7 MDO Architectures

Multidisciplinary design and optimization (MDO) is a field of growing recognition in both academic and industrial circles. Multidisciplinary design by itself is by no means a recent insight: the Wright brothers realized that they needed to simultaneously consider multiple disciplines in order to successfully design a powered airplane. Aircraft are prime examples of multidisciplinary systems where the interaction between the different disciplines is extremely important.

In the last few decades have numerical techniques that predict the performance of engineering systems have been developed and are now mostly mature areas of research. Numerical optimization made use of these techniques to further automate the design process by automatically searching the space of possible designs and by providing a mathematical definition of what constitutes an optimum design.

While single-discipline optimization is in some cases quite mature, the design and optimization of systems that involve more than one discipline is still in its infancy. Although there have been hundreds of research papers written on the subject, MDO has still not lived up to its full potential.

In the opinion of some MDO researchers, industry won't adopt MDO because they don't realize their utility. Some engineers in industry think that researchers are making a big deal out of a concept that they have always used in their work.

There is some truth to each of these prospectives. Real-world aerospace design problem may

involve thousands of variables and hundreds of analyses and engineers. This kind of problem has so far been of a much larger scale than what has been studied by researchers.

An overview of the structure of an aircraft company shows that its organization involves a number of different levels — a hierarchy — as well as complex coupling between the divisions.

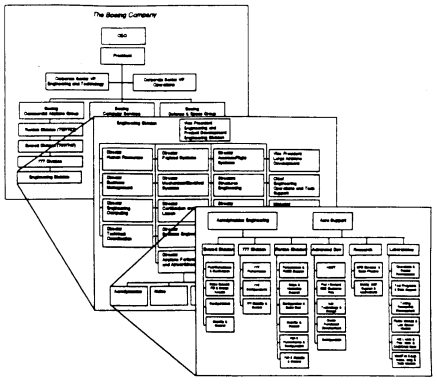


Figure 1: Organization of an aircraft company.

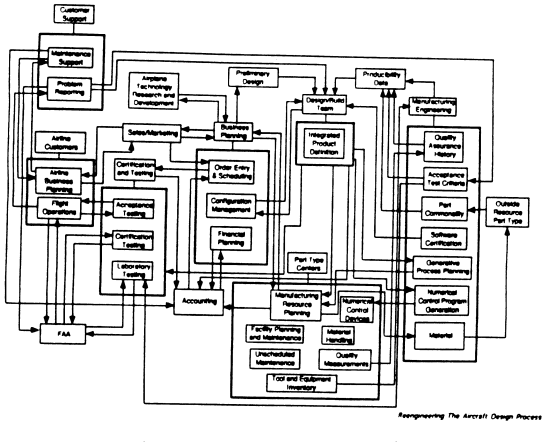


Figure 2: Flow chart for an aircraft design procedure.

MDO is concerned with the development of strategies that utilize current numerical analyses and optimization techniques to enable the automation of the design process of a multidisciplinary system. One of the big challenges is to make sure that such a strategy is scalable and that it will work in realistic problems.

Traditionally, designers have resorted to a series of parametric studies to make design decisions. This involves plotting a figure of merit or constraint versus one or three design parameters. This studies are limited because of the inherent difficulty of visualizing data that has more than three dimensions. In addition, the computational cost of such studies is proportional to p^n where p is the number of points evaluated in each direction and n is the number of design variables.

Why MDO?

- Parametric trade studies are subject to the "curse of dimensionality".
- Iterated procedures for which convergence is not guaranteed.
- Sequential optimization that does not lead to the true optimum of the system

Objectives of MDO:

- Avoid difficulties associated with sequential design or partial optimization.
- Provide more efficient and robust convergence than by simple iteration.
- Aid in the management of the design process.

Difficulties of MDO:

- Communication and translation.
- Time.
- Scheduling and planning.

Nomenclature

In addition to the usual nomenclature we have used thus far, such as f for the objective function and x for the design variables, and c for the vector of constraints, we will use a two more sets of variables.

