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Determination of Internal Heat Power during the Induction Heating Based on Solution of Linear Inverse Heat Conduction Problem

Diligenskaya Anna [1, a *], Mandra Andrey [1, b]

[1] 244, Molodogvardeyskaya str., Samara State Technical University, Samara, 443100, Russia.

[a] adiligenskaya@mail.ru, [b] amandra@mail.ru

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Abstract. In this paper an inverse heat conduction problem for estimating the constant value of power of internal heat source during induction heating of paramagnetic material is developed. The experimentally measured temperature data are used as the input for the inverse heat transfer model. The problem is formulated as a problem of optimal control over an object with distributed parameters, the internal heat power is considered as the control action. The problem is reduced to a problem of mathematical programming; the special optimisation method based on the alternance properties of the sought optimum solutions is applied to estimate the internal heat power.

Introduction

The problem of determination of the internal heat source power based on the experimentally measured temperature data is encountered at induction heating study. Inverse heat conduction problem (IHCP) methodology [1-3] is a powerful tool to identify parameters and characteristics of thermophysical processes including induction heating processes [4, 5]. In this case, the heterogeneous equation of heat transfer which consists of the power of internal heat sources in its right side without modeling of the electromagnetic field [6-8] is considered as a basic mathematical model of induction heating process.

In many practical applications of induction heating of paramagnetic materials, when power supplies have appreciably limited frequency adjustability ($\zeta = const$), the heat source distribution along the length or the radius of billet has fixed dependence. In this case, stabilizing an optimal coil voltage represents one of the simplest operational modes of induction heating process and yields a fixed value of total consumed heating power $P(t) = P = const$. The value P depends upon the electrical efficiency and thermal efficiency of induction heating coil, and so on.

The common inverse problem of induction heating, which is solved at the stage of plant operation, is determination of the constant value of internal heat source power from the measured temperature of surface $x^* = R$ or in inner point $x^* \in [0, R)$ of solid.

Statement of the IHCP

Let us consider the features of identification of power of the internal heat source during the induction heating of paramagnetic materials when the dimensional component data, thermophysical properties and boundary conditions are known functions. This problem can be based on solution of IHCP.

The following one-dimensional linear heterogeneous equation of heat transfer is a basic mathematical model of induction heating process of cylindrical body:

$$C\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda\frac{\partial T(x,t)}{\partial x}\right] + \frac{1}{x}\frac{\partial T(x,t)}{\partial x} + F(t,x);\ \ 0 < x < R;\ 0 < t \le t^0 \qquad (1)$$

with initial

$$T(x,0) = T_0(x) \tag{2}$$

and boundary conditions

$$\frac{\partial T(0,t)}{\partial x} = 0;\ \lambda \frac{\partial T(R,t)}{\partial x} = q(t)\,. \tag{3}$$

Here $T(x,t)$ is the temperature field dependent on the time t and the space coordinate x; $F(x,t)$ is a function of distribution of internal heat source density induced by eddy currents per unit time in a unit volume; $q(t)$ represents a flow of heat loss from a surface of the heated body; $T_0(x)$ is the initial temperature profile within the workpiece; $C = const$ is the volumetric heat capacity; $\lambda = const$ is the thermal conductivity.

The internal heat source function $F(x,t)$ has the standard form for a cylindrical workpiece of infinite length:

$$F(x,t) = \frac{P(t)}{R} W(x)\,, \tag{4}$$

where

$$W(x) = \frac{ber'^2\left(\zeta \frac{x}{R}\right) + bei'^2\left(\zeta \frac{x}{R}\right)}{ber\zeta\, ber'\zeta + bei\zeta\, bei'\zeta}\zeta\,. \tag{5}$$

Here,

$P(t)$ is active power absorbed by unit surface of heated body;

ber(y), bei(y), ber'(y), bei' (y) are Kelvin's functions and their first derivatives;

ζ is a specific parameter defined as $\zeta = R\sqrt{\mu\omega\gamma}$;

μ, γ are the permeability and the electric conductivity of material respectively;

$\omega = 2\pi f$, where f is a frequency of coil current.

The following statement of the problem is considered.

