AAS 12-167

OPTIMIZATION OF DEBRIS REMOVAL PATH FOR TAMU SWEEPER

Jonathan Missel* and Daniele Mortari†

This paper provides a path optimization strategy for debris removal satellites, focusing on the proposed *TAMU Sweeper* mission. The optimized solution is a set of n satellite maneuvers, n debris captures, and n debris ejections. Ejected debris are sent to lower perigee orbits or to re-enter the atmosphere. Optimization is performed using an evolutionary algorithm that solves the combinatory problem of selecting the debris interaction order, transfer trajectories, and sequence timing, while optimizing fuel cost and effectiveness towards debris mitigation. For a fixed time interval and number of debris interactions, the most efficient and effective sequence is sought. The broader goal of this work is to evaluate feasibility of such missions. Our early findings show that the *TAMU Sweeper* technique directly removes 81% of the debris encountered through re-entry, and significantly lowers the perigees of the rest. It does so while using 40% less fuel than "traditional" successive rendezvous approaches.

INTRODUCTION

Orbital debris is a well established and universal concern for space flight, and forecasts predict the problem will grow exponentially worse. China's successful anti-satellite test in 2007, and the collision of Cosmos 2251 and Iridium 33 in 2009 have illuminated the issue in the public eye [1, 2]. In LEO alone, 500,000 pieces of manmade clutter larger than 0.04 inches endanger human and craft alike [3] Addressing this issue is nontrivial. Traditional satellites and mission structures are not efficient enough; successively transferring orbits to collect debris consumes excessive fuel. Several ideas have been proposed to interact with debris at a distance (such as lasers and ion guns); however, they are often viewed as potential weapons, eliminating them as options due to political sensitive [3, 4].

To remedy this situation, the recently proposed *TAMU Sweeper* mission structure plans to put a twist on traditional missions that will improve their fuel economy to make them feasible. The full explanation of this debris removal technique is presented in Ref. [5]. In the same way that gravity assists take advantage of existing momentum in the broader system to extend the capabilities of a spacecraft, *TAMU Sweeper* steals momentum from the debris field to save fuel. The key to unlocking the advantages of these opportunistic methods is executing specific and well timed maneuvers. Our objective in this paper is to establish the cornerstone of this technique: a trajectory sequence optimization method that effectively and efficiently interacts with debris for removal.

*PhD Student, 301B Reed McDonald, Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, Tel.: (585) 298-1483. E-mail: JWMISSEL@TAMU.EDU

†Professor, 746C H.R. Bright Building, Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, Tel.: (979) 845-0734, Fax: (979) 845-6051, AIAA Associate Fellow. E-mail: MORTARI@TAMU.EDU

Consider the initial orbit of a spacecraft using the *TAMU Sweeper* mission structure. Here, the satellite captures and ejects debris, lowering their perigee altitudes. The momentum from these interactions is exploited to save fuel costs on successive transfers. Capture is executed not only by a large-scale intersection of orbits, but also a small-scale intersection of one of the satellite's collectors and the debris. To eject the debris (as specified by the optimization results), the satellite arm lengths change to control its spin rate, and the associated tangential ejection speed of the debris. The ejection angle is a matter of timing the debris' release from the ends of the spinning satellite. Debris is ejected into a new orbit with reduced perigee, or one that will eventually decay into the atmosphere.

Upon capture and ejection of each particle, the satellite will experience a reaction impulse. These impulses are considered in the optimization problem, and will work toward saving on fuel requirements. The remaining task is to quantify the effectiveness of the initial orbit over an assigned maximum time interval, $T_{\max}$, as a total of n objects are captured and ejected (see Figure 1). This constitutes the focus of this paper.

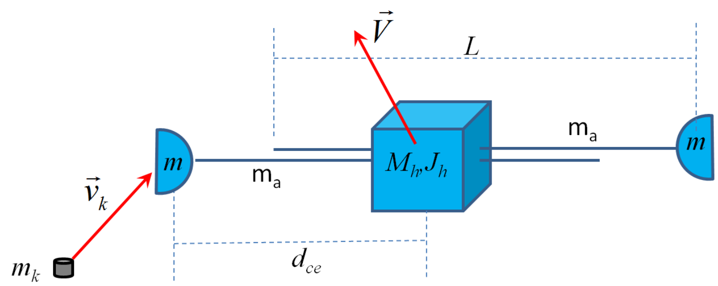

Figure 1. System Definition

To analyze effectiveness, an optimality gain function, G, is built as the ratio between what we want to maximize, the debris de-orbiting and/or perigee lowering, and what we want to minimize, the total fuel spent to accomplish the mission, $\Delta V_{\text{tot}} = \sum_{k=1}^{n} |\, \Delta \mathbf{V}_k \,|$.

