

Economic Evaluation of Scale Dependent Technology Investments

Phillip J. Lederer • Tushar D. Mehta

William E. Simon Graduate School of Business Administration, University of Rochester, Rochester NY 14627, USA
 lederer@ssb.rochester.edu • tushar@tushar-mehta.com

We study the effect of financial risk on the economic evaluation of a project with capacity decisions. Capacity decisions have an important effect on the project's value through the up-front investment, the associated operating cost, and constraints on output. However, increased scale also affects the financial risk of the project through its effect on the operating leverage of the investment. Although it has long been recognized in the finance literature that operating leverage affects project risk, this result has not been incorporated in the operations management literature when evaluating projects.

We study the decision problem of a firm that must choose project scale. Future cash flow uncertainty is introduced by uncertain future market prices. The firm's capacity decision affects the firm's potential sales, its expected price for output, and its costs. We study the firm's profit maximizing scale decision using the CAPM model for risk adjustment.

Our results include that project risk, as measured by the required rate of return, is related to the inverse of the expected profit per unit sold. We also show that project risk is related to the scale choice. In contrast, in traditional discounted cash flow analysis (DCF), a fixed prescribed rate is used to evaluate the project and choose its scale. When a fixed rate is used with DCF, a manager will ignore the effect of scale on risk and choose suboptimal capacity that reduces project value. S/he will also misestimate project value.

Use of DCF for choosing scale is studied for two special cases. It is shown that if the manager is directed to use a prescribed discount rate that induces the optimal scale decision, then the manager will greatly undervalue the project. In contrast, if the discount rate is set to the risk of the optimally-scaled project, the manager will undersize the project by a small amount, and slightly undervalue the project with the economic impact of the error being small. These results underline the importance of understanding the source of financial risk in projects where risk is endogenous to the project design.

Key words: capacity planning; project risk; technology investments

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1. Introduction

This paper studies how scale decisions affect the financial risk and therefore the value of a project. For many years, finance scholars have recognized that cost structure affects the financial risk of a firm through its "operating leverage," which is defined as the ratio of variable profits (revenue minus variable costs) to operating profits (variable profits minus fixed operating costs). In markets where claims to these cash flows are traded, financial risk affects the riskiness of the firm's future cash flows and thus, the price investors are willing to pay for a claim to these cash flows. Using finance valuation models, such as the Capital Asset

Pricing Model (CAPM), analytic results about the relationship between operating leverage and project risk, and thus firm value, have been derived. Although this approach has been used to value firm-wide cash flows, it is also possible to use it in capital budgeting contexts to value projects of firms whose stocks are traded on financial markets.

However, use of financial risk adjustment models that incorporate the risks associated with operating leverage have rarely been applied to valuing and designing operations projects. A rare example of an operations paper that explores this issue is Lederer and Singhal (1988). In that paper, the authors show that

cost structure affects the risk of projects, but do not address the effect of project choice on revenues. In general, when scale (or other operations decisions that affect revenue or costs) changes, the repercussions include changes in the cost structure, the production volume, the operating leverage, the project cash flow, the riskiness of the project, and finally, present value of risk adjusted cash flow. If decisions are made that ignore the interaction of scale and risk, then cash flows are misvalued and suboptimal operations decisions are made.

The relationship between a firm's operating leverage and risk, as measured by CAPM's beta parameter, was analytically documented in Rubinstein (1973), Brenner and Schmidt (1978), and Gahlon and Gentry (1982). Lev (1974) and Mandelker and Rhee (1984) empirically showed that operating leverage computed as the ratio of total firm contribution divided by total firm profit is directly related to firm risk, confirming these results. However, all of these studies limit use of this theory to show total firm value is related to its risk through operating leverage, and do not apply this theory to projects and other firm investment decisions.

The interaction of scale and risk is usually ignored when using the most popular financial evaluation technique: discounted cash flow analysis (DCF). In DCF analysis, a fixed discount rate is typically applied to the cash flows. Because this rate is usually not changed when scale changes, the effect of scale on value is misestimated and generally, suboptimal scale decisions are made.

Although finance theory indicates that projects should be evaluated according to the rate specific to each project, there is considerable evidence that firms use one rate or, at most, a small set of discount rates in DCF computations. Chadwell-Hatfield et al. (1996) report that only 21% of the managers they surveyed used project-specific rates, while others reported that the discount rate varied between 5% and 25%, with one-third of the managers stating they used rates between 10% and 12%. These management-prescribed rates implicitly categorize projects into different risk groups based on firm criteria.

We show that a fixed discount rate for capacity decisions (or other decisions that affect project risk) can cause large errors in project scale choice and valuation. There are, of course, two special prescribed rates: one that induces the optimal capacity decision, the other that equals the rate for an optimally designed project. We show that even if the prescribed rate induces the optimal capacity decision, the project will generally be greatly undervalued. Alternatively, if the pre-set rate equals the discount rate for the optimally designed project, the project scale will be larger than optimal. Between these two, the latter results in a

smaller deviation from the optimal profit, and, consequently, is preferred as the lesser of two evils.

In this paper, we assume that the firm is publicly owned with shares that trade on a stock exchange. This assumption allows use of tools of modern finance theory to value risky cash flows. Thus, risk adjustment need not be based upon risk aversion of decision makers, but instead on risk adjustment of financial markets.

Our work is related to one component of project risk, namely non-diversifiable or stock market risk. As discussed in Lederer and Singhal (1988), there are three non-mutually exclusive sources of risk associated with investment in any capital project: implementation risk, industry risk, and business risk. Implementation risk is the risk associated with the time and cost to complete the project and uncertainty about operating costs and output rates for the project ex-post. Industry risk is that risk associated with future market prices and market shares within an industry. Business risk is the risk associated with the variation of project cash flows with the general state of the economy. We consider the scenario where scale can affect both the expected market price and the maximum sales rate. These, in turn, affect expected revenues and the riskiness of the cash flows with respect to the general economy. Thus, we consider business risk adjustment in capacity decisions and project valuation. A major result of modern finance theory is that investors will only be concerned about risk that cannot be diversified away by holding a portfolio of assets. In this paper, we assume that project and industry risk can be diversified away, and focus on the business risk.

Through a single-period problem of capacity investment, we highlight the critical issues in a way that would be masked by the complexity of a multi-period model, i.e., the risk associated with a project is unique to it and the use of a management-prescribed discount rate leads to suboptimal decisions. Our goal is to highlight a basic problem of capacity choice that exists independent of the number of periods considered.

Risk is injected into our one-period problem through price uncertainty. The model assumes that the firm knows the product demand curve and thus, the expected price received as a function of sales volume. However, there is some uncertainty in the demand function, and the firm must decide on how much capacity to install in the face of this uncertainty. We assume that the capacity sets the future production rate. This assumption is reasonable in many large capital process industries (such as paper and petrochemical refining). Van Miegham and Dada (1999) describe this firm strategy as *Price Postponement with Clearance*.

The assumption that capacity choice fixes the future production rate allows use of the Capital Asset Pricing

Model (CAPM) to adjust for risk. Allowing production rates to vary according to future market conditions requires use of more complex options pricing models whose valuation estimates cannot be presented in simple closed-form solutions as in the work by Mehta and Lederer (2002). That paper shows that the results we find here are general to industries where capacity merely caps output rates, but does not mandate output rates. Using an approach similar to that found here, multi-period analysis of capacity expansion can be modeled with options pricing, but the resulting analysis is complex and does not lead to *analytical* expressions of risk adjustment. However, numerical analysis of multi-period problems yields identical insights to those found in this paper. An empirical study that yields results consistent with the present paper is Hendricks and Singhal (2005) who study the long term consequences of a firm's announcement of a disruption in its supply chain. One of their results is that disruption causes firm operating risk to rise. If one interprets underinvestment as a leading cause of supply disruptions, this result agrees with our analysis.

