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Paraconsistent Multivalued Logic and *Coincidentia Oppositorum*: Evaluation with Complex Numbers

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Abstract Paraconsistent logic admits that the contradiction can be true. Let p be the truth values and *P* be a proposition. In paraconsistent logic the truth values of contradiction is $v(P \wedge \neg P) = p(1-p) = 1 \Rightarrow p - p^2 - 1 = 0$ $\Rightarrow p^2 - p + 1 = 0$. This equation has no real roots but admits complex roots $p = e^{\pm i\frac{\pi}{3}}$. This is the result which leads to develop a multivalued logic to complex truth values. The sum of truth values being isomorphic to the vector of the plane, it is natural to relate the function V to the metric of the vector space R^2. We will adopt as valuations the norms of vectors. The main objective of this paper is to establish a theory of truth-value evaluation for paraconsistent logics with the goal of using in analyzing ideological, mythical, religious and mystic belief systems.

Keywords: *belief systems, circle of truth, coincidentia oppositorum, contradiction, complex number, denier, logic coordinations, paraconsistency, propositions, truth values*

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1. Introduction

"Philosophy is a collection of big mistakes, but mistakes so seemingly close to an aspect of truth, that they require serious consideration as premises, at least until their consequences and revelations become temporarily exhausted." (Florencio Asenjo, 1985)

"If … we construe [this statement] as [a statement] about a sun which is actually all it is able to be, then we see clearly that this sun is not at all like the sensible sun. For while the sensible sun is in the East, it is not in any other part of the sky where it is able to be. [Moreover, none of the following statements are true of the sensible sun:] "It is maximal and minimal, alike, so that it is not able to be either greater or lesser"; "It is everywhere and anywhere, so that it is not able to be elsewhere than it is"; "It is all things, so that it is not able to be anything other than it is"—and so on." (Nicholas of Cusa)

What we need initially is an answer to the question "What is a Contradiction?" That can only be had with a definition of the term. The English term itself derives from the Latin verb *contradictio* (contradicere), "I speak against" ("to speak against"). But the initial definition of "contradiction" comes to us from Aristotle. In the Greek the term Aristotle used was *antiphasis*. That term is composed of two Greek words. The term anti is a preposition. In this use, it means "against." The second term, *phasis*, comes from the verb *phēmi*, which means "to say, speak or tell." It connotes the act of expressing opinion, thought or belief, and, thus, of having an opinion, thought or belief. The term *phasis* itself means a "saying, speech, sentence, affirmation or assertion." A fair etymological definition of the term *antiphasis*, then, is that it means a "saying, speech, sentence, affirmation or assertion against." So Latin and Greek provide the same basic meaning. But both leave us with the question against what? And we shall answer that in due course. For now, however, we need to look at Aristotle's own definition of the term.

In the current paradigm of consciousness, duality is perceived be a binary state of mutual exclusion. One sees this notion reflected in human thought and language where something must be "*either X or Y*", but not "*both X and Y*". A new paradigm of consciousness is required that no longer operates in a "*dualistic*" notion of "*either/or*", but one that conveys a "*holistic*" notion of "*both/and*". We currently view duality as a disjunctive rather than a conjunctive aspect of being. The difference between this dualistic and holistic paradigm of consciousness can be symbolically expressed in the language of logic. Current dualistic paradigm of consciousness $X \neq Y; X \bigcup Y$ (something is either X or Y); a disjunctive exclusion. Emerging holistic paradigm of consciousness $X = Y; X \bigcap Y$ (something is both X and Y); a conjunctive inclusion. This conceptual paradigm of viewing the world in such an exclusionary and disjunctive dualistic state has been programmed into us by an outdated Cartesian philosophical worldview and a Newtonian scientific view of the universe. This modern paradigm of dualistic thought has been prevalent ever since René Descartes proclaimed. That tradition clings to the principle of non-contradiction, but understood as *rejection of contradiction* hereinafter abbreviated as RC. Well, is paraconsistent any approach that rejects this same RC, ie admit that certain contradictions can be true (not necessarily all, of course). In particular, today is paraconsistent a treatment of problems such as a philosophy of religion that accommodates certain antinomian assertions and in doing so, offered as underlying logic to build theory, not classical logic is a logic of Aristotelian stamp, but one of the denominated precisely paraconsistent logics.

It seems to us that the philosophy of paraconsistency can propose a concept of modern rationality which will enable us to restore and gradually elaborate in never ending self-criticism "*the vision of the whole*" as a co-evolutionary unity of mankind and Nature. To the basics of this modern rationality would belong of the non-exclusive relation between analytical and dialectical thinking, their developmental unity. The desirable unifying can be conceived of in various ways. It follows from this paper that we are skeptical about the proposal to unify analytical and dialectical thinking through a kind of reduction of the latter to the first by applying the idea of paraconsistency. It would mean to reduce the whole to a part. What we propose is to conceive analytical thinking as a part of and a derivative from something more complex and more fundamental.

The main objective of the authors is to establish a theory of truth-value evaluation for paraconsistent logics, unlike others who are in the literature (Asenjo, 1966; Avron, 2005; Belnap, 1977; Bueno, 1999; Carnielli, Coniglio and Lof D'ottaviano, 2002; Dunn, 1976: Tanaka et al, 2013), with the goal of using that paraconsistent logic in analyzing ideological, mythical, religious and mystic belief systems (Nescolarde-Selva and Usó-Doménech, 2013[a,b], 2014[a,b,c]; Nescolarde-Selva, Usó-Doménech and Lloret-Climent, 2014; Usó-Doménech and Nescolarde-Selva, 2012).

2. Historical Perspective

Numerous paraconsistent logical calculi have been constructed which allow the formula $P \wedge \neg P$ to be true (derivable) under some special conditions and thus tolerate $P \wedge \neg P$ without becoming trivial. To provide some grounds for this theory let us take a look at Aristotle's theory of contrariety from the point of view of modern dialectic. This is detailed account of what Aristotle calls *Antiphasis* is to be found in *Metaphysics Book 4*. Aristotle's examples of the four kinds of opposites are: *double* and *half*, *bad* and *good*, *blindness* and *sight* and *he sits* and *he does not sit*. Aristotle was deeply interested in investigating the modes of opposition and their ontological relevance in the early, middle and late period of his philosophizing. He ascribed to the opposites an important role in almost all fields of reality, in Nature, in society as well as in thought, but disagreed with that ontological overestimation of the role of opposites, which he found in many preceding Greek thinkers. The second is his misinterpretation of Heraclitus in the sense of Protagoras' relativism hereby not only the sophistic relativism, but also the Heraclitian anticipations of dialectical ontology.

Aristotle is right in insisting that the denial of this principle would lead to a kind of total trivialization of human thinking and people would become prisoners of a helpless tenet "*which prevents a thing from being made definite by thought*. Now let us compare three following allegedly synonymous formulations. Aristotle took all three as stating the same principle and in different places mutually argues the truth of each of them from the presupposed evidence of each of them.

1) "*Contradictory propositions are not true simultaneously*". This statement is, as already mentioned, acceptable and respected on the new ontology.

2) "*Contradictories cannot be predicated at the same time*". This statement would be unacceptable if interpreted in the following way: (in the European tradition translated as "*contradictio*") is for Aristotle sometimes the conjunction of two sentences (or statements, propositions) of which one affirms what the other denies; sometimes either part of this conjunction; sometimes the negation of any given subject, property, relation, action etc. (e. g. *man - not-man*, *changing - unchanging*).

3) "*Contraries cannot at the same time belong to the same subject*" if taken, as Aristotle did, as a general principle valid for all entities.

These opposites are, for Heraclitus, to be taken in unity, as constituting in their opposition and unity something identical. If sometimes in the dialectical tradition Heraclitus' position was characterized as claiming not only the unity, but even the identity of opposites, never was the Leibnizian identity meant, allowing us to replace one of the identical expressions and/or concepts by the other mutually and thus to remove completely the opposition.

A closely related, but more general, acceptation that figures in most literary manuals defines paradox as an apparent contradiction which, upon examination, actually reveals a hidden, startling truth. It is important to stress, however, that the apparent contradiction of paradox occurs only in the surface meaning of the opposing statements, each of which is found to be true in some sense or to a certain degree. Paradox therefore uses the language of apparent nonsense to express startling «truths» that exceed the bounds of logic and propositional discourse. A third meaning of paradox is technically called «*antinomy*»: an insoluble contradiction in which asserting the truth of a particular proposition necessarily entails asserting that proposition's falsity (Quine. 1966). The prototypical antinomy is the Paradox of the Liar, which asserts, in effect: "*This statement is false*". Clearly, if that statement if false, it is also true, in the same sense and to the same degree. For our purposes here, it is important to bear in mind that what renders antinomies insoluble is their wholly internal reference and their self-contained quality. Beyond their utterly fixed terms, there remains no logical or semantic space, and no other level of abstraction, which permit an assertion of even partial truth or falsity. One of the chief sources of paradox literature in the West is Plato's *Parmenides*. Through a barrage of paradoxical utterances, the dialogue not only treats of such eminently philosophical questions as "*unity and diversity*", "*likeness and unlikeness*" and "*being and non-being*". It also provides a model of Plato's rhetorical art, including a *practical* model for the training of novice orators.

