Performance Analysis of Topology-Unaware TDMA MAC Schemes for Ad-Hoc Networks with Topology Control ⋆

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Abstract

Traditional omni-directional antennas result in increased mutliuser interference and are known to limit the performance of Medium Access Control (MAC) protocols for ad-hoc networks. *Topology control* is the capability of a node to control the set of neighbor nodes and in this paper, the impact of using smart antennas and/or power control for topology control is investigated. The performance of TDMA MAC schemes with common frame for which the assignment of time slots to a node is not aware of the time slots assigned to the neighbor nodes (*topology-unaware* schemes like the Deterministic Policy and the Probabilistic Policy), is studied as well. A comparison based on analytical models reveals the advantages of topology control, as well as its dependence on the *mobility* of the nodes and its *resolution*. It is shown that topology control with "high resolution" in highly mobile environments may not be effective and conditions are established under which topology control is beneficial. Simulation results for a variety of network topologies support the claims and the expectations of the aforementioned analysis and show that the system throughput achieved under topology control can be higher under both policies and especially under the Probabilistic Policy. Simulation results also show how mobility affects system throughput and that topology control may not be suitable for highly mobile environments.

Key words: Ad-Hoc, TDMA, MAC, Topology Control, Mobility.
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1 Introduction

Ad-hoc networks require no infrastructure and nodes are free to enter, leave or move inside the network without prior configuration, thus making the design of an efficient Medium Access Control (MAC) a challenging problem. CSMA/CA-based MAC protocols have been proposed, [1], whereas others have additionally employed handshake mechanisms like the Ready-To-Send/Clear-To-Send (RTS/CTS) mechanism, [2], [3], [4], [5], to avoid the *hidden/exposed terminal* problem. TDMA-based MAC protocols have also been proposed for ad-hoc networks. S-TDMA, proposed by Kleinrock and Nelson, [6], is capable of providing *collision-free* scheduling based on the exploitation of noninterfering transmissions in the network. Other collision-free protocols, mechanisms or algorithms have been proposed recently, [7], [8], [9], [10], [11], [12].

Topology-Unaware TDMA MAC schemes, under which the assignment of time slots to nodes does not consider the time slots assigned to the neighbor nodes (nodes that a direct transmission is possible), have also been proposed, [13], [14], [15], [16], [17]. In particular, Farago proposed the *Deterministic Policy*, [13], whereas the *Probabilistic Policy* has been proposed and analyzed in [15], [16] and [17]. This analysis has shown that the Probabilistic Policy outperforms the Deterministic Policy under certain conditions. The aforementioned analysis was based on traditional *omni-directional* antennas, where the transmitting node did not have any *topology control* capabilities. Topology control is a node's capability of controlling the set of neighbor nodes and it may be achieved by adjusting the *angle* of the transmission beam and/or the transmission power and thus, the interference caused to neighbor nodes when transmitting.

The use of *directional antennas* for topology control is not a new idea and has been proposed in the past, [18]. Nowadays, more sophisticated *smart antennas* is possible to be used to adjust the angle of the transmission beam and even be incorporated into portable devices. Several MAC protocols have been proposed for ad-hoc networks that exploit the capabilities of smart antennas. The majority of them is based on random access schemes (i.e. ALOHA or CSMA/CA) and enhancements of the RTS/CTS mechanism, [19], [20], [21]. *Power control* may also be used for topology control. The transmission power is possible to be adjusted according to the location of the receiver and

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reduce the interference caused to neighbor nodes by the transmitting node, [22], [23], [24], [25], [26]. *Resolution* is an important factor of topology control. The higher the resolution of the topology control, the narrower the transmission beam of the smart antennas and/or the smaller the transmission range corresponding to a particular transmission.

In this work both the Deterministic Policy and the Probabilistic Policy are considered when topology control is applied (use of smart antennas and/or power control) and their performance is compared against that induced when no topology control is present (use of traditional omni-directional antennas). This comparison is based on an analytical approach and is supported by simulation results. The nodes' *mobility* is also taken into account, since it is expected to impact the performance especially under topology control. The (mobility) conditions under which topology control (for a given resolution) improves performance, are also established here.

Topology control is presented in Section 2. In Section 3, an ad-hoc network is described and some key definitions are introduced. The Deterministic Policy and the Probabilistic Policy are presented in Section 4. In Section 5, expressions for the *system throughput* under both policies are derived with and without topology control. The mobility aspects are considered in Section 6, where the conditions under which topology control with a certain resolution is beneficial for the system performance are also established. Simulation results for network topologies with different characteristics are presented in Section 7. These results support the claims and the expectations introduced by the analytical comparison and show that the system throughput achieved under the Probabilistic Policy and under topology control can be rather high. On the other hand, it is shown that mobility degrades the system throughput especially under topology control with high resolution and therefore, topology control may not be desirable under certain conditions. Finally, Section 8 presents the conclusions.

2 Topology Control

Traditional omni-directional antennas transmit and receive from all directions. Consequently, the receiver is not benefited by the entire power of the transmitter since this power is scattered in the 360^o pattern. Furthermore, as it will be seen in the following section, the interference caused by neighbor nodes may be high and spatial reuse of the network resources becomes a difficult task. In Figure 1(a) an omni-directional antenna example is shown.

Directional antennas have been introduced with fixed transmission and reception directions. The advantage is that the power of the transmitter is "directed" to the receiver. In Figure 1(b) a directional antenna example is presented while in Figure 1(c) several directive antennas are used to cover the 360^o pattern.

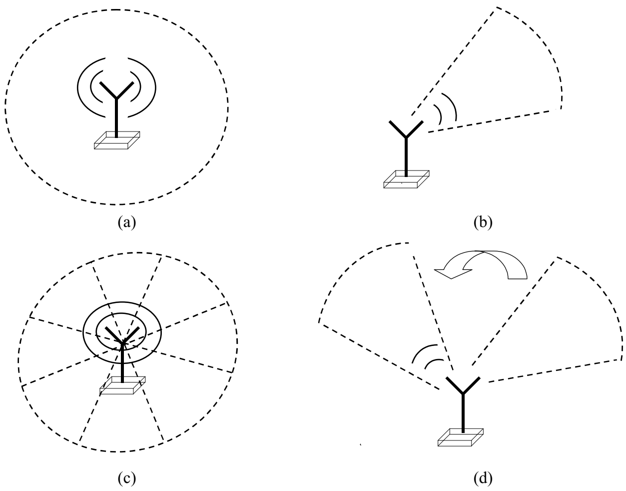

Fig. 1. Various antenna types.

Smart antennas are considered as one of the more promising technologies for reducing interference and increasing the utilization of the network resources. They are composed of an array of antennas and their "smartness" is due to the efficient combination of incorporated Digital Signal Processing (DSP) capabilities and an antenna array, [28]. A special subset of smart antennas are the *adaptive array antennas* which are capable of focusing the main lobe of the transmission power towards a certain direction (the receiver's direction). This case is depicted in Figure 1(d). Another subset of smart antennas are the *switched-beam antennas* which choose to switch between predefined directions. For the rest, when smart antennas are considered, it is assumed that they can adjust the transmission angle towards the receiver, as it is depicted in Figure 1(d).