We make the distinction between *local* design variables, i.e., variables that only affect one discipline and *global* or *shared* design variables z, which are variables that affect more than one discipline. We also need to consider the variables that are exchanged between disciplines (the coupling variables), which we will denote by y. both x and y are discipline specific and we will use the subscript i to denote the discipline.

The coupling variables for the i^{th} system can be written as

$$y_i = y_i\left(x_i, y_j, z\right), \tag{1}$$

where j is the set of indices that corresponds to the disciplines that the i^{th} discipline depends on and therefore $j \neq i$.

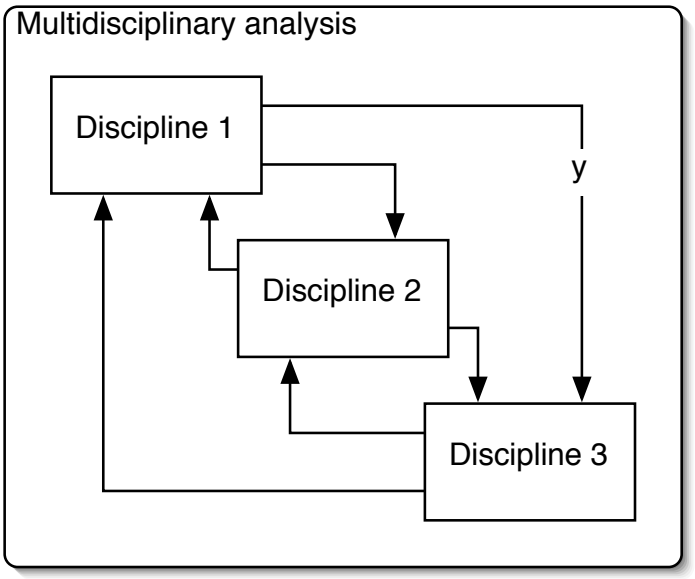


Figure 3: Nomenclature for multidisciplinary analysis.

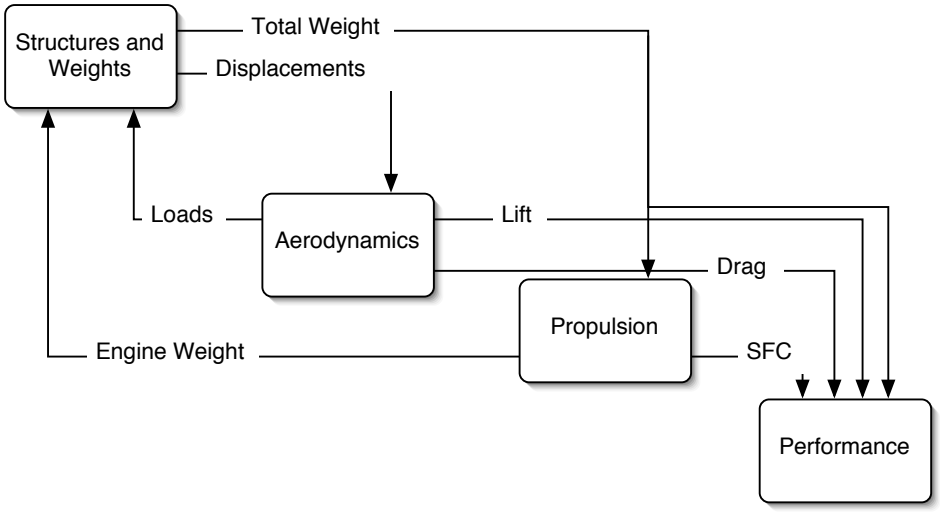


Figure 4: Example of the analyses for the design of a supersonic business jet.

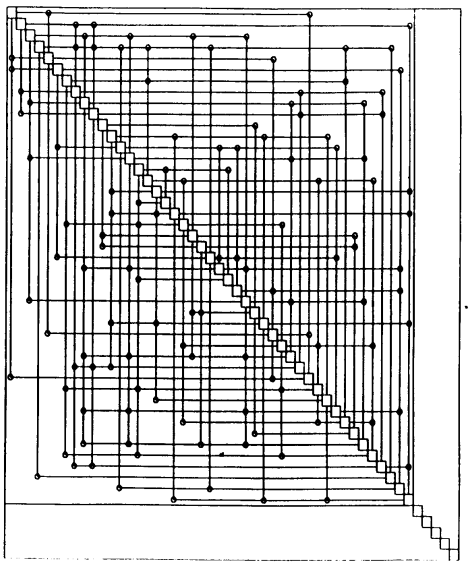


Figure 5: Coupling matrix of the original multidisciplinary analysis.