Parmenides says to the young Socrates: *There is an art which is called by the vulgar «idle talking», and which is often imagined to be useless; in that art you must train yourself, now that you are young, or truth will elude your grasp* (Plato, 1973). Parmenides goes on to demonstrate that, simply put; this art consists in arguing opposite sides of a question. "*Truth"* is thus shown to lie, not so much between, as beyond extremes, each of which is in some way deficient, at once partially true and partially false. Further, truth is also shown to prove elusive and paradoxical, as set forth in the dialogue's startling "*conclusion*" about what the truth *seems* to be: *[Parmenides]: Let this much be said; and further let us affirm what seems to be the truth, that, whether [the] one is or is not, [the] one and the others [plurality] in relation to themselves and one another, all of them, in every way, are and are not, appear to be and appear not to be. [Socrates]: Most true*. (Plato, 1973).

In his *Of Learned Ignorance* (*De docta ignorantia*), written in 1440, Nicholas of Cusa adopted an equally paradoxical approach to questions of truth, albeit within a Christian intellectual framework. A clear echo of Socrates' knowing only that he knows nothing, and the Pauline distinction between worldly and godly wisdom, the title of Cusa's first chapter reads: "*How Knowledge is Ignorance*" (Cusanus, 1986). Yet such ignorance becomes increasingly «learned», hence increasingly unknowing, through reflective contemplation of the created order, which unfolds in time as a perplexing admixture of unity and plurality, likeness and unlikeness, being and non-being-a *coincidentia oppositorum*, or an alternately conflicting and harmonious blend of contraries. A source of paradox literature is Erasmus' *The Praise of Folly*. Modeled after the classical type of "*paradoxical encomium*", Erasmus' work seems often to conclude that many species of "*folly*" are wisdom, and that folly is at once good and bad, laudable and despicable. Yet the viciously circular text resists anything akin to univocal interpretation, owing primarily to its status as an exaltation of folly pronounced by Folly herself. Indeed, the character's declamation presents an extreme case of self-reference, self-praise, self-love (*philautia*) and a lack of self-knowledge, which casts even its seemingly «truthful» utterances in doubt.

The doctrine of *coincidentia oppositorum*, the interpenetration, interdependence and unification of opposites has long been one of the defining characteristics of *mystical* (as opposed to philosophical) thought. Mystics of various persuasions have generally held that such paradoxes are the best means of expressing within language, truths about a whole that is sundered by the very operation of language itself. Any effort, it is said, to analyze these paradoxes and provide them with logical sense is doomed from the start because logic itself rests upon assumptions, such as the principles PNC and PEM, that are violated by the mystical ideas. The *coincidentia oppositorum* is a common trope in many religious traditions, particularly those with a mystical or initiatory aspect:

"*For I am the first and the last, I am the honored one and the scorned one; I am the whore and the holy one….*" (Thunder, Perfect Mind*)*

"*Being and non-being produce one another. Hard depends on easy, long is tested by short, High is determined by low.*" (Tao Te Ching)

"*The way up and the way down are one and the same*." (Heraclitus)

A modified version of the *coincidentia* occurs in a negative form, in which conjoined pairs of opposites are asserted and then rejected as the locus of ultimate truth, which is held to be transcendent of the objects of discriminating consciousness:

"*The Self is to be described as* not this, not that. *It is incomprehensible, for it cannot be comprehended; undecaying, for it never decays; unattached, for it never attaches itself, unfettered, for it is never bound. He who knows the Self is unaffected, whether by good or by evil*." (Brihadaranyaka Upanishad)

"*I prostrate to the perfect Buddha, the best of all teachers, who taught that that which is dependently-arisen is without cessation, without arising; without annihilation, without permanence; without coming, without going; without distinction, without identity and peaceful — free from fabrication.*" (Nagarjuna)

The problem of the apparent irrationality of the *coincidentia* in Buddhist philosophy was an area of intense exegetical concern for Tibetan interpreters of Indian Madhyamaka, and the attempt to interpret apparently incoherent statements by Nagarjuna and Chandrakirti was a central preoccupation of many of Tibet's great scholars. Alternately, in Ch'an and Zen, the problem of awareness operating beyond logical categories was fervently embraced and the paradoxical quality of the language was emphasized.

Whereas western mystics have often held that their experience can only be described in terms that violate PNC western philosophers have generally maintained that this fundamental logical principle is inviolable. Nevertheless, certain philosophers, including Meister Eckhardt and G.W.F. Hegel have held that presumed polarities in thought do not exclude one another but are actually necessary conditions for the assertion of their opposites. Hegel (1948) was the last great philosopher to hold that the identity of opposites could be demonstrated rationally. His view that coincidentia oppositorum yields a logical principle was treated with such scorn by later generations of philosophers that the idea of finding a rational/philosophical parallel to the mystic quest became an anathema to serious philosophers. Even W. T. Stace (1950, p 213), who was highly sympathetic to mysticism eventually, came to the view that in trying to make logic out of the coincidence of opposites Hegel fell "*into a species of chicanery*." According to Stace, "*every one of [Hegel's] supposed logical deductions was performed by the systematic misuse of language, by palpable fallacies, and sometimes…by simply punning on words."* Stace, who early on wrote a sympathetic, and now much maligned, book on Hegel's system, gave up the idea that coincidentia oppositorum could be shown to be a rational principle, holding that "*the identity of opposites is not a logical, but definitely an alogical idea.*"

The paradoxical nature of the *coincidentia oppositorum* receives primary attention in much contemporary exegetical literature in the West. It is often glossed by interpreters as an intentional confound to conventional logic, which is based on the *law of the excluded middle*, an axiom of reasoning that holds any given thing must either by X or not-X, where X is any possible predicate. The emphatic assertion of coincident opposites appears on the surface to be a direct challenge to Boolean or Aristotelian logic. In the XX century the physicist Neils Bohr commented that superficial truths are those whose opposites are false, but that "*deep truths*" are such that their opposites or apparent contradictories are true as well. The psychologist Carl Jung (1955/56) concluded that the "*Self*" is a *coincidentia oppositorum*, and that each individual must strive to integrate opposing tendencies (anima and animus, persona and shadow) within his or her own psyche. And the self *is* in Jung's view exactly such an ultimate '*central archetype of order'* (Urban 2005) and clearly in his view to a *coincidentia oppositorum*. Jung is *not* saying that the self is divine but rather that it is an expression of the Divine: '*Divineness*' or Divinity (*die Göttlichkeit*) expresses or shapes or moulds (*ausdrückt*) the self in the form of a *coincidentia oppositorum*. Jung is not *equating* self with Divinity in this passage or saying that the self is divine; he is bringing the terms into another kind of relation. The idea is that *die Göttlichkeit* forms and shapes (*ausdrückt*) the self into the pattern of *coincidentia oppositorum*. In other words, the psychological self is shaped and conditioned by something with a theological title, *die Göttlichkeit.* The self mirrors the Divinity (at least to some degree). This is a non- or post-traditional restatement of the familiar Biblical idea that humankind is created in the image of God (Genesis 1:27—'*So God created man in his own image, in the image of God he created him; male and female he created them*') and therefore embodies (whether psychologically, mentally, spiritually, or even according to some theologians, physically) the *imago Dei.* Jung clearly states a strong and intimate relation between the self and *die Göttlichkeit* (both as defined by the phrase, *coincidentia oppositorum)*, but they are not identical (Stein, 2008).

More recently, postmodern thinkers such as Derrida (1967, 1980) have made negative use of the *coincidentia oppositorum* idea, as a means of overcoming the privileging of particular poles of the classic binary oppositions in western thought, and thereby deconstructing the foundational ideas of western metaphysics. Opposition wave-particle cornerstone of quantum mechanics is a good example of *coincidentia oppositorum.*

In religious texts is often very difficult to resort to paraphrase well (if they are paraphrases, that's another problem, as they may be explanations and not mere paraphrase). It is often swaged with this emphasis, the same God and the same context, in relation to the same entities and under the same aspect, which is benign and malevolent, sweet and angry, fearful and kind, inaccessible and accessible, indulgent and severe -not to mention thousands of other contradictions that perhaps could be susceptible to other reinterpretations, less literal and more charitable-, which is implausible given such utterances the meaning of mere nuances implicit of gradualness

The more or less obvious meaning of many of these texts and discourses seems that God has each pair of opposite determinations within a special way, owning one as well as another in a high degree. Using the Meinong theory of objects (Peña, 1985) will admit as non illogical the principle of characterization attributable to Meinong:

Principle of Characterization: *An entity who comes presented as being in one way or another, has effectively a determination to be in one form or another; but with the precision of which does not necessarily follow, that an entity that has the property to be in one form or another, that entity is in one form or another*

When this principle is complied is produced named *rule of characterization.* If we have a system with the principle but not the rule of characterization, the system wills a *weak Meingonian system*. Instead a system with the principle and with a restricted version of the rule will be a *strong Meingonian system*. What cannot be is a system that having both and unrestricted both the full version of the principle as that of the rule; this system would be deliquescent, in it each sentence (syntactically well formed) would be both a thesis (asserted). But it is possible to have a treatment with both, rule and principle in its full and unrestricted versions. But it does not follow that do not fit but both treatments containing restricted or qualified versions. E.g. be a version according to which, if having the property A implies have the property B, then each entity will have one another. This may seem a simple tautology, but it is not demonstrable in the systems of first-order logic. Moreover, the treatment Meingonian weakly we are surmising could contain some much nuanced version of the rule of characterization, a version that would prevent the emergence of contradictions. Within a treatment so, arguably, if God is instituted or presented as having such and such characteristics, then He has effectively such features; but there will not continue necessarily having the features in question (although it may follow that in many cases).