Theory shows why insights from the single period model ought to match those of a multiperiod dynamic model. Constantinides (1978) showed that if returns on a future cash flow are a function of the value of an asset following a Wiener Process, $dx/x = \mu dt + \sigma dw$, and if the instantaneous covariance of the asset's value and the return on the stock market is $\text{Cov}(x, \tilde{R}_m)$, then the risk-adjusted returns for the cash flow can be obtained by modifying the Wiener Process by using the alternative process $dx'/x' = \mu' dt + \sigma dw$; where $\mu' = \mu - \lambda \text{Cov}(x, \tilde{R}_m)$. The risk adjusted stochastic process can be evaluated as a dynamic option pricing model to compute project value. But, the reduction in project value at any time t due to risk adjustment of the instantaneous return is just $[-\lambda \text{Cov}(x, \tilde{R}_m)(dV_t/dx)]/V_t$, where V_t is the project's value at t , and (dV_t/dx) is the derivative the project value at t with respect to the asset value. This reduction in project value due to risk is identical to that found in the static one-period model in this paper¹. How market risk adjustment interacts with the operating leverage of the instantaneous project is technically identical to our analysis in this paper. Thus, all of the issues of how cost and demand elements affect financial risk and project value also apply to more complex dynamic models.

Our work is related to at least two important literatures. First, problems with DCF analysis are outlined in several papers. Kaplan (1986) points out that relevant cash flows are sometimes ignored because they are hard to quantify. Similarly, Ayers and Miller (1981), Gerwin (1982), Gold (1982), Thompson and Paris (1982), Kaplan (1983), Jelinek and Goldhar

(1984), and Meredith (1987a;b) provide a discussion of the strategic benefits of capital projects that are often ignored. An extensive survey of evaluation issues about investments in technology is provided by Singhal et al. (1987). Our paper contributes to this literature by pointing out the missing relationship under DCF analysis between investment decisions (such as a plant where scale is chosen) and project risk.

Second, there is a considerable literature on capacity choice in risky situations. Others who have researched capacity choice and project risk adjustment include Dixit and Pindyck (1994) and Birge (2000). Dixit and Pindyck's Chapter 11 presents option pricing models that study incremental investment and irreversible capacity choices. In this multi-period model, they assume a fixed time-independent and exogenously specified discount rate. Alternatively, another procedure for risk adjustment is proposed. They suggest that if an efficient market exists where all assets are traded and valued, the future price of the risky cash flows can be determined. Thus, the problem of risk adjustment is addressed by using exogenous market prices for the risk. In a similar vein, Birge models capacity choice in a multi-period context. The model uses real option pricing with an exogenous and fixed profit margin together with the ability to hold a hedge in an efficient market to value capacity choice. Again, this research assumes that markets exist for risky assets (or, in this case, demand) and the markets determine the required rate of return on the project. In our paper, a firm makes a capacity decision in one period. Some papers in the literature consider learning as a way to reduce risk. For example, Miller and Park (2005) develop a two period model wherein the investment in period 2 is contingent on the firm making an investment in period 1. The initial investment creates an option that lets the firm learn more about investment opportunities. However, they do not consider how investments affect operating risk.

The rest of the paper is organized as follows. In the introduction to Section 2, we introduce the model and its notation, while in Section 2.1, we study how to adjust for project risk. The analysis of optimal project scale under risk is presented in Section 2.2, and in Section 2.3, we introduce decision-making with a prescribed discount rate. In Section 2.4, project risk when using prescribed discount rates is studied and in Section 2.5, we analyze project value estimation errors resulting from using prescribed rates that induce a manager to make the optimal scale decision. Concluding Section 2, in 2.6, we analyze the magnitude of errors resulting from using prescribed rates. Section 3 includes illustrative examples demonstrating some of our results, and Section 4 summarizes the results and suggests directions for new research.

¹ See equations (2-5) and (2-6) for the analog of this paper.

Table 1 Variables and Parameters in the Model

Variable	Meaning	
\tilde{p}	Uncertain price (\$/unit)	
p	Expected price (\$/unit)	
p_{ce}	Certainty equivalent of the uncertain unit price, \tilde{p}	
d	Production rate (units), also referred to as <i>scale</i> [Decision Variable]	
K	Capacity of the project (units per period) [Decision Variable]	
R	Required rate of return (%)	
\tilde{R}	Uncertain rate of return for the project (%)	
R^*	Discount rate when project is scaled to produce d^*	
R_p	Management mandated (or prescribed) rate; set <i>a priori</i> to project scale or production decisions	
$\tilde{\varepsilon}$	Uncertain error term in inverse demand function (\$)	
Parameter	Meaning	Comment
λ	Market price for risk (%)	$\frac{E\tilde{R}_m - R_f}{\sigma_m^2}$ Estimated to be 2.25
σ_m^2	Variance in market returns	Estimated to be 0.04
R_f	Risk free rate of return	Estimated to be 3%
\tilde{R}_m	Uncertain rate of market return	
ER_m	Expected one period stock market return	Estimated to be 12%
i_0, i_1	Fixed and per unit initial investment (\$, \$/unit)	
I	Total initial investment (\$)	
f_0, f_1	Fixed and per unit period cost (\$, \$/unit)	
F	Total period cost (\$)	
a_0, a_1	Parameters of inverse demand function	
c	Variable cost per unit (\$/unit)	
τ	Vector describing a technology in terms of its cost structure	
π	Net Present Value of project	
PV_{proj}	Present value of project related future cash flows	
d^*	Production quantity that maximizes project value	
π^*	Net present value when project is scaled to produce d^*	

2. The Model

We study the choice of capacity and evaluation of value of a production technology. Table 1 lists the variables and parameters we use in this paper.

Here are the assumptions we make in our model:

ASSUMPTION A1. The firm manufactures and sells a single product at a price that is a function of output. The inverse demand function that the firm faces is:

$$\tilde{p} = a_0 - a_1 d + \tilde{\varepsilon}$$

The uncertainty term $\tilde{\varepsilon}$ has a symmetrical, bell-shaped distribution with compact support and with mean zero and known variance. Further, a_0 is sufficiently large that $\tilde{p} > 0$.

ASSUMPTION A2. The project exists for a single period. At the beginning of the period the firm chooses its capacity. The price uncertainty is resolved at the end of the period. The firm liquidates the project at the end of the period. The salvage value of the technology is zero.

ASSUMPTION A3. The firm will choose an output rate equal to its production capacity.

ASSUMPTION A4. The cost structure for technology can be characterized by the parameters (i_0, i_1, f_0, f_1, c) .

ASSUMPTION A5. The initial investment is incurred at the beginning of the period. The fixed and variable

operating cost and the revenue are realized at the end of the period.

A6. The firm is an all equity firm where the equity holders contribute the initial investment.

A7. All taxes are zero.

The first assumption states that the firm produces a single product and faces an inverse demand function that has a random additive term. The inverse demand function is given by $p = a_0 - a_1 * d$, where p is the *expected* market clearing price, d is the quantity brought to the market, and a_0 and a_1 are positive constants. Higher production will reduce the expected price the firm receives. While the firm knows the expected price it will receive at the end of the period, the realized price is uncertain and market-driven. We model this relationship with a stochastic term: price is specified by $\tilde{p} = p + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is the uncertainty. Our assumption that $\tilde{p} > 0$ allows the use of the CAPM model². Positivity of price is realistic and consistent with observation that goods have positive market prices. The second assumption specifies the timing of

² If \tilde{p} could be negative, a more complex options model is required for risk adjustment. While the model becomes more complex, the essential results remain the same.

events and resolution of uncertainties. The next assumption fixes output at the production capacity, i.e., $d = K$. This assumption is explained in the introduction.

The fourth assumption characterizes the cost of capacity in terms of the initial investment and period-fixed operating costs. The timings of cash flows are done for convenience. Although broader definitions of technology are possible, here we define a technology by the five elements of the cost structure: $\tau = (i_0, i_1, f_0, f_1, c)$. The first two components are the installation costs: a fixed cost, i_0 , that is strictly a function of the selected technology, and a per-installed-unit cost, i_1 . Given the firm's decision on capacity, it will incur a total installation cost of $I = i_0 + i_1d$. The third and fourth components are 'per-period' costs: a fixed charge, f_0 , and a constant installed-unit cost, f_1 . Consequently, it will incur a total period cost of $F = f_0 + f_1d$. The fifth component of the cost structure is the variable production cost, c . We assume it is constant, independent of quantity produced.