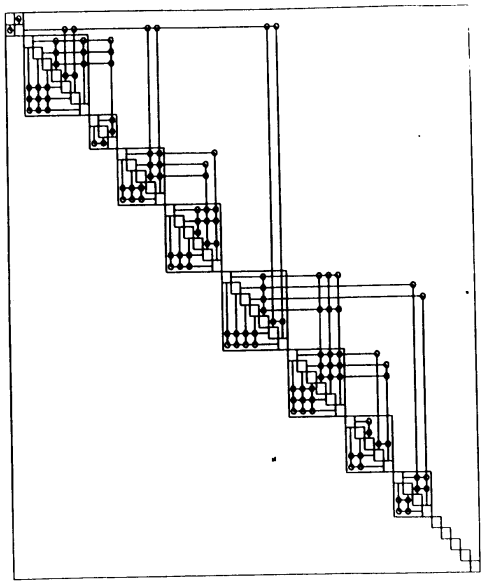
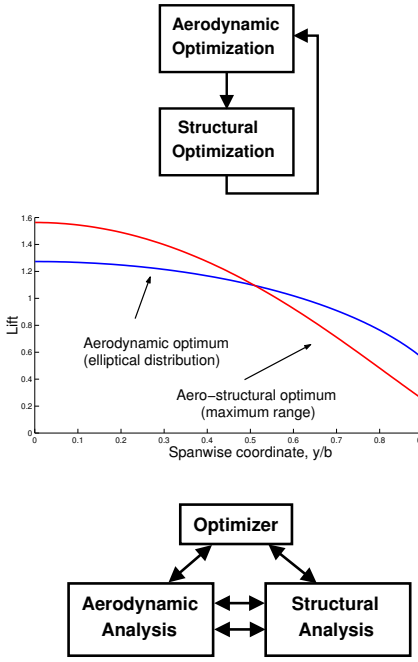


Figure 6: Coupling matrix after decomposition and re-ordering.

Example: Trade-off Between Aerodynamics and Structures



Minimizing drag and weight sequentially does not lead to the true optimum.

A more representative objective function for aircraft would be

$$\text{Range} \propto \frac{L}{D} \ln\left(\frac{W_i}{W_f}\right) .$$

Optimize aerodynamic shape and structural variables simultaneously.

The result a better overall design that represents a compromise between disciplines.

7.1 Monolithic Optimization

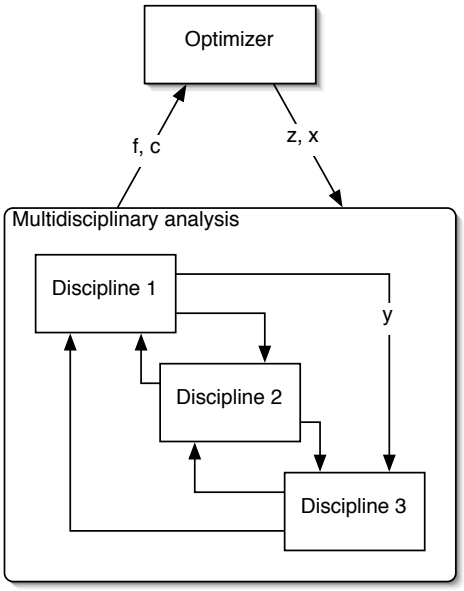
One approach to MDO is to consider the different disciplines as a single monolithic analysis. This is conceptually very simple, and once all disciplines are coupled to form one single multidisciplinary analysis module, one can use the same techniques that are used in single discipline optimization.

One of the disadvantages of this approach is that the solution of the one system might be very costly and does not exploit the potentially weak coupling between some of the disciplines that would enable the division into different analyses modules that might run in parallel.

The only opportunity for parallelizing the optimization procedure would be the use of different processes for each member of the population when using a genetic algorithm or running the analyses for different design points when calculating gradients by finite differencing or when evaluating the points for a response surface.