Then the sentence P = "*X has the property of being in one form or another and has the property of not being in one form or another*" is no real contradiction in the classical sense and does not move in terms of truth and falsity absolute. That phrase appears only real contradiction if we add the rule of characterization. This last rule makes adopt a Meingonian weak approach.

Some of the adherents of this trend in contemporary logic investigate explicitly also its philosophical presuppositions and implications (Bueno, 2010; Carnielli and Marcos, 2001). Among other problems, the question of the relationship between the idea of paraconsistency and the traditional and/or contemporary forms of dialectical thinking is being examined. It seems to us that in the philosophy of paraconsistency a differentiation can be observed today. One of the tendencies, represented by Arruda (Arruda et al, 1980), da Costa (da Costa and Wolf, 1980), Quesada (1989) while assessing highly important philosophical implications of the logic of paraconsistency, insists upon the view that paraconsistency is closely linked with the theory of logical calculi. The philosophizing logicians of this tendency give, as a rule, only modest hypothetical accounts of the relationship between paraconsistency and dialectic. The other tendency, represented by G. Priest (Priest, 1989, 1995, 1998; Priest, Routley, and Norman, 1989), dares to defend vehemently more radical and ambitious assumptions about the philosophical and scientific implications of paraconsistent logic, concerning not only the relation to dialectic, but also the conception of rationality in general. Let us have a closer critical look at some main claims of the philosophy of paraconsistency from a special point of view, namely, from the point of view of secular (ontopraxeological) dialectic which aims at elaborating a theory of modern rationality taking inspiration from Hegel's critique of Kant and Marx's critique of Hegel. Needless to say, no simple reception of any philosophy of the past is able to cope with our contemporary problems of rationality. References to Kant and Hegel remain mostly mere decoration. Priest's use of the calculus-oriented notion of inconsistency in his interpretation of the so-called Kant/Hegel thesis about the inherently inconsistent nature of human reason seems to us to be misleading. Supposing we accept Kant's argumentation in his "*Transcendental Dialectics*" as a justification of the statement that our thinking is in its very nature (apparently, but necessarily) inconsistent: then Hegel's critique of Kant's antinomies should be taken as an attempt at a new consistency which corrects the antinomic dialectic of (apparent, but necessary) inconsistency of human reason in a section of its usage. The dictum of a unitary "*Kant/ Hegel thesis*" hides this difference.

Following Priest, we will say that a logical system is *paraconsistent*, if and only if its relation of logical consequence is not "*explosive*", i.e., iff it is not the case that for every formula *P* and *Q*, *P* and not-*P* entails *Q*; and we will say a system is *dialectical*, iff it is paraconsistent and yields (or "endorses") *true contradictions*, named "*dialetheias*". A paraconsistent system enables to model theories which in spite of being (classically) inconsistent are not trivial, while a dialectical system goes further, since it permits *dialetheias*, namely contradictions as true propositions. Still following Priest, semantics of dialectical systems provide truth-value *gluts* (its worlds or set-ups are overdetermined); however, truth-value *gaps* (opened by worlds or set-ups which are underdetermined) are considered by Priest to be irrelevant or even improper for dialectical systems. Besides that, sometimes the distinction is drawn between weak and strong paraconsistency, the latter considered as equivalent with dialectics. A reader of recent literature in this field may have an impression that dialectics as strong paraconsistency is more a question of ontology than of logic itself, namely that it states the existence of "*inconsistent facts*" (in our actual world) which should verify dialetheias. One more introductory remark has to be put here: in recent literature of paraconsistency there are no quite unanimous, among paraconsistent logicians generally accepted distinction between paraconsistent and dialectical logical systems. But it remains an open question whether; semantically paradoxes express any "*inconsistent facts*".

2.1. *Coincidentia oppositorum* in Mysticism: The Early Kabbalah Case

The Kabbalists use the term, *achdut hashvaah*, to denote that *Ein-sof*, the Infinite God, is a "*unity of opposites*," (Figure 1).

Or as Scholem (1974, p. 88) at one point translates the term a *"complete indistinguishability of opposites,"* one that reconciles within itself even those aspects of the cosmos that are opposed to or contradict one another. *Sefer Yetzirah*, an early (3rd to 6th century) work which was of singular significance for the later development of Jewish mysticism, had said of the *Sefirot* (the ten archetypal values through which divinity is said to constitute the world) *"their end is imbedded in their beginning and their beginning in their end."* (Kaplan, 1997, p. 57). According to *Yetzirah*, the *Sefirot* are comprised of five pairs of opposites: "A depth of beginning, a depth of end. A depth of good, a depth of evil. A depth of above, a depth of below, A depth of east, a depth of west. A depth of north, a depth of south. The 13th century Kabbalist Azriel of Gerona was perhaps the first Kabbalist to clearly articulate the doctrine of *coincidentia oppositorum*. For Azriel "Ein Sof …is absolutely undifferentiated in a complete and changeless unity…He is the essence of all that is concealed and revealed." (Azriel. 1966). According to Azriel, *Ein-sof* unifies within itself being and nothingness, "*for the Being is in the Nought after the manner of the Nought, and the Nought is in the Being after the manner [according to the modality] of the Being… the Nought is the Being and Being is the Nought.* (Scholem, 1987, p. 423). For Azriel, *Ein-sof* is also *"the principle in which everything hidden and visible meet, and as such it is the common root of both faith and unbelief."* (Scholem, 1987, pp. 441-442). Azriel further held that the very essence of the *Sefirot*, the value archetypes through with *Ein-sof* is manifest in a finite world, involves the union of opposites, and that this unity provides the energy for the cosmos. The nature of *sefirah* is the synthesis of everything and its opposite. For if they did not possess the power of synthesis, there would be no energy in anything. For that which is light is not dark and that which is darkness is not-light. Further, the coincidence of opposites is also a property of the human psyche; "*we should liken their (the Sefirot) nature to the will of the soul, for it is the synthesis of all the desires and thoughts stemming from it. Even though they may be multifarious, their source is one, either in thesis or antithesis."* (Azriel. 1966, p. 94). Azriel was not the only Kabbalist to put forth a principle of *coincidentia oppositorum*. The early Kabbalistic *Source of Wisdom* describes how God's name and being is comprised of thirteen pairs of opposites (derived from the 13 traits of God enumerated in Chronicles), and speaks of a Primordial Ether (*Avir Kadmon*), as the medium within which such oppositions are formed and ultimately united.

Figure 1. Representation of *Ein-sof*

The concept of *achdut hashvaah* figures prominently in the Lurianic Kabbalah, which became the dominant theosophical and theological force in later Jewish mysticism and Chasidism. Isaac Luria (1534-72) wrote very little, but his chief expositor, Chayyim Vital (1543-60) records: *Know that before the emanation of the emanated and the creation of all that was created, the simple Upper Light filled all of reality…but everything was one simple light, equal in one hashvaah, which is called the Light of the Infinite.* (Zohar III, 113a. , Vol. 5, p. 153). While Vital's account suggests a unity of opposites in the godhead only *prior* to creation, a close examination of the Lurianic Kabbalah reveals a series of symbols that are applicable to God, the world and humanity, and which overcome the polar oppositions of ordinary (and traditional metaphysical) thought. Indeed, each of the major Lurianic symbols expresses a coincidence of opposites between ideas that are thought to contradict one another in ordinary thought and discourse. For example, Luria held that the divine principle of the cosmos is both *Ein-sof* (without end) and *Ayin* (absolute nothingness), that creation is both a *hitpashut* (emanation) and a *Tzimtzum*(contraction), that *Ein-sof* is both the creator of the world and is itself created and completed through *Tikkun ha-Olam*, the spiritual, ethical and "world restoring" acts of humanity, and, finally, that the *Sefirot* are both the originating elements of the cosmos and only fully realized when the cosmos is displaced and shattered (via *Shevirat ha-Kelim*, the Breaking of the Vessels).