From the above discussion is clear that the transmitting node is able to "control the topology" if smart antennas are used. Topology control may also be achieved by adjusting the transmission power. The transmission power plays an important role regarding the existence of a link between two nodes as well as the *quality* of the link, [27]. In general, the higher the power of a transmission the more likely a node to receive it successfully. Let $Power_t$ be the transmission power at the transmitting node u and a be the distance between node u and node v. In order for node v to be able to receive successfully a transmission from node u, the reception power $Power_r$ has to be above a certain threshold. It is shown that $Power_r \sim \frac{Power_t}{a^n}$, where n is a positive constant that depends on the particular environment, [27]. This relation reveals the fact that an exponential increment of the transmitting power is required as the distance between two nodes increases.

An example is depicted in Figure 2, where u is the transmitting node and v the receiver. If the transmission power of node u is $Power_t(a_1)$ ($Power_t(a_2)$), then a transmission is possible at a distance a_1 (a_2). If $a_2 = 2a_1$ and assuming that $n = 3$, [27], $Power_t(a_2)$ is 8 times higher than $Power_t(a_1)$. For both cases depicted in Figure 2, node u is able to transmit to node v as well as to other nodes. It is clear that as $Power_t$ increases, node u is able to transmit to a higher number of nodes, (the *transmission range* of node u increases). On the other hand, the number of nodes that *overhear* node's u transmission increases, resulting in increased corrupted transmissions originated from other nodes. For the case that the transmission power is equal to $Power_t(a_1)$, those nodes that overhear the transmission from node u are gray-colored, while for $Power_t(a_2)$, the black-colored nodes also overhear the particular transmission.

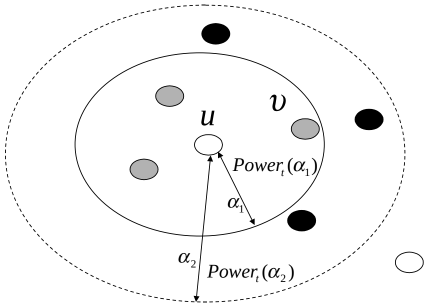

Fig. 2. Different levels of $Power_t$ corresponding to different number of neighbor nodes.

It is expected that *mobility* affects the performance under topology control. In particular, when smart antennas are used the transmission angle is adjusted based on power sensing. The receiver from its side, is able to determine the direction of the transmitter by processing the information received from the array of antennas. The problem is how the transmitter initially determines the correct angle towards the receiver. A proposed approach is to send at the beginning a "beacon" signal, [28]. Other approaches also exist, [29], [30], [31], [32], [33], but it should be mentioned that this is still an open problem. For the case when power control is used, the issue is how the transmitter becomes aware of the minimum transmission power needed in order for the receiver to receive correctly. This may be achieved by the use of feedback information from the receiver, [27]. In any case, when nodes move outside the transmission range, adaptation of the transmission power has to take place. Section 6 provides more information on mobility issues.

A factor that increases the performance improvement under topology control is the *resolution* of the angle and the power of the transmission. The higher the resolution, the closer the angle of transmission to the *ideal angle* of transmission and/or the closer the transmission power to the *ideal power* of transmission. On the other hand, the higher the resolution under topology control, the higher the (negative) impact of nodes' mobility on the system throughput.

For the rest of this work it is assumed that the receiver receives using a traditional omni-directional antenna (which cannot adjust its transmission power). *Under topology control* it is assumed that smart antennas and/or power control is possible at the transmitter. *Under no topology control* it is assumed that traditional omni-directional antennas are used for transmission purposes. How the aforementioned characteristics can be modeled and then analyzed is the subject of the following section.

3 Network Definition

An ad-hoc network may be viewed as a time varying multihop network and may be described in terms of a graph $G(V, E)$, where V denotes the set of nodes and E the set of links between the nodes at a given time instance. Let $|X|$ denote the number of elements in set X and let $N = |V|$ denote the number of nodes in the network. Let S_u denote the set of neighbors of node u, $u \in V$. These are the nodes v to which a direct transmission from node u (*transmission* $u \to v$) is possible. Let D denote the maximum number of neighbors for a node; clearly $|S_u| \leq D$, $\forall u \in V$.

Suppose that omni-directional antennas are used and that node u wants to transmit to a particular neighbor node v in a particular time slot i. In order for transmission $u \to v$ to be successful (uncorrupted), two conditions should be satisfied. First, node v should not transmit in the particular time slot i, or equivalently, no transmission $v \to \psi$, $\forall \psi \in S_v$ should take place in time slot i. Second, no neighbor of v - except u - should transmit in time slot i, or equivalently, no transmission $\zeta \to \chi$, $\forall \zeta \in S_v - \{u\}$ and $\chi \in S_\zeta$, should take place in time slot i. Consequently, transmission $u \to v$ is corrupted in time slot i if at least one transmission $\chi \to \psi$, $\chi \in S_v \cup \{v\} - \{u\}$ and $\psi \in S_\chi$, takes place in time slot i. Let $S^O_{u \to v}$ denote the set of nodes $\chi \in S_v \cup \{v\} - \{u\}$; a simultaneous transmission by any node in $S^O_{u \to v}$ corrupts transmission $u \to v$.

The transmission(s) that corrupts transmission $u \to v$ may or may not be successful itself. Specifically, in the presence of transmission $u \to v$, transmission $\chi \to \psi$, $\chi \in S_v \cup \{v\} - \{u\}$ and $\psi \in S_\chi \cap (S_u \cup \{u\})$, is corrupted. If $\psi \in S_\chi - (S_\chi \cap (S_u \cup \{u\}))$, then transmission $\chi \to \psi$ is not affected by transmission $u \to v$.

Let $\Phi^O_{u \to v}$ be the set of transmissions which corrupt transmission $u \to v$ and at the same time they are themselves corrupted by transmission $u \to v$ as well. Let $\Theta^O_{u \to v}$ be the set of transmissions which corrupt transmission $u \to v$ but are not corrupted themselves by it. Note that transmissions that belong in $\Theta^O_{u \to v}$ may still be corrupted by a transmission other than transmission $u \to v$. It is evident that $\Phi^O_{u \to v} \cup \Theta^O_{u \to v} = S^O_{u \to v}$ is the set of transmissions that corrupts transmission $u \to v$. Obviously $\Phi^O_{u\to v} \cap \Theta^O_{u\to v} = \emptyset$. Transmission sets $\Phi^O_{u\to v}$ and $\Theta^O_{u\to v}$ are given by equations (1) and (2) respectively.

$$\Phi^O_{u\to v} = \Big\{ \chi \to \psi : \chi \in S_v \cup \{v\} - \{u\}, \psi \in S_\chi \cap (S_u \cup \{u\}) \Big\}, \tag{1}$$

$$\Theta^O_{u\to v} = \Big\{ \chi \to \psi : \chi \in S_v \cup \{v\} - \{u\}, \psi \in S_\chi - (S_\chi \cap (S_u \cup \{u\})) \Big\}. \tag{2}$$

Figure 3 depicts an example topology of 27 nodes. Transmission $8 \to 13$ is denoted by a white arrow between nodes 8 and 13 and transmissions that belong in $\Phi^O_{8\to 13}$ ($\Theta^O_{8\to 13}$) are denoted by black dense (dotted black) arrows.