This approach is also known as multidisciplinary-feasible design (MDF) or single-level optimization. The method is summarized in the following statement,

$$\begin{aligned} \text{minimize} \quad & f\left(z, x, y\left(x, y, z\right)\right) \\ \text{w.r.t.} \quad & z, x \\ \text{s.t.} \quad & c\left(z, x, y\left(x, y, z\right)\right) \leq 0 \end{aligned}$$



7.2 Optimizer-Based Decomposition (OBD)

The main idea of this method is to use the optimizer to enforce inter-disciplinary compatibility. Instead of iterating the multidisciplinary analysis to converge the coupling variables y, these coupling variables are given by the optimizer as a guess, y'.

The new optimization problem can be written as

$$\begin{aligned} \text{minimize} \quad & f\left(z, y\left(x, y', z\right)\right) \\ \text{w.r.t.} \quad & z, x, y' \\ \text{s.t.} \quad & c\left(z, y\left(x, y', z\right)\right) \leq 0 \\ & y' - y\left(x, y', z\right) = 0. \end{aligned}$$

The number of design variables has increased, and is equal to the number of original design variables plus the number o coupling variables. This increases the size of the optimization problem, but conveniently decouples all the analyses, which can now be solved in parallel without intercommunication. Note that when using gradient-based optimization, the gradients $\partial f/\partial y'$ and $\partial c/\partial y'$ must also be calculated.

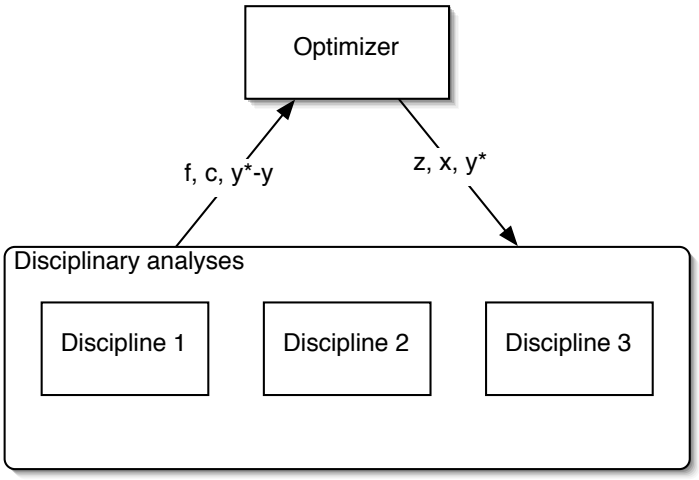


Figure 7: Optimizer-based decomposition.

OBD is sometimes called the individual discipline feasible (IDF) method. It is particularly advantageous for cases where there is a small number of coupling variables.

7.3 Collaborative Optimization (CO)

The collaborative optimization architecture is designed to promote disciplinary autonomy while achieving interdisciplinary compatibility. The optimization problem is decomposed into optimization subproblems corresponding to the different disciplines. Each subproblem is given control over its own set of local design variables, is responsible for satisfying its own set of local constraints and does not know about the other disciplines' design variables or constraints. The objective of each subproblem is to agree on the values of the coupling variables with the other disciplines. A system-level optimizer is used to coordinate this process while minimizing the overall objective.

The system level optimization problem can be stated as,

$$\begin{aligned} \text{minimize} \quad & f(z, y) \\ \text{w.r.t.} \quad & z, y \\ \text{s.t.} \quad & J_i^* \left((z, z^*, y, y^* \left(x_i^*, y, z_i^* \right) \right) = 0, \qquad i = 1, \ldots, N \end{aligned}$$

where N is the number of disciplines and J_i^* represents a measure of the interdisciplinary discrepancy for the i^{h} discipline after solving the disciplinary subproblem:

$$\begin{aligned} \text{minimize} \quad & J_i \left(z, z_i, y, y_i \left(x_i, y, z_i \right) \right) = \sum \left(z - z_i \right)^2 + \sum \left(y - y_i \right)^2 \\ \text{w.r.t.} \quad & z, y_i, x_i \\ \text{s.t.} \quad & c \left(x_i, z_i, y_i \left(x_i, y, z_i \right) \right) \leq 0 \end{aligned}$$

where c is the vector of constraints for the i^{h} discipline.

