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ABSTRACT

Presented are abstracts and analyses of eleven research reports related to mathematics education. Five deal with aspects of learning theory, three with classroom practices, and one each on student characteristics, cognitive development, and mathematics anxiety. Research related to mathematics education which was reported in RIE and CIJE between April and June 1981 is also listed. (MP)

INVESTIGATIONS IN MATHEMATICS EDUCATION

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An editorial comment . . .

"Higher Quality" Research

Joe Dan Austin
Rice University

Investigations in Mathematics Education is now 14 years old. During these years the stated purposes of the journal have changed somewhat [see the editorial by Suydam (1978, 11, n4)] but the purposes still include the hope that the journal reviews will help researchers and writers plan instruction, develop curriculum, and design research. Assuming empirical research is useful in doing these tasks, IME would seem potentially very valuable since it is the only regular publication giving a review and evaluation of the empirical research in mathematics education. The only other regular source of reviews of mathematics education research, the yearly reports in the Journal for Research in Mathematics Education, has of necessity brief reviews that do not include evaluations of the research. For planning instruction and developing curriculum, the editorial by Gawronski (1981, 18, n1) considers how empirical research can directly and indirectly be important. In this editorial I will consider some ideas on how IME can perhaps be useful in the designing of research studies.

It may be reasonable to consider first what would be desirable for future research in mathematics education. In terms of specific research topics there are, of course, numerous books and publications on needed research in mathematics education. When considering the likely influence of IME, however, a somewhat more general perspective seems necessary. Specifically it seems reasonable to consider how IME might contribute to future research being of "higher quality" than existing research. I would like to interpret "higher quality" in a positive way if possible and to interpret it to mean research that is built on what has been learned from the existing research. Such an interpretation would then include replicating studies where questions were raised about the design, measurement, population, etc., and "new" studies where questions were raised about generalizing results established by existing research.

One way in which IME could contribute to future research being of "higher quality" than existing research would be if IME evaluations were used in research design courses or in other courses where mathematics education research was considered. Specifically, in such courses students are typically asked to critique published research by summarizing a study. The student is also usually asked to consider the strengths and weaknesses of the study as well as implications of the study and additional questions raised by it. After the students complete their reviews, the instructor could distribute the IME evaluation and ask the students to respond to the points raised in the IME evaluation either individually or as a group. The IME evaluation need not be considered the final evaluation of the study critiqued as it represents just one person's evaluation. However, the IME evaluation - usually written by someone experienced in educational research - can serve as an effective starting point for class discussion and for the instructor's evaluation of the study. Using this procedure with a systematic study of different research designs might help the future researchers and users of research to understand through specific examples the strengths and weaknesses of commonly used research designs. All empirical research involves some compromises as to what the researchers can control, measure, and manipulate. Therefore the weaknesses of existing research should probably not be the major focus of the analysis. Rather, it may be more beneficial to stress when possible the implications and research questions raised by the study [see Torrance and Harmon (1961) for a discussion of this point]. The students could also profit from a general overview of what research has been done and what additional research seems needed in the particular area considered in the study. These are sometimes briefly considered in the article and in the IME review but one might consider using some other publication, such as Begle (1979), for a general overview of existing research and some suggestions for needed research.

The previous discussion on the use of IME considers "preservice" researchers. There seems to be no reason that practicing researchers cannot also profit from IME reviews. A second opinion on existing research may be valuable in altering or suggesting research questions. I have no direct knowledge that any researchers have initiated or altered any research studies because of the IME evaluations of published research. However, I would hope that some researchers have so used the IME reviews. Certainly the reviews of manuscripts submitted for publication often are instrumental in producing an improved - if considerably shortened - version for publication. Perhaps it is reasonable to hope for a similar effect in the planning stage for research studies, particularly when the research studies are based on existing research that has been considered in the IME reviews.

