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The projective synchronization of drive-reponse complex dynamical networks

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Abstract: This paper investigates the projective synchronization of drive-response complex dynamical networks. Based on the stability theory for fractional-order differential equations, controllers are designed torealize the projective synchronization for complex dynamical networks. Morover, some simple synchronization conditions are proposed. Numerical simulations are presented to show the effectiveness of the proposed method.

Introduction

As is well known, complex networks permeate every aspect of our daily lives, existing in various fields of the real world, such as social networks, biological networks, organizational networks, neural networks, and many others[1-2]. In recent years, several types of synchronization have been investigated, such as phase synchronization and complete synchronization [3-4], projective synchronization [5-6].

In fact, many systems consist of a large number of connected fractional-order dynamical nodes. By summarizing the previous results, it is interesting to ask if that the drive system is an integer-order system, and the response dynamical networks consisting of nodes with fractional-order dynamics during the synchronization.

Model description

In this paper, the Caputo definition is adopted for derivatives which is introduced briefly below[7]:

$$D_*^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-n+1} f^{(n)}(\tau) d\tau , \tag{1}$$

for $n-1 \le \alpha < n$, where $\Gamma(\cdot)$ is the Gamma function.

$$\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt. \tag{2}$$

Consider the following fractional order system

$$D^q \mathbf{X} = f(\mathbf{X}) \text{or } D^q \mathbf{X} = \mathbf{A}\mathbf{X}. \tag{3}$$

Where $\mathbf{X} \in \mathbf{R}^n$, $\mathbf{A} \in \mathbf{R}^{n \times n}$, $0 < q \le 1$.

Lemma 1 If $\mathbf{A}$ is a constant matrix, then the autonomous linear fractional-order system (3) is asymptotically stable if $|\arg(\lambda_i(\mathbf{A}))| > q\pi/2$.

Consider the complex dynamical networks as follows:

$$\begin{aligned} &\dot{s}(t) = f(s(t)), \\ &D_*^q x_i(t) = f(x_i(t)) + \sum_{j=1}^{n} c_{ij} \Gamma x_j(t) + u_i \qquad i = 1,2,...,N. \end{aligned} \tag{4}$$

Where $0 < q \le 1$ is the fractional order, $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{in})^T \in R^n$ denotes the state vector of the ith node, $f(x_i(t))$ are $n \times 1$ continuous differentiable functions which describes the dynamics of the individual nodes, Γ is the inner coupling matrix, $\mathbf{C} = (c_{ij})_{N \times N}$ is the outer coupling matrix representing the topological structure of the network.

Define the error

$$\mathbf{e}_i(t) = \mathbf{x}_i(t) - \Lambda \mathbf{s}(t), \quad i = 1,2,...,N . \tag{5}$$

Where $\mathbf{\Lambda}(t) = diag(\lambda(t),...,\lambda(t))^T$ denotes scaling matrix. For simplicity, we define another matrix $\mathbf{P}(t) = \mathbf{\Lambda}(t)\mathbf{s}(t)$.

So we obtain the error dynamical system as following:

$$D_*^q \mathbf{e}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}\mathbf{x}_j(t) - D_*^q \mathbf{P}(t) + u_i(t) \qquad i = 1,2,...,N \tag{6}$$

The vector function $f(x_i(t))$ is linearized as follows in the neighborhood of the goal value via Taylor expansions:

$$f(x_i(t)) = f(\mathbf{P}(t)) + \frac{\partial f}{\partial \mathbf{P}}(x_i - \mathbf{P}(t)) + \cdots, \qquad i = 1,2,...,N. \tag{7}$$

Substituting in Eq.(5), we obtain

$$D_*^q e_i(t) = f(\mathbf{P}(t)) + \sum_{j=1}^{N} c_{ij}\mathbf{e}_j(t) - D_*^q \mathbf{P}(t) + \mathbf{H}e_i(t) + \mathbf{u}_i(t) \tag{8}$$

Synchronization scheme

Theorem 1. For a certain fractional-order $q \in (0,1]$ and scaling factor Λ, the mixed drive-response network (5) can achieve function projective synchronization via the following controllers.

$$u_i(t) = D_*^q \mathbf{P}(t) - f(\mathbf{P}(t)) - \mathbf{M}e_i(t) \qquad i = 1,2,...,N. \tag{9}$$

Proof: According to error signal $\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{\Lambda}(t)\mathbf{s}(t) = x_i(t) - \mathbf{P}(t)$, then we get error dynamical system as following

$$D_*^q \mathbf{e}_i(t) = f(\mathbf{x}_i(t)) + \sum_{j=1}^{N} c_{ij}\mathbf{e}_j(t) - D_*^q \mathbf{P}(t) + u_i(t) \qquad i = 1,2,...,N \tag{10}$$

The vector function $f(x_i(t))$ is linearized as follows in the neighborhood of the goal value via Taylor expansions:

$$f(x_i(t)) = f(P(t)) + \frac{\partial f}{\partial P}(x_i - P(t)) + \cdots, \qquad i = 1,2,...,N. \tag{11}$$