A closer examination of two key elements in the Lurianic system, *Tzimtzum* (concealment/contraction) and *Shevirat ha-kelim* (the Breaking of the Vessels) can provides further insights into the Lurianic conception of the coincidence of opposites.

In the symbol of *Tzimtzum* (the withdrawal, concealment and contraction of the infinite that gives rise to the world) there is a coincidence of opposites between the positive acts of creation and revelation and the negative acts of concealment, contraction and withdrawal. For Luria, God does not create the world through a forging or emanation of a new, finite, substance, but rather through a contraction or concealment of the one infinite substance, which prior to such contraction is both *"Nothing"* and *"All."* Like a photographic slide, which reveals the details of its subject by selectively filtering and thus concealing aspects of the projector's pure white light (which is both *"nothing"* and *"everything"*), *Ein-sof* reveals the detailed structure of the finite world through a selective concealment of its own infinite luminescence. By concealing its absolute unity *Ein-sof* gives rise to a finite and highly differentiated world. Thus in the symbol of *Tzimtzum* there is a coincidence of opposites between addition and subtraction, creation and negation, concealment and revelation. In order to comprehend the notion of *Tzimtzum*, one simultaneously think two thoughts, for example, one thought pertaining to divine concealment and a second pertaining to (this concealment as) creation and revelation.

For Luria, the further realization of *Ein-sof* is dependent upon a second coincidence of opposites; between creation and destruction, symbolized in the *Shevirat ha-Kelim*, the "Breaking of the Vessels." *Ein-sof*is only fully actualized as itself, when the ten value archetypes which constitute the *Sefirot* are *shattered* and are subsequently restored by humankind (*Tikkun ha-Olam*). While *Ein-sof* is the source and *"creator"* (Zohar III, 113a. Vol. 5, p. 153). of all,*Ein-sof* paradoxically only becomes itself, through a rupture which results in a broken and alienated world in need of humanity's *"restoration"* and repair (*Tikkun*). For Luria, *Ein-sof* is propelled along its path from "nothing" (*Ayin*) to *"something"* (*Yesh*), through the creative and restorative acts of humankind; for it is only humanity acting in a broken and displaced world, that can undertake the *mitzvoth*, the creative, intellectual, spiritual and ethical acts that fully actualize the values and traits that exist only *potentially* within God. It is for this reason that the *Zohar* proclaims *"He who 'keeps' the precepts of the Law and 'walks' in God's ways…'makes' Him who is above."* (Zohar III, 113a. Vol. 5, p. 153). Thus, just as humanity is dependent for its existence upon *Ein-sof*, *Ein-sof* is dependent for its actual being upon humanity. The symbols of *Ein-sof*, *Shevirah* (rupture) and *Tikkun* (Repair) thus express a coincidence of opposites between the presumably opposing views that God is the creator and foundation of humanity and humanity is the creator and foundation of God.

3. Contradiction and Deniers

Let F be a non-countable set whose algebraic structure is at least that of a semi-ring. F is a semi-ring, a ring or a group.

We lay the following definitions and fundamental axioms:

Axiom 1: *Any proposition P has a truth value p, element of a set E which is a part, not countable and stable for multiplication of the set F.*

Axiom 2: *Any proposition P is endowed with a valuation* $v \in [0,1]$ *such that* $v = V(p)$ *, V reciprocal application of E on* $[0,1]$ *subject to the following conditions:*

1) $V^{-1}(0) = 0,$

2) $V(p_1, p_2) = V(p_1)V(p_2)$

being p_1 and p_2 two truth values.

Axiom 3: *Truth value* p^* *denotes the negation or contradiction of P denoted as* $\neg P$ *and* $V\left(p + p^*\right) = 1$.

Let P_i be n propositions, i= 1,2,…, n of p_i and p_i^* be the truth values of their contradictories. Then:

Definition 1: *A compound proposition (or logical coordination or logical expression) of order n is a proposition whose truth value c is a function* f_n *of p_i and* p_i^*.

$$c = f_n\left(p_1, p_1^*, p_2, p_2^*, ... p_n, p_n^*\right)$$

f_n values in F; it determines a truth value if $c \in E$. The condition of existence of a compound proposition defined by f_n is $c \in E$, or what is equivalent, $V(c) \in [0,1]$.

Axiom 4: *f_n is a polynomial in which each index 1, 2,...,n must be at least once and that all coefficients are equal to unity.*

Definition 2: *Let $p+p^{*}=u$ be. u is a denier of the proposition P if the following three conditions are fulfilled* (Usó-Doménech, Nescolarde-Selva and Pérez-Gonzaga, 2014)*:*

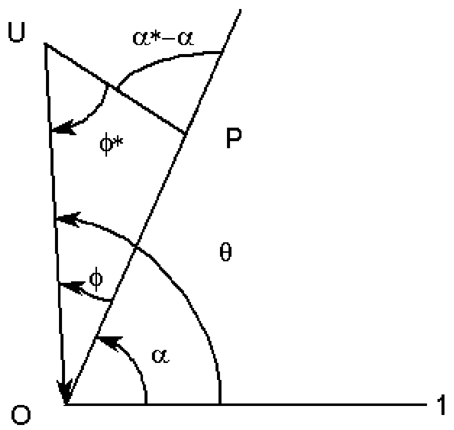
a). $u \in E$

b). $V(u)=1$; *u unitary truth value (from axiom 3)*

c). $u-p=p^{*} \in E$ *(from axiom 1)*

Paraconsistent logic admits that the contradiction can be true. Then $v(P \wedge \neg P)=p(1-p)=1 \Rightarrow p-p^{2}-1=0$ $\Rightarrow p^{2}-p+1=0$. This equation has no real roots but admits complex roots $p=e^{\pm i\frac{\pi}{3}}$. This is the result which leads to develop a multivalued logic to complex truth values. The sum of truth values being isomorphic to the vector of the plane, it is natural to relate the function V to the metric of the vector space R^2. We will adopt as valuations the norms of vectors. E is the set of complex numbers of modulus less or equal to 1 and the function V is such that $V(p)=|p|^{2}$ and it has satisfied axiom 2.

Let P be a proposition of fixed truth value $p=|p|e^{i\alpha}$. If p = 0, α is indeterminate, we agree to take $\alpha=0$ .According to definition 2, here a denier is a unitary complex number $u=e^{i\omega}$ such that $|u-p| \le 1$. Putting $\varphi=\theta-\alpha$ (Figure 2) this inequality entails $\cos\varphi \ge \frac{|p|}{2}$, be $|\varphi| \le \theta,\ \theta=\left|\arccos\frac{|p|}{2}\right|$.



$p=O\vec{P}\quad u=O\vec{U}\quad p^{*}=P\vec{U}$

Figure 2. Circle of truth

In summary

$$\varphi \in [-\theta,\theta];\quad \cos\theta=\frac{|p|}{2};\quad \theta \in \left[\frac{\pi}{3},\frac{\pi}{2}\right] \tag{1}$$

Deniers u of P form a continuous set: the sector of the *circle of truth* (trigonometric circle) of angle 2θ whose vector p is collinear to the bisector (Figure 3).

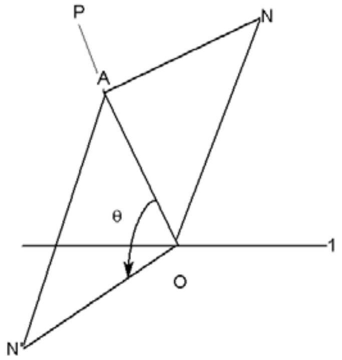


Figure 3. Location of U: arc NPN'; NN' mediatrix of OA

A denier is determined by the angle ϑ. $u(\varphi)$ is a bijective function.

Contradictory proposition ¬P provides, fixed p, a continuous set of truth values $p^{*}(\varphi),\ p^{*}=|p^{*}|e^{i\alpha^{*}}$ and then:

$$\begin{aligned}&|p^{*}| \in [1-|p|,1];\varphi=0 \Leftrightarrow |p^{*}|=1-|p|;\\&\varphi=\pm\theta \Leftrightarrow |p^{*}|=1\end{aligned} \tag{2}$$

Putting $\omega-\alpha^{*}=\varphi^{*},\ \varphi=-\theta \Rightarrow \varphi^{*}=\pi-2\theta;\ \varphi=0 \Rightarrow$ $\varphi^{*}=0 \Rightarrow \varphi^{*}=2\theta-\pi;$ in summary,

$$\varphi^{*} \in [2\theta-\pi,\pi-2\theta] \Rightarrow \varphi-\varphi^{*} \in [\theta-\pi,\pi-\theta] \tag{3}$$

On the other hand we have $\alpha^{*}-\alpha=\varphi-\varphi^{*}$.