If one interprets "higher quality" of future research as building on what has been learned from existing research, then it is reasonable to consider where should the building eventually lead? One answer, and the most defensible answer in my opinion, is that the building of research should lead to results that can be applied, primarily in the classroom. The actual building process may be painfully slow. For example, consider the time it took to begin to see classroom applications of Piaget's research efforts. However, it seems an important goal eventually to relate research to teaching and/or learning. Here the reviews in IME may be of some limited assistance in that some reviews do suggest additional research questions or limitations of the study in regard to classroom applications. However, it may be that the most successful way to move future research toward classroom applications is to involve more directly the school research groups in publishing research. Very few major school districts do not now have a research or evaluation group. While the group may be mainly concerned with system-wide testing, it would be extremely useful if these groups would more actively publish research since the concerns of these groups are almost always applied. There are many questions related to mathematics education and education in general that these research groups seem well suited to study. For example, no university ever feels its teacher program has weaknesses, but schools complain about poorly qualified teachers or teacher applicants. Can the school research groups study teacher effectiveness in terms of the teachers' preservice teacher preparation? Few university researchers have a broad enough data base to consider this important research area. Other questions relate to school organization. For example, many schools have introduced a number of different pre-algebra courses with varying levels of difficulty. Do these courses facilitate student performance in algebra and/or increase the number of students who eventually enroll in algebra? Many questions relate to classroom questions. For example, can we identify what classroom placements of handicapped children are effective and then identify what the schools or teachers did to make such placements effective? Other questions are simply whether a theory applies to the classroom. There are, of course, many other important research questions. With the increasing difficulty in obtaining access to students for educational research, researchers in school research groups would seem uniquely able to study some very important applied questions. How IME can help more school researchers to publish research is unclear, at least to me. However, I believe the involvement of school personnel by IME in the writing of research evaluations and editorials is an important first step.

Finally, when considering future research in mathematics, it may be useful to consider the number of published studies. IME has very limited influence in this area since it publishes no articles. However, I would argue that more studies need to be published as there are so many questions that need to be addressed. However, I find it professionally very disappointing when the list of published articles and dissertations in the Journal for Research in Mathematics Education each year shows the number of published studies is much smaller than the number of dissertations. For example, in the 1981 listing there were 195 research articles or reviews of research and 359 dissertations. Whether the university program or the major professor failed to motivate the student to publish or to do a publishable study is not clear. What is clear is that many dissertations remain unpublished, and that many graduates never publish a research study. Perhaps universities need to re-examine the merits of their graduate programs in light of this problem.

This editorial has attempted to list some of the contributions that IME can possibly make in bringing about "higher quality" research in mathematics education. Just as no one research study answers all questions relating to a particular topic, perhaps no one journal can hope to improve all aspects of research in mathematics education. However, each research study (hopefully) contributes something to what we know about teaching and/or learning. Perhaps it is then not unreasonable to hope and expect that this journal can and does effect some movement in the complex process of extending and improving research in mathematics education.

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A letter . . .

Zalman Usiskin
Associate Professor of Education
Department of Education
The University of Chicago

I am writing to express my disagreement with some points raised in the editorial "Toward the Goal", by Jane D. Gawronski, in the Winter 1981 issue of IME.

The editorial states that NCTM's Agenda for Action "was not prepared in the isolation of the mathematics education research community. Rather, it reviewed the available research data bases and built on this reseach knowledge to present viable, well-founded recommendations."

Where is this review? It is not in the Agenda for Action document. Such a review was not the purpose of the Priorities in School Mathematics (PRISM) Project, which to my knowledge was the only research study designed to assist in formulating the NCTM recommendations. If the PRISM survey results are allowed to substitute for research reviews, then we are in the position of reasoning from opinion to research rather than vice-versa, a very dangerous direction indeed.

Where is the evidence that the Agenda for Action recommendations were built upon research knowledge? There are no references to any research study or curriculum effort or even any previous policy decisions by NCTM in the Agenda for Action document. In the PRISM Executive Summary, it is noted that several of the NCTM recommendations were not supported by a majority of some of the groups sampled.

Where is the evidence that the Agenda for Action recommendations are viable? At the same time that one of the recommendations calls for more mathematics to be required of students, NCTM itself is alerting people to a nation-wide shortage of qualified mathematics teachers. At the same time that one of the recommendations calls for taking full advantage of the power of calculators and computers at all grade levels, the support for the use of calculators in place of paper-and-pencil algorithms (their most common use) was below 25% in all samples studied by PRISM.