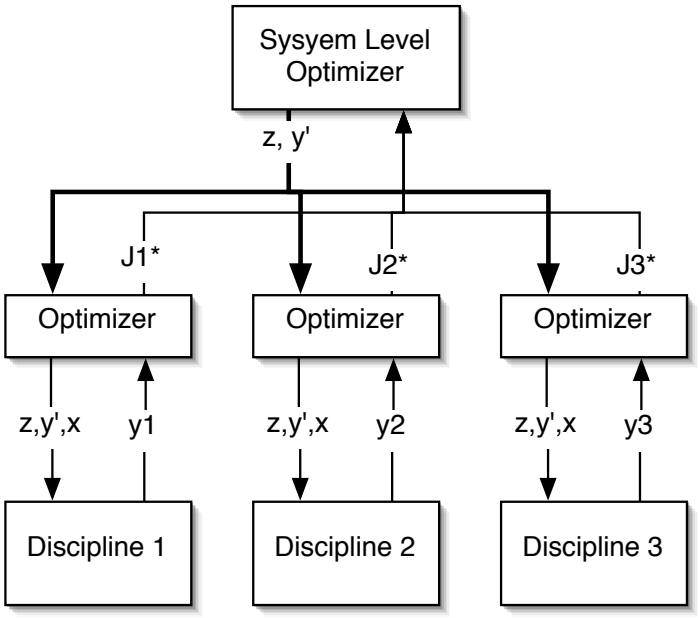


Figure 8: Collaborative Optimization.

7.4 Concurrent Subspace Optimization (CSSO)

The CSSO method is also a decomposition-based strategy that allows for the disciplines to run decoupled from each other. Again, the multiple subspace optimization problems are driven by a system-level optimizer that provides overall coordination.

Each subproblem in CSSO uses approximations to non-local disciplinary coupling variables to estimate the influence of these variables on the system-level objective and constraints.

The subspace optimization problem for the i^{h} discipline is given by,

$$\begin{aligned} \text{minimize} \quad & f\left(\left(z, y_i\left(x_i, \tilde{y}_j, z_i\right), \tilde{y}_j\right)\right. \\ \text{w.r.t.} \quad & z, x_i, y_i, \tilde{y}_j \\ \text{s.t.} \quad & c\left(x_i, z, y_i\left(x_i, \tilde{y}_j, z_i\right)\right) \leq 0, \end{aligned}$$

where $\tilde{y}_j = \tilde{y}_j(z, x_j)$ is are the approximations to the other disciplines' coupling variables, or states.

The system-level optimizer solves the following problem,

$$\begin{aligned} \text{minimize} \quad & f\left(\left(z, \tilde{y}\right)\right. \\ \text{w.r.t.} \quad & z, \tilde{y} \\ \text{s.t.} \quad & c\left(x_i, z, y_i\left(x_i, \tilde{y}_j, z_i\right)\right) \leq 0, \end{aligned}$$

After each iteration of the system-level optimizer, a multidisciplinary analysis is performed to update the model which gives the approximate response of all coupling variables $\tilde{y}$.