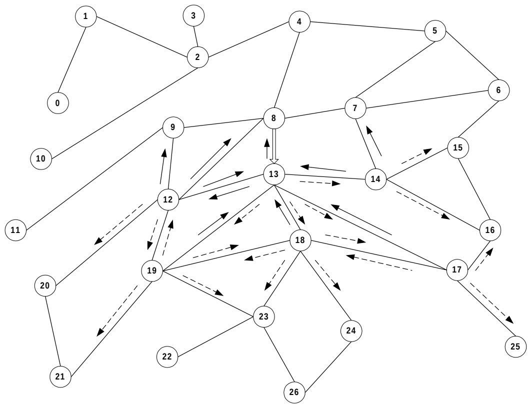

Fig. 3. Transmission sets $\Phi^O_{8\to 13}$ and $\Theta^O_{8\to 13}$ for an example of a network of 27 nodes.

It is assumed that an *acknowledge message* (ACK) is returned by the receiver after the successful reception of a transmission. In particular, a fixed part at the end of each time slot may be used for this purpose (to be referred to as the *ACK part* of the time slot), [12]. If transmission $8 \to 13$ takes place in time slot i and it is not corrupted, at the end of time slot i transmission $13 \to 8$ will take place (ACK message in the ACK part of the time slot) and it can be seen that $13 \to 8$ will also be uncorrupted, for the traditional omnidirectional antenna case, [17]. Under smart antennas and/or topology control it is possible the ACK message (transmission $13 \to 8$) to be corrupted. Consequently, more sophisticated error control schemes (like Selective Repeat ARQ) are required in order for the transmitter to become aware of the successfully transmitted packets. For the rest it is assumed that a successful transmission is instantaneously acknowledged, [34].

Let $S^S_{u\to v}$ be that set of nodes that their transmissions affect transmission $u \to v$ when smart antennas are used and $S^P_{u\to v}$ when power control is applied. For the case in which topology control is achieved via smart antennas, transmissions $\chi \to \psi \in \Phi^O_{u\to v}$ corrupt transmission $u \to v$ but not all transmissions $\chi \to \psi \in \Theta^O_{u\to v}$ corrupt transmission $u \to v$. Only a subset of $\Theta^O_{u\to v}$ corrupts transmission $u \to v$ and this subset is denoted by $\Theta^S_{u\to v}$ ($\subseteq \Theta^O_{u\to v}$). Clearly, $|\Theta^S_{u\to v}| \leq |\Theta^O_{u\to v}|$ and $S^S_{u\to v} \subseteq S^O_{u\to v}$.

For the case that topology control is achieved via power control, as it is already shown, the set of neighbor nodes of node u (S_u) changes according to the transmission power. Assuming that the traditional omni-directional antennas transmit at the maximum power it can be concluded that when power control is used, the transmission power will not exceed that maximum transmission power. Consequently, $S^P_{u\to v} \subseteq S^O_{u\to v}$.

For the rest of the paper, $S^T_{u\to v}$ will denote the set of nodes whose transmissions influence transmission $u \to v$ when topology control is applied (corresponding either to $S^S_{u\to v}$ or $S^P_{u\to v}$); in view of the above, $|S^T_{u\to v}| \leq |S^O_{u\to v}|$.

4 Scheduling Policies

Under the policy proposed in [13], each node $u \in V$ is randomly assigned a unique polynomial f_u of degree k with coefficients from a finite Galois field of order q ($GF(q)$). Polynomial f_u is represented as $f_u(x) = \sum_{i=0}^{k} a_i x^i (\text{mod } q)$, [14], where $a_i \in \{0, 1, 2, ..., q-1\}$; parameters q and k are calculated based on N and D, according to the algorithm presented either in [13] or [14]. For both algorithms it is satisfied that $k \geq 1$ and $q > kD$ or $q \geq kD + 1$ (k and D are integers) and $q^{k+1} \geq N$ (in order the number of unique polynomial, q^{k+1} to be greater than the number of nodes N).

The access scheme considered is a TDMA scheme with a frame consisted of q^2 time slots. If the frame is divided into q subframes s of size q, then the time slot assigned to node u in subframe s, ($s = 0, 1, ..., q-1$) is given by $f_u(s)\text{mod } q$, [14]. Let the set of time slots assigned to node u be denoted as Ω_u. Consequently, $|\Omega_u| = q$. The deterministic transmission policy, proposed in [13] and [14], is the following.

The Deterministic Policy: Each node u transmits in a slot i only if $i \in \Omega_u$, provided that it has data to transmit.

The assignment of the unique polynomials, or equivalently the assignment of the time slot sets Ω_χ to any node χ, is random in the sense that neither the node nor its neighbors are taken into account in order to assign the polynomial. The polynomial assignment is similar to the MAC identification number (MAC ID) assignment: either it is already in the device or it is distributed by the time a node becomes part of the network.

Depending on the particular random assignment of the polynomials, it is possible that two nodes be assigned overlapping time slots (i.e., $\Omega_u \cap \Omega_v \neq \emptyset$). Let $C^O_{u\to v}$ ($C^T_{u\to v}$) be the set of overlapping time slots between those assigned to node u and those assigned to any node $\chi \in S^O_{u\to v}$ ($\chi \in S^T_{u\to v}$), when traditional omni-directional antennas are used (under topology control). $C^O_{u\to v}$ and $C^T_{u\to v}$ are given by (3), ($C^K_{u\to v}$ for $K \in \{O, T\}$).

$$C^K_{u\to v} = \Omega_u \cap \left(\bigcup_{\chi \in S^K_{u\to v}} \Omega_\chi \right). \tag{3}$$

Note that since $|S^T_{u\to v}| \leq |S^O_{u\to v}|$, $|C^T_{u\to v}| \leq |C^O_{u\to v}|$.

Let $R^O_{u\to v}$ ($R^T_{u\to v}$) denote the set of time slots i, $i \notin \Omega_u$, over which transmission $u \to v$ would be successful, using omni-directional antennas (topology control). Equivalently, $R^O_{u\to v}$ ($R^T_{u\to v}$) contains those slots not included in set $\bigcup_{\chi \in S^O_{u\to v} \cup \{u\}} \Omega_\chi$ ($\bigcup_{\chi \in S^T_{u\to v} \cup \{u\}} \Omega_\chi$). Consequently ($K \in \{O, T\}$),

$$|R^K_{u\to v}| = q^2 - \left| \bigcup_{\chi \in S^K_{u\to v} \cup \{u\}} \Omega_\chi \right|. \tag{4}$$

Note that since $|S^T_{u\to v}| \leq |S^O_{u\to v}|$, $|R^T_{u\to v}| \geq |R^O_{u\to v}|$.