$$G = \frac{\displaystyle\sum_{k=1}^{n_r} \Delta \bar{r}_{pk} + \sum_{k=1}^{n-n_r} \Delta r_{pk}}{\displaystyle\sum_{k=1}^{n} |\, \Delta \mathbf{V}_k \,|} \tag{1}$$

where n is the total number of debris captured and ejected in $T_{\max}$ (n is also the number of impulses provided, see Figure 2), $\Delta \bar{r}_{pk}$ is the debris' perigee distance to the atmosphere, $n_r \leq n$ is the number of debris whose perigee reduction was $\Delta r_{pk} \geq \Delta \bar{r}_{pk}$, and $(n - n_r)$ is the remaining number of debris whose perigee reduction was $\Delta r_{pk} < \Delta \bar{r}_{pk}$. By this classification, the most attractive solution is that which maximizes G.

The dimensionality of the optimization problem involves finding $6n$ variables. These are, (see Figure 2):

- n impulse, n capture, and n ejection times (t_{vk}, t_{ck}, t_{ek},)
- n debris selection indices (I_k) contained in the debris catalog
- n arm length variations (from capture length d_{ck} to ejection length d_{ek})
- n debris ejection angles (ejection angle α_{ek})

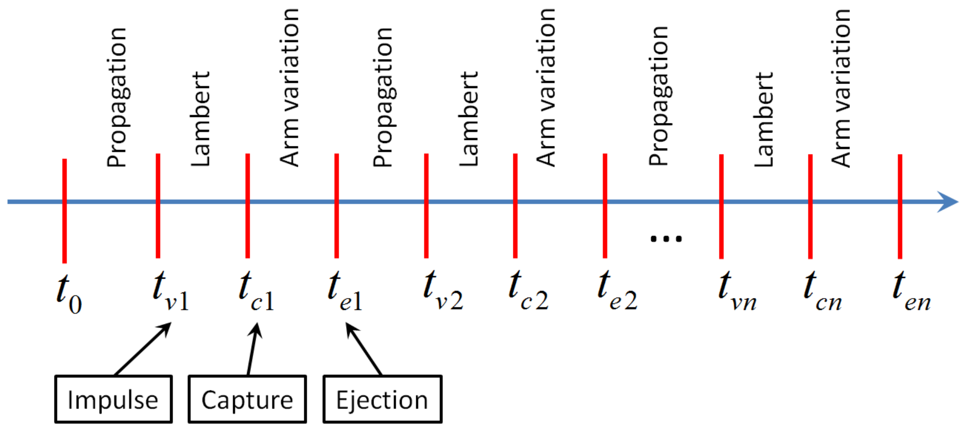

Figure 2. Times Sequence

Put simply, the optimization problem loops n times through the following activities: 1) solves the combinatory problem that identifies the debris to capture (I_k), 2) identifies the time t_{vk} of the $\Delta \mathbf{v}_k$ impulse of a rendezvous maneuver to 3) capture the debris at time t_{ck}, and 4) eject it at time t_{ek} into a trajectory lowering its perigee (angle α_{ek}).

Debris Selection by Estimated Cost

We want to minimize the mission fuel cost (sum of n rendezvous impulses $|\Delta \mathbf{V}_k|$) as part of our optimality function. However, calculating and searching through the fuel cost for all possible combinations of sequences, timings, and trajectories is a massive and wasteful exercise. To make this reasonable, the process must first be streamlined to focus on the most promising candidates. We do this by carefully choosing which debris to encounter by assigning higher probability to to be selected to “closer” debris, in term of approximated cost to rendezvous, $\Delta V_{k\text{tot}}$.

Since computing the exact cost for all the debris is computationally tasking, an estimated value of $\Delta V_{k\text{tot}}$ is used. It is based on two approximations. The first considers the debris and satellite orbits as circular, using information on the angular momenta, $\mathbf{h}_k = h_k\,\hat{\mathbf{h}}_k$ and $\mathbf{h} = h\,\hat{\mathbf{h}}$, respectively. The second approximation considers the total position rendezvous cost as only a plane change plus a Hohmann transfer maneuver. The cost to match velocities in the rendezvous is disregarded since we are capturing with plastic collisions, and relative velocities are desired.