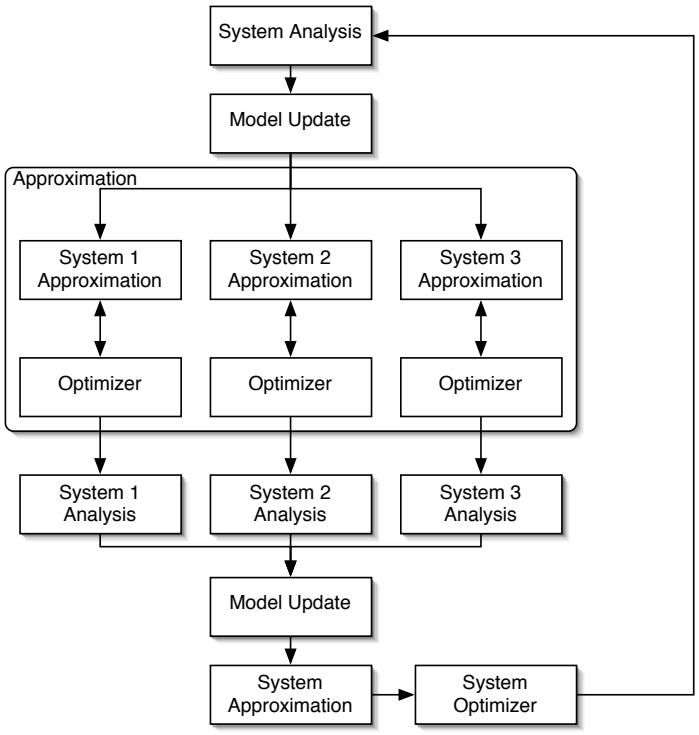


Figure 9: Concurrent Subspace Optimization.

7.5 Coupled-Sensitivity Analysis

Coupled-sensitivity analysis is not an MDO architecture on its own but is essential in the understanding of some MDO methods. It can also be used to perform a monolithic optimization, since the optimization problem in that case requires total derivatives of the multidisciplinary system.

Although the analytical sensitivity analysis methods that we covered in Section 5.5 are applicable to multidisciplinary systems (provided that the governing equations for all disciplines are included in $\mathcal{R}_k$) we now explicitly discuss the sensitivity analysis of multidisciplinary systems, using aero-structural optimization as an example. This example illustrates the fundamental computational cost issues that motivate our choice of strategy for sensitivity analysis. The following equations and discussion can easily be generalized for cases with additional disciplines.

In the aero-structural case we have coupled aerodynamic ($\mathcal{A}_k$) and structural ($\mathcal{S}_l$) governing equations, and two sets of state variables: the flow state vector, w_i, and the vector of structural displacements, u_j. In the following expressions, we split the vectors of residuals, states and adjoints into two smaller vectors corresponding to the aerodynamic and structural systems

$$\mathcal{R}_{k'} = \begin{bmatrix} \mathcal{A}_k \\ \mathcal{S}_l \end{bmatrix}, \quad y_{i'} = \begin{bmatrix} w_i \\ u_j \end{bmatrix}, \quad \Psi_{k'} = \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix}. \tag{2}$$

Figure 10 shows a diagram representing the coupling in this system.

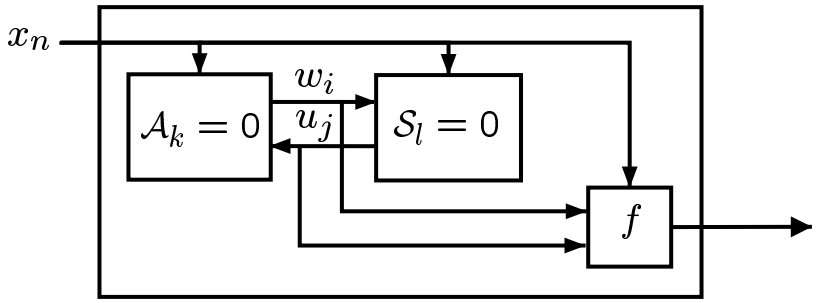


Figure 10: Schematic representation of the aero-structural system.

Using this new notation, the direct sensitivity equation for an aero-structural system can be written as

$$\begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial w_i} & \frac{\partial \mathcal{A}_k}{\partial u_j} \\ \frac{\partial \mathcal{S}_l}{\partial w_i} & \frac{\partial \mathcal{S}_l}{\partial u_j} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}w_i}{\mathrm{d}x_n} \\ \frac{\mathrm{d}u_j}{\mathrm{d}x_n} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial x_n} \\ \frac{\partial \mathcal{S}_l}{\partial x_n} \end{bmatrix} . \tag{3}$$