Temperature field is measured at a certain fixed control point $x^* \in [0, R]$ on the identification interval $t \in [0, t^0]$ and the temperature dependence $T^*(t)$ is the result of measurement.

In accordance with Eq. 1 – 4, it is necessary to recover the constant value of power of internal heat source $P(t)$ when the law of its spatial distribution $W(x)$ is known.

To solve the formulated problem we used the analytical method of parametric optimization [9] when the set of initial control actions $u(t) = P(t)$ is limited to the class of constant functions $P(t) = P = const$. The initial IHTP is formulated as the problem of the search for the optimum control action $u^* = P^*$ that ensures the minimum error of uniform approximation of temperature field $T(x^*, t)$ to assigned dependence $T^*(t)$ on the interval $[0, t^0] \ni t$ [10].

To determine the constant internal heating power P, the unknown value of $P = P^* \le P_{\max}$, $t \in (0, t^0)$ is viewed as single parameter of optimum control u^*. Thereupon we can formulate the problem of parametric optimization:

$$I(u) = \max_{t \in [0, t^0]} \left| T(x^*, t, u) - T^*(t) \right| \to \min_u\,. \tag{6}$$

Here, $T(x^*,t,u)$ is the exact solution of boundary value problem, Eq. 1 – 3, corresponding to the sought function $u^*(t)$. It has the form of expansion of the temperature field $T(x,t)$ in an infinite series in the eigenfunctions $J_0\left(\eta_m \frac{x}{R}\right)$ of the thermal problem, where $J_0(y)$ is a Bessel function of the first kind and of the zero order.

Solution of problem (Eq. 6) based on special optimization method [9] allows yielding the error of uniform approximation $T(x^*,t,u^*)$ to assigned temperature measurements $T^*(t)$ as low as possible. In accordance with properties of discrepancy $T(x^*,t,u^*)-T^*(t)$, at certain points $t_j \in (0,t^0)$, $j=\overline{1,2}$ on the identification interval $[0,t^0]$ we obtain alternating deviations that are maximum in absolute value. Thus we can write the following system:

$$\begin{aligned} &T(x^*,t_j,u^*)-T^*(t_j)=(-1)^j\cdot\psi\cdot I(u^*), \qquad j=1,2; \quad \psi=\{1\bigcap -1\} \\ &\frac{\partial}{\partial t}\left[T(x^*,t_1,u^*)-T^*(t_1)\right]=0. \end{aligned} \tag{7}$$

Solution of problem (Eq. 6) gives P^* that approximates the sought P_0 with minimum value of uniform deviation of estimated temperature field $T(x^*,t)$ from assigned dependence $T^*(t)$.

Numerical Results and Discussion

The problem on identification of the constant internal heating power during the induction heating of a cylindrical workpiece of infinite length with the radius of R=0.27 m made from the titanium was solved with the following initial data.

At first the direct boundary value problem for nonlinear heterogeneous equation of heat transfer

$$C(T)\frac{\partial T(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T(x,t)}{\partial x}\right]+\frac{1}{x}\frac{\partial T(x,t)}{\partial x}+F(t,x);\ 0<x<R;\ 0<t\le t^0 \tag{8}$$

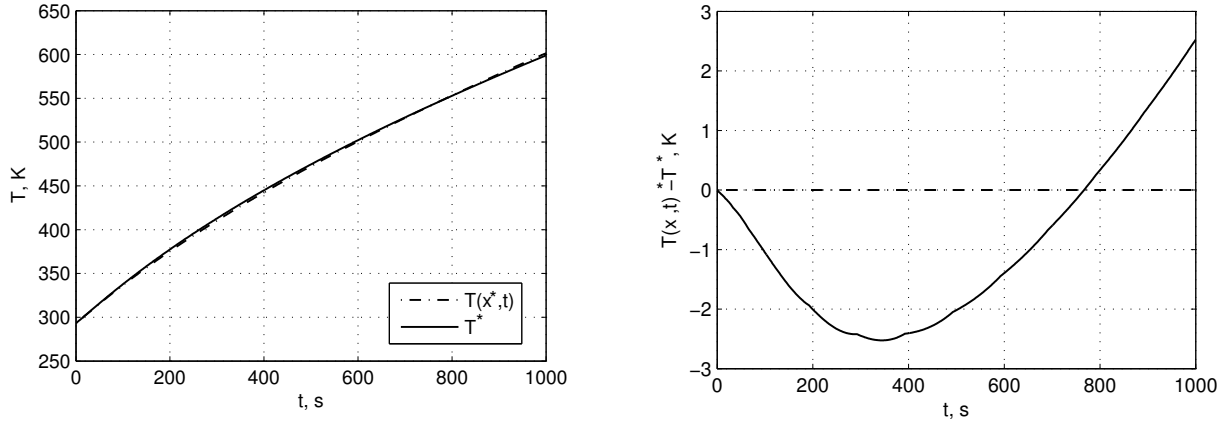
with initial (Eq. 2) and boundary (Eq. 3) conditions at the assigned power $P=P_0=106000\ \mathrm{W/m^2}$ was solved using numerical methods.