The fifth assumption specifies the timing of cash flows so that they can be valued. The sixth assumption allows use of the CAPM to adjust for risk and uncertainty. The final assumption frees us from issues of taxes and tax shields due to depreciation.

These assumptions can be generalized. A firm can be allowed to produce multiple products, but the demand functions must be specified. The inverse demand function can be generalized to a non-linear function. The disturbance term can be non-normally distributed, but must remain symmetrical for CAPM to be used to evaluate risk. Under more general distributions, option pricing models could be used to adjust for risk.

Although we assume the project exists for a single period, this may be generalized. We assume that capacity is chosen first, and it cannot be increased or decreased. Models which allow capacity to be adjusted over time can be constructed, but that is not the aim of this paper. We are concerned with the risk associated with inflexible investments. Production rates could be allowed to be adjusted to maximize profits and thus, differ from capacity. Both can be done using an options model as has been done in Mehta and Lederer (2002). The cost structure assumed is very general. It is actually far more general than most capital budgeting models.

Issues of debt financing could be introduced and capital structure studied. However, we do not study these issues here. Taxes could be introduced, and the cash flow consequences of depreciation tax shields and tax payments could easily be incorporated into the model. Issues of bankruptcy would complicate the model. See Lederer and Singhal (1994) for models incorporating taxes and bankruptcy considerations.

2.1. Project Risk Adjustment and Discount Rates

For a given technology, $\tau = (i_0, i_1, f_0, f_1, c)$ and scale decision, d , the expected net present value of the project is

$$\pi = E \left[\frac{(\tilde{p} - c)d - (f_0 + f_1d)}{(1 + R)} \right] - i_0 - i_1d \quad (2-1)$$

where R is the risk adjusted discount rate (derived later in this section). The discount rate will be shown to be a function of d .

For fixed production and scale decisions, the expected value of the project can be computed as follows. Initial investments i_0 and i_1d are certain. Future cash flows are uncertain and CAPM can value them³. According to the *certainty equivalent* form of CAPM, the present value of uncertain cash flows occurring at the end of period 1 is

$$PV_{proj} = \frac{(p - c)d - (f_0 + f_1d) - \lambda d \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{(1 + R_f)}$$

Thus the *net* present value of the project is

$$\pi = \frac{(p - c)d - (f_0 + f_1d) - \lambda d \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{(1 + R_f)} - i_0 - i_1d. \quad (2-2)$$

It will be useful to define the certainty equivalent of the unit price as the price adjusted for future risk, allowing for a simpler representation of profit:

$$p_{ce} = p - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m) \quad (2-3)$$

and

$$\pi = \frac{(p_{ce} - c)d - (f_0 + f_1d)}{(1 + R_f)} - i_0 - i_1d.$$

Both (2-1), using the appropriate risk-adjusted discount rate, R , and (2-2), using the risk-free rate, R_f , correctly value the project, i.e., they yield the same result. We can derive the discount rate R by equating these two expressions, but choose an alternate method to develop intuition about the risk adjustment process.

Consider the project's uncertain end-of-period cash flow denoted by $C\tilde{F} = (\tilde{p} - c)d - (f_0 + f_1d)$. Viewing this future cash flow as an asset, the uncertain return of this cash flow is

$$\begin{aligned} \tilde{R} &= \frac{C\tilde{F}}{PV(C\tilde{F})} \\ &= \frac{(\tilde{p} - c)d - (f_0 + f_1d)}{\frac{(p - c)d - (f_0 + f_1d) - \lambda d \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{(1 + R_f)}}. \end{aligned} \quad (2-4)$$

³ See Brealey and Myers (1991) for discussion of CAPM.

The uncertain return is an inverse function of the risk-adjusted end-of-period cash flow divided by the number of units sold, $[(p - c)d - (f_0 + f_1d) - \lambda d \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)]/d$. More simply described, the uncertain return is also an inverse function of the per unit expected operating profit which is often called the “operating margin”. The lower either of these ratios, the higher the risk. In particular, an increase in fixed period cost will cause higher risk. Thus project risk depends directly on all the cost parameters for non-initial costs, the chosen demand rate and the expected price. In addition, the presence of d in (2-4) means that the risk also depends, albeit indirectly, on the per-unit installation cost, i_1 .

We next compute the risk adjusted return for (2-4). By the Capital Asset Pricing Model (CAPM), the risk-adjusted return on an investment is related to the return on the market portfolio, R_m , and R_f by

$$R_{proj} = R_f + (E\tilde{R}_m - R_f)\beta, \quad (2-5)$$

where β measures the risk of the uncertain cash flows. Using the definition of β (Brealey and Myers, 1991), $\beta_{proj} = (1/\sigma_m^2) \text{cov}(\tilde{R}, \tilde{R}_m)$, and (2-4), we find

$$\beta_{proj} = \frac{(1 + R_f) \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{\sigma_m^2((p_{ce} - c)d - (f_0 + f_1d))/d}. \quad (2-6)$$

Equation (2-6) indicates that project risk (and thus the proper risk-adjusted discount rate) to use in (2-1) is proportional to the inverse of the operating profit per unit sold. The operating profit per unit sold considers revenues and period costs but excludes the upfront investment. The equation can be factored and rewritten in a more familiar way, relating to operating leverage:

$$\beta_{proj} = \frac{(1 + R_f) \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)d}{\sigma_m^2(p_{ce} - c)d} \frac{(p_{ce} - c)d}{(p_{ce} - c)d - (f_0 + f_1d)}.$$

The last fraction is often referred to as “operating leverage.” All things being equal, higher operating leverage implies higher risk of an investment. The first term can be interpreted as a measure of the variability of contribution: the “contribution beta”. For example, if the contribution beta is equal to 0.5, and a broad stock market index were to rise 20%, then the firm’s contribution would be forecast to rise $0.5 * 20\% = 10\%$. Thus the riskiness of the investment is a function of the variability of the contribution times the operating leverage.

The firm’s problem is to choose scale, d , to maximize the net present value of the project, i.e., $\max_d \pi = [(p_{ce} - c)d - (f_0 + f_1d)]/[1 + R_f] - i_0 - i_1d$. We next study this decision.

2.2. Optimum Scale and Related Measures

The firm’s profit maximizing scale choice, d^* , satisfies $\text{Arg}_d [\partial \pi(d)]/\partial d = 0$ or

$$d^* = \frac{a_0 - (c + f_1) - i_1(1 + R_f) - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{2a_1}. \quad (2-7)$$

The strict concavity of the profit function ensures us that the optimal scale maximizes profit.

The certainty equivalent of price at the optimal scale is $p_{ce}(d^*) = a_0 - a_1 * d^* - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$, and the profit, π^* that corresponds to the optimal scale of d^* is $\pi^* = \pi(d^*) = \{[(p_{ce}(d^*) - c)d^* - (f_0 + f_1d^*)]/[1 + R_f]\} - i_0 - i_1d^*$. Substituting for $p_{ce}(d^*)$ and simplifying yields

$$\pi(d^*) = \frac{1}{(1 + R_f)} \{a_1d^{*2} - f_0 - i_0(1 + R_f)\}. \quad (2-8)$$

It follows immediately that for the firm to earn a positive profit, a necessary condition is $d^* \geq \sqrt{[f_0 + i_0(1 + R_f)]/a_1}$. The inequality defines the breakeven production quantity and simply states that the revenues from producing d^* must exceed the fixed project costs. In turn, it leads to the weaker condition:

$$d^* \geq \sqrt{\frac{f_0}{a_1}}. \quad (2-9)$$

This inequality states that the firm must produce a quantity so that its revenues are sufficient to cover the fixed period cost.