The truth contradiction $v(P \wedge \neg P)=V|pp^{*}|=1$ requires $|p|=1$ where $\theta=\pm\frac{\pi}{3}$ and also $|p^{*}|=1$ where $\varphi=\pm\frac{\pi}{3}$ and $\varphi^{*}=\pm\frac{\pi}{3},\ \ \varphi-\varphi^{*}=\pm\frac{2\pi}{3}=\alpha^{*}-\alpha$.

Solutions of $v(P \wedge \neg P)=1$ are finally (Figure 4): $p=e^{i\alpha}$, α anyone; $p^{*}=e^{i\left(\alpha\pm\frac{2\pi}{3}\right)}$.

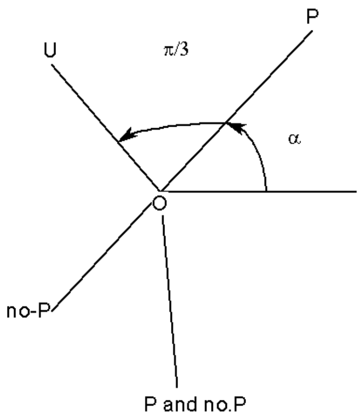


Figure 4.

Then, multivalued logic with complex truth values is paraconsistent.

Another characteristic of paraconsistent logic is that the negation of the negation does not necessarily leads back to the original proposition as Hegel said. If $u' \ne u, v(\neg\neg P)=V(u'-p^{*}) \ne V(p); v(\neg\neg P) \ne v(P)$.

4. Conditions

4.1. Condition 1

It is written $|u_1u_2 - p_1p_2| \le 1$ with the above notation:

$$\left|e^{i(\varphi_1+\varphi_2)} - |p_1||p_2|\right| \le 1$$

Condition 1 back to

$$\cos(\varphi_1+\varphi_2) \ge \frac{|p_1||p_2|}{2}$$

There is a continuous set of deniers $u_1(\varphi_1)$ and other deniers $u_2(\varphi_2)$ that that satisfy (eg $\varphi_1+\varphi_2=0$). As well the $\neg P_1 \vee \neg P_2$ incompatibility exists, provides p_1 and p_2 fixed, on a continuous set of truth values $u_1u_2 - p_1p_2$ and we have:

$$v(\neg P_1 \vee \neg P_2) = v\neg(P_1 \wedge P_2)$$

Similarly $P_1 \vee P_2$ does exist, provided that:

$$\cos\left(\varphi_1^* + \varphi_2^*\right) \ge \frac{|p_1^*||p_2^*|}{2}$$

satisfied, for example if $\varphi_1+\varphi_2=0$.

Similarly the implication $P_1 \Rightarrow P_2$ on condition that:

$$\cos\left(\varphi_1^* + \varphi_2^*\right) \ge \frac{|p_1||p_2|}{2}$$

4.2. Condition 2

Posing $\alpha_1 - \alpha_2 = \alpha$, that $|p_1+p_2| = \left||p_1|e^{i\alpha} + |p_2|^2\right|$ it result that $\forall p_1 \ne 0, \forall p_2 \ne 0,$ $\left|\alpha \ge \frac{2\pi}{3}\right| \Rightarrow |p_1+p_2| \le 1$; then $|p_1+p_2| > 1 \Rightarrow |\alpha| < \frac{2\pi}{3}$.

If $|p_1+p_2| > 1$, which requires non-zero p_1 and p_2, complementarity $P_1 \Im P_2$ does not exist and $\neg P_1 \Im \neg P_2$ must exist.

Theorem 1: Can be found deniers u_1 and u_2, such that $\left|p_1^* + p_2\right| \le 1$.

Proof

Just for this inequation is satisfied that $\left|\alpha_1^* - \alpha_2^*\right| \ge \frac{2\pi}{3}$.

After (4) $\alpha_1^* - \alpha_2^* = \left(\varphi_1 - \varphi_1^*\right) - \left(\varphi_2 - \varphi_2^*\right) + \alpha$.

Let $0 \le \alpha = \frac{2\pi}{3} - \beta$ be. After (3) the maximum value of $\varphi_1 - \varphi_1^*$ is $\pi - \theta_1 > 0$ and the one of $-\left(\varphi_2 - \varphi_2^*\right)$ is $\pi - \theta_2 > 0$ and therefore

$$\sup\left|\alpha_1^* - \alpha_2^*\right| = 2\pi - (\theta_1+\theta_2) + \frac{2\pi}{3} - \beta > \frac{5\pi}{3} - \beta$$

because $\theta_1 + \theta_2 < \pi$.

The result is $\sup\left|\alpha_1^* - \alpha_2^*\right| \ge \pi$ since $0 \le \beta \le \frac{2\pi}{3}$; or sufficient condition is $\left|\alpha_1^* - \alpha_2^*\right| \ge \frac{2\pi}{3}$.

There is a continuous set of values of $\left|\alpha_1^* - \alpha_2^*\right|$ and therefore of deniers $u_1(\varphi_1)$ and $u_2(\varphi_2)$ which satisfy this condition.

4.3. Condition 3

It is written $\left|p_1p_2 + p_1^*p_2^*\right| \le 1$. As condition 2, it is sufficient for it is fulfilled that the angle of non-zero vectors p_1p_2 and $p_1^*p_2^*$ is $\ge \frac{2\pi}{3}$ therefore $\left|\alpha_1 + \alpha_2 - \left(\alpha_1^* + \alpha_2^*\right)\right| \ge \frac{2\pi}{3}$ or else after (4) that:

$$\left|\varphi_1^* - \varphi_1 + \varphi_2^* - \varphi_2\right| \ge \frac{2\pi}{3} \tag{5}$$

or in according (3)

$$\sup\left|\varphi_1^* - \varphi_1 + \varphi_2^* - \varphi_2\right| = 2\pi - (\theta_1+\theta_2) > \pi \tag{6}$$

4.4. Condition 4

It is written $\left|p_1p_2^* + p_1^*p_2\right| \le 1$. Studied by the same method it proves to be satisfied if (sufficient condition)

$$\left|\varphi_1 - \varphi_1^* + \varphi_2 - \varphi_2^*\right| \ge \frac{2\pi}{3} \tag{7}$$

It is an inequation whose solution is the same of (5).

The result is that concordance $P_1 \Xi P_2$ and discordance $P_1 \mathrm{X} P_2$ can exist together; then one is the negation of the other by denier u_1u_2 .

5. Propositional Paraconsistent Algebra

Propositional algebra can be built on the set of complex truth values. The main normal binary propositions are the following:

1. *Conjunction:*

$$v(P_1 \wedge P_2) = |p_1p_2|^2 = |p_1|^2|p_2|^2 \tag{8}$$

2. *Incompatibility:*

$$\begin{cases} v(\neg P_1 \wedge \neg P_2) = |u_1u_2 - p_1p_2|^2 = \left|e^{i(\varphi_1+\varphi_2)} - |p_1||p_2|\right|^2 \\ v(\neg P_1 \vee \neg P_2) = v\left(\neg(P_1 \wedge P_2)\right), \ \ denier \ \ u_1u_2 \end{cases} \tag{9}$$

3. *Disjunction:*

$$\begin{cases} v(P_1 \vee P_2) = |u_2p_1 + u_1p_2 - p_1p_2|^2 \\ = \left||p_1|e^{i\varphi_2} + |p_2|e^{\varphi_1} - |p_1||p_2|\right|^2 \\ v(P_1 \vee P_2) = v\left(\neg(\neg P_1 \wedge \neg P_2)\right), \ \ denier \ \ u_1u_2 \end{cases} \tag{10}$$

4. *Implication:*

$$\begin{cases} v\left(P_1 \Rightarrow P_2\right) = \left|u_1 u_2 - p_1\left(u_2 - p_2\right)\right|^2 \\ = \left|e^{i(\varphi_1+\varphi_2)} - \left|p_1\right|\left(e^{i\varphi_2} - \left|p_2\right|\right)\right|^2 \\ v\left(P_1 \vee P_2\right) = v\left(\neg\left(P_1 \wedge \neg P_2\right)\right), \ \ denier \ \ u_1 u_2 \end{cases} \quad (11)$$

5. *Concordance:*

$$\begin{cases} v\left(P_1 \Leftrightarrow P_2\right) = \left|u_1 u_2 - u_2 p_1 - u_1 p_2 + 2 p_1 p_2\right|^2 \\ = \left|e^{i(\varphi_1+\varphi_2)} - \left|p_1\right| e^{i\varphi_2} - \left|p_2\right| e^{i\varphi_1} + 2\left|p_1\right|\left|p_2\right|\right|^2 \\ v\left(P_1 \Leftrightarrow P_2\right) = v\left(\neg\left(P_1 \mathrm{X} P_2\right)\right), \ \ denier \ \ u_1 u_2 \end{cases} \quad (12)$$

