COMBINING HARMONIC FUNCTIONS AND RANDOM SAMPLING IN ROBOT MOTION PLANNING [1]

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Abstract: This paper presents a robot motion planning method that combines harmonic functions and random sampling. The approach uses harmonic functions efficiently computed over a non-regular grid decomposition of Configuration Space. Random sampling allows the use of the harmonic functions approach without the explicit knowledge of the robot's Configuration Space. The combination of both results in a resolution and probabilistic complete planner.

Keywords: Robotics, path planning, robot navigation, random searches, harmonic functions.

1. INTRODUCTION

Robot motion planning is usually done in the robot's Configuration Space ($\mathcal{C}$-space), were the robot is mapped to a point and the obstacles in the workspace are enlarged accordingly ($\mathcal{C}$-obstacles). The use of potential field methods is a practical approach to robot motion planning, despite its drawbacks as the creation of minima other than the goal and the difficulty to cope with large $\mathcal{C}$-spaces. To cope with the local minima problem, the use of harmonic functions was proposed by Connolly *et al.* (1990). In order to efficiently compute harmonic functions in large $\mathcal{C}$-spaces, a method was proposed based on a non-regular grid decomposition of $\mathcal{C}$-space (Rosell and Iñiguez, 2002).

The random sampling scheme for path planning (Barraquand *et al.*, 1997) is based on the random sampling of the robot's Configuration Space to avoid its complex analytical computation. This scheme gives rise to probabilistic complete planners applicable to any type of robots. This scheme has been applied on a potential-field planner in order to escape from local minima (Barraquand *et al.*, 1992), and on a roadmap planner (Kavraki and Latombe, 1994). This latter approach has been called PRM (Probabilistic Roadmap Method) and has given very good results in path planning problems with many degrees of freedom.

The motivation of this paper is to take advantage of both motion planning methods by combining them. The use of random sampling allows the use of the harmonic functions approach without the explicit knowledge of the robot's Configuration Space. The combination of both results in a resolution and probabilistic complete planner. A similar proposal is that of Yang and Valle (2000) that define a navigation function over a collection of spherical balls that cover the free $\mathcal{C}$-space.

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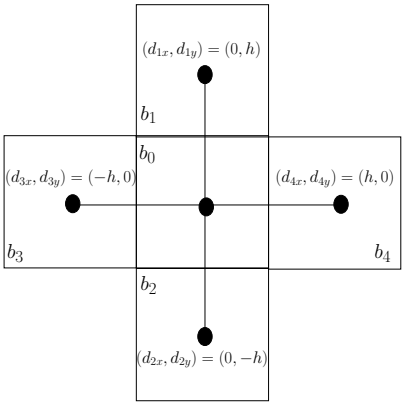

Fig. 1. Cell b_0 with neighbor cells b_1 to b_4.

The paper is structured as follows. After this introduction, Sections 2 and 3 review, respectively, the use of harmonic functions and the use of random sampling schemes for path planning. Section 4 presents the approach that combines both methods, Section 5 explores future improvements and, finally, Section 6 summarizes the work.

2. HARMONIC FUNCTIONS FOR PATH PLANNING

2.1 *Harmonic Functions*

An harmonic function ϕ on a domain $\Omega \subset \Re^n$ is a function that satisfies Laplace's equation (Connolly and Grupen, 1993):

$$\nabla^2 \phi = \sum_{i=1}^{n} \frac{\partial^2 \phi}{\partial x_i^2} = 0 \qquad (1)$$

Numerical solutions to Laplace's equation can be found using finite difference methods (Kreyszig, 1990). For $n = 2$, let u, u_x, u_y, u_{xx}, u_{yy} and u_{xy} be, respectively, a regular sampling on a grid (with spacing h) of ϕ and its derivatives. Let also f_m represent the evaluation of any given function f at the center (x_{b_m}, y_{b_m}) of a cell b_m of the grid, i.e. $f_m = f(x_{b_m}, y_{b_m})$. Then, being b_0 and b_i two neighbor cells with the center of b_i located at (d_{ix}, d_{iy}) with respect to the center of b_0 and using Taylor's theorem, u_i is expressed as:

$$u_i = u_0 + d_{ix}u_{x0} + d_{iy}u_{y0} + \frac{1}{2}d_{ix}^2 u_{xx0} + \frac{1}{2}d_{iy}^2 u_{yy0} + d_{ix}d_{iy}u_{xy0} \qquad (2)$$

