educational Computing Research, v3 n3, 269-86, 1987.

EJ 358 376 Wepner, Gabriella. Evaluation of a Postsecondary Remedial Mathematics Program. Journal of Developmental Education, v11 n1, 6-9, September 1987.

EJ 358 459 Leder, Gilah C. Teacher Student Interaction: A Case Study. Educational Studies in Mathematics, v18 n3, 255-71, August 1987.

EJ 358 462 Dekker, Rijkje. Roos and Jose, Two Children in a Mixed Ability Group. Educational Studies in Mathematics, v18 n3, 317-34, August 1987.

EJ 358 662 Straker, Neil. Mathematics Teacher Shortages in Secondary Schools: Implications for Mathematics Departments. Research Papers in Education, v2 n2, 126-52, June 1987.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN
RESOURCES IN EDUCATION
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