From the angular momentum modulus we obtain the equivalent radii and velocities of circular orbits for the satellite and all the debris. These are:

$$\begin{cases} \text{debris}: & r_k = h_k^2/\mu \quad v_k = \mu/h_k \\ \text{spacecraft}: & R = h^2/\mu \quad V = \mu/h \end{cases} \tag{2}$$

Using these equivalent radii, the Hohmann transfer cost

$$\Delta V_{hk} = \sqrt{\frac{\mu}{r_k}} \left| \sqrt{\frac{2R}{R + r_k}} - 1 \right| + \sqrt{\frac{\mu}{R}} \left| \sqrt{\frac{2r_k}{R + r_k}} - 1 \right| \tag{3}$$

is evaluated. From the direction of the angular momentum, the angle between satellite's and debris' orbital planes is computed, $\cos\vartheta_k = \hat{\mathbf{h}} \cdot \hat{\mathbf{h}}_k$. This intersection yields two points in the orbit where the satellite can change planes. The furthest one has lower cost requirements, so it is used to calculate plane change cost as

$$\Delta V_{ik} = \begin{cases} = 2V \sin\left(\frac{\vartheta_k}{2}\right) & \text{if} \quad R > r_k \\ = 2v_k \sin\left(\frac{\vartheta_k}{2}\right) & \text{if} \quad R < r_k \end{cases} \tag{4}$$

The approximated cost $\Delta V_{k\text{tot}} = \Delta V_{hk} + \Delta V_{ik}$ lets us assign probability. Since the selection probability must be greater for those debris characterized by smaller $\Delta V_{k\text{tot}}$, the following probability (p_k) and its cumulative probability ($\bar{p}_k$) functions,

$$p_k = \frac{1}{\Delta V_{k\text{tot}} \displaystyle\sum_{i=1}^{n} \frac{1}{\Delta V_{i\,\text{tot}}}} \qquad \text{and} \qquad \bar{p}_k = \sum_{i=1}^{k} p_i, \tag{5}$$

are introduced. Eq. (5) guarantees the probability constraint $\displaystyle\sum_{k=1}^{n} p_k = 1$.

Handling Debris for Ejection

Between capture and ejection, the satellite has two objectives: estimate the debris mass and prepare for ejection. Once each debris is captured, the spin rate and center of mass of the satellite will change. This change can be observed, and estimates of the debris mass can be backed out. An analysis of the sensitivity of these estimates to the physical design parameters of the hardware is currently underway. After the dynamics are measured for mass estimation, the satellite must prepare for ejecting the debris; it does so by varying its arm lengths to achieve the desired tangential ejection velocity. This section details the representation of how the satellite prepares for ejection in the context of the mission simulation. To start off, we will be clear on the assumptions made.

Assumptions:

- Capture and ejection happen in planar motion.
- There is only one arm length change per cycle (n total).
- The arm length changes instantaneously half way (time) between capture and ejection.
- Both arms change in unison.

In reference to Figure 1, the simulation adopted the following values.

$$\begin{aligned} M_h = 200 \text{ kg}, \qquad m = 10 \text{ kg}, \qquad L = 5 \text{ m}, \\ m_a = 5 \text{ kg}, \qquad \text{and} \qquad J_h = \frac{M_h d_h}{24} \text{ kg m}^2 \end{aligned} \tag{6}$$

The notation selected to indicate the velocities directly before and after capture and ejection is shown in Figure 3. In particular, the upper-case (V) indicates the satellite, lower-case (v) the debris, and an apostrophe indicates "after capture" or "before ejection."

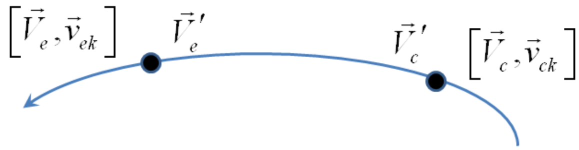

Figure 3. Capture and Ejection

The velocity after capture is derived from the linear momentum equation

$$m_k\,\mathbf{v}_{ck} + M\,\mathbf{V}_c = (M + m_k)\,\mathbf{V}_c' \tag{7}$$

where $M = M_h + 2m + 2m_a$ is the total mass of the spacecraft. The total satellite MOI (with arms elongated at distance d_c) is

$$\begin{aligned} J(d_c) &= J_h + 2md_c^2 + \int_{d_c-2L}^{d_c} m_a x dx + \int_{-d_c}^{2L-d_c} m_a x dx = \\ &= J_h + 2d_c^2(m + \frac{m_a}{3}) \end{aligned} \tag{8}$$