Summing the values of u_i for the four Manhattan neighbors of b_0 shown in Figure 1, the discrete form of Laplace's equation is obtained:

$$h^2(u_{xx0} + u_{yy0}) = u_1 + u_2 + u_3 + u_4 - 4u_0 = 0 \qquad (3)$$

The solution of this equation is found by relaxation methods, like the Jacobi iteration method that simultaneously replaces, at iteration k, any non-boundary node of the grid with the average of its $2n$-Manhattan neighbors computed at iteration $(k-1)$:

$$u_0^k = \frac{1}{4}[u_1^{k-1} + u_2^{k-1} + u_3^{k-1} + u_4^{k-1}] \qquad (4)$$

where u_0^k is the harmonic function value of cell b_0 computed at iteration k. The generalization of equation (4) to more d.o.f. is:

$$u_0^k = \frac{1}{2n}\sum_{i=1}^{2n} u_i^{k-1} \qquad (5)$$

The initialization phase is done using the Dirichlet boundary condition, that fixes the $\mathcal{C}$-obstacle cells at a high value and the goal cell at a low one.

Harmonic functions give rise to a practical path planning approach with the following main features:

- A potential field with a unique minimum.
- An efficient update of the potential field.
- Completeness up to the discretization error in the environment.
- A robust and reactive behavior.

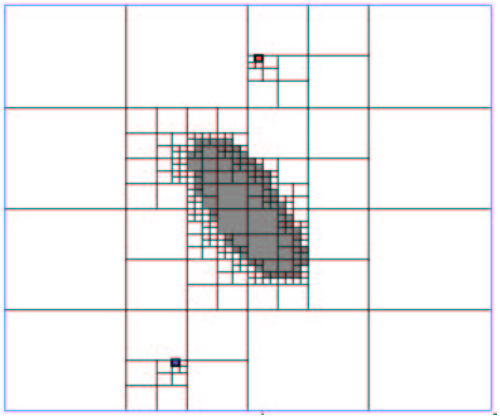

Fig. 2. Hierarchical decomposition of $\mathcal{C}$-space.

2.2 *Hierarchical discretization*

Using a non-regular grid decomposition of $\mathcal{C}$-space, the relaxation methods to compute the harmonic functions converge quicker and less memory resources are required, which allows the use of this path planning approach to higher dimensional $\mathcal{C}$-spaces.

Let $\mathcal{G}_k$ be a grid at a given level k in the hierarchy and b_k be a cell with sides of size s_k. A coarse regular grid, $\mathcal{G}_0$, is first considered with $s_0 = \Delta 2^m$ and $m \in \aleph^+$. Then, the cells over the $\mathcal{C}$-obstacle border are refined by hierarchically halving the spacing, until b_m cells are reached, with $s_m = \Delta$ (Figure 2).

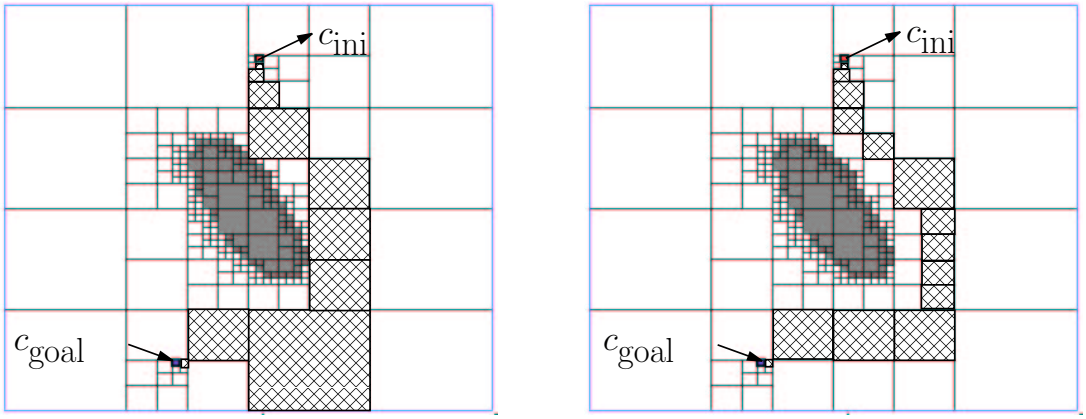

Fig. 3. Path from c_{ini} to c_{goal} computed using harmonic functions (left). Modified path without cells neighboring to $\mathcal{C}$-obstacles (right).