By (2-8) the optimum profit is a quadratic function of the demand parameters, period cost parameters, and the risk free rate. The discount rate corresponding to this optimum profit is found by substituting (2-7) into (2-6) and (2-5):

$$R^* = R_f + \frac{(1 + R_f)\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{\left[\frac{a_0 - c - f_1 + i_1(1 + R_f) - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{2} \right] - \left[\frac{2a_1f_0}{a_0 - c - f_1 - i_1(1 + R_f) - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)} \right]} \quad (2-10)$$

From (2-10), demand and technology parameters affect project risk. By differentiation, the appropriate risk-adjusted discount rate is a

- decreasing function in a_0 ;
- increasing function of a_1 , and f_0 ;
- increasing function of f_1 , c and $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$ as long as operating profit per unit is positive;
- decreasing function of i_1 as long as operating profit per unit is positive, and
- independent of i_0 .

Except for the fourth bullet, all of these results are intuitive: when operating costs rise, then risk rises,

and when market size falls, or when demand elasticity rises, then risk rises as well. But the result about i_1 , the scale dependent initial investment cost, is not intuitive. The result is explained by realizing that when the scale dependent initial investment cost rises, the project scale falls and project risk falls with scale. From (2-10), as long as the firm earns a positive profit, $(\partial R^*/\partial d^*) = \{[(1 + R_p)\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)] / [(a_0 - a_1 d^* - c - f_1 - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m) - (f_0/d^{*2}))](a_1 - (f_0/d^{*2}))\} > 0$. Since d^* is monotonically decreasing in i_1 , $(dR^*/di_1) = \{(\partial R^*/\partial d^*)(\partial d^*/\partial i_1)\} < 0$. This is an interesting contrast to the partial derivatives for f_1 , c and $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$: an increase in any of these parameters causes scale to fall but the margins fall enough to increase risk.

These results lead to some interesting insights. Consider two alternative technologies to serve a market, with cost structures $\tau_1 = (i_0, i_1, f_0, f_1, c)$ and $\tau'_1 = (i'_0, i'_1, f'_0, f'_1, c')$, respectively, with $(f'_0, f'_1, c') \geq (f_0, f_1, c)$ and $i'_1 \leq i_1$. Then, the first alternative, τ_1 has a lower risk when it is optimally scaled than the latter, and should be evaluated at a lower discount rate. Of course, alternative technologies may also affect risk through favorable effects on demand through improvements in lead time, quality, or customer service, and not just cost. If two alternative technologies affect market demand such that $a'_0 \leq a_0$ and $(a'_1, \text{cov}(\tilde{\varepsilon}', \tilde{R}_m)) \geq (a_1, \text{cov}(\tilde{\varepsilon}, \tilde{R}_m))$, then the first alternative project properly scaled has a lower risk than the latter and should be evaluated at a lower discount rate.

We conclude that technology parameters (i_1, f_0, f_1, c) , and demand parameters (a_0, a_1) affect project risk, and should be considered in any decision on project scale.

2.3. DCF Analysis Using A Management Prescribed Discount Rate

Typically, in DCF analysis a firm *prescribes* the discount rate to be used by a manager. This discount rate, also called a *hurdle rate*, is set *a priori* of scale decisions and is used by the manager for the economic evaluation of the project. In practice, we know of no procedure that is actually used to adjust for scale. However, (2-6) clearly indicates that project risk is a function of the project scale. Effectively, when the hurdle rate is established before the project scale is set, the use of such a prescribed discount rate becomes a “second best” procedure. In much of the remainder of Section 2, we explore the effect of this second best procedure.

In the rest of this section, we look at Δd^* , the error in production scale, and $\Delta \pi^*$, the reduction in profit, resulting from the use of a fixed discount rate. Suppose a manager is told to choose scale and evaluate a project given a *prescribed rate* R_p . The estimated project value profitability, based upon the prescribed rate, is

$$\pi_p = \frac{(a_0 - a_1 d - c)d - f_0 - f_1 d}{(1 + R_p)} - i_0 - i_1 d. \quad (2-11)$$

It is an estimate of profitability since the risk adjustment is not necessarily correct. If a manager uses this measure of expected profitability, the profit maximizing scale solves $\pi_p^* = \max_d \pi_p$. Differentiating and solving for the zero results in

$$d_p(R_p) = \frac{a_0 - c - f_1 - i_1 * (1 + R_p)}{2 * a_1}. \quad (2-12)$$

The estimate of optimal profit can be written as $\pi_p^*(R_p) = \{(a_1 [d_p(R_p)]^2 - f_0 - i_0(1 + R_p)) / (1 + R_p)\}$. Since there is a one-to-one correspondence between the prescribed rate and the optimal production quantity, the estimated project profit can be written indifferently as either $\pi_p(R_p)$ or $\pi_p(d_p)$.

Note that the optimal production scale is a monotonically decreasing function in the prescribed rate. If the firm uses a fixed rate prescribed by management, R_p , it will fail to account for how scale affects risk. It will choose scale $d_p(R_p)$ rather than the quantity d^* in (2-7). The error in scale is

$$\Delta d^* = \frac{i_1 * (R_p - R_f) - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{2 * a_1}. \quad (2-13)$$

Actual profit, given the firm selects the quantity that maximizes $\pi_p(R_p)$, is based upon that quantity applied to equation (2-11)

$$\begin{aligned} & \frac{1}{4a_1} [a_0 - (c + f_1) - i_1(1 + R_p)]^2 \\ & + \frac{1}{4a_1} [2i_1(R_p - R_f) - 2\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)] \\ & * [a_0 - (c + f_1) - i_1(1 + R_p)] - f_0 \\ \pi(d_p(R_p)) = & -i_0 + \frac{ - f_0}{(1 + R_p)}. \end{aligned}$$

The firm loses an amount given by $\Delta \pi^* = \pi(d^*) - \pi(d_p(R_p))$, yielding the expression

$$\Delta \pi^* = \frac{1}{4a_1(1 + R_p)} [i_1(R_p - R_f) - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)]^2. \quad (2-14)$$

Use of a prescribed discount rate, R_p , leads to a non-optimal scale, $d_p(R_p)$, being chosen that, in turn, leads to a reduction in profit given by (2-16). If R_p is too large, then the chosen production level is too small; the converse also holds. Figure 1 shows the scale choices relative to the optimum as a function of the prescribed rate. For values of R_p below the zero-deviation value, the firm over-produces, while for higher discount rates, it under-produces. The zero variation value corresponds to the prescribed rate that induces the manager to choose the optimal scale. We refer to that prescribed rate as R_p^* , and discuss it further in Section 2.5.

Figure 1 Deviation from optimal production quantity as a function of the prescribed discount rate; the under(over) production is linear in the prescribed rate. $R_p^* = R_f + (\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m))/i_1$.

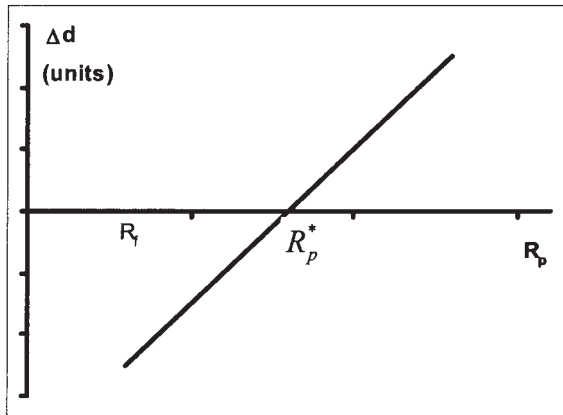


Figure 2 Deviation from optimal profit as a function of the prescribed discount rate; the deviation is quadratic in the prescribed rate and symmetric around R_p^* .

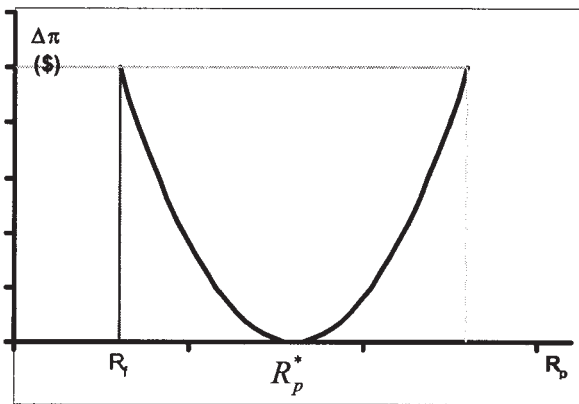


Figure 2 plots the loss in net present value with R_p . The loss function is a quadratic in R_p about R_p^* .

