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Transient Thermal Stress Analysis of a Plate with Double-Sided FGM Coatings

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Abstract. In this paper, the transient heat conduction and transient thermal stresses in an infinite plate with double-sided functionally graded coatings (FGM coatings) under the convective boundary condition are investigated. The thermo-physical properties of the two symmetrical FGM coatings are assumed to have distributions of power forms along the thickness direction of the plate, the effects of which on the thermal shock resistance of the FGM coated plate are analyzed via numerical calculations. And consequently some design rules for the double-sided FGM coatings are put forward, which provide a guidance for the development of FGM coated cutting tools.

Introduction

Coating technology gained a growing importance in cutting tool applications. Coated tools have a considerably higher edge life compared to uncoated ones and may produce a higher quality of the machined surface due to reduced friction [1]. As the improvements to the monolayer coatings which are susceptible to thermal residual stresses induced failures such as coating delamination and cracking, multilayer and gradient coatings are increasingly applied. Several researchers investigated the effects of varying architecture and compositional gradient on the thermal shock resistance of FGM coatings experimentally [2] or analytically [3]. However, the transient temperature field and transient thermal stresses of a plate with double-sided FGM coatings under the convective boundary condition have not been previously studied, which are investigated in the present paper via an analytical approach and may direct us to design FGM coated cutting tools with high thermal shock resistance.

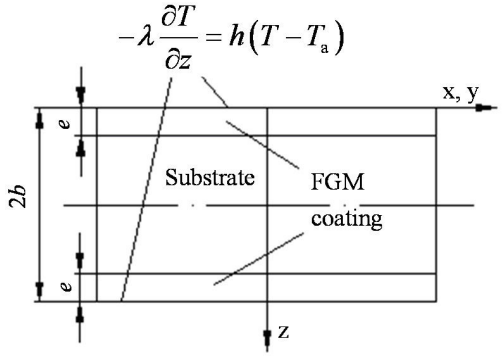

Fig. 1 Plate with double-sided FGM coatings

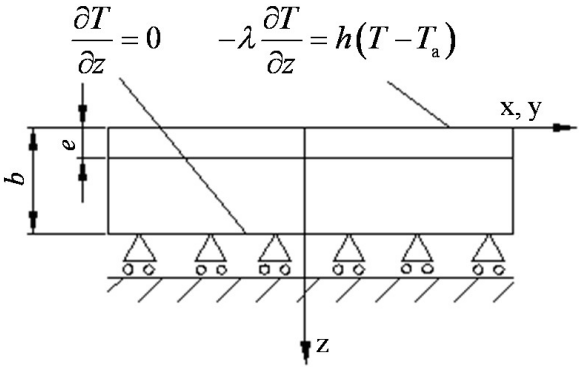

Fig. 2 Simplified model

Analytical Models

Transient temperature field. As shown in Fig. 1, consider an infinite plate with double-sided FGM coatings. It is assumed that initially the plate is at an uniform temperature T_0, and at time t=0 the coating surfaces (at z=0 and z=2b) are suddenly exposed to a convective medium of temperature T_a. The material properties are continuous across the interface between the coating and the substrate. Because of the symmetries for both plate structure and thermal loading, the plate can be simplified into the model shown in Fig. 2 with only the upper half being studied. Then the following one-dimensional transient heat conduction equations govern the temperature field in the plate

$$c(z)\rho(z)\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\left[\lambda(z)\frac{\partial T}{\partial z}\right],\quad 0\le z\le e;\quad c_s\rho_s\frac{\partial T}{\partial t}=\lambda_s\frac{\partial^2 T}{\partial z^2},\quad e\le z\le b. \tag{1}$$

where T is temperature, t is time, $c(z)$, $\rho(z)$ and $\lambda(z)$ are the specific heat, the mass density and the thermal conductivity of the coating respectively, and c_s, ρ_s and λ_s are the corresponding properties of the substrate.