$R^O_{u\to v}$ is the set of *non-assigned eligible* time slots for transmission $u \to v$, that if used by transmission $u \to v$, the probability of success for the particular transmission would be increased. As it was shown in [15], [16], $|R^O_{u\to v}| \geq q(k-1)D$. It is obvious that for $k > 1$, $|R^O_{u\to v}| \geq qD$. Consequently, the number of non-assigned eligible slots may be quite significant for the cases where $k > 1$ (this case corresponds to "large networks," [14]). Even for the case where $k = 1$, $|R^O_{u\to v}| \geq 0$, that is, $|R^O_{u\to v}|$ can still be significantly greater than zero, as it may be seen from Theorem 1.

Theorem 1 *It is satisfied that* $|R^O_{u\to v}| \geq q^2 - q(|S_v| + 1)$.

PROOF. Given that for any two nodes χ and ψ, $|\Omega_\chi| = q$ and $|\Omega_\psi| = q$, then $\Omega_\chi \cup \Omega_\psi \leq 2q$. Therefore, $\left|\bigcup_{\chi \in S^O_{u\to v} \cup \{u\}} \Omega_\chi\right| \leq q|S^O_{u\to v} \cup \{u\}| = q(|S_v| + 1)$. Consequently, $|R^O_{u\to v}| \geq q^2 - q(|S_v| + 1)$. $\square$

In order to efficiently use those slots i, $i \in R^O_{u\to v}$, the Probabilistic Policy has been introduced in [15].

The Probabilistic Policy: Each node u always transmits in slot i if $i \in \Omega_u$ and transmits with probability p in slot i if $i \notin \Omega_u$, provided it has data to transmit.

The Probabilistic Policy does not require specific topology information (e.g., knowledge of $R^O_{u\to v}$, etc.) and, thus, induces no additional control overhead. The access probability p is a simple parameter common for all nodes. Under the Probabilistic Policy, all slots $i \notin \Omega_u$ are potentially utilized by node u: both those in $R^O_{u\to v}$, for a given transmission $u \to v$, as well as those not in $\Omega_u \cup R^O_{u\to v}$ that may be left by neighboring nodes under non-heavy traffic conditions. On the other hand, the probabilistic transmission attempts induce interference to otherwise collision-free transmissions, [15], [16]. The Probabilistic Policy is capable of utilizing the non-assigned eligible time slots under topology control, and potentially benefit more than under traditional omni-directional antennas since $|R^T_{u\to v}|$ is higher than $|R^O_{u\to v}|$.

5 System Throughput

In [15], [17], both policies were analyzed for the case of traditional omni-directional antennas, and heavy traffic conditions; that is, there is always data available for transmission at each node, for every time slot. Let $P^O_{D,u\to v}$ ($P^O_{P,u\to v}$) be the *probability of success* for transmission $u \to v$ in a time slot, averaged over a frame, under the Deterministic (Probabilistic) Policy, when omni-directional antennas are used ($K \equiv O$) and $P^T_{D,u\to v}$ ($P^T_{P,u\to v}$) under topology control ($K \equiv T$).

$$P^K_{D,u\to v} = \frac{q - |C^K_{u\to v}|}{q^2}, \tag{5}$$

$$P^K_{P,u\to v} = \frac{q - |C^K_{u\to v}| + p|R^K_{u\to v}|}{q^2}(1-p)^{|S^K_{u\to v}|}. \tag{6}$$

Theorem 2 *It is satisfied that* $P^T_{D,u\to v} \geq P^O_{D,u\to v}$ *and* $P^T_{P,u\to v} \geq P^O_{P,u\to v}$.

PROOF. Since $|C^T_{u\to v}| \leq |C^O_{u\to v}|$, it is easily obtained that $P^T_{D,u\to v} \geq P^O_{D,u\to v}$. Additionally, given that $|S^T_{u\to v}| \leq |S^O_{u\to v}|$, $(1-p) \leq 1$ and $|R^O_{u\to v}| \leq |R^T_{u\to v}|$, it is easily obtained that $P^T_{P,u\to v} \geq P^O_{P,u\to v}$. □

Theorem 2 clearly shows an improvement of the performance under topology control. This is also depicted in Figure 4 concerning transmission $8 \to 13$ depicted in Figure 3. The Deterministic Policy is unaffected by changes of the value of p as expected. For $p = 0$, $P^O_{P,u\to v} = P^O_{D,u\to v}$ and $P^T_{P,u\to v} = P^T_{D,u\to v}$. As p increases, $P^O_{P,u\to v}$ and $P^T_{P,u\to v}$ increase until a certain maximum assumed at $p^O_{0,u\to v}$ and $p^T_{0,u\to v}$ for either case, and then they start decreasing until $p = 1$, where $P^T_{P,u\to v} = 0$ and $P^O_{P,u\to v} = 0$.

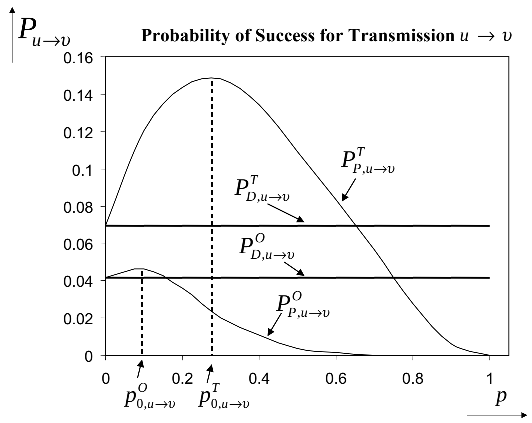

Fig. 4. Probability of success for transmission $u \to v$ in one frame ($P_{u\to v}$) as a function of p.

In [15] it was shown that $P^O_{P,u\to v} \geq P^O_{D,u\to v}$ is satisfied when $|R^O_{u\to v}| > (q - |C^O_{u\to v}|)|S^O_{u\to v}|$ and the maximum is assumed at $p^O_{0,u\to v} = \frac{|R^O_{u\to v}|-(q-|C^O_{u\to v}|)|S^O_{u\to v}|}{|R^O_{u\to v}|(|S^O_{u\to v}|+1)}$. Following a similar analysis, it may be concluded that $P^T_{P,u\to v} \geq P^T_{D,u\to v}$ is satisfied when $|R^T_{u\to v}| > (q - |C^T_{u\to v}|)|S^T_{u\to v}|$ and the maximum is assumed at $p^T_{0,u\to v} = \frac{|R^T_{u\to v}|-(q-|C^T_{u\to v}|)|S^T_{u\to v}|}{|R^T_{u\to v}|(|S^T_{u\to v}|+1)}$.