Use of a prescribed discount rate can potentially cause large errors in the scale decision that reduces firm profit. Examining (2-16), the magnitude of the opportunity loss in profit is a quadratic function of the scale dependent initial cost, i_1 . When this parameter is zero, there are no losses because the firm can choose scale to be arbitrarily large. But in realistic situations which always have a scale dependent upfront cost, losses can be large. We next study how scale decisions made using a prescribed discount rate affect the project risk. In subsequent sections, we explore the effect of a prescribed rate on project evaluation and project scale from the manager’s perspective.

2.4. The Project Risk Induced by a Prescribed Discount Rate

If a prescribed rate is used to choose scale, then project scale will be set as (2-13) and the actual project risk is $R(R_p) = R_f + \{[(1 + R_p)\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)]/[p_{ce}(d_p) - c - f_1 - (f_0/d_p)]\}$, or substituting for p_{ce} and d_p at optimality,

$$R(R_p) = R_f + \frac{(1 + R_f)\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{\left[\frac{a_0 - c - f_1 + i_1(1 + R_p)}{2} - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m) \right] \frac{2a_1f_0}{a_0 - c - f_1 - i_1(1 + R_p)}} \tag{2-15}$$

Risk under the prescribed rate is affected by the cost and demand parameters just as R^* was, as described in Section 2.2. In particular, risk is a decreasing function of i_1 as long as operating profit per unit is positive for the same reason as before: larger scale dependent investment cost causes the project to be small, reducing fixed cost.

Figure 3 plots function (2-17), $R(R_p)$, which is the actual risk induced by the prescribed rate, R_p . The figure also reports the identity function $R(R_p) = R_p$. The two vertical asymptotes are the solutions to the quadratic formed by setting the denominator of (2-17) to zero. The denominator can be interpreted as the risk-adjusted operating profit per unit. The asymptotes then correspond to where the project breaks even ignoring the initial investment. Thus, within the asymptotes, the firm’s risk-adjusted period operating profit is positive. The risk function and the identity function intersect at two points, R_p', R_p'' . At these two prescribed rates, project risk will equal the prescribed rates.

But, within the interval $[R_p', R_p'']$, the project’s actual risk is lower than the prescribed rate. This implies that DCF analysis for a project that uses a prescribed rate in this interval will undervalue the project as the real risk is lower than the risk applied to choose scale. The opposite is also true: if the prescribed rate is chosen outside of the interval then the manager will overvalue the project. Choosing a prescribed rate at the boundary points, R_p', R_p'' , will cause the manager to choose a suboptimal scale, but will properly estimate the value of the project.

Setting any prescribed rate aside from a specific one that induces the optimal scale will cause the project to be suboptimally scaled. In the next section, we study the prescribed rate that induces the optimal scale.

2.5. Inducing Managers to Choose Optimal Scale Causes Managers to Undervalue Projects

By (2-16) there is a prescribed rate, R_p^* , that will cause the project to be scaled correctly:

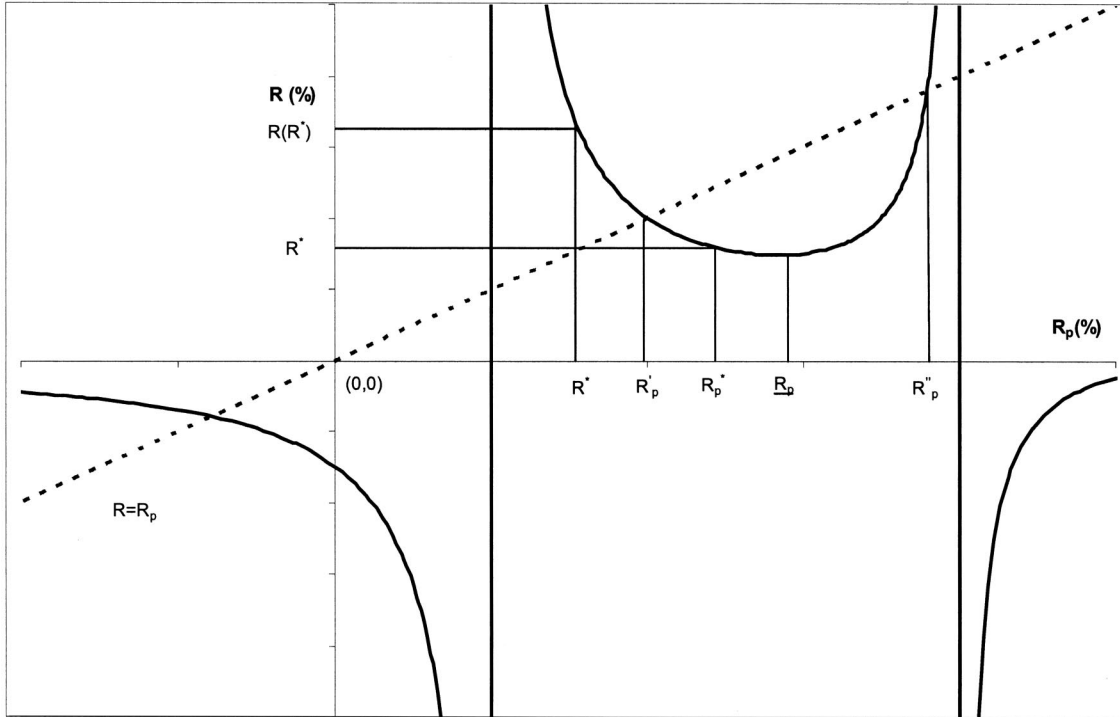
$$R_p^* = R_f + \frac{\lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{i_1} \tag{2-16}$$

Direct computation of $R(R_p^*)$ yields the following result.

PROPERTY 1.0. Use of a prescribed discount rate, R_p^* , that induces the correct scale leads the manager’s estimate to strictly undervalue the project, i.e., $R(R_p^*) < R_p^*$.

Proof. See the Appendix. □

Figure 3 Scale-dependent risk, R , as a function of management prescribed rate, R_p , as defined by (2-17). The risk-adjusted operating profit is positive between the bolded vertical asymptotes. Between R'_p and R''_p , the scale-dependent discount rate is less than the prescribed rate; R^* is the profit maximizing scale-adjusted discount rate; \underline{R}_p is the risk minimizing prescribed rate, and R_p^* is the rate that induces the correct production decision.



Thus, if a manager is given the discount rate that induces optimal scale, his estimate will undervalue the project as R_p^* lies in the interval $[R'_p, R''_p]$. When the DCF analysis is used to decide whether the project is “go” or “no go,” this undervaluation of the estimate may cause the project to be rejected when it is a positive NPV project.

Equation (2-14) directly implies that opportunity losses that occur with arbitrary prescribed rates can be arbitrarily large. We next study the losses that occur when the project discount rate at the optimal scale is chosen as the prescribed rate. We know that such a choice will lead to a suboptimal capacity choice. In the next section we show that the opportunity loss of that choice will generally be small. To explore this issue we will need the next technical result about where R^* lies on R_p domain. We show that it is to the left of the minimum point of $R(R_p)$ on the interval $[R'_p, R''_p]$. This property is useful in understanding the effect of using the discount rate of the optimally designed project as the prescribed rate.

PROPERTY 1.1. Suppose the project is profitable at the optimal scale. Consider Figure 3. $R_p^* \in [R'_p, R''_p]$ where $\underline{R}_p = \text{Arg min}_{R_p \in [R'_p, R''_p]} R(R_p)$.

Proof. See the Appendix. \square

The next section studies the magnitude and sign of errors made when prescribed discount rates are used.