Just before capture, the satellite is spinning with angular velocity $\omega_{ck} = \omega_{e(k-1)}$ (in this simulation no arm length variation is considered between previous ejection and new capture). The angular velocity after capture (ω_{ck}') is obtained from the angular momentum equation (planar motion)

$$J(d_{ck})\,\omega_{ck} = \left[J(d_{ck}) + m_k d_{ck}^2\right]\,\omega_{ck}' \tag{9}$$

where it is assumed that the debris' relative velocity to the basket at capture is zero. This will be enforced by a controller, which was already successfully simulated. Through actuation of the arm lengths it controls the spin of the satellite to ensure the debris and basket intersect for capture. It also specifies an intersection with zero relative impact velocity, which helps ease concerns of vibration. Though we have completed this work, the controller was not incorporated in this simulation for simplicity.

While the arms are varying in length, preservation of angular momentum allows us to write

$$\left[J(d_{ck}) + m_k d_{ck}^2\right]\omega_{ck}' = \left[J(d_{ek}) + m_k d_{ek}^2\right]\omega_{ek}' \tag{10}$$

With the debris onboard, the distance from the center of the satellite to the combined center of mass is $L_{cm} = \dfrac{m_k d_{ek}}{M + m_k}$. With reference to Figure 4, the relative debris velocity is

$$\Delta\mathbf{v}_k = \omega_{ek}'(d_{ek}' - L_{cm})\hat{\mathbf{i}}_\alpha \tag{11}$$

in the body frame. Rotated into the inertial frame, this becomes

$$\Delta\mathbf{v}_k = -\omega_{ek}'(d_{ek} - L_{cm})\sin(\alpha)\hat{\mathbf{i}} + \omega_{ek}'(d_{ek} - L_{cm})\cos(\alpha)\hat{\mathbf{j}} \tag{12}$$

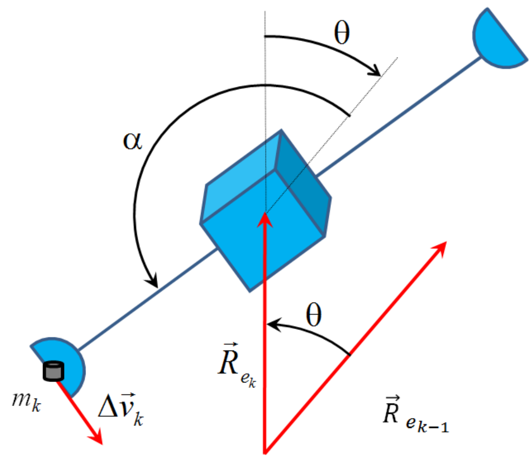

Figure 4. Ejection Angle

where

$$\alpha_k = \alpha_{k-1} + \int_{t_{k-1}}^{t_k} \omega dt + \cos^{-1}\left(\frac{\mathbf{R}_{e_k-1} \cdot \mathbf{R}_{e_k}}{\|\mathbf{R}_{e_k-1}\|\,\|\mathbf{R}_{e_k}\|}\right) \tag{13}$$

is the ejection angle defined from the radial direction at the time of the previous ejection. Our assumption that the arms change lengths instantaneously and at the half way point of the time between capture and ejection makes the integration trivial.

Right after ejection, the new debris velocity is

$$\mathbf{v}_{ke} = \mathbf{V}'_e + \Delta\mathbf{v}_k \tag{14}$$

while the new satellite velocity ($\mathbf{V}_e$) can be computed from linear momentum as

$$(M + m_k)\,\mathbf{V}'_e = M\,\mathbf{V}_e + m_k\,\mathbf{v}_{ke} \tag{15}$$

Angular momentum conservation allows the angular velocity after ejection, ω_{ek}, to be computed as

$$\omega_{ek} = \omega'_{ek}\left(1 - \frac{m_k^2 d_e^2}{J_{de}\,(M + m_k)}\right) \tag{16}$$

Capturing and ejecting the debris provides the satellite with the first two of three impulses needed to intersect with the next debris targeted in the sequence. The cycle is repeated n times over the course of the mission.