Where is the evidence that the recommendations are well-founded? The recommendation which has been given most attention is that (quoting from An Agenda for Action) "problem solving be the focus of school mathematics in the 1980's". If this recommendation is "well-founded", upon what is it founded? Do we have data concerning what will happen if students are put through such a school mathematics curriculum? Do we even agree upon what is meant by "focussing school mathematics upon problem solving?"

I do not mean to argue here either for or against any of the recommendations in An Agenda for Action. Time or research may determine the extent to which some of the recommendations (those that can be researched) are appropriate. In that sense, the recommendations are useful to the research community in that they provide fodder for future research.

However, the editorial asserts that these recommendations were the result of examining the research bases, i.e., that research on these issues preceded the recommendations. If this is the case, then it would seem to have been most appropriate for the Task Force on Recommendations, of which Dr. Gawronski was a member, to have published a short summary of the research upon which the recommendations were based. Without this knowledge, the reader of An Agenda for Action is left with the belief that the recommendations are merely the untested opinions of those on the writing committees and that, as usual, any research in mathematics education that might have affected the decisions was ignored.

A response . . .

J. D. Gawronski, Director
Planning, Research and Evaluation
Department of Education
San Diego County

I am writing a response to Dr. Zalman Usiskin's letter of June 24, 1981, regarding my editiorial "Toward the Goal".

Dr. Usiskin's primary difficulty seems to be concerned with the lack of documentation or references in NCTM's An Agenda for Action. First, let me note that this document was not intended as a research review. Rather, it was intended as a popular document which would influence community and school decision makers, the lay public, and educators. Research reviews, such as those apparently favored by Dr. Usiskin, do not influence these groups and, in fact, would not be read by most of them.

However, An Agenda for Action did, in fact, rely on available research data bases, including PRISM and the NSF case studies. Furthermore, individual members of the task force made use of the research in preparing and supporting their recommendations. ERIC staff were also available for consultation and assistance.

Finally, recommendations represent a synthesis of research, opinion, and needs. The research describes what we know about some aspects of mathematics education as some people have practiced them. Opinion describes what practitioners and lay people think should be accomplished in mathematics education. Needs, expressed in terms of skills, describe what children must obtain from the mathematics curriculum.

The task force on recommendations addressed all of these areas. Such a synthesis does not lend itself to the format of a "research paper" but may be built on research knowledge, viable and well-founded.

I am pleased that Dr. Usiskin has taken such interest in my editorial.

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Fall 1981

Bright, G.; Wheeler, M.; and Harvey, J. GAME CONSTRAINTS, PLAYER VERBALIZATIONS, AND MATHEMATICAL LEARNING. Journal of Experimental Education, 49: 52-55; Fall 1980.

Abstract and comments prepared for I.M.E. by SAMUAL P. BUCHANAN, University of Central Arkansas.

1. Purpose

The purpose of the study was to investigate "the effects of game constraints on the quantity and quality of verbalizations about the content of mathematical instructional games during the playing of those games" (p. 52). It was also the purpose of this experiment to study the effects that game constraints have on learning.

2. Rationale

The investigators refer to a previously reported study where they state that the literature in this area was "fragmented and was not clearly related to any theoretical base" (p. 52). It had been observed also that the quantity of student verbalization was greater during game-playing than during regular classroom instruction. The investigators were interested in the nature of that verbalization and its effect on learning mathematics.

3. Research Design and Procedures

The investigation consisted of two phases, clinical and empirical.

A. Clinical Phase

After three preliminary observations in elementary schools, the investigators received the cooperation of a teacher of 26 fourth-grade students. The teacher was asked to have the class spend 15 minutes a day for five weeks playing mathematical games to familarize the students with the game and to encourage student verbalization. The games used were Remainder Game, MULTIG, My Number - Your Number, Moon Shot, Get to 999 First, and Shapescrabble. All the games are from Developing Mathematical Processes.

Groups of three or four students were video-taped while playing the games on four different dates. In each session, a game was explained and several rounds were played. The constraints, changed between each round of play, consisted of team vs. individual play, mode of recording information, and mode of calculation.

The video tapes were viewed by the investigators on two different dates. Because of the fragmented nature of the student utterances, the verbalization was not codified.