Let P^O_D (P^O_P) denote the probability of success of a transmission (averaged over all transmissions) under the Deterministic (Probabilistic) Policy (to be referred to as the *system throughput* for both policies) assuming that each node u may transmit to only one node $v \in S_u$ in one frame. According to equations (5) and (6), it can be concluded that P^O_D (P^T_D) and P^O_P (P^T_P) are given by the following equations ($K \in \{O, T\}$).

$$P^K_D = \frac{1}{N} \sum_{\forall u \in V} \frac{q - |C^K_{u\to v}|}{q^2}, \tag{7}$$

$$P^K_P = \frac{1}{N} \sum_{\forall u \in V} \frac{q - |C^O_{u\to v}| + p|R^K_{u\to v}|}{q^2} (1-p)^{|S^K_{u\to v}|}, \tag{8}$$

where $v \in S_u$.

Following an approach similar to that in Theorem 2, it can easily be proved that $P^T_D \geq P^O_D$ and $P^T_P \geq P^O_P$. This is clearly depicted in Figure 5 regarding the network depicted in Figure 3.

From Equation (8) it can be concluded that P^O_P is influenced by $|S^O_{u\to v}|$ in an exponential manner. This has been extensively studied in [15], [16], [17], and an efficient *topology density* metric, capable of capturing the density of the topology, was introduced. The topology density is denoted by $\overline{|S|}/D$, where $\overline{|S|} = \frac{1}{N}\sum_{\forall u \in V} |S^O_{u\to v}|$, $v \in S_u$. Under topology control, P^T_P is obviously influ-

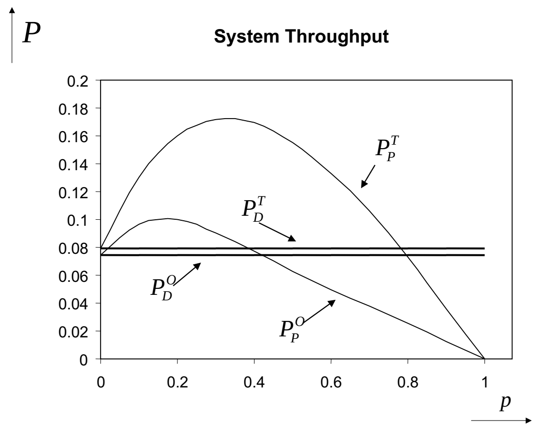

Fig. 5. System throughput P as a function of p.

enced by $|S^T_{u\to v}|$ (see Equation (8)). Since $|S^T_{u\to v}| \leq |S^O_{u\to v}|$, it is evident that P^T_P is influenced by $\overline{|S|}/D$ but not as strongly as P^O_P does.

On the other hand, mobility impacts on the system throughput achieved under topology control more than under no topology control. The following section provides a discussion on how mobility affects the system throughput under topology control or no topology control.

6 Mobility

Mobility impacts significantly on the performance of ad-hoc networks and has not been investigated so far under either the Deterministic Policy or the Probabilistic Policy. Mobility is shaped by the *relative* node movement and consequently, captures changes of the network topology. For the case of a specific transmission $u \to v$ and no topology control (traditional omni-directional antennas), it may be seen from equations (5) and (6) that $P^O_{D,u\to v}$ depends on $|C^O_{u\to v}|$, while $P^O_{P,u\to v}$ additionally depends on $|R^O_{u\to v}|$ and $|S^O_{u\to v}|$. Consequently, the movement of the nodes does not leave unaffected the system throughput. Let $\overline{P}^O$ ($\overline{P}^T$) denote the system throughput, averaged over a large number of frames F for the traditional omni-directional antenna case (under topology control) for either of the MAC policies assuming that no frames are lost due to nodes' movement.

In addition to changing the topology metrics $|C^K_{u\to v}|$, $|R^K_{u\to v}|$, $|S^K_{u\to v}|$, $K \in \{O,T\}$, mobility also leads to *link failures* during a transmission. Consider the example in Figure 6(a) where transmission $u \to v$ is in progress when node v starts moving in the direction of the white arrow. After some time, node v will be outside the transmission range of node u, a link failure will take place between node u and node v and transmission $u \to v$ will be interrupted (failed). Consider a similar example under topology control and in particular when power control is applied, as it is depicted in Figure 7(a). In this case, the transmission range of node u is adapted to the minimum transmission range required in order for transmission $u \rightarrow v$ to be feasible. If node v moves away, then a link failure is also possible. The only difference is that for this case the transmission range is smaller and thus, a link failure between u and v is more likely to happen than for the case depicted in Figure 6. The same applies for smart antennas as depicted in Figure 7(c). In general, under topology control the effect of the mobility factor increases. Mobility was not considered in previous related work, [15], [16], [17], and it is interesting to describe how the network behaves after a link failure under topology control or under no topology control.

Consider the traditional omni-directional antenna case when a node v moves outside the transmission range of node u. Transmission $u \rightarrow v$ is not feasible any more and the *routing protocol* is responsible for determining a new neighbor node v' to receive and forward packets from u towards the final destination (under heavy traffic conditions nodes have always data available for transmission), as it is depicted in Figure 6. This process is not instantaneous and requires a number of frames before transmission $u \rightarrow v'$ actually takes place.

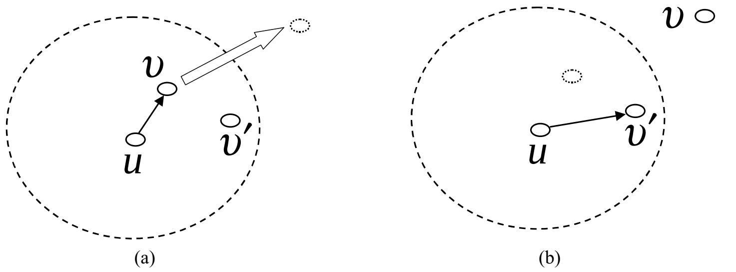

Fig. 6. Node v is moving (white arrow) under no topology control (omni-directional antenna). Transmission $u \rightarrow v$ (black arrow) is terminated and, after a certain number of frames, transmission $u \rightarrow v'$ is started.

Under topology control and when the movement of node v is the cause of a link failure, adaptation either of the transmission power and/or of the angle of the transmission beam, may allow transmission $u \rightarrow v$ to continue. This is the case depicted in Figure 7. Certainly, a number of frames are required before this adaptation takes place. Note that if node v moves out of the maximum transmission range of node u, then the routing protocol is again responsible to identify another node v', as it is the case depicted in Figure 6.