2.6. Magnitude of Errors When Using Prescribed Discount Rate to Make Decisions

We have established that the use of a prescribed discount rate leaves a firm with apparently unpleasant choices. The prescribed rate will either cause the manager’s estimate to undervalue the project, or induce a suboptimal scale, or both. This is the inevitable result of choosing an inflexible discount rate before setting the project scale. Given that an error of one sort or another is unavoidable, we next ask: when are errors using prescribed discount rates large? We explore two situations. They are:

- i. The project manager is told to use the prescribed rate that will induce the optimal scale decision.
- ii. The manager is told to use the prescribed rate equal to the risk-adjusted rate for the optimally scaled project.

We restrict the analysis to these two instances only since any *other* choice of a discount rate will lead to a decision where the firm will scale the project suboptimally, earn a suboptimal profit, and usually estimate the project risk incorrectly (note that at points R' , R'' , the project risk is estimated correctly). Indeed, Figure 2 shows that the opportunity loss due to suboptimal scale choice can be arbitrarily large.

A reasonable reader might well ask the question of how the firm decides on the discount rate that corre-

sponds to cases (i) and (ii) above. The question for case (i) is analytically answered in Section 2.3 above. The rate that corresponds to case (ii) could be obtained analytically but also could be empirically derived from observing the risk of an earlier project that had been scaled correctly or derived from information about an industry rival. The analysis of this section, as well as the other sections, relies on one basic tenet of DCF analysis: with a higher discount rate the discounted value of future cash flows will be lower. In our case, $R(R_p) < R_p \Rightarrow \pi(R_p) > \pi_p(R_p)$, of course, the inverse is also true.

In the first case, although the optimal production decision is made, the manager's DCF analysis will value the project below its true economic worth. In this case, an important question is: when is the difference between the actual and estimated DCFs large? By underestimating the true value of the project, the manager is more likely to decide against undertaking a project that would actually be in the firm's best interest! This can be significant since we show that the manager's error tends to be large.

In the second case, a manager makes a suboptimal production quantity decision and also evaluates the project at a suboptimal discount rate. Two questions arise: does the suboptimal decision cost the firm much? Does the manager undervalue the project as designed by much?

We can show that in general, the losses of the first case are large, and the losses due to suboptimal decisions of the second case are small, and net present value misestimation is also small. Thus, it is generally better to give a manager a prescribed rate set equal to the real project risk (and accept a suboptimal decision) rather than give the manager a rate to induce optimal production.

In the first case, $R_p = R_p^*$ and the difference between real and estimated net present value is

$$\pi^* - \pi_p(d_p^*) = \frac{\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m)(a_1 d^{*2} - f_0)}{i_1(1 + R_f) \left(1 + R_f + \frac{\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{i_1}\right)},$$

where $d_p^* = d_p(R_p^*)$. This follows from (2-8) and (2-11). As a fraction of actual profit, the manager underestimates project value by

$$\frac{\pi^* - \pi_p(d_p^*)}{\pi^*} = \frac{\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m)(a_1 d^{*2} - f_0)}{i_1 \left(1 + R_f + \frac{\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m)}{i_1}\right) (a_1 d^{*2} - f_0 - i_0(1 + R_f))}.$$

Note that this fraction is large, often close to 1. Projects are undervalued and economic losses arise because managers reject profitable projects.

On the other hand, using $R_p = R^*$ as the prescribed rate induces a fall in actual project value:

$$\pi^* - \pi(R^*) = \frac{(1 + R_f)(\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m))^2}{4a_1} \times \left(1 - \frac{i_1 d^{*2}}{a_1 d^{*2} - f_0 + i_1(1 + R_f)d^{*2}}\right)^2.$$

This follows from replacing R_p in (2-16) by R^* . In fractional terms:

$$\frac{\pi^* - \pi(R^*)}{\pi^*} = \frac{(1 + R_f)(\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m))^2}{4a_1} \times \left(1 - \frac{i_1 d^{*2}}{a_1 d^{*2} - f_0 + i_1(1 + R_f)d^{*2}}\right)^2 \frac{1}{a_1 d^{*2} - f_0 + i_0(1 + R_f)}.$$

This *real* loss is small when $1/[a_1 d^{*2} - f_0 + i_0(1 + R_f)]$ is small. When $a_1 d^{*2} - f_0 > 0$, per period operating profit is positive. Therefore, real fractional errors can be large only when operating profits and the initial fixed investment are both small. Managers do not make economically significant errors when using the optimal project risk.

Using R^* as the prescribed rate, the manager's estimate overvalues the project (compared to the optimally scaled one):

$$\pi^* - \pi_p(R^*) = - \frac{i_1^2 (\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m))^2 (1 + R_f) d^{*2}}{4a_1^3 \left(d^{*2} - \frac{f_0}{a_1}\right)^2}.$$

This is computed as follows: π^* is computed from (2-8), and $\pi_p(R^*)$ from (2-10).

As a fraction of optimal profit, the error is:

$$\frac{\pi^* - \pi_p(R^*)}{\pi^*} = - \frac{i_1 (\lambda \operatorname{cov}(\tilde{\varepsilon}, \tilde{R}_m)) (1 + R_f)^2 d^{*2}}{\left(d^{*2} - \frac{i_0(1 + R_f) + f_0}{a_1}\right) \left(d^{*2} + \frac{(1 + R_f)d^{*2} - f_0}{a_1}\right)}.$$

Thus, the fractional error behaves approximately as $1/(d^{*2} - (f_0/a_1))^3$, which is small as long as $d^* - (f_0/a_1)$ is sufficiently large.

Finally, we can deduce that if R^* is set as the prescribed rate then the manager will choose too large a scale and his estimate of project value will overvalue the project *at the chosen scale*. It should be noted that while the manager overvalues the project, in reality it will earn less than what it would have earned if it had used R_p^* . The argument is as follows. Property 1.1 showed that function R rises to the left of R_p^* and this implies that $R(R_p') > R^* = R(R_p^*)$. Looking at Figure 3 shows that this implies that $R_p' > R^*$. By (2-13), $d_p(R^*) > d_p(R_p^*) = d^*$. At this scale, the manager's estimate will overvalue the project (i.e., $\pi(R^*) < \pi_p(R^*)$) because to the left of R_p' , $R(R_p) > R_p$. See Figure 3.

3. Examples

Having studied the relationship between the performance metrics, the decision variables and the model parameters, we present some examples based upon the estimates of financial parameters recorded in Table 1. We show first that project risk varies according to relationships found in this paper as described in Section 2.2, and differences in project risk can be large. We then demonstrate the errors in scale choice and evaluation that occur when prescribed discount rates are used.

The values of the parameters of the demand function are based upon an empirical study of a local manufacturer of prototype lenses. In what we call the base case, the intercept of the inverse demand function, a_0 , is 200. The slope of the function, a_1 , is 0.85. The covariance of the error term and the market return is 5, i.e., $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 5$. The cost structure of the base case is $\tau = (i_0 = 0, i_1 = 15, f_0 = 1,000, f_1 = 10, c = 1)$.

First, we compare the risk of the base case with the risk of three different technologies, each of which is incrementally different from the base case (for a total of four technologies). Each of the three new technologies differs from the base case by a single parameter: $i_1 = 20, f_0 = 100$, or $f_1 = 1$, respectively. We study the effect of these four technologies for three different demand functions—characterized by a_0 values of 100, 200, and 500, respectively.

Second, we study the effect of the correlation between the product price and market return for each of these 12 combinations. We do so through the covariance term, $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$, with values of 1, 2, 5, and 10.

Table 2 presents the risk-adjusted discount rate for the 48 possible combinations. All of the results are intuitively understood by equation (2-4) linking riskiness of returns to the operating margin of the project. Compared to the base case for each demand function and covariance (the three columns labeled ‘Base’ in Table 2), as the fixed period costs f_0 and f_1 decrease (the columns labeled $f_1 = 1$ and $f_0 = 100$), the expected operating margins increase. Consequently, by (2-4), the risk associated with the project decreases, and the project discount rate decreases. These cost parameters

can produce a large effect: if $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 2$ and $a_0 = 100$, then a shift of f_1 from 1 to 10 causes the discount rate to fall from 21.3% to 12.3%.

If the firm earns a positive profit, an increase in i_1 leads to a decrease in risk, a result that is consistent with Section 2.2. The one exception is when the covariance is 10 and the demand function intercept is 100. In this case, the firm earns a negative profit in the base case itself, and an increase in i_1 leads, as expected, to an increase in project risk.