6. *Discordance:*

$$\begin{cases} v\left(P_1 \Updownarrow P_2\right) = \left|u_2 u_1 + u_1 p_2 - 2 p_1 p_2\right|^2 \\ = \left|\left|p_1\right| e^{i\varphi_2} + \left|p_2\right| e^{i\varphi_1} - 2\left|p_1\right|\left|p_2\right|\right|^2 \\ v\left(P_1 \Updownarrow P_2\right) = v\left(\neg\left(P_1 \Leftrightarrow P_2\right)\right), denier \, u_1 u_2 \end{cases} \quad (13)$$

7. *Complementarity:*

$$v\left(P_1 \Im P_2\right) = \left|p_1 + p_2\right|^2 = \left|\left|p_1\right| e^{i(\alpha_1-\alpha_2)} + \left|p_2\right|\right|^2 \quad (14)$$

8. *Inverse complementarity:*

$$\begin{aligned} v\left(\neg P_1 \Im \neg P_2\right) &= \left|u_1 + u_2 - p_1 - p_2\right|^2 \\ &= \left|\left(e^{i\varphi_1} - \left|p_1\right|\right) e^{i(\alpha_1-\alpha_2)} + e^{i\varphi_2} - \left|p_2\right|\right|^2 \end{aligned} \quad (15)$$

9. *Equivalence:*

$$v\left(P_2 \wp P_1\right) = \left|p_1 + u_2 - p_2\right|^2 = \left|\left|p_1\right| e^{i(\alpha_1-\alpha)} + e^{i\varphi_2} - \left|p_2\right|\right|^2 \quad (16)$$

Here intervenes the angle $\alpha_1 - \alpha_2$ of vectors p_1, p_2.

Seek what deniers should be chosen so that if $v\left(P_1\right) = v\left(P_2\right)$, that is to say, if $\left|p_1\right| = \left|p_2\right| = \left|p\right|, \alpha_1 \neq \alpha_2$, we have: $v\left(P_2 \wp P_1\right) = 1 = v\left(P_1 \wp P_2\right)$.

We then: $v\left(P_2 \wp P_1\right) = \left|\left|p\right|\left(e^{i\alpha} - 1\right) + e^{i\varphi_2}\right|^2$ where $\alpha = \alpha_1 - \alpha_2$. So that $v\left(P_2 \wp P_1\right) = 1$, the necessary and sufficient condition is:

$$\sin\left(\frac{\alpha}{2} - \varphi_2\right) = \left|p\right| \sin\frac{\alpha}{2} \quad (17)$$

Similarly, for $v\left(P_1 \wp P_2\right) = 1$ the necessary and sufficient condition is:

$$\sin\left(\frac{\alpha}{2} + \varphi_1\right) = \left|p\right| \sin\frac{\alpha}{2} \quad (18)$$

of which $\varphi_1 = \varphi_2, \varphi_2$ solution of (17).

Figure 5 shows the geometric representation:

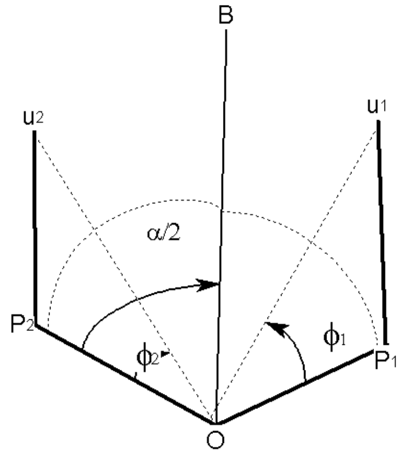


Figure 5. The geometric representation

5.1. Normal Propositions of Order n

1. *Conjunction:*

$$v\left(P_1 \wedge P_2 \wedge \ldots \wedge P_n\right) = \left|p_1 p_2 \ldots p_n\right|^2 \quad (19)$$

2. *Incompatibility:*

$$\begin{aligned} v\left(\neg P_1 \vee \neg P_2 \vee \ldots \vee \neg P_n\right) &= \left|u_1 u_2 \ldots u_n - p_1 p_2 \ldots p_n\right|^2 \\ v\left(\neg P_1 \vee \neg P_2 \vee \ldots \vee \neg P_n\right) &= v(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \end{aligned} \quad (20)$$

3. *Disjunction:*

$$\begin{aligned} v\left(P_1 \vee P_2 \vee \ldots \vee P_n\right) &= \left|u_1 u_2 \ldots u_n - p_1^* p_2^* \ldots p_n^*\right|^2 \\ v\left(P_1 \vee P_2 \vee \ldots \vee P_n\right) &= v\left(\neg P_1 \wedge \neg P_2 \wedge \ldots \wedge \neg P_n\right) \end{aligned} \quad (21)$$

4. *Complementarity*:

$$v\left(P_1 \Im P_2 \Im \ldots \Im P_n\right) = \left|p_1 + p_2 + \ldots + p_n\right|^2 \quad (22)$$

5. *Inverse complementarity*:

$$v\left(\neg P_1 \Im \neg P_2 \Im \ldots \Im \neg P_n\right) = \left|\begin{matrix} u_1 + u_2 + \ldots + u_n \\ -(p_1 + p_2 + \ldots + p_n) \end{matrix}\right|^2 \quad (23)$$

6. Paraconsistent Boolean Logic

It is the Boolean reduction of strong paraconsistent logic; modules of complex truth values there can be only 0 or 1. The circle of truth is there reduced to its center and its circumference. Although Boolean, this logic differs radically from the classical logic: it remains paraconsistent. The contradiction can be true there. We may have verified all the normal binary propositions that the propositional algebra of the paraconsistent Boolean logic contains well beyond the classical logic as a special case.

Since $v\left(P\right) = 0 \Rightarrow v\left(\neg P\right) = 1$, p^* indeterminate and $v\left(P \wedge \neg P\right) = 0$.

Since $v\left(P\right) = 1 \Rightarrow \theta = \pm\frac{\pi}{3}$. Since $\left|p^*\right|$ must be Boolean φ can only take two values:

$$\begin{aligned} \varphi = 0 &\Leftrightarrow v\left(\neg P\right) = 0 \Rightarrow v\left(P \wedge \neg P\right) = 0 \\ \varphi = \pm\frac{\pi}{3} &\Leftrightarrow v\left(\neg P\right) = 1 \Rightarrow v\left(P \wedge \neg P\right) = 1 \end{aligned} \quad (24)$$

It has always: $\varphi^* = -\varphi \Rightarrow \alpha^* - \alpha = 2\varphi$.

1. *Conjunction:* The truth table of the *conjunction* is identical to that of classical logic.

2. *Disjunction:* From (10), it is false if $v(P_1) = v(P_2) = 0$ and true if $v(P_1) = 0$ and $v(P_2) = 1$ or if $v(P_1) = 1$ and $v(P_2) = 0$.

Condition 1 is written $\cos(\varphi_1^* + \varphi_2^*) \geq \frac{1}{2}; \varphi^* = -\varphi$ thus the disjunction exists only if $\varphi_1 + \varphi_2 = -\frac{\pi}{3}, 0, \frac{\pi}{3}$; we have: $v(P_1 \vee P_2) = \left|e^{i\varphi_2} + e^{i\varphi_1} - 1\right|$ then:

$$v(P_1) = v(P_2) = 1 \Rightarrow \begin{cases} v(P_1 \vee P_2) = 0 \; if \; \varphi_1 = \pm\frac{\pi}{3}, \varphi_2 = \pm\frac{\pi}{3} \\ v(P_1 \vee P_2) = 1 \; \begin{matrix} if \;\; \varphi_1 + \varphi_2 = \pm\frac{\pi}{3}, \\ or \; \varphi_1 = \varphi_2 = 0 \end{matrix} \end{cases} \quad (25)$$

Hence the truth table of disjunction (Table 1):

Table 1. Truth table of disjunction $P_1 \vee P_2$

	P_1		
P_2		1	0
	1	1/0*	1
	0	1	0

Not conform evaluation to the classical logic is indicated by *.

It coincides with that of classical logic, in case $v(P_1) = v(P_2) = 1$, is chosen φ_1 and φ_2 such that $\varphi_1\varphi_2 = 0$.

3. *Implication:* From (11) is true if $\forall p_2, v(P_1) = 0$. If $v(P_1) = v(P_2) = 1$ the condition of existence is $\cos(\varphi_1 - \varphi_2) \geq \frac{1}{2}$; such as $v(P_1 \Rightarrow P_2) = \left|e^{i(\varphi_1+\varphi_2)} - e^{i\varphi_2} + 1\right|^2$, were $v(P_1 \Rightarrow P_2) = 1$ in all cases permitted by the condition of existence except where $\varphi_1 = \varphi_2 = \pm\frac{\pi}{3}$ for which $v(P_1 \Rightarrow P_2) = 0$. If $v(P_1) = 1$ and $v(P_2) = 0$ were $v(P_1 \Rightarrow P_2) = \left|e^{i\varphi_1} - 1\right|; v(P_1 \Rightarrow P_2) = 0$ if $\varphi_1 = 0, v(P_1 \Rightarrow P_2) = 1$ if $\varphi_1 = \pm\frac{\pi}{3}$.