Keeping the first-order terms in Eq.(12) and substituting in Eq.(11), we obtain

$$\begin{aligned} D_*^q e_i(t) &= f(P(t)) + \sum_{j=1}^{N} c_{ij}e_j(t) + D_*^q P(t) + \mathbf{H}e_i(t) - f(P(t)) - \mathbf{M}e_i(t) - D_*^q \mathbf{P}(t) \\ &= \mathbf{H}e_i(t) + \sum_{j=1}^{N} c_{ij}e_j(t) - \mathbf{M}e_i(t). \end{aligned} \tag{12}$$

Where $\mathbf{H} = \frac{\partial f}{\partial P}$ and $\mathbf{M} = diag(m_1, m_2, ..., m_n)$ is gain matrix to be determined.

Numerical simulation

We take the integer-order Liu system as the drive system, which can be described by

$$\begin{cases} \dot{s}_1(t) = a(s_2 - s_1) \\ \dot{s}_2(t) = bs_1 - ks_1s_3 \\ \dot{s}_3(t) = -cs_3 + \sigma s_1^2. \end{cases} \tag{13}$$

We consider the response dynamical network consisting of 5 identical fractional-order Lorenz systems. The single fractional-order Lorenz system in the ith node can be described by:

$$\begin{cases} D_*^q x_{i1}(t) = d(x_{i2} - x_{i1}) \\ D_*^q x_{i2}(t) = ex_{i1} - x_{i1}x_{i3} - x_{i2} \\ D_*^q x_{i3}(t) = x_{i1}x_{i2} - fx_{i3} \end{cases} \qquad i = 1,2,...,5. \tag{14}$$

When the parameters are chosen as d=10,e=28,f=8/3,$q=0.96$, system (16) displays a chaotic attractor in Fig.2.

Then the mixed drive-response complex network with 5 nodes can be described by:

$$\begin{cases} \dot{s}_1(t) = a(s_2 - s_1) \\ \dot{s}_2(t) = bs_1 - ks_1 s_3 \\ \dot{s}_3(t) = -cs_3 + \sigma s_1^2. \end{cases}$$
$$\begin{bmatrix} D_*^q x_{i1}(t) \\ D_*^q x_{i2}(t) \\ D_*^q x_{i3}(t) \end{bmatrix} = \begin{bmatrix} d(x_{i2} - x_{i1}) \\ ex_{i1} - x_{i1}x_{i3} - x_{i2} \\ x_{i1}x_{i2} - fx_{i3} \end{bmatrix} + \sum_{j=1}^{N} c_{ij}\Gamma x_j(t) + u_i(t) \qquad i = 1,2,...,5. \tag{15}$$

Fig.1. shows that the state trajectories of the drive system and nodes of the response network with different initial conditions. Time evolution of the synchronization errors are depicted in Fig.2.

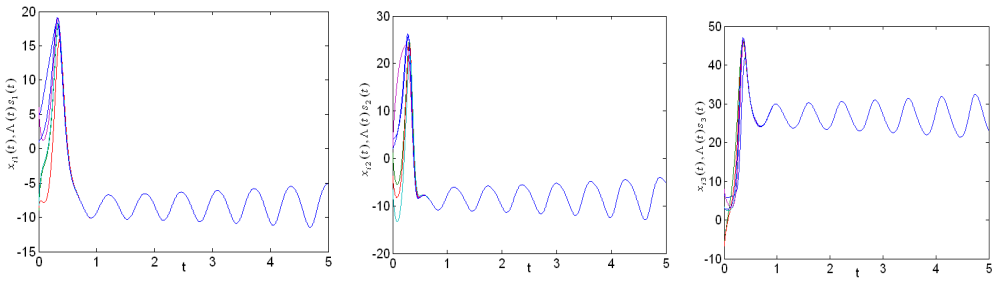

Fig.1. The state synchronization variables between the drive-response network (15)

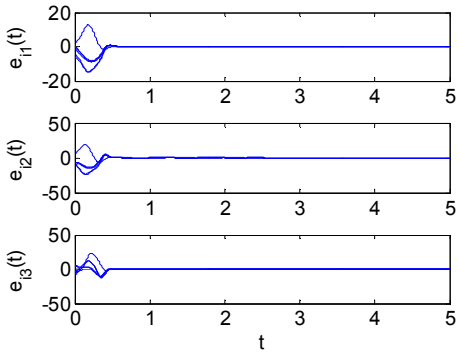

Fig.2. Synchronization errors of the mixed drive-response complex network (15)

Conclusion

This paper studied the projective synchronization of drive-response complex dynamical networks. Based on the stability of fractional-order system, an effective controller has been designed and analyzed. Simulated examples have been presented by using the fractional-order chaotic Lorenz system as nodes of the dynamical network to show the effectiveness of the proposed control scheme.

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