As the demand function intercept increases from $a_0 = 100$ to $a_0 = 500$, the influence of the error term (ε) on the price decreases. As this uncertainty decreases, the risk of the project decreases. Going from left to right in the table below, the risk of the same technology decreases. For example, if $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 2$ then the risk of the base case falls as a_0 increases from 21.3% ($a_0 = 100$), to 8.2% ($a_0 = 200$) to 4.88% ($a_0 = 500$).

As the covariance term increases, the volatility increases and the firm compensates with a lower certainty-equivalent of price, $p_{ce} = p - \lambda \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$. As this risk-adjusted expected price decreases, expected future revenues decline, as do the expected unit margin and the expected operating margin. The result is increased risk. For the case, $a_0 = 100$, using base cost parameters, the risk rises with $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$: 11.5% (for $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 1$), 21.3% (for $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 2$), 63.0% (for $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 5$), and 305.0% (for $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m) = 10$).

A final effect worth noting is the increasing rate-of-change relative to the covariance term. The risk of a project is inversely proportional to the risk-adjusted operating margin. A key adjustment is the reduction in price by the $-\lambda \cdot d \cdot \text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$ term. Consequently, as the variability increases through the covariance term, the operating margin approaches zero and the risk increases sharply.

We now turn our attention to two cases in which a manager uses a prescribed rate for the scale decision and valuation. In the first case (Table 3), the prescribed rate is set equal to the optimal scale-dependent discount rate. When a manager treats this rate as a fixed, scale-independent rate, the result is an increase in the

Table 2 Project Risk in Percent as a Function of Technology Parameters Under Different Demand Parameters

R	$a_0 = 100$				$a_0 = 200$				$a_0 = 500$			
	Base	$f_1 = 1$	$f_0 = 100$	$i_1 = 20$	Base	$f_1 = 1$	$f_0 = 100$	$i_1 = 20$	Base	$f_1 = 1$	$f_0 = 100$	$i_1 = 20$
1	11.5%	9.7%	7.8%	11.3%	5.5%	5.4%	5.3%	5.5%	3.94%	3.92%	3.92%	3.93%
2	21.3%	17.2%	12.8%	20.9%	8.2%	7.9%	7.7%	8.0%	4.88%	4.85%	4.86%	4.86%
5	63.0%	45.5%	29.4%	62.6%	16.5%	15.7%	15.1%	16.1%	7.77%	7.68%	7.71%	7.72%
10	305.0%	138.0%	64.6%	358.8%	32.1%	30.3%	28.8%	31.3%	12.77%	12.59%	12.63%	12.67%

Base case definition is $a_1 = 0.85, f_1 = 10, f_0 = 1,000, i_0 = 0$, and $i_1 = 15$.

Note that project risk varies greatly in the cases: from 3.92% to 385.8%. For any particular combination of a_0 and demand uncertainty, $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$, a decrease in the fixed operating cost f_0 , or in the per-unit operating cost f_1 , decreases project risk. A non-intuitive result is that for a profitable project an increase in the per-unit installation cost i_1 , decreases project risk. An increase in product demand (an increase in a_0) decreases project risk and an increase in demand uncertainty, $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$, increases project risk.

Table 3 If the Prescribed Discount Rate is Equal to the Actual Risk of the Project at the Profit Maximizing Scale, a Manager will Undervalue the Project and Choose Scale Larger than Optimal

cov(ε , R_m)	Scale dependent rate				Analysis with pre-set rate				
	(a) Optimal quantity (d^*)	(b) Optimum profit (π^*)	(c) Discount rate at optimality (R^*)	(d) Optimal quantity (d_p^*)	(e) Perceived profit (π_p^*)	(f) Profit given pre-set rate ($\pi(d_p^*)$)	(g) Overproduction error ($d_p^* - d^*$)	(h) Overestimate in profit ($\pi_p^* - \pi(d_p^*)$)	(i) Loss from optimal profit ($\pi(d_p^*) - \pi^*$)
1	100.76	2,158	5.54%	101.86	2,159	2,157	1.09%	0.10%	-0.06%
2	99.44	1,940	8.15%	101.63	1,943	1,936	2.20%	0.38%	-0.18%
5	95.47	1,301	16.45%	100.90	1,322	1,277	5.69%	3.56%	-1.84%
10	88.85	294	32.07%	99.52	368	200	12.01%	83.78%	-32.04%

This example shows that generally, the errors in project valuation are small, as is the loss due to suboptimal scale decisions unless Cov(ε , R_m) is large. In this example, $a_0 = 200$, $a_1 = 0.85$, $f_0 = 1,000$, $f_1 = 10$, $i_0 = 5,250$, $i_1 = 15$. The table records the optimal scale (column a), profit (column b) and the risk-adjusted discount rate at these decisions (column c). It also reports the quantity decision if the discount rate in column c is used to set scale (column d) and the actual project value given the suboptimal scale decision (column e). Column g reports the size of errors in production, column h reports overestimation of profit, and column i reports the loss due to suboptimal scale choice, all expressed in percentage of the respective optimal decisions. Errors made in the last three columns rise as Cov(ε , R_m) increases.

project scale. Consequently, the firm chooses a scale larger than optimal and overvalues the project. In addition, the firm ignores the volatility of the demand function in its analysis, and both effects are exaggerated as the covariance increases. For example, when $cov(\varepsilon, \tilde{R}_m) = 5$, the manager will choose a scale 5.69% too large, estimate profits as being 3.56% too high, and actually reduce project value by 1.84% compared to optimal-scale decisions. All of these effects increase in value to 12.01%, 83.78% and -32.04%, respectively, when $cov(\varepsilon, \tilde{R}_m) = 10$.

In the second case (see Table 4), the prescribed rate is set to induce a manager to choose the optimal scale. Asked to use such a prescribed rate, a manager greatly undervalues the project although the scale decision is optimal. In this example, even if demand variability is its smallest value ($cov(\varepsilon, \tilde{R}_m) = 1$), the undervaluation error exceeds 43%! This error only increases with increasing demand variability.

4. Summary

This paper studied the effect of operating leverage on operations decisions. We studied this problem in the context of choice of scale, where scale affects initial investment and fixed operating costs. We also allowed the firm to have market power, which means that a firm's scale decision affects the units sold as well as market price. We showed that these decisions affect operating leverage, which in turn affects the risk adjustment for the project.

Using the CAPM, we computed the risk-adjusted value of projects, and the risk-adjusted profit maximizing scale. We also explicitly showed the direct effect of project scale on risk. Among the results is that project risk is inversely proportional to the expected operating margin.

Our paper highlights an important deficiency of the usual application of discounted cash flow analysis, where the discount rate used is most often held to be scale (and other important operations decisions) independent. We showed that using DCF in this way can cause serious errors in project design and economic evaluation.

Within the specific model we analyzed, a firm that sets a discount rate prior to deciding the project scale will generally choose suboptimal scale and incur a loss that is quadratic in its deviation from the optimal discount rate. Even if it chooses a rate that induces the correct capacity decision (with a fixed discount rate), the firm will undervalue the project. Project undervaluation can cause serious economic losses through failure to adopt value-creating investments.