The initial and boundary conditions for the temperature field are given as

$$T(z,0)=T_0, 0\le z\le b;\quad -\lambda\left.\frac{\partial T}{\partial z}\right|_{z=0}=h(T-T_a),\quad \left.\frac{\partial T}{\partial z}\right|_{z=b}=0. \tag{2}$$

where h is the heat transfer coefficient of the coating surface (z=0).

The temperature and the flux satisfy the continuity conditions at the interface between the coating and the substrate. For the purpose of simplicity, the following dimensionless variables are introduced

$$\left.\begin{aligned}
&U=\frac{T-T_0}{T_a-T_0},\quad \bar{z}=z/b,\quad \tau=\frac{\kappa(0)}{b^2}t,\\
&\bar{c}(\bar{z})=c(z)/c(0),\quad \bar{c}_s=c_s/c(0),\\
&\bar{\rho}(\bar{z})=\rho(z)/\rho(0),\quad \bar{\rho}_s=\rho_s/\rho(0),\\
&\bar{\lambda}(\bar{z})=\lambda(z)/\lambda(0),\quad \bar{\lambda}_s=\lambda_s/\lambda(0).
\end{aligned}\right\} \tag{3}$$

where $c(0)$, $\rho(0)$, $\lambda(0)$ and $\kappa(0)$ are the specific heat, the mass density, the thermal conductivity and the thermal diffusivity of the coating surface.

The closed-form solutions to the temperature fields of the plate with double-sided FGM coatings are very difficult to derive because of the variable coefficients in the partial differential equation, whereas the asymptotic solutions to the dimensionless temperature fields of the coating and the substrate can be obtained respectively as follows by reference to the algorithm of Jin [3].

$$\begin{aligned}
U(\bar{z},\tau)=&\frac{B}{A(\tau)+C(\tau)D(\tau)}\left\{\left[\bar{\rho}(\bar{z})\bar{c}(\bar{z})\bar{\lambda}(\bar{z})\right]^{-1/4}erfc\left[\frac{1}{2\sqrt{\tau}}\int_0^{\bar{z}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}\right]\right.\\
&\left.+C(\tau)\left[\frac{\bar{\rho}_s\bar{c}_s\bar{\lambda}_s}{\bar{\rho}(\bar{z})\bar{c}(\bar{z})\bar{\lambda}(\bar{z})}\right]^{1/4}erfc\left[\frac{1}{2\sqrt{\tau}}\left(\int_{\bar{z}}^{\bar{e}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}+(1-\bar{e})(\bar{\kappa}_s)^{-1/2}\right)\right]\right\},\quad 0\le\bar{z}\le\bar{e}.
\end{aligned} \tag{4a}$$

$$\begin{aligned}
U(\bar{z},\tau)=&\frac{B}{A(\tau)+C(\tau)D(\tau)}\left\{\left[\bar{\rho}_s\bar{c}_s\bar{\lambda}_s\right]^{-1/4}erfc\left[\frac{1}{2\sqrt{\tau}}\left(\int_0^{\bar{z}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}+(\bar{z}-\bar{e})(\bar{\kappa}_s)^{-1/2}\right)\right]\right.\\
&\left.+C(\tau)erfc\left[\frac{1}{2\sqrt{\tau}}\left((1-\bar{z})(\bar{\kappa}_s)^{-1/2}\right)\right]\right\},\quad \bar{e}\le\bar{z}\le 1.
\end{aligned} \tag{4b}$$

where B=$hb/\lambda(0)$ is the Biot number of the coating surface. $A(\tau)$, $C(\tau)$ and $D(\tau)$ are given by

$$A(\tau)=B-\frac{1}{4}\cdot\left.\frac{\mathrm{d}\left[\bar{\rho}(\bar{z})\bar{c}(\bar{z})\bar{\lambda}(\bar{z})\right]}{\mathrm{d}\bar{z}}\right|_{\bar{z}=0}-\frac{1}{\sqrt{\pi\tau}}. \tag{5}$$

$$C(\tau)=\left(\bar{\rho}_s\bar{c}_s\bar{\lambda}_s\right)^{-1/4}\exp\left[-\frac{1}{4\tau}\left(\int_0^{\bar{e}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}+(1-\bar{e})(\bar{\kappa}_s)^{-1/2}\right)^2\right]. \tag{6}$$