From the above discussion it may be concluded that under certain mobility conditions, the system throughput achieved under topology control may be less than the system throughput achieved under no topology control for the same mobility conditions. In order to "visualize" this argument, average values

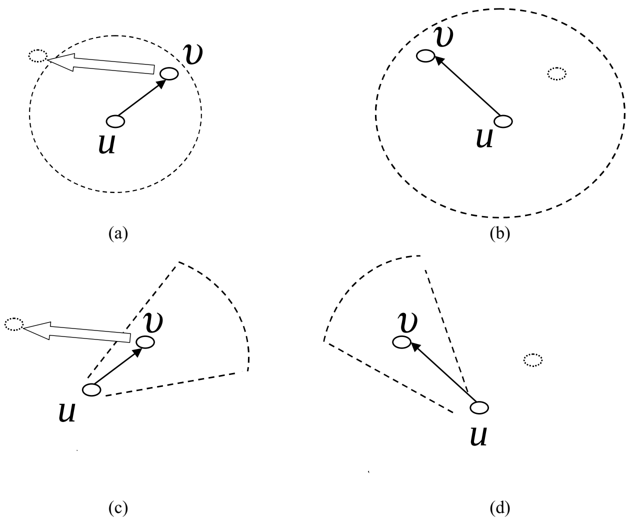

Fig. 7. Node v is moving (white arrow) under topology control. Adaptation of the transmission power, (a) and (b), takes place or adaptation of the transmission angle, (c) and (d). Transmission $u \rightarrow v$ (black arrow) continues and it is not terminated.

of parameters that influence the system throughput are considered next.

Let l^O denote the (average) *probability that a link failure occurs* for a transmission in one frame because the receiver node has moved outside the maximum transmission range of the transmitting node (case depicted in Figure 6). Let f^O denote the (average) *number of frames* required until a new destination is determined by the routing protocol. Let F be a large number of frames ($F >> \frac{1}{l^O}$). The number of frames spent by the routing protocol over a horizon of F frames by the process of determining a new destination after a link interruption is equal to $Fl^O f^O$ and the corresponding number of successful transmissions that would have taken place otherwise, is $Fl^O f^O \overline{P}^O$. Finally, $F(1 - l^O f^O)\overline{P}^O$ is the average number of successful transmissions in F slots. $l^O f^O < 1$ is required.

Under topology control, the probability that a link failure occurs increases (for the same mobility conditions) since a link failure appears not only when a node is outside the maximum transmission range (denoted by probability l^O) but also (a) when a node is outside the transmission range of the transmitting node (provided that this range is not the maximum and therefore power control is required) and/or (b) outside the boundaries of the transmission beam. Let l^T denote the *probability of a link failure* due to (a) and/or (b). Consequently, the probability that a link failure occurs under topology control is equal to $l^O + l^T$. Let f^T denote the (average) *number of frames* required for the adaptation of the transmitter towards the new location of the receiver (adjustment of the transmission beam and/or the transmission power). The corresponding number of frames needed for this adaptation, for a period of F frames, is equal to $Fl^T f^T$ and the corresponding number of successful transmissions that would have taken place otherwise, is $Fl^T f^T \overline{P}^T$. Note that $Fl^O f^O$ frames are also needed by the routing protocol due to movement outside the maximum transmission range. Finally, $F(1 - l^T f^T - l^O f^O)\overline{P}^T$ is the average number of successful transmissions in F slots. $l^T f^T + l^O f^O < 1$ is also required. Note that under topology control the probability that a link failure takes place $(l^O + l^T)$ is higher (on average) than the probability that a link failure takes place under no topology control (l^O).

The condition under which $F(1-l^T f^T - l^O f^O)\overline{P}^T \geq F(1-l^O f^O)\overline{P}^O$ is satisfied, is equivalent to

$$l^T f^T \leq (1 - l^O f^O) \frac{\overline{P}^T - \overline{P}^O}{\overline{P}^T}. \tag{9}$$

It is clear that small values of l^T and f^T are required in order for the system throughput to be increased under topology control. Additionally, large values of $\overline{P}^T$ allow for large values of l^T and f^T to satisfy the aforementioned condition.

For a given value of l^O, l^T may take different values as long as the aforementioned requirements are satisfied. If the resolution is high then higher system throughput may be achieved for the case that nodes are not moving. If nodes are moving (for a given value of l^O), l^T increases as the resolution increases resulting in smaller throughput. Consequently, there is a trade-off between the resolution and the mobility of the nodes in a network regarding the system throughput.

For the example network depicted in Figure 3, system throughput simulation results were obtained in order to illustrate a number of interesting results. Further information regarding the simulator can be found in the following section. Figure 8 presents system throughput simulation results under the Probabilistic Policy as a function of p (for $p = 0$ the system throughput corresponds to that under the Deterministic Policy), while $f^O = f^T = 1$. Figure 8(a) depicts the system throughput under no topology control. $l^O = 0$ corresponds to no movement ($l^T = 0$ as well) and as l^O increases the system throughput decreases. Figure 8(b) depicts the system throughput under topology control for a fixed value of l^O (0.05) and for different values of l^T. As l^T increases, the system throughput decreases.

Figures 9(a) and 9(b) depict the system throughput for the same example network of Figure 3, as a function of l^T, while figures 9(c) and 9(d) depict system throughput as a function of l^O. Figures 9(a) and 9(c) depict the system throughput under the Deterministic Policy, while figures 9(b) and 9(d) depict the system throughput under the Probabilistic Policy (p is set to a value, 0.2,

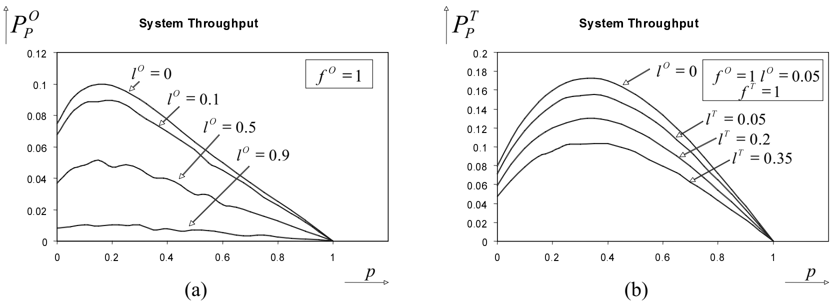

Fig. 8. System throughput under the Probabilistic Policy for the network depicted in Figure 3 (a) under no topology control and (b) under topology control.

for which P_P^O is close to the maximum as it may be observed from Figure 8(a)).

From figures 9(a) and 9(b) it can be observed that as l^T increases, the system throughput under topology control decreases and it becomes smaller than the system throughput under no topology control. It is interesting to note that the actual value of l^T for which $P_D^T = P_D^O$ (l^T close to zero) is smaller than the value of l^T for which $P_P^T = P_P^O$ (l^T close to 0.4). This is due to the fact that the system throughput under the Probabilistic Policy is increased compared to that under the Deterministic Policy.