We did show that if the decision maker somehow uses a discount rate that corresponds to the true risk of

Table 4 If the Prescribed Rate is Set to Induce Optimal Scale Decisions, a Manager will Greatly Undervalue the Project

$\text{cov}(\varepsilon, \tilde{R}_m)$	(a) Optimal quantity (d^*)	(b) Optimum profit (π^*)	(c) Discount rate at optimality (R^*)	(d) Rate to ensure optimal decision R_p ($d_p^* = d^*$)	(e) Profit given pre-set rate (π_p^*)	(f) Loss from optimal profit ($\pi^* - \pi_p^*$)
1	100.76	2,158	5.5%	18.0%	1,217	-43.6%
2	99.44	1,940	8.2%	33.0%	318	-83.6%
5	95.47	1,301	16.5%	78.0%	-1,459	-212.2%
10	88.85	294	32.1%	153.0%	-2,993	-1117.0%

The degree of undervaluation is generally large and increases with the $\text{Cov}(\varepsilon, \tilde{R}_m)$. In this example $a_0 = 200$, $a_1 = 0.85$, $f_0 = 1,000$, $f_1 = 10$, $i_0 = 5,250$, $i_1 = 15$. Column a reports the optimal scale, column b reports the profit and column c reports the risk-adjusted discount rate. Column d reports the prescribed discount rate that induces a manager to make optimal decisions. Column e reports the manager’s estimate of profit, and column f reports the size of the manager’s estimating error as a percentage of actual optimal profits.

the optimally designed project, both design and evaluation error will tend to be small. Interestingly, with this proper discount rate, the economic impact of scale choice error will be small, as will be the underestimation of the true value of the project at the chosen scale. This showed that effective use of DCF is critically dependent on the proper choice of discount rates because of the effect they have on managerial decision making about the design and adoption of investments.

There are many possible extensions to this research. There are a host of other important operations decisions that affect the operating leverage of projects with future payoffs. Operating leverage can be affected by cost as well as revenue dependencies of projects. Thus profit maximizing design of any such project type can be reexamined. In this paper we studied projects where operations decisions are made once and are irreversible. The findings in this paper can be extended to include a price structure that is sensitive to the time needed to deliver customized products (see Mehta and Lederer 2002a). The findings in this paper can also be applied in the context of financial evaluation of dynamically changing projects. This can be done using options pricing models and the ideas of Constantinides (1978) to exploit the relationship between operating leverage and project risk. A beginning in this effort is shown in Mehta and Lederer (2002).

5. Appendix-Proofs

PROPERTY 1.0. Use of a prescribed discount rate, R_p^* , that induces the correct scale leads to a strict undervaluation of the project, i.e., $R(R_p^*) < R_p^*$.

Proof. From (2-17) and (2-3)

$$R(R_p^*) - R_p^* = -\lambda \text{cov}(\varepsilon, \tilde{R}_m) \left[\frac{p_{ce} - c - f_1 - i_1(1 + R_f) - \frac{f_0}{d_p^*}}{i_1 \left(p_{ce} - c - f_1 - \frac{f_0}{d_p^*} \right)} \right]$$

The numerator is the DCF divided by the scale of the optimally designed project, and the denominator $p_{ce} - c - f_1 - (f_0/d_p)$ is the average risk adjusted operating profit. Both will be positive if the project at a scale of d_p has a positive DCF. Consequently, $R(R_p^*) < R_p^*$. QED. □

PROPERTY 1.1. Suppose the project is profitable at the optimal scale. Consider Figure 3. $R_p^* \in [R', R_p]$ where $\underline{R}_p = \text{Arg min}_{R_p \in [R', R_p^*]} R(R_p)$.

Proof. R_p^* lies in the interval $[R_p', R_p'']$ if the project is profitable. We next show that the risk minimizing prescribed rate (denoted by \underline{R}_p) is larger than R_p^* . The minimum of the project risk occurs at a value of \underline{R}_p which induces scale decision d_p that is the solution to

$$\begin{aligned} \frac{\partial \beta}{\partial d_p} &= \frac{(1 + R_f) \text{Cov}(\varepsilon, \tilde{R}_m)}{\sigma_m^2} \\ &\quad \times \frac{(a_0 - a_1 d - c - f_1)d - f_0 - d(a_0 - 2a_1 d - c - f_1)}{[(a_0 - a_1 d - c - f_1)d - f_0]^2}, \\ &= \frac{(1 + R_f) \text{Cov}(\varepsilon, \tilde{R}_m)}{\sigma_m^2} \frac{a_1 d^2 - f_0}{[(a_0 - a_1 d - c - f_1)d - f_0]^2} \end{aligned}$$

Thus, $d_p = \sqrt{(f_0/a_1)}$, and we can conclude that $d^* > d_p$ by (2-9), which implies, by (2-13), that $\underline{R}_p > R_p^*$. QED. □

References

Ayers, R. U., S. Miller. 1981. Robotics, CAM, and industrial productivity. *National Productivity Review* 1 42–60.

Birge, J. R. 2000. Option Models for Incorporating Risk into Linear Capacity Planning Models. *Manufacturing & Service Operations Management* 2(1) 19–31.

Brealey, R. A., S. C. Myers. 1991. Principles of Corporate Finance, 4th edition. McGraw-Hill, Inc., 199–200.

Brenner, M., S. Schmidt. 1978. Asset characteristics and systematic risk. *Financial Management* 7 33–39.

Chadwell-Hatfield, P., B. Goitein, P. Horvarth, A. Webster. 1996. Financial criteria, capital budgeting techniques, and risk analysis of manufacturing firms. *Journal of Applied Business Research* 13(1) 95–104.

- Constantinides, G. M. 1978. Market risk adjustment in project valuation. *Journal of Finance* 33 603–616.
- Dixit, A. K., R. S. Pindyck. 1994. *Investment under uncertainty*. Princeton University Press. ISBN 0-691-03410-9.
- Gahlon, J. M., J. A. Gentry. 1982. On the relationship between systematic risk and the degree of operating and financial leverage. *Financial Management* 11 15–23.
- Gerwin, D. 1982. Do's and don'ts of computerized manufacturing. *Harvard Business Review* 60(2) 107–116.
- Gold, B. 1982. CAM sets new rules for production. *Harvard Business Review* 60(6) 88–94.
- Hendricks, K. B., Singhal, V. R. 2005. An empirical analysis of the effect of supply chain disruptions on long-run stock price and equity risk of the firm. *Production and Operations Management* 14(1) 35–52.
- Jelinek, M., J. D. Goldhar. 1984. The strategic implications of the factory of the future. *Sloan Management Review* 25(4) 29–37.
- Kaplan, R. S. 1983. Measuring manufacturing performance: A new challenge for managerial accounting research. *The Accounting Review* 68 686–705.
- Kaplan, R. S. 1986. Must CIM be justified by faith alone? *Harvard Business Review* 64(2) 87–95.
- Lederer, P. J., V. R. Singhal. 1988. Effect of Cost Structure and Demand Risk in the Justification of New Technologies, *Journal of Manufacturing and Operations Management* 1 339–371.
- Lederer, P. J., V. R. Singhal. 1994. The effect of financing decisions on the choice of manufacturing technologies, with V. R. Singhal. *International Journal of Flexible Manufacturing Systems* 6.
- Lev, B. 1974. On the association between operating leverage and risk. *Journal of Financial and Quantitative Analysis* 9 627–641.
- Mandelker, Gershon N., S. Ghon Rhee. 1984. The Impact of Operations and Financial Leverage on Systematic Risk of Common Stock. *Journal of Financial and Quantitative Analysis* 19 45–57.
- Mehta, T. D., P. J. Lederer. 2002. Technology choice with risk and dynamic capacity. *Working Paper Simon School of Business, University of Rochester*.
- Mehta, T. D., P. J. Lederer. 2002a. Time competition, risk, and capacity choices. *Working Paper Simon School of Business, University of Rochester*.
- Meredith, J. R. 1987a. The strategic advantages of the factory of the future. *California Management Review* 29(3) 27–41.
- Meredith, J. R. 1987b. The strategic advantages of new manufacturing technologies for small firms. *Strategic Management Journal* 8 249–258.
- Miller, L. T., C. S. Park. 2005. A learning real options framework with application to process design and capacity planning. *Production and Operations Management* 14(1) 5–20.
- Rubinstein, M. E. 1973. A mean variance synthesis of corporate financial theory. *Journal of Finance* 28 267–182.
- Singhal, Kalyan, Fine, C. H., Meredith, J. R., Suri, R. 1987. Research and models for automated manufacturing. *Interfaces*. November–December 1987, 5–14.
- Thompson, H., M. Paris. 1982. The changing face of manufacturing technology. *Journal of Business Strategy* 2 45–52.
- Van Miegham, J., M. Dada. 1999. Price versus production postponement: Capacity and competition. *Management Science* 45(12) 1631–1649.

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