Procedure Summary

The cost function can be summarized as follows:

1. generation of all the subsequent times (impulse, capture, ejection) ($3n$ genes). This is done using a cumulative vector which is then normalized to $T_{\max}$;

2. debris selection (n genes): if $\bar{p}_{k-1} \leq X \leq \bar{p}_k$ then the k-th debris is selected for encounter;
3. satellite orbit propagation to the impulse time and selected debris orbit propagation to the capture time;
4. use of Lambert solver to perform transfer to the selected debris at capture time (evaluate the Δv_k at impulse time);
5. evaluation of the capture relative velocity: if the relative velocity is too high the capture is not admissible and the cost function exits with a default high value;
6. selection of the ejection arm length and angle ($2n$ genes);
7. satellite orbit is propagated to the ejection time, and
8. the cost function is updated.

This is done to capture and eject n debris, that is n times.

SIMULATION RESULTS

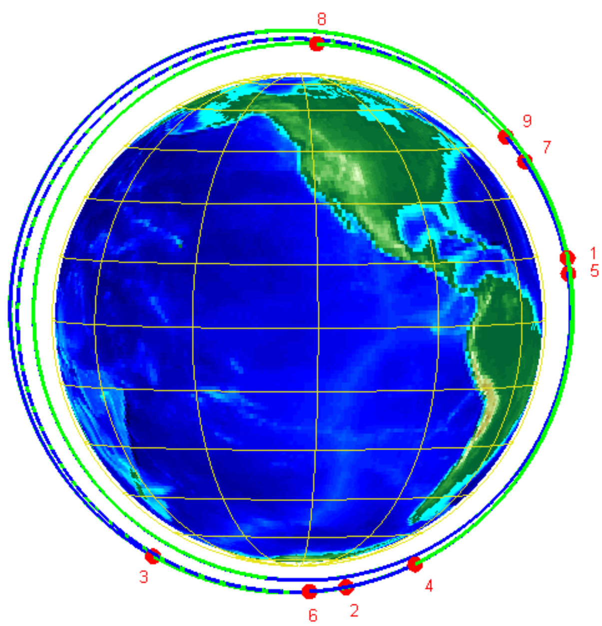

Figure 5. Example Sequence Solution

Following the procedure detailed above, we built a full mission segment simulation. For a specified time span and number of debris interactions, the optimal debris sequence and satellite path is determined. This simulation was limited to the Iridium-33 debris cloud, and uses a 50 generation Genetic Algorithm (GA) with a population of 50, crossover fraction of 0.6, and a mutation rate of 0.2. The two-line element catalogues from NASA's Debris Assessment Software (DAS) were

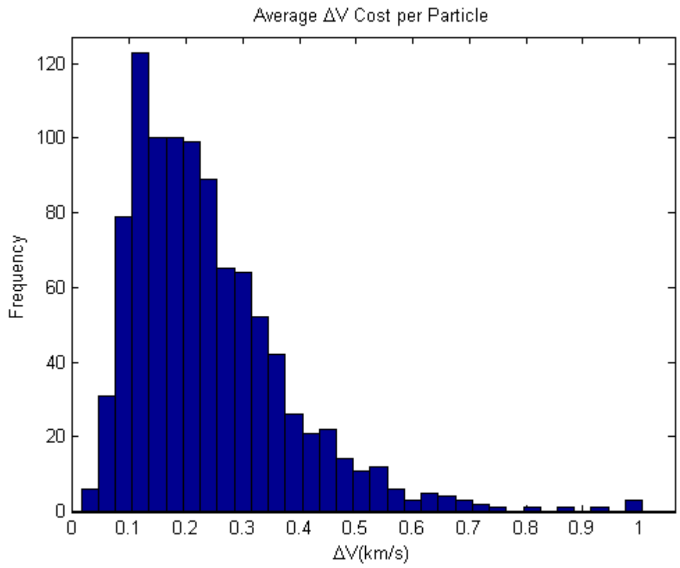

Figure 6. ΔV Cost per Particle Distribution

used to simulate the debris fields [6]. The GA has random components, and may generate different solutions each time. As an illustrative example, Figure 5 shows a representative orbit generated to interact with 3 debris in 1 day. Numbered are the sequential events of impulse, capture and eject for all 3 interactions.