The harmonic function over $\mathcal{G}_k$ is computed by sequentially applying the relaxation methods to the cells of the initial regular grid $\mathcal{G}_0$. When an s_0-cell is encountered that has been subdivided, the harmonic function is computed on the descendant cells, following a depth-first procedure, up to the smallest subdivision (Rosell and Iñiguez, 2002). A similar approach is followed in (Zelek, 2002).

2.3 *Path planning*

The solution path is obtained in two steps. First, a sequence of free cells (channel) is found from the cell containing the initial configuration to the cell containing the goal one. Then any path contained within the interior of the channel is a valid solution free path.

Since the harmonic function has been computed such that the goal is the unique minimum, the channel is searched by selecting, at each step from the initial cell to the goal one, the neighbor cell with minimum harmonic function value (Figure 3 left). If the path contains cells that neighbor on $\mathcal{C}$-obstacles, those cells can be hierarchically subdivided, the harmonic function recomputed and the solution path searched again (Figure 3 right). As a result a channel of free cells with non-zero clearance is obtained.

3. THE RANDOM SAMPLING SCHEME FOR PATH PLANNING

The random sampling scheme for path planning was introduced to cope with large dimensional configuration spaces, since in these cases robot path planning is a hard problem with exponential time complex solutions in the number of dimensions, that preclude any useful application. Using the random sampling scheme probabilistic complete planners can be obtained.

The general random sampling path planning scheme works as follows (Barraquand *et al.*, 1997). First, samples of $\mathcal{C}$-space are randomly obtained. Then, those that lie in free $\mathcal{C}$-space are stored as nodes of a graph and adjacent [2] configurations are connected by links of the graph. If the initial and the goal configuration belong to the same connected component of the graph, there exists a free path between them. Otherwise, more samples are needed.

Two planners that follow the random sampling scheme are the potential-field based planner introduced by Barraquand *et al.* (1992), and the roadmap planner introduced by Kavraki and Latombe (1994).

The potential-field based planner alternates *down motions* that follow the negated gradient of the potential field and *escape motions* randomly computed in order to flee from local minima.

The probabilistic roadmaps method (PRM) randomly samples the robot's configuration space and, with the aid of a local planner, generates a roadmap with the nodes corresponding to free configurations. The initial and the goal configurations are connected to the roadmap and then the roadmap is searched for a feasible path. This method has been successfully applied to many degrees of freedom path planning problems (Kavraki *et al.*, 1996), including problems with moving obstacles (Kindel *et al.*, 2000) or contact motion planning (Ji and Xiao, 2001).

The probabilistic completness of the PRM approach has been studied in (Kavraki *et al.*, 1998).

[2] Two configurations are adjacent if the straight line joining them lies entirely in free $\mathcal{C}$-space.

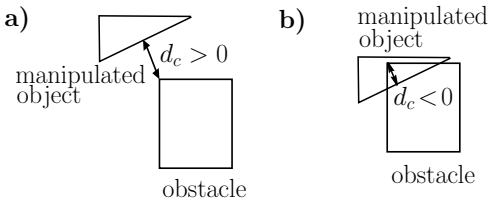

Fig. 4. Distances from the robot to the obstacles in the workspace: a) positive distance b) negative (penetrating) distance.

4. COMBINATION OF APPROACHES

This Section describes how $\mathcal{C}$-space is randomly sampled and how the obtained information is used to compute an harmonic function over a non-regular grid that partitions the $\mathcal{C}$-space. The approach is illustrated with an example.

4.1 *Distance detection*

For any given configuration c of the robot, the minimum distance d_c to the obstacles in the workspace is computed in physical space by means of a distance checker. If c is a free configuration then $d_c > 0$ (Figure 4a), otherwise it is a penetrating distance and $d_c \leq 0$ (Figure 4b).

4.2 *Cell classification*

In order to classify a given cell b_k of the grid that partitions the $\mathcal{C}$-space, the robot is set at the configuration represented by the center of the cell and the distance to the obstacles in the workspace is computed, and mapped into $\mathcal{C}$-space as d_{b_k}. Then, the cell is classified as:

- Free cell: If the whole cell is inscribed inside the circumference of radius d_{b_k} and $d_{b_k} > 0$ (cell 1 in Figure 5).
- Obstacle cell: If the whole cell is inscribed inside the circumference of radius d_{b_k} and $d_{b_k} < 0$ (cell 2 in Figure 5).
- Partially free cell: If the cell is not inscribed inside the circumference of radius d_{b_k} (cell 3 in Figure 5).