$$\begin{aligned}
D(\tau)=&\left\{B-\frac{1}{4}\cdot\left.\frac{\mathrm{d}\left[\bar{\rho}(\bar{z})\bar{c}(\bar{z})\bar{\lambda}(\bar{z})\right]}{\mathrm{d}\bar{z}}\right|_{\bar{z}=0}\right\}\left[\bar{\rho}_s\bar{c}_s\bar{\lambda}_s\right]^{1/4}erfc\left[\frac{1}{2\sqrt{\tau}}\left(\int_0^{\bar{e}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}+(1-\bar{e})(\bar{\kappa}_s)^{-1/2}\right)\right]\\
&+\frac{1}{\sqrt{\pi\tau}}\left[\bar{\rho}_s\bar{c}_s\bar{\lambda}_s\right]^{1/4}\exp\left[-\frac{1}{4\tau}\left(\int_0^{\bar{e}}\left(\bar{\kappa}(\bar{z})\right)^{-1/2}\mathrm{d}\bar{z}+(1-\bar{e})(\bar{\kappa}_s)^{-1/2}\right)^2\right].
\end{aligned} \tag{7}$$

Transient thermal stress field. The transient thermal stress of the coated plate can be calculated

by using the generalized Hooke's law as

$$\sigma(\bar{z},\tau)=\frac{E(\bar{z})\Delta T}{1-\nu(\bar{z})}\left[\psi(\tau)-\alpha(\bar{z})U(\bar{z},\tau)\right],\qquad 0\le\bar{z}\le 1 \tag{8}$$

where E, ν and α are Young's modulus, Poisson's ratio and thermal expansion coefficient respectively. ΔT is the temperature difference. The variable $\psi(\tau)$ is calculated as

$$\psi(\tau)=\frac{\int_0^{\bar{e}}E^*(\bar{z})\alpha(\bar{z})U(\bar{z},\tau)\,\mathrm{d}\bar{z}+E_s^*\alpha_s\int_{\bar{e}}^1 U(\bar{z},\tau)\,\mathrm{d}\bar{z}}{\int_0^{\bar{e}}E^*(\bar{z})\,\mathrm{d}\bar{z}+(1-\bar{e})E_s^*},\qquad E^*=E/(1-\nu). \tag{9}$$

The following dimensionless thermal stress is introduced for the universality

$$\bar{\sigma}(\bar{z},\tau)=\frac{\sigma(\bar{z},\tau)\left[1-\nu(0)\right]}{\alpha(0)E(0)\Delta T} \tag{10}$$

where $E(0)$, $\nu(0)$ and $\alpha(0)$ are the Young's modulus, Poisson's ratio and thermal expansion coefficient of the coating surface respectively.

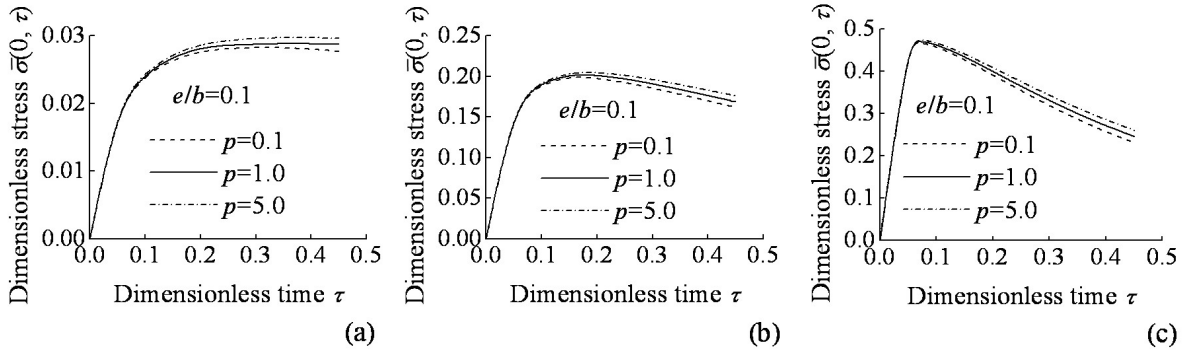

Fig. 3 Dimensionless transient thermal stresses as a function of dimensionless time under different Biot numbers: (a) B=0.1, (b) B=1.0 and (c) B=5.0