At this point it should be noted that the entire mission would not last only one day, and collect three particles. Rather, projecting a short time into the future is part of a strategy. Currently, the debris masses are unknown. For the Iridium cloud, the total mass is known (the mass of the intact satellite), and the majority of the particles are being tracked (466 pieces). Based on this we can guess at a mass distribution, and estimate the probability of the mass of a specific particle that will be encountered. *TAMU Sweeper* provides a way to determine the mass of debris once it is captured. This information can then be used to update the estimated mass distribution for improved accuracy on later encounters. It is a bit like chess. A good player will think three moves ahead, but much beyond that is a waste of brain power. Throughout the game new information is used to adapt the projected strategy. Similarly, this optimization can be rerun every time the estimated mass distribution is updated. This improves future solutions, and greatly simplifies the optimization process by only considering a few interactions at a time.

By nature, the optimisation technique used here guesses, evaluates and adapts solutions; the mathematical optimal solution is not guaranteed. In addition, this simulation was run on a consumer laptop which restricts how aggressively the GA can be set up (i.e. population size). Accordingly, we were interested in the statistical accuracy of our results. To investigate this, we conducted 1,000 runs with the same initial conditions to see how the results were distributed.

Figure 6 is a histogram showing the average ΔV per debris distribution for each run. The lowest bins represent the most cost effective solutions; for this reason, these would be the candidate sequences for flight. When the GA is run with a higher population size, the standard deviation is

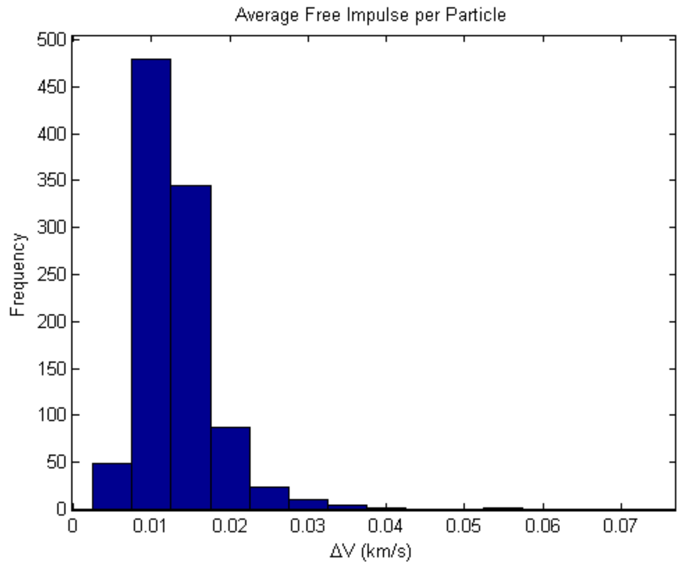

Figure 7. Distribution of Free ΔV from Capture and Ejection

much smaller, and the mean shifts towards the most efficient solution. However, the trade-off is increased computational time. Figure 7 shows the distribution of the average free impulse (per particle) provided by the combined capture and ejection of each particle. The best solution of those plotted yielded an average required impulse of 28.1 m/s per debris, with an average free impulse of 10.8 m/s. That means the mission required 28% less ΔV, not include the impulses saved by avoiding velocity matched rendezvous.

In terms of a sample mission, lets assume we have a purpose built satellite with a total ΔV of 4.3 km/s available for enacting these transfers. Using the results above, limited to the Iridium-33 debris field, and ambitiously specified to interact with three particles per day, the total mission would last 51 days, and interact with 153 particles. That is 1/3 of the total 466 particle Iridium debris field. Though these are example figures based on estimates from our preliminary findings, they are promising.

To further stretch the fuel economy, the allowable mission time can be extended. Figure 8 shows the ratio of fuel burned to mission time for almost 1,700 different simulations. With the exception of a few rogue results (where the optimization failed), the relation is fairly flat, suggesting a predictable decrease in fuel cost for increased mission times. Greater time broadens the range of trajectories, likely including more efficient paths. Also, extending the mission time does not statistically influence the free impulse magnitudes; therefore, a higher percentage of the mission is free! Furthermore, the mass and geometry of the satellite are directly related to the momentum exchanges in the free impulses, and can be designed to maximize this effect. The hardware in this simulation was somewhat arbitrarily specified, so even more promising results are expected.