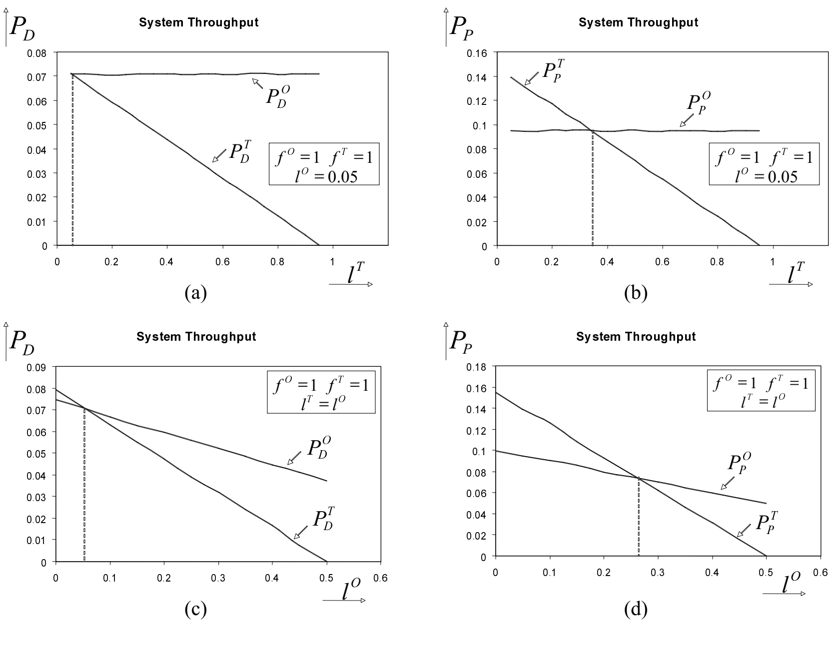

Fig. 9. System throughput as a function of l^T, (a), (b), and as a function of l^O, (c), (d).

From figures 9(c) and 9(d) it may be observed that as l^O increases ($l^T = l^O$) the system throughput decreases. It is clear that the value of l^O for which $P_D^T = P_D^O$ (l^O close to 0.05), is smaller than that value of l^O for which $P_P^T = P_P^O$ (l^O close to 0.27). Also, for $l^O + l^T = 1$ it can seen that the system throughput under topology control is zero. Further simulation results for a number of topologies with different characteristics are presented in the following section.

7 Simulation Results

In the previous section, preliminary simulation results demonstrated the impact of mobility on the system throughput under topology control. The aim of this section is to provide simulation results for four different network topologies with different characteristics with respect to the topology density. Traditional omni-directional antennas will be considered under no topology control, while smart antennas will be considered under topology control. For any transmission $u \to v$, any transmission $\chi \to \psi \in \Theta^O_{u\to v}$ (for any two nodes χ and ψ) is considered that it does not corrupt transmission $u \to v$ under topology control, or $\Theta^S_{u\to v} = \emptyset$, resulting in $S^T_{u\to v} = S^S_{u\to v} \leq S^O_{u\to v}$.

Four different networks of 100 nodes are considered during the simulations, for $D = 10$ and $\overline{|S|}/D = 0.212, 0.424, 0.614$ and 0.866, respectively. The algorithm presented in [14] is used to derive the sets of scheduling slots and the system throughput is calculated averaging the simulation results over 100 frames. Unique polynomials, that correspond to time slot sets Ω_χ, are assigned randomly to each node χ, for each particular topology. The particular assignment is kept the same for each topology category throughout the simulations. Heavy traffic conditions have also been assumed in the sense that data are always available for transmission at each node in the network, for each time slot.

The destination node of a transmitting node is randomly selected among the neighbor nodes of the transmitting node and it remains the same for the 100 frames simulation time. Mobility is also taken into consideration in the sense that the number of successful transmissions in a frame is zero with probability l^O under no topology control and with probability $l^O + l^T$ under topology control. It is also assumed that $f^O = f^T = 1$.

Figure 10 presents simulation results under no topology control. For $l^O = 0$ the nodes are not moving. As l^O increases it can be observed that the system throughput decreases (irrespectively or the topology density). For a given pair of p and l^O, as $\overline{|S|}/D$ increases it is evident that the system throughput decreases. The dependence of the system throughput on the topology density is also shown in Figure 11 for the case that there is no movement in the network. It can be seen that the system throughput (under no topology control) under the Deterministic Policy (P^O_D) decreases almost linearly while under the Probabilistic Policy (P^O_P) decreases almost exponentially. For each topology, p is set to the corresponding value that maximizes P^O_P (Figure 10).

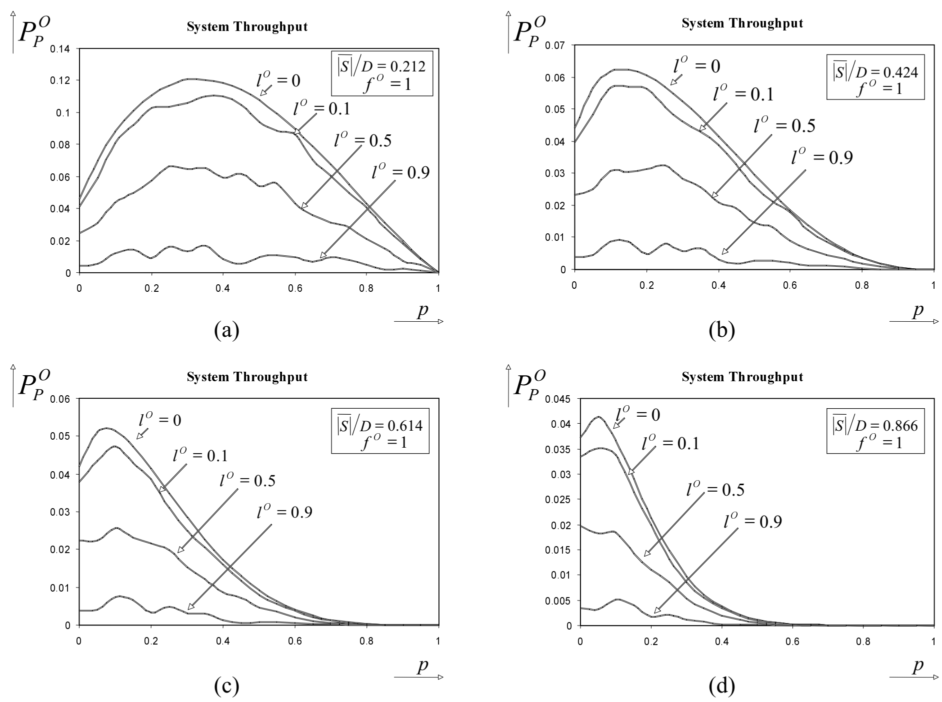

Fig. 10. System throughput (P) simulation results as a function of p for different values of l^O under no topology control (traditional omni-directional antennas).

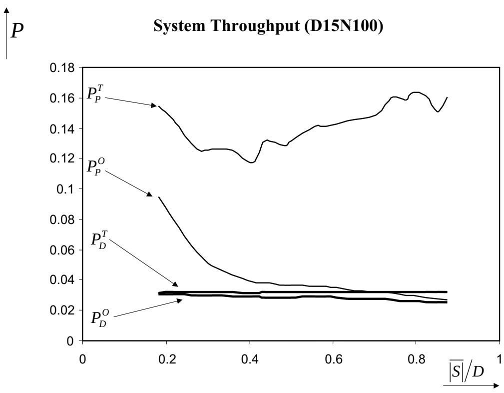

Fig. 11. Maximum system throughput (P) simulation results for both policies, for different values of the topology density $\overline{|S|}/D$, when traditional omni-directional and smart antennas are used (nodes are not moving).