Hence the truth table of implication (Table 2):

Table 2. Truth table of implication $P_1 \Rightarrow P_2$

	P_1		
P_2		1	0
	1	1/0*	1
	0	0/1*	1

It coincides with that of classical logic if one rejects the case $v(P_1) = v(P_2) = 1$ the choice $\varphi_1 = \varphi_2 = \pm\frac{\pi}{3}$ and the case $v(P_1) = 1$ and $v(P_2) = 0$ the choice $\varphi_1 = \pm\frac{\pi}{3}$.

These rejections are required to conduct a rigorous deduction in paraconsistent Boolean logic: the fundamental articulation of the deduction is indeed true implication denoted $\Rightarrow$, that if P_1 is true requires true P_2.

4. *Concordance:* From (12) is true if $v(P_1) = v(P_2) = 0$. If $v(P_1) = 1$ and $v(P_2) = 0$ then $v(P_1 \Xi P_2) = \left|e^{i\varphi_1} - 1\right|^2$ therefore $v(P_1 \Xi P_2) = 0$ if $v(P_1 \Leftrightarrow P_2) = 1$ $\varphi_1 = 0$ and $v(P_1 \Xi P_2) = 0$ if $\varphi_1 = \pm\frac{\pi}{3}$; same if $v(P_1) = 0$ and $v(P_2) = 1$ we have $v(P_1 \Leftrightarrow P_2) = 0$ if $\varphi_2 = 0$ and $v(P_1 \Leftrightarrow P_2) = 1$ if $\varphi_2 = \pm\frac{\pi}{3}$. If $v(P_1) = v(P_2) = 1$ the concordance does not exist when $\varphi_1 = \pm\frac{\pi}{3}, \varphi_2 = \pm\frac{\pi}{3}$, but it is true in all other cases.

Hence the truth table of concordance (Table 3):

Table 3. Truth table of concordance $P_1 \Leftrightarrow P_2$

	P_1		
P_2		1	0
	1	1	0/1*
	0	0/1*	1

It coincides with that of the equivalence of classical logic if when $v(P_1) = 1$ and $v(P_2) = 0$ is chosen $\varphi_1 = 0$ and when $v(P_1) = 0$ and $v(P_2) = 1$ is chosen $\varphi_2 = 0$.

Importantly, to conduct a rigorous reasoning with these choices, the concordance becomes identical to the deductive equivalence.

7. Reflections

The argument concerning belief systems may be circumvented if one claims that ordinary belief is not deductively closed. That is, at least, controversial, but an ideal reasoner should aspire to closure. Considering the case of a paraconsistent system being used as a meta-language to analyze a belief system, it is also the task of paraconsistent logic to define paraconsistent contradictions, that is, contradictions that are so threatening to this belief system that they really compromise rational inference-making within the belief system. This "*bad*" kind of inconsistency can be quantitative (too many classic contradictions may be a sign that even paraconsistency cannot save the belief system) or qualitative - that is, the classic contradiction in question is so strong (for example, a proof that all statements of the belief system can be proved both true and false) that it is also a paraconsistent contradiction, a contradiction that even a paraconsistent logician cannot accept. This argument implies the idea that the set of paraconsistent contradictions is a subset of the set of classic contradictions and that is indeed a rather intuitive idea. But we cannot think of any conclusive argument against the existence of a paraconsistent contradiction that is not a classic contradiction, so this idea is only a plausible conjecture.

To appreciate the significance for metaphilosophy of the recent developments in paraconsistent logic showing how, within formal systems, contradictory propositions can be held simultaneously without trivialization. The scientificist conception of the search for truth is partly motivated and partly justified by the ancient rejection of all contradictions. But this rejection is no longer a logical imperative. Indeed, it cannot be endorsed without, at least, severe qualifications that rob it from its argumentative bite. Thus the way will be open to adopt a novel understanding of the search for truth. And we shall present a model that conceives it as the determination of the range of legitimate answers to a given question (without precluding answers that, to an extent, contradict each other).

Rescher's objection to syncretism in metaphilosophy stems from his belief that because of its readiness to embrace all different answers to a given question, it is bound to hold contradictory answers. But this, according to Rescher, is not rationally acceptable. It is tantamount to destroying the cognitive nature of philosophy, its aspiration to constitute a search for truth: *"To accept a plurality of answers is not to have answers at all; an unending openness to a variety of possibilities, a constant yes-and-no leaves us in perpetual ignorance"*. (Rescher, 1995. p. 350; cf p. 344.) The variety of metaphilosophical pluralism here outlined rejects this view. And, in order to understand why it does, we must now evaluate the validity of two logical arguments that have traditionally being used to uphold it.

According to the first, the inclusion of contradictory propositions within the same conceptual space is ruled out by the principle of contradiction. The second is a formal argument known since the Middle Ages (*ex absurdo sequitur quodlibet*), which in contemporary symbolic logic becomes the proof that from the simultaneous assertion of two contradictory sentences everything can be deduced. (Hilbert and Ackermann, 1928) During the 20th century, however, a growing body of formal developments called paraconsistent logic, which in the last decade became a leading topic in logico-mathematical research, has critically undermined this view.

Deductive logical systems can incorporate some contradictions. And they can be articulated without thereby necessarily causing the disruption of the inferential structure as a whole. A paraconsistent logical system can serve as the underlying logic that allows the formalization of a theory including some inconsistencies within its postulates (or, indeed, in the consequences derivable from them) without thereby trivializing itself. Although paraconsistent logical systems are as consistent as the classical ones, they can support some contradictions when they formalize a non-purely logical theory. A recent research line within paraconsistent logic that seeks the unification of mutually inconsistent theories (such as, paradigmatically, classical and quantum mechanics) is especially important for present purposes. The basic strategy of this research line comprises two steps. In the first, the aim is to formalize each theory presenting some extralogical postulates that can characterize them, using a logical system (which can be classical logic) as the underlying logic. In the second step, the aim is to articulate these deductive systems in a global theory that will contain their extralogical postulates, but using as underling logic a paraconsistent system. Although no attempt will be made here to formalize philosophical theories (we are, after all, merely sketching a research program the development of which would include it), this application of paraconsistent logic to the metatheoretical reflection about science suggests applying a similar strategy to metaphilosophy. The rejection of positions, such as Multilevel Pluralism (MLP) that are prepared to hold several answers to a question can no longer be sustained on logical grounds alone. Speaking about logical impossibilities requires specifying the logic we are talking about, for such impossibility does not hold for all deductive systems.

In this way, the logical objection to a metaphilosophical pluralism that is prepared to accept a diversity of answers at the individual level is completely dissolved. Pluralism, therefore, becomes equally acceptable both at the disciplinary and the individual level.

Paraconsistent logic was in some sense born of the realization that consistency, in its classical sense, was not a good enough criterions to discriminate between good and bad belief system, exactly because our actual reasoning is, it seems, much more able to cope with inconsistent premises than classical logic. Indeed, it has become a motto in many circles of non-classical logic that classical logic simply is not an accurate model of human rationality.

References

[1] Arruda A. I., Chuaqui R., da Costa N. C. A. (eds.) 1980. *Mathematical Logic in Latin America*. North-Holland Publishing Company, Amsterdam, New York, Oxford.

[2] Asenjo, F.G. 1966. A calculus of antinomies. *Notre Dame Journal of Formal Logic* 7. Pp 103-105.

[3] Avron, A. 2005. Combining classical logic, paraconsistency and relevance. *Journal of Applied Logic*. 3 (1). pp 133-160.

[4] Azriel. 1966. *The Explanation of the Ten Sefirot*. In Joseph Dan, *The Early Kabbalah,* texts trans. by Ronald C. Kieber. New York: Paulist Press.

[5] Belnap, N.D. 1977. How a computer should think. In *Contemporary aspects of philosophy*, ed. G. Ryle, 30-55. Oriel Press. Boston.

[6] Bueno, O. 1999. True, Quasi-true and paraconsistency. *Contemporary mathematics*. 39. pp 275-293.

[7] Bueno, O. 2010. Philosophy of Logic. In Fritz Allhoff. *Philosophies of the Sciences: A Guide*. John Wiley & Sons. p. 55.

[8] Carnielli, W.A., Coniglio and M. Lof D'ottaviano, I.M. 2002. *Paraconsistency: The Logical Way to the Inconsistent*. Marcel Dekker, Inc. New York.

[9] Carnielli, W. and Marcos, J. 2001. Ex contradictione non sequitur quodlibet. *Proc. 2nd Conf. on Reasoning and Logic* (Bucharest, July 2000).

[10] Cusanus, N. 1986. *Of Learned Ignorance*, Germain Heron, tr. Yale University Press. New Haven.

[11] Da Costa N. C. A., Wolf R. G. 1980 Studies in Paraconsistent Logic I: The Dialectical Principle of the Unity of Opposites. Philosophia, *Phil. Quaterly of Israel*, 9, 189-217.