This equation was first written for a multidisciplinary system by Sobieski [6]. In his paper, Sobieski also presents an alternative approach to the problem, which he shows is equivalent to (3), i.e.,

$$\begin{bmatrix} \mathcal{I} & -\frac{\partial w_i}{\partial u_j} \\ -\frac{\partial u_j}{\partial w_i} & \mathcal{I} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}w_i}{\mathrm{d}x_n} \\ \frac{\mathrm{d}u_j}{\mathrm{d}x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial w_i}{\partial x_n} \\ \frac{\partial u_j}{\partial x_n} \end{bmatrix} , \tag{4}$$

where $\mathcal{I}$ denotes the identity matrix. Solving either of these equations (3, 4) yields the total sensitivity of the state variables with respect to the design variables. This result can then be substituted into the aero-structural equivalent of the total sensitivity equation,

$$\frac{\mathrm{d}f}{\mathrm{d}x_n} = \frac{\partial f}{\partial x_n} + \frac{\partial f}{\partial u_j}\frac{\mathrm{d}u_j}{\mathrm{d}x_n} + \frac{\partial f}{\partial w_i}\frac{\mathrm{d}w_i}{\mathrm{d}x_n}. \tag{5}$$

The biggest disadvantage of these direct approaches, as we discussed earlier, is that the sensitivity equation must be solved for each design variable x_n. For large iterative coupled systems, the cost of computing the total sensitivities with respect to many design variables becomes prohibitive, and this approach is impractical.

In the alternate direct approach (4), the partial derivatives of the state variables of a given system with respect to the variables of the other system ($\partial w_i/\partial u_j$, $\partial u_j/\partial w_i$) and the partial derivatives of the state variables with respect to the design variables ($\partial w_i/\partial x_n$, $\partial u_j/\partial x_n$) have a different meaning from the partial derivatives we have seen so far. In this formulation, the partial derivatives of the state variables of a given system take into account the solution of that system. Although the solution of the coupled system is not required, this is in contrast with the partial derivatives of the residuals in the formulation (3), which do not require the solution of even the single discipline.

The adjoint approach to sensitivity analysis is also applicable to multidisciplinary systems. In

the case of the aero-structural system, the adjoint equations can be written as

$$\begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial w_i} & \frac{\partial \mathcal{A}_k}{\partial u_j} \\ \frac{\partial \mathcal{S}_l}{\partial w_i} & \frac{\partial \mathcal{S}_l}{\partial u_j} \end{bmatrix}^T \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix} = - \begin{bmatrix} \frac{\partial f}{\partial w_i} \\ \frac{\partial f}{\partial u_j} \end{bmatrix} . \tag{6}$$

Note that the matrix in the coupled adjoint equation is the same as in the coupled direct method (3). In addition to containing the diagonal terms that appear when we solve the single discipline adjoint equations, this matrix includes off-diagonal terms that express the sensitivity of one discipline to the state variables of the other.

Finally, for completeness, we note that there is an alternative formulation for the coupled-adjoint method which is parallel to the alternate direct equations (4),

$$\begin{bmatrix} \mathcal{I} & -\frac{\partial w_i}{\partial u_j} \\ -\frac{\partial u_j}{\partial w_i} & \mathcal{I} \end{bmatrix}^T \begin{bmatrix} \bar{\psi}_i \\ \bar{\phi}_j \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial w_i} \\ \frac{\partial f}{\partial u_j} \end{bmatrix} , \tag{7}$$

where the partial derivatives have the same meaning as in the alternate direct sensitivity equations (4) and therefore, as previously discussed, the uncoupled solution of each discipline is required. This is a disadvantage relative to the standard coupled-adjoint approach (6), where none of the partial derivatives require the solution of governing equations. The alternate direct and adjoint formulations are, however, not without their advantages. They are the only suitable approach when disciplinary solvers are available just as black boxes, that is, if one has access

only to the input and output. Furthermore, the size of the matrix of state variable sensitivities can be reduced since one only needs to consider the coupling variables.

The alternate adjoint vector, $\bar{\psi}_k$ differs from the standard adjoint and therefore requires a different total sensitivity equation,

$$\frac{\mathrm{d}f}{\mathrm{d}x_n} = \frac{\partial f}{\partial x_n} + \bar{\psi}_i \frac{\partial w_i}{\partial x_n} + \bar{\phi}_j \frac{\partial u_j}{\partial x_n}, \tag{8}$$

where the partial derivatives of the state variables ($\partial w_i / \partial x_n, \partial u_j / \partial x_n$) also require the solution of the corresponding governing equations.