Numerical Results and Discussions

Compositional distributions of the coated plate. A plate of TiC-Al_2O_3 FGM coating/Al_2O_3 substrate/Al_2O_3-TiC FGM coating system is considered. The thermo-physical properties of the materials are ρ=4.93g/cm^3, λ=20W/(m·K), c=0.56J/(g·K), α=7.6×10^{-6}/K, E=440GPa, ν=0.19 for TiC, and ρ=3.99g/cm^3, λ=30W/(m·K), c=0.77J/(g·K), α=8.5×10^{-6}/K, E=380GPa, ν=0.19 for Al_2O_3. The volume fraction of Al_2O_3 in the FGM coating is assumed to follow a power function

$$V(\bar{z})=(\bar{z}/\bar{e})^p,\qquad 0\le\bar{z}\le\bar{e} \tag{11}$$

where p is the power exponent. The thermo-physical properties c, ρ, λ, E, ν and α of the FGM coating are obtained by using the well-established theories [4], and then fitted into power forms along the thickness direction of the plate via the least-square method.

Transient thermal stress calculations. The dimensionless transient thermal stresses $\bar{\sigma}(0,\tau)$ at the coating surface (e/b=0.1) for the volume fraction profiles p=0.1, 1.0 and 5.0 (see Eq. (11)) under different Biot numbers (B=0.1, 1.0 and 5.0) in cold shock are plotted against dimensionless time τ in Fig. 3a-c, respectively. For a given value of p under a given Biot number, the thermal stress $\bar{\sigma}(0,\tau)$ increases with time, reaches a maximum value $\bar{\sigma}_{\max}(0,\tau_c)$ at a particular time τ_c and then decreases with further increase of time.

Data of $\bar{\sigma}_{\max}(0,\tau_c)$ and τ_c for the three values of p under different Biot numbers are listed in Table 1, from which it can be seen that the maximum thermal stress $\bar{\sigma}_{\max}(0,\tau_c)$ decreases with decreasing

values of both p and B. Whereas the value of τ_c decreases with decreasing value of p and increasing value of B. Because p=5.0 approximately models a homogeneous TiC coating on a Al_2O_3 substrate, the graded TiC-Al_2O_3 coating with a lower value of p has lower $\bar{\sigma}_{max}(0,\tau_c)$ and hence higher thermal fracture resistance.

It can be concluded form the above results that both the thermal conductivity λ and the thermal expansion coefficient α of the FGM coating surface (e.g. TiC) should be lower than that of the substrate (e.g. Al_2O_3). While the Young's modulus E of the FGM coating surface should be higher than that of the substrate. And furthermore the power exponent p should be assigned a lower value. These rules provide an approach to the design of FGM coated cutting tools with high thermal shock resistance.

Table 1 Data of the maximum dimensionless thermal stresses and their times of occurrence

B	p=0.1		p=1.0		p=5.0	
	τ_c	$\bar{\sigma}_{max}(0,\tau_c)$	τ_c	$\bar{\sigma}_{max}(0,\tau_c)$	τ_c	$\bar{\sigma}_{max}(0,\tau_c)$
0.1	0.3045	0.0283	0.3427	0.0288	0.3809	0.0297
1.0	0.1658	0.1990	0.1747	0.2016	0.1836	0.2049
5.0	0.0668	0.4685	0.0690	0.4717	0.0713	0.4749

Summary

The transient heat conduction and thermal stresses in an infinite plate with double-sided FGM coatings under the convective boundary condition are investigated in this paper. The effects of the thermo-physical property distributions of the coatings on the thermal shock resistance of the plate are analyzed via numerical calculations, from which some design rules for the FGM coated cutting tools with high thermal shock resistance are put forward.

Acknowledgements

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