[12] Derrida, J. 1967. *De la Grammatologie*. Paris: Minuit.

[13] Derrida, J. 1980. Difference. In *Writing and Difference*. University of Chicago Press.

[14] Dunn, J.M. 1976. Intuitive semantics for first-degree entailments and coupled trees. *Philosophical Studies* 29. pp 149-168.

[15] Hegel, G.W.F. *Wissenschaft der Logik*. Verlag von Felix Meiner. Leipzig. 1948. (In German)

[16] Hilbert, D. and Ackermann, W. 1928. *Grundzuege der Theoretischen Logik*. Berlin: Julius Spinger Verlag. (In German).

[17] Jung, C.G. 1955/56. *Mysterium Conjunctionis*. GW 14/CW 14.

[18] Kaplan, A. 1997. *Sefer Yetzirah: The Book of Creation*. Rev ed. York Beach, Maine: Samuel Weiser.

[19] Nescolarde-Selva, J. and Usó-Doménech, J. 2013[a]. Topological Structures of Complex Belief Systems. *Complexity*. 19 (1). pp. 46-62.

[20] Nescolarde-Selva, J. and Usó-Doménech, J. 2013[b]. Topological Structures of Complex Belief Systems (II): Textual materialization. *Complexity*. 19 (2). pp. 50-62.

[21] Nescolarde-Selva, J. and Usó-Domènech, J. L. 2014[a]. Semiotic vision of ideologies. *Foundations of Science*. 19 (3). pp. 263-282.

[22] Nescolarde-Selva, J. and Usó-Domènech, J. L. 2014[b]. Reality, Systems and Impure Systems. *Foundations of Science*. 19 (3). pp. 289-306.

[23] Nescolarde-Selva, J. and Usó-Doménech, J.L. 2014[c]. Myth, language and complex ideologies. *Complexity*. 20 (2), 63-81.

[24] Nescolarde-Selva, J.; Usó-Doménech, J.L.; Lloret-Climent, M. 2014. Mythical Systems: Mathematic and Logical Theory. *International Journal of General Systems.*

[25] Peña, L. 1985. *El ente y su ser: un estudio lógico-metafísico*. Servicio de Publicaciones de la Universidad de León. León. (In Spanish).

[26] Plato. 1973. *Parmenides*. In *The Republic and Other Works*, B. Jowett, tr. Doubleday. New York.

[27] Priest G. 1987. In Contradiction. A Study of the Transconsistent, Martinus Nijhoff Publishers, Dordrecht, Boston, Lancaster.

[28] Priest, G. 1995. *Beyond the Limits of Thought*. Cambridge University Press.

[29] Priest, G. 1998. *Dialetheism*. Stanford Encyclopedia of Philosophy.

[30] Priest G., Routley R., Norman, J. (eds.). 1989. *Paraconsistent Logic. Essays on the Inconsistent*. Philosophia Verlag, Munchen, Hamden, Wien.

[31] Priest G. and Woods, J. 2007. *Paraconsistency and Dialetheism. The Many Valued and Nonmonotonic Turn in Logic*. Elsevier.

[32] Quesada F.M. 1989. Paraconsistent Logic: Some Philosophical Issues. In: *Priest et al*., pp. 627-651.

[33] Quine, W.V. 1966. The Ways of Paradox. In *The Ways of Paradox and Other Essays.* Random House. New York., pp. 3-20.

[34] Rescher, N. 1995. *La lucha de los sistemas: un ensayo sobre los fundamentos e implicaciones de la diversidad filosófica.* México. UNAM. Instituto de Investigaciones Filosóficas (In Spanish)

[35] Scholem,G. 1974. *Kabbalah*. Jerusalem: Keter.

[36] G. Scholem. 1987. *Origins of the Kabbalah.* Trans. by R.J. Zwi Werblowski. Princeton: Princeton University Press.

[37] Stace, W.T. 1960. *Mysticism and Philosophy*. MacMillan Press. London.

[38] Stein, M. 2008. Divinity expresses the self… An investigation. *Journal of Analytical Psychology.* 53, 305-327.

[39] Tanaka, K., Berto, F. Mares, E. and Paoli, F. (Editors). 2013. *Paraconsistency: Logic and Applications.* Logic, Epistemology and Unity of Science, 26. Springer Dordrecht. Heidelberg, London, New York.

[40] Usó-Domènech, J.L. and Nescolarde-Selva, J.A. 2012. *Mathematic and Semiotic Theory of Ideological Systems. A systemic vision of the Beliefs.* Lap Lambert Academic Publishing. Saarbrücken. Germany.

[41] Usó-Doménech, J.L. and Nescolarde-Selva, J. 2014. Revealing the face of Isis. *Foundations of Science.* 19 (3). pp. 311-318.

[42] Usó-Domènech, J.L, Nescolarde-Selva, J.A. and Pérez-Gonzaga. 2014. Truth Values in t-norm based Systems Many-Valued Fuzzy Logic. *American journal of Systems and Software.* Vol 2. No 6. 139-145.

[43] Urban, F. 2005. Fordham, Jung and the self: a re-examination of Fordham's contribution to Jung's conceptualization of the self. *Journal of Analytical Psychology.* 50, 5, 571-594.

[44] *The Zohar.* Sperling and Simon.

ANNEX A

We will represent in the following table a comparison between three logics: classical (CL), quasi-paraconsistent (QPL) (Usó-Doménech, Nescolarde-Selva and Pérez-Gonzaga, 2014) and strong paraconsistent (SPL).

Table 4. Truth table of principal normal binary propositions

Notation	Name	CL truth values $p_1, p_2 \in \{0,1\}$	QPL truth values $p_1, p_2 \in [0,1]$	SPL truth values $p_1, p_2 \in [0,1]$
$P_1 \wedge P_2$	Conjunction	$p_1 p_2$	$p_1 p_2$	$\lvert p_1\rvert^2 \lvert p_2\rvert^2$
$\neg P_1 \vee \neg P_2$	Incompatibility	$1 - p_1 p_2$	$u_1 u_2 - p_1 p_2$	$\left\lvert e^{i(\varphi_1+\varphi_2)} - \lvert p_1\rvert\lvert p_2\rvert\right\rvert$
$P_1 \vee P_2$	Disjunction	$p_1 + p_2 - p_1 p_2$	$u_2 p_1 + u_1 p_2 - p_1 p_2$	$\left\lvert \lvert p_1\rvert e^{i\varphi_2} + \lvert p_2\rvert e^{\varphi_1} - \lvert p_1\rvert\lvert p_2\rvert\right\rvert^2$
$P_1 \Rightarrow P_2$	Implication	$1 - p_1 + p_1 p_2$	$u_1 u_2 - p_1(u_2 - p_2)$	$\left\lvert e^{i(\varphi_1+\varphi_2)} - \lvert p_1\rvert\left(e^{i\varphi_2} - \lvert p_2\rvert\right)\right\rvert^2$
$P_1 \Leftrightarrow P_2$	Concordance	$1 - p_1 - p_2 + 2p_1 p_2$	$u_1 u_2 - (u_2 p_1 + u_1 p_2) + 2p_1 p_2$	$\left\lvert e^{i(\varphi_1+\varphi_2)} - \lvert p_1\rvert e^{i\varphi_2} - \lvert p_2\rvert e^{i\varphi_1} + 2\lvert p_1\rvert\lvert p_2\rvert\right\rvert^2$
$P_1 \Updownarrow P_2$	Discordance	$p_1 + p_2 - 2p_1 p_2$	$u_2 p_1 + u_1 p_2 - 2p_1 p_2$	$\left\lvert \lvert p_1\rvert e^{i\varphi_2} + \lvert p_2\rvert e^{i\varphi_1} - 2\lvert p_1\rvert\lvert p_2\rvert\right\rvert^2$
$P_1 \Im P_2$	Complementarity	$p_1 + p_2$	$p_1 + p_2$	$\left\lvert \lvert p_1\rvert e^{i(\alpha_1-\alpha_2)} + \lvert p_2\rvert\right\rvert^2$
$\neg P_1 \Im \neg P_2$	Inverse complementarity	$2 - p_1 - p_2$	$p_1^* + p_2^*$	$\left\lvert \left(e^{i\varphi_1} - \lvert p_1\rvert\right) e^{i(\alpha_1-\alpha_2)} + e^{i\varphi_2} - \lvert p_2\rvert\right\rvert^2$
$P_2 \wp P_1$	Equivalence	$1 + p_1 - p_2$	$p_1 + u_2 - p_2$	$\left\lvert \lvert p_1\rvert e^{i(\alpha_1-\alpha\ )} + e^{i\varphi_2} - \lvert p_2\rvert\right\rvert^2$
$P_1 \wp P_2$	Inverse equivalence	$1 - p_1 + p_2$	$u_1 - p_1 + p_2$	$\left\lvert \lvert p_2\rvert e^{i(\alpha_1-\alpha\ )} + e^{i\varphi_1} - \lvert p_1\rvert\right\rvert^2$