B. Empirical Phase

Students from two fourth-grade and two fifth-grade classes were used as subjects for the empirical studies. Each class was used to test one of the following hypotheses:

Hypothesis 1: Using different devices for generating information available from which the student can choose does not affect achievement.

Hypothesis 2: Recording or not recording information in written form does not affect achievement.

Hypothesis 3: Incorporating the instructional objective into the rules of game does not affect achievement.

A pretest on the mathematical content of the game was as administered to the students. The subjects were randomly assigned to one of two groups. One group played a published game while the other group played an altered form of the game. The games were altered by varying "the device used for generating information while playing the game" (p. 54); by varying "written record generated"; or by varying "the relationship of the instructional objective to the rules of the game" (p. 54). The games were played by the two groups for 15 minutes a day for nine days and a posttest was administered to measure achievement in the mathematical topics being taught by the game.

A t-test was used to determine if learning had occurred within the groups and an ANOVA was used to "measure differential effects of the treatments" (p. 54).

4. Findings

A. Clinical Phase

The investigators found in the clinical phase of the study that skill games produced more verbalization than concept games. However, the mathematical verbalization was concentrated on computational answers and basic facts. It was also observed that more utterances were found during the playing of games where information was generated by the repeated use of a single device than during the playing of games where information was generated at once. Although playing on teams tended to generate more student utterance than individual play, most of the verbalization was not concerned with mathematics.

B. Empirical Phase

The data gathered from the empirical study resulted in nonrejection of of all three of the hypothesis.

5. Interpretations

"Although there seemed to be some differences in the amount of verbalization with certain combinations, there were no corresponding effects on achievement" (p. 55). The investigations suggest that, since all treatments were at the posttest level in the empirical phase of the study, further insight in this area might be gained from a similar study at the "pre- and co-instructional level" (p. 55).

Abstractor's Comments

This experiment might well have been reported as two different studies. There was no obvious connection between the clinical phase and the empirical phase of this report. The clinical phase is concerned with the amounts of student verbalization created by varying game constraints, while the empirical phase investigates student achievement in mathematics. The clinical phase reported conclusions based upon taped dialogues which were not codified. No mention is made in the report of any effort to quantify the students' utterances. Some of the published conclusions of the clinical phase seemed to be extraneous to the stated purpose of the paper. The empirical phase could be best described as an exploratory study. The investigators acknowledge no theoretical base exists for believing that varying game constraints would alter mathematical achievement.

Caldwell, Janet H. and Goldin, Gerald A. VARIABLES AFFECTING WORD PROBLEM DIFFICULTY IN ELEMENTARY SCHOOL MATHEMATICS. Journal for Research in Mathematics Education 10: 323-336; November 1979.

Abstract and comments prepared for I.M.E. by LOYE Y. HOLLIS, University of Houston.

1. Purpose

To compare the relative difficulties for elementary children of four types of word problems: abstract factual (AF), abstract hypothetical (AH), concrete factual (CF), and concrete hypothetical (CH).

2. Rationale

The variables were selected because of their importance from the standpoint of cognitive-developmental theory. Students at the formal operations thought stage of development can construct systems and theories and can draw conclusions from pure hypotheses as well as from actual observations. Students at the preceding stage of concrete operations do not have the ability to deal with abstract situations or to think in a hypothetical-deductive manner. For elementary school students at the concrete operations stage, concrete and factual problems should be less difficult than abstract problems and hypothetical problems, respectively. For older students, the differences should tend to disappear.

3. Research Design and Procedures

The subjects for the study consisted of 399 students in grades four, five, and six in two predominantly white middle- and upper-middle-class suburban elementary schools. These two schools were selected from seven because they were deemed to be most representative in terms of mathematics achievement.

The word problems used in the study consisted of five sets of four problems each. Each set of four problems contained one problem of each type AF, AH, CF, and CH. An abstract word problem was defined as a problem involving a situation that describes only abstract or symbolic objects. A concrete word problem was one describing a real situation with real objects. A factual problem was defined to be one that merely describes a situation.