4.3 *The Path Planning Algorithm*

Let us introduce the following nomenclature:

G: Set of partially free cells (gray).
B: Set of obstacle cells (black).
W: Set of free cells (white).
b_k: Cell being evaluated, with sides of size s_k.
d_{b_k}: Minimum distance in $\mathcal{C}$-space from the configuration corresponding to the center of b_k to the $\mathcal{C}$-obstacles.
D_{b_k}: Distance from the center of b_k to one of its vertices.

The following functions are used in the path planning algorithm:

Compute Distance (b_k): Computes distance d_{b_k}.
Partition (b_k): Subdivides a partially free cell b_k into subcells.
Compute Harmonic Function (S): Computes the harmonic function over a set S of free cells of a non-regular grid.
Find path (H,c_{ini}, c_{goal}): Finds a path from the cell containing c_{ini} to the cell containing c_{goal} following the negated gradient of the harmonic function H.
Random Select (S): Randomly selects an element of the set S.

The algorithm Path Planning combines the random sampling of algorithm Explore, detailed below, with the path planning performed using harmonic functions. For the computation of the harmonic function, the obstacle cells and the partially free cells are considered as $\mathcal{C}$-obstacles, and the free cells as free $\mathcal{C}$-space. If no path is found between c_{ini} and c_{goal}, more samples are required and the algorithm restarts. This is done until a path is found or a maximum predefined number of cells has been explored ($N_{\max}$).

Path Planning(c_{ini}, c_{goal})
  $G \leftarrow$ cells of $\mathcal{G}_0$
  $B \leftarrow \emptyset$
  $W \leftarrow \emptyset$
  DO
    $\{G, W, B\} \leftarrow$ Explore($\{G, W, B\}$)
    $H \leftarrow$ Compute Harmonic Function (W)
    $p \leftarrow$Find path(H, c_{ini}, c_{goal})
    IF $p \neq \emptyset$ RETURN p
  WHILE $p = \emptyset$ or $n < N_{\max}$
  RETURN $\emptyset$
END

The algorithm Explore first randomly samples from G a predefined number of cells ($S_{\max}$), classifies them as free cells or obstacle cells, when possible, or otherwise subdivides them. In this latter case, the generated subcells are classified as partially free cells and are stored in G to be explored later. Cells in G are of different size. Currently, random sampling is weighed taking into account this fact: The probability of a cell being selected increases with its volume. This leads to a rapid characterization of the $\mathcal{C}$-obstacles border, since the uncertainty of big partially free cells is elucidated earlier.

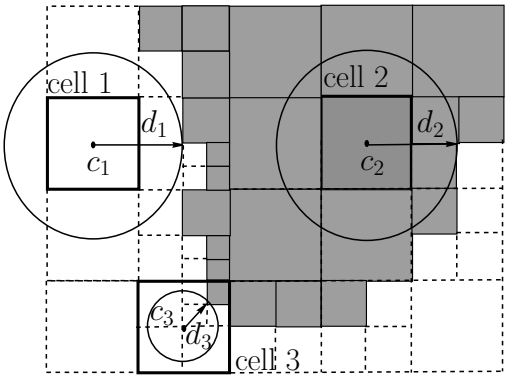

Fig. 5. Cell classification depending on the distance of its center to the $\mathcal{C}$-obstacles. Cell 1 is a free cell, cell 2 is an obstacle cell and cell 3 is a partially free cell.

Explore($\{G, W, B\}$)
  FOR $s = 1$ TO $S_{\max}$
    $b_k \leftarrow$ Random Select(G)
    $d_{b_k} \leftarrow$Compute Distance(b_k)
    IF $|d_{b_k}| \geq D_{b_k}$ THEN
      IF $d_{b_k} > 0$ THEN $W \leftarrow b_k$
      ELSE $B \leftarrow b_k$ END IF
    ELSE $G \leftarrow$Partition(b_k) END IF
  END FOR
  RETURN $\{G, W, B\}$
END

4.4 *Examples*

Figure 6 (left) shows the result of randomly sampling the $\mathcal{C}$-space of Figure 2. White cells are free cells, black cells are obstacle cells and gray cells are partially free cells to be evaluated later, if necessary. In this simple uncluttered environment a path (crossed cells) is found at the first iteration of the algorithm Path planning, using few $\mathcal{C}$-space samples ($S_{\max}$=205 corresponding to the 5% of the cells of the maximum-resolution regular grid 64×64).