Figure 12 presents simulation results under topology control. For $l^O = 0$ the nodes are not moving, while $l^O = 0.05$ is considered to be the case for any other simulation scenario. As l^T increases, it can be observed that the system throughput decreases. It is interesting to observe that the system throughput is not (strongly) affected by an increase in the topology density. This may also be observed from Figure 11 where the curves corresponding to P_D^T and P_P^T appear not to be affected by $\overline{|S|}/D$. Consequently, the system throughput under topology control is not strongly affected by the topological characteristics like the topology density.

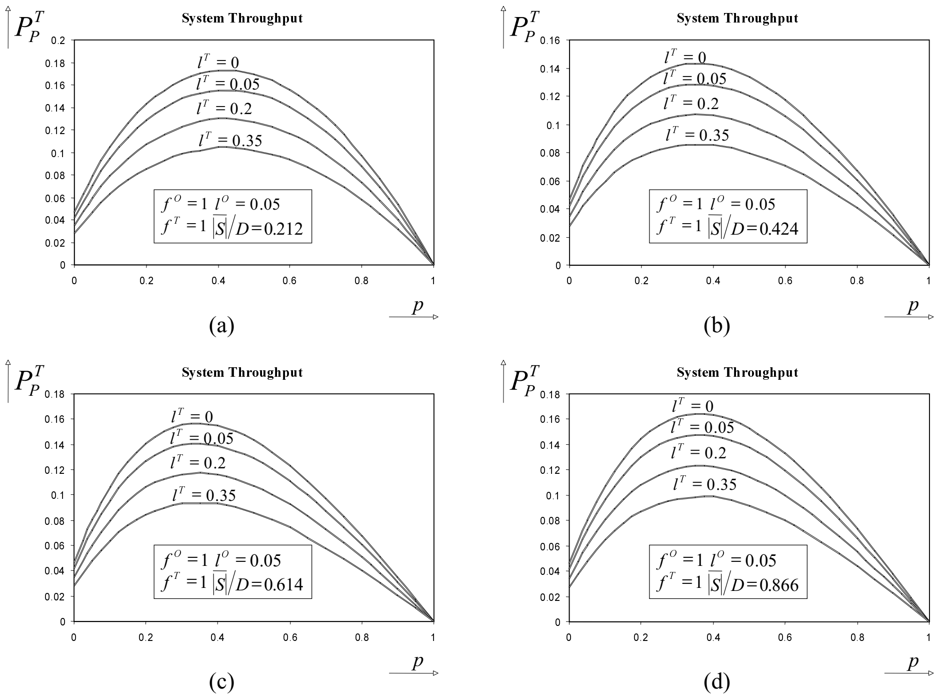

Fig. 12. System throughput (P_P^T) simulation results as a function of p for different values of l^T under topology control (smart antennas).

Figure 13 presents simulation results as a function of l^T and $l^O = 0.05$. For each topology, p is set to the corresponding value that maximizes P_P^O (Figure 10). Under both policies the system throughput is higher under topology control for small values of l^T. As l^T increases, P_D^O and P_P^O are not affected but P_D^T and P_P^T decrease towards zero. $P_D^T = P_P^T = 0$ for $l^O + l^T = 0.9$. It is evident that high values of l^T result in small system throughput. Consequently, under topology control, system throughput improvement (compared to that achieved under no topology control) is possible under low mobility.

8 Conclusions

In this paper the performance of topology-unaware TDMA MAC policies (the Deterministic Policy and the Probabilistic Policy) is studied under sophisticated antennas and power control (topology control) and compared to that under the traditional omni-directional antennas (no topology control). The mobility factor is also taken into consideration and it is shown that topology control (although more vulnerable to mobility than under traditional omni-directional antennas) still improves the system performance under certain mobility conditions. Simulation results validate the claims and expectations.

Initially, topology control is defined and its effect on node transmissions is

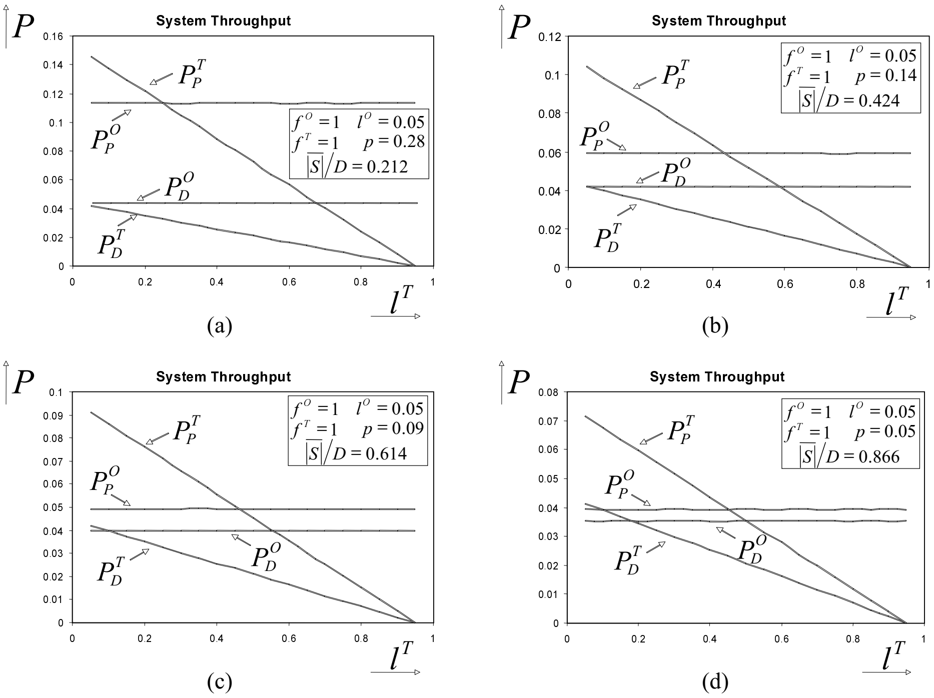

Fig. 13. System throughput (P) simulation results as a function of l^T for both policies with or without topology control.

investigated. In the sequel, both topology-unaware policies are presented and it is shown that, under topology control, increased system throughput is achieved when nodes are static. A discussion regarding mobility has shown that link failures are more likely under topology control than under no topology control, for given mobility conditions, leading to a possibly smaller system throughput. Analytical expressions based on average values are obtained and the conditions are established determining the mobility conditions under which topology control is beneficial. It is also shown that there exists a trade-off between the resolution of the topology control, the mobility and the system throughput achieved. Preliminary simulation results for an example network support the aforementioned arguments.

Simulation results are obtained for four network topologies corresponding to different values of the topology density ($\overline{|S|}/D$). The results demonstrate that the Probabilistic Policy outperforms the Deterministic Policy, even for dense topologies. When smart antennas are used, higher maximum system throughput is achieved and it is observed that the Probabilistic Policy is not so greatly affected by the topology density as under no topology control. However, the simulation results demonstrate the fact that under certain mobility conditions (high mobility) topology control may lead to smaller system throughput, depending on the resolution of the topology control as it is also indicated by the earlier analysis.

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