A hypothetical problem is one that not only describes a situation but also describes a possible change in the situation. Examples of each type are:

"a. There is a certain given number. Three more than twice this given number is equal to 15. What is the value of the given number? (Abstract factual).

b. There is a certain number. If this number were 4 more than twice as large, it would be equal to 18. What is the number? (Abstract hypothetical).

c. Susan has some dolls. Jane has 5 more than twice as many, so she has 17 dolls. How many dolls does Susan have? (Concrete factual).

d. Susan has some dolls. If she had 4 more than twice as many, she would have 14 dolls. How many does Susan really have? (Concrete hypothetical)" (p. 325).

Tests were administered to all the students on two consecutive days. Each day the students were asked to solve 10 word problems in 30 minutes, followed by a five-item computational skills test to be completed in 10 minutes. The word problems were sequenced so as to eliminate any difficulty with the order in which students worked the problems.

The researchers state, "The basic experimental design was a multifactorial analysis of variance with repeated measures on two experimental factors. Factors included in the analysis were: (A) grade level--4, 5, or 6; (B) Sex--M or F; (C) test order--Part I first or Part II first; (D) performance in the computational skills test--pass or fail; (E) first experimental factor--abstract or concrete; and (F) second experimental factor--factual or hypothetical" (p. 328).

4. <u>Findings</u>

The findings were as follows:

a. There was a significant interaction between grade level and performance on the computational skills test.

b. The factor of grade alone was barely on the verge of statistical significance.

c. There was no significance between subject sex differences.

d. There were no differences with respect to the order of test administration.

e. Significantly more concrete problems than abstract problems were solved.

f. Significantly more hypothetical problems than factual problems were solved.

g. The order of difficulty of the four problem types, from easiest to most difficult, was CF, CH, AH, AF.

h. Female students solved somewhat more abstract problems and fewer concrete problems than did male students.

5. Interpretations

The researchers believe, "The findings of this study confirm that for elementary school children concrete verbal problems are substantially less difficult than abstract ones, when other relevant variables are controlled" (p. 334). They make the point that all the problems studied were abstract since no real objects were used, and thus the study does not argue against the increased use of abstract verbal problems with elementary school children.

The researchers found that with abstract problems the hypothetical versions were consistently less difficult than the factual. They conclude, "This finding seems to contradict some prevalent conceptions about pre-formal operational thought and indicates the great caution with which easy inferences from developmental theory should be viewed in the context of school mathematics" (p. 334).

Abstractor's Comments

This study deals with a most important area of the mathematics curriculum. More needs to be known about problem types and their relative difficulty for students. The logical defining and structuring of problem types was an important contribution of this study.

The major difficulty with the study was no test being administered to determine the developmental stage of the subjects. Children in grades four, five, and six are likely to be in either the "concrete operations" stage or the "formal operation thought" stage. It would be very difficult to draw conclusions about the relationship of problem types to developmental level without knowing the latter.

Since the variables selected were based on their importance from the standpoint of cognitive-developmental theory, it would seem reasonable that some measure of cognitive development would have been included. This undoubtedly restricted the conclusions that could be drawn by the researchers.

Charles, Randall. EXEMPLIFICATION AND CHARACTERIZATION MOVES IN THE CLASSROOM TEACHING OF GEOMETRY CONCEPTS. Journal for Research in Mathematics Education 11: 10-21; January 1980.

Abstract and comments prepared for I.M.E. by JOHN W. GREGORY, University of Florida.

1. Purpose

The investigation sought to ascertain the feasibility of training preservice elementary teachers to use exemplification (E) moves (give examples and nonexamples) and characterization (C) moves (point out relevant or irrelevant attributes) in teaching concepts. Obtaining data for analyses of student concept achievement taught by trained and untrained teachers served as a secondary purpose of the study.

2. Rationale

The author reports findings of other investigations which support the facilitating effect of providing examples along with characterization moves when teaching concepts. For the most part, the previous studies have employed programmed materials to mediate the instruction. This study would attempt to provide supporting evidence for the presentation of examples accompanied by characterization moves via teacher verbal production.

3. Research Design and Procedures

Being a two-phase investigation, there were two samples. Eighteen preservice elementary teachers enrolled in a mathematics methods program were subjects for the training phase. Groups comprised of four randomly selected second-grade students provided data relative to the effect of frequencies of E and C moves on achievement.