The following thermal properties of the material $\lambda(T)=13.4+0.0058\cdot T, \mathrm{W/(m\cdot K)}$, $C(T)=2.276\cdot 10^6+493.1\cdot T, \mathrm{J/(m^3\cdot K)}$, corresponding to titanium, were considered. The initial temperature distribution is uniform $T_0(x)=T_0=293K$. The heat flux density on the assigned interval is $q(t)=1.9\cdot 10^4\left(1-\exp(-7.5\cdot 10^{-4}t)\right)$.

The electromagnetic properties of material are $\mu=1.2567\cdot 10^{-6}, \gamma=0.556\cdot 10^6\ \mathrm{S}$ and the inductor current frequency is f=50 Hz, that corresponds to the value $\zeta=4$ in Eq. 5. The time interval is $t\in[0,1000]$ with a $\Delta=1s$ step and the spatial step is $h=2.7\cdot 10^{-3}\ \mathrm{m}$.

Then the solution $T(x^*,t)$ of the direct problem (Eq. 8, 2, 3) in point $x^*=0.9R=0.243\mathrm{m}$ was taken as the result of temperature measurement, and the linear inverse problem, Eq. 1 – 3, was solved at averaged thermal properties $\lambda(T)=\lambda=16.05\ \mathrm{W/(m\cdot K)}$, $C(T)=C=2.56\cdot 10^6, \mathrm{J/(m^3\cdot K)}$.

Some of the obtained results of numerical solution of the linear inverse heat conduction problem are presented in Fig. 1.



a b

Fig. 1. The exact solution $T(x^*,t)$ and the assigned temperature measurements $T^*(t)$ (a); the error of approximation of the temperature field $T(x^*,t)-T^*(t)$ (b)

A number of facts such as dimension of the solid, the number and location of measurement points influence the accuracy of defining of P_0. Some of the key issues are the effects of the duration of identification process and the location of measurement point on approximation error. Tables 1, 2 present the results of influence of these factors on the inverse heat conduction problem solution of P^*.

Table 1. Relative error $\varepsilon = \left|P_0 - P^*\right| / P_0$ for different duration t^0 of identification process at $x^* = 0.243\,\text{m}$

t^0,s	800	850	900	950	1000	1050	1100	1200	1400
ε,%	3.364	3.224	3.084	2.951	2.820	2.751	2.630	2.397	1.986

Table 2. Relative error $\varepsilon = \left|P_0 - P^*\right| / P_0$ for different location of measurement point x^* at $t^0 = 1000s$

x^*	R	$0.9R$	$0.85\,R$	$0.8\,R$	$0.75\,R$	$0.7\,R$	$0.65\,R$	$0.6\,R$
ε,%	1.884	2.820	2.822	3.273	3.640	3.751	4.022	4.493

The results obtained show that the accuracy of restoration of P essentially depends on location of measurement point and the duration of identification process.

Conclusion

This paper demonstrates the possibility of analytical identification of the constant value of internal heat source power during the induction heating processes of paramagnetic materials from the measured temperature.

The estimation of P is the solution of linear IHCP with the parameterization of sought function $u^* = P^*$ and farther use of special method taking account of the alternance properties of temperature discrepancies.

The results obtained demonstrate that the solution of inverse heat conduction problem on identification of the constant internal heating power has accuracy sufficient for problems in practical heat engineering.

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