The main goal of the *TAMU Sweeper* concept is to stretch the on-board fuel to make debris capturing missions feasible. It does this by eliminating the cost to complete a full velocity matched rendezvous, exploiting the momentum exchange from capture and ejection, and keeping the net

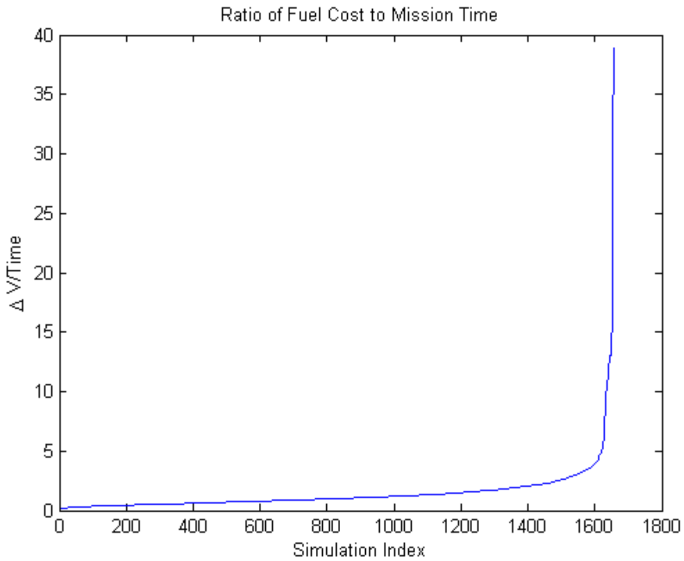

Figure 8. Ratio of Mission Cost to Mission Time

mass of the satellite down by ejecting captured debris. As a measure of success, this method would ideally be compared with the "traditional" concept of rendezvousing with debris, collecting them and repeating. However, the comparison is not straight forward. Each technique has different optimal sequences, and performance is dependant on many unshared factors. That being said, our results suggest that the average *TAMU Sweeper* mission only uses (about) 60% of the fuel burned by the "traditional" mission collecting the same number of debris, over the same time frame.

Finally, the most fundamental objective of this work is obviously to remove debris. By design, the ejection process serves two purposes: to provide a free impulse to the satellite, and to remove or lower the perigee of the debris. However, an efficient mission is useless if it does not mitigate the debris problem. The Monte Carlo simulation showed an average perigee reduction of 723 km; a satisfying result considering it put 81% of the debris encountered on direct trajectories to burn up in the atmosphere. The rest have greatly reduced perigees, and would most likely re-enter after one more encounter.

CONCLUSIONS

TAMU Sweeper introduced the idea that an optimal path exists where the momentum of a debris field can be used to help fuel a satellite designed to remove it, the only trick left was to find it. The goal of this paper was to develop a trajectory optimization process that would search for this path, and demonstrate the benefits of *TAMU Sweeper*. The results presented above are merely preliminary, but they are already showing the feasibility of this technique. As the optimization methods are refined, the results can only improve. More demanding optimization can be executed through super-computing, or possibly parallel computing (with come clever coding). The number of attractive trajectory combinations is vast, yet even in our limited simulation, that was restricted to one debris cloud, we were able to significantly improve the fuel economy of the over all mission.

Conservative results were acquired by choosing an active mission plan (3 debris per day), and still showed that 28% of the total impulse was free. Compared to a traditional mission, this optimization required only 60% of the fuel for the total mission. This is promising evidence supporting the feasibility *TAMU Sweeper*.

REFERENCES

[1] J. Pearson, J. Carroll, E. Levin, and J. Oldson, "EDDE: ElectroDynamic Debris Eliminator for Active Debris Removal," *NASA-DARPA International Conference on Orbital Debris Removal*, 2009.

[2] T. Kelso, "The Parameterization of the Rotation Matrix using Redundant Attitude Coordinates," *American Astronautical Society*, Vol. 135, 2010, pp. 1–2.

[3] D. Kushner, "Cluttered Space," *Popular Science*, Vol. Aug. 2010, 2010, pp. 60–64.

[4] C. Bombardelli and J. Pelaezy, "Ion Beam Shepherd for Contactless Space Debris Removal," *Journal of Guidance, Control and Dynamics*, Vol. Dec. 2010, 2010, pp. 1–13.

[5] J. Missel and D. Mortari, "Sling Satellite for Debris Removal with Aggie Sweeper," *Advances in the Astronautical Sciences*, Vol. 140, 2011, pp. 60–64.

[6] *Debris Assessment Software*. Houston, Texas: NASA Orbital Debris Program Office, 2.0.1 ed., 2011.