Given that the aerodynamic analysis in our case (CFD) is rather costly, and that the number of coupling variables (all surface pressures and all nodal forces) is $\mathcal{O}(10^4)$, the standard coupled-adjoint approach (6) is adopted.

Solving the coupled-adjoint equations (6) by factorizing the full matrix would be extremely costly, so our approach is to decouple the two disciplines, much like we did for the aero-structural solution. To solve the coupled-adjoint equations iteratively, the adjoint vectors are *lagged* and the two different sets of equations are solved separately. For the calculation of the adjoint vector of one discipline, we use the adjoint vector of the other discipline from the previous iteration, i.e., we solve

$$\underbrace{\frac{\partial \mathcal{A}_k}{\partial w_i} \psi_k = - \frac{\partial f}{\partial w_i}}_{\text{Aerodynamic adjoint}} - \frac{\partial \mathcal{S}_l}{\partial w_i} \tilde{\phi}_l, \tag{9}$$

$$\underbrace{\frac{\partial \mathcal{S}_l}{\partial u_j}\phi_l = -\frac{\partial f}{\partial u_j}}_{\text{Structural adjoint}} - \frac{\partial \mathcal{A}_k}{\partial u_j}\tilde{\psi}_k, \tag{10}$$

where $\tilde{\psi}_k$ and $\tilde{\phi}_l$ are the lagged aerodynamic and structural adjoint vectors respectively. Upon convergence, the final result given by this system is the same as that given by the original coupled-adjoint equations (6). We call this the *lagged-coupled adjoint* (LCA) method for computing sensitivities of coupled systems. Note that these equations look like the single discipline adjoint equations for the aerodynamic and structural solvers, with the addition of forcing terms in the right-hand side that contain the off-diagonal terms of the residual sensitivity matrix. This allows us to use existing single-discipline adjoint sensitivity analysis methods. Note also that, even for more than two disciplines, this iterative solution procedure is nothing more than the well-known block-Jacobi method.

Once both adjoint vectors have converged, we can compute the final sensitivities of the objective function by using the following expression

$$\frac{\mathrm{d}f}{\mathrm{d}x_n} = \frac{\partial f}{\partial x_n} + \psi_k \frac{\partial \mathcal{A}_k}{\partial x_n} + \phi_l \frac{\partial \mathcal{S}_l}{\partial x_n}, \tag{11}$$

which is the coupled version of the total sensitivity equation.

The approach for solving the coupled system of sensitivity equations by lagging can also be used to solve the direct equations (3, 4) but the disadvantages of these methods for problems that require iterative methods and are parameterized with a large number of design variables remain the same.

For the aero-structural optimization problem at hand the aerodynamic portion is usually characterized by a single objective function, a few aerodynamic constraints, and a large number of design variables. On the other hand, the structural portion of the optimization problem involves a large number of constraints dictating that the stress in each element of the finite-element model is not to exceed the material yield stress for a set of load conditions. Constrained gradient optimization methods generally require that the user provide the gradient of both the cost function and each nonlinear constraint with respect to all of the design variables in the problem. Using the adjoint approach, the evaluation of the gradient of each constraint would require an independent coupled solution of a large adjoint system. Since the number of structural constraints is similar to the number of design variables in the problem ($\mathcal{O}(10^3)$ or larger), the usefulness of the adjoint approach is questionable in this case.

The remaining alternatives — the direct and finite-difference methods — are not advantageous either since they both require a number of solutions that is comparable to the number of design variables. In the absence of other choices that can efficiently evaluate the gradient of a large number of constraints with respect to a large number of design variables, it is necessary to reduce the size of the problem either through a reduction in the number of design variables or through a reduction in the number of nonlinear constraints.

The reason for the choice of the KS functions to lump the structural constraints now becomes clear. By employing KS functions, the number of structural constraints for the problem can be reduced from $\mathcal{O}(10^3)$ to just a few. In some problems, a single KS function may suffice. If this constraint lumping methodology is effective, an adjoint method would be very efficient for computing MDO sensitivities.

[5, 4, 3, 1, 2, 7, 8, 6]

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