Figure 6 (right) shows another example. In this case, the more cluttered environment requires 4 iterations of the path planning algorithm, using $S_{\max}$=164 samples in each exploration (4% of the cells of the maximum-resolution regular grid 64×64).

5. FUTURE IMPROVEMENTS

Due to the need of finding paths trough narrow passages, further sampling schemes are to be evaluated to bias the sampling in order to increase the success probability and the quality of the path found. Some proposals have been introduced for the PRM approach, like for instance the increase of the number of samples on the border of the obstacles (Boor *et al.*, 1999) or where the manipulability is low (Leven and Hutchinson, 2002). Other PRM variants use the quasi-random sampling technique (Branicky *et al.*, 2001), that produces low-dispersion samples, which gives a better performance than the classical PRM approach. Also, in order to reduce planning time, single-query strategies can be considered, as done in (Sánchez and Latombe, 2002) and (Song *et al.*, 2001) where the initial and goal configurations are taken into account to bias the sampling not to explore the entire $\mathcal{C}$-space. With this aim, lazy evaluation approaches (Bohlin and Kavraki, 2000) are useful too, since they delay collision checking until it is absolutely needed. Our current research is focused in using the harmonic function to guide the random sampling.

6. SUMMARY

The combination of random sampling and harmonic functions has proven to be an interesting approach to robot path planning. On one hand, harmonic functions allow the availability of a potential navigation function with an unique minimum at the goal configuration. On the other hand, random sampling permits path planning methods without the explicit knowledge of the robot's Configuration Space. In this paper, harmonic functions are computed on a non-regular grid decomposition of Configuration Space using a hierarchical, simple and computationally efficient method, both in time and memory. Random sampling is done over the partially free cells of the non-regular grid. Using a distance checker, selected cells are classified as free cells, obstacle cells or partially free cells. In this latter case they are hierarchically partitioned and the generated subcells are set to be explored later. The path planning method alternatively performs the exploration and the generation of the harmonic function. The successful application to several 2D tasks has been reported.

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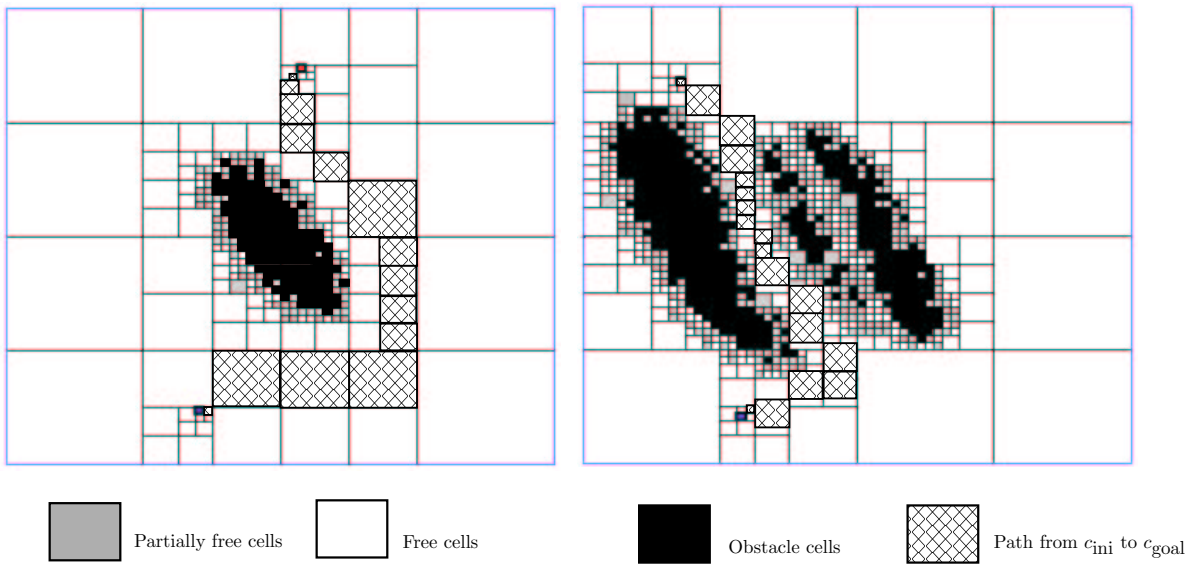

Fig. 6. Examples of paths from c_{ini} to c_{goal} computed using harmonic functions over two $\mathcal{C}$-spaces computed by random sampling.

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