The teachers were randomly divided into two groups: trained and control. Treatments for the two groups were identical except for an investigator-led lesson on the use of E and C moves as a portion of the trained-teacher treatment. Pairs of teachers (two trained or two control) prepared twenty-minute lessons for each concept of bilateral symmetry and rotational symmetry.

Student subjects were pre- and posttested for their ability to discriminate between examples and nonexamples of the two concepts, where the examples and nonexamples were different from those used by the teachers in the study (reliability coefficients for these measures were greater than .77). The pretests were administered the day preceding the lesson presentation on bilateral symmetry. The lesson on rotational symmetry was given the day following the bilateral symmetry lesson, followed on the fourth day by the posttests.

Codings of the lessons were achieved utilizing various research-acceptable techniques. Indices of agreement (a form of observer reliability) were at least .86 between those providing variable counts for numbers of examples, nonexamples, total E moves, C moves for relevant attributes, C moves for irrelevant attributes, total C moves and teacher clarity ratings (the latter being based upon identifiable mathematics objective, lesson planning and execution, effective use of models and illustrations, and "flow of ideas from the instructor to pupil was understood"; evaluation was ongoing).

4. Findings

Training Results: Analysis of variance led to the identification of a significant difference in the use of C moves for irrelevant attributes and total number of C moves favoring the trained teachers for both lesson types. In the lesson on bilateral symmetry the mean frequency of nonexamples for trained teachers was significantly greater than that of the control teachers.

Significant positive Pearson product-moment correlation coefficients were found to exist between the clarity rating and all three C move frequencies for both lesson types. Clarity was also related to the number of examples used in the rotational symmetry lessons, but negatively.

The only significant correlations between E and C moves was for nonexamples and C moves for irrelevant attributes (and subsequently all C moves) in the bilateral symmetry lesson.

Student Achievement: The posttest on bilateral symmetry provided the only significant difference between the trained and control teacher groups, favoring the trained group. Post hoc analyses identified the number of nonexamples as the significant contributor to this performance difference.

The number of examples was found to account for 26.7 percent of the variance in posttest scores for the rotational symmetry lessons (r = .52). No other variables were significant.

5. Interpretations

Both concepts taught are unidimensional, that is, have only one relevant attribute. This could help explain the nonsignificant differences between teacher groups for the use of C moves related to relevant attributes (being a "natural" behavior; control teachers used as high a frequency as the trained). The investigator's interpretation of the results is that training preservice elementary teachers in the use of E and C moves does facilitate student concept acquisition for the two concepts taught. The investigator suggests that teaching concepts having a greater number of relevant attributes could lead to different findings.

The inconsistent findings relative to the use of nonexamples (related to learning the concept of bilateral symmetry but not to rotational symmetry learning) led the investigator to cite work of others regarding the influence of various sequences of example-nonexample moves. Similarly, sequencing of E and C moves (e.g., ECE vs. CEC) in teaching concepts is suggested as a possible influencing factor not assessed in this study.

Another alternate hypothesis offered is the possiblity of differential effects of use of nonexamples for learning "difficult" vs. "easy" concepts, having found that mean performance on all tests for rotational symmetry was consistently higher than that for bilateral symmetry.

Abstractor's Comments

The investigation reaffirms the complex nature of the teaching act. Once again we find that explanation of learning cannot be found by considering only one or two variables. This is not to say that research of this type is not important. On the contrary, such research can lead to the identification of "tools" of teaching which are worthy of being learned by teachers for subsequent use in the development of personal strategies which must be variable enough to accommodate the variety of mathematical content as well as inherent differences in student populations to be taught. Instead of developing a pure science of teaching, the findings of investigations like the one under consideration will enrich the art of teaching by providing teachers with a broader selection of tools from which to choose in their teaching composition.

There are some questions raised by this investigation that may extract more information from the results as well as suggest alternate methods for future studies:

1. If teachers were given predesigned lesson plans containing the examples, nonexamples, and characterization moves to be used, might not greater teacher variance be achieved and thus more significant findings relative to student learning be possible?

The design of the study is sound. But the design is effective only if the groups of teachers provide significantly different frequencies in instructional variables. The training program fell short on this point. The investigator appropriately sought correlational information under the constraints of the training results.

Since E and C moves are planned prior to lesson presentation, the investigator could have more than likely ascertained training program effect by analyzing the lesson plans. Further, for purposes of having student treatments be consistently different, much would be gained by providing the teachers with the lesson plans containing different frequencies (and possibly, sequences) of E and C moves. There is little doubt that the subsequent presentation of these lesson plans would differ greatly from the plans with regard to these variable frequencies.

2. If students are consistently taught under one strategy (say, with examples but not nonexamples), might the effect of a different strategy presentation on a one- or two-shot basis be lost?

There is evidence to suggest that students "get used to" or accommodate a particular teaching style of their current teachers in an effort to learn. This investigation inserted two strange individuals (and possibly strange strategies) into the learning environment of young learners. The brief encounter may not have allowed the strategy of using E and C moves to take effect. A study of longer duration, using the same teacher, may lead to totally different results. Again, for control purposes, lesson plans given to the teachers would be suggested.

3. The Pearson product-moment correlation coefficient seeks a linear model for data. Might there not be an optimal frequency of E and/or C moves for learning and thus a non-linear model be found?

Other investigations attempting to relate teacher behaviors to student achievement have found the existence of an "inverted U" in data graphics, suggesting an optimal frequency for a variable. Looking at the mean frequencies reported, if an optimal frequency exists with regard to E and C moves, it might have been surpassed. For the trained teachers, there was an average of 27.11 total E moves and 58.89 total C moves in one 20-minute lesson. That is about 2.3 E moves and 6 C moves per minute -- an astounding number of moves.

4. If E moves are used to illustrate (i.e., clarify) the concept being taught, why were the clarity ratings either negatively or not related at all to the use of E moves?

Reviewing the categories upon which clarity ratings were based, it appears that the use of examples would enhance the rating. One category ("The models and illustrations were effectively used") seems to be synonomous with "uses examples". It may very well be the case that although one plans specific examples, the presentation of these examples verbally in the classroom is not clear. Considering also that positive correlations existed between the clarity ratings and use of C moves, either the raters were sensitized to C moves and not E moves or just the inherent inadequacies of rating systems are reflected. More systematic observation (such as the use of an interval system) employing better defined variables of clarity of presentation (e.g., structuring, use of conditionals, wait-time, etc.) may shed greater light on the quality of the presentation of E and C moves. There is little disagreement with the statement that poorly presented lessons, no matter how high the quality of the plans, will not facilitate student learning.

Cohen, Martin P. and Carpenter, John. THE EFFECTS OF NON-EXAMPLES IN GEOMETRICAL CONCEPT ACQUISITION. International Journal of Mathematical Education in Science and Technology 11: 259-263; April-June 1980.

Abstract and comments prepared for I.M.E. by DAVID L. STOUT and RICHARD J. SHUMWAY, The Ohio State University.

1. Purpose

Two questions were investigated:

"(1) What are the different effects of an instructional sequence of both examples and non-examples and a sequence of all examples on the acquisition of the concept of semi-regular polyhedra?

"(2) Does the order in which specific non-examples are presented have an effect on the acquisition of the concept of semi-regular polyhedra?" (p. 260).

2. Rationale

The usefulness of non-examples in concept learning has been debated via experimental findings and classroom practice. There remains "...the question of why there is so much variance between experimental findings and current practice" (p. 260). The authors state that a common classroom "...procedure is for the teacher to define the concept and then to point out the critical dimensions to the students through a sequence of examples and non-examples" (p. 260). However, in experimental research, subjects are generally presented with a task of inferring the concept from a sequence of instances and are not informed of critical dimensions. The authors point out that a statement presented with the instances, explaining the presence or absence of critical attributes, has been found to improve concept learning.

The author's research is somewhat related to that of Markle and Tiemann, Shumway and Tennyson, Steve, and Bratwell, who concluded that non-examples were useful in concept learning.

3. Research Design and Procedures

The subjects were 54 high-ability geometry students from three classes. A majority of the subjects were middle class and the remainder were inner city. The same number of subjects in each class were randomly assigned to one of three treatments: C (control) groups, E_1 (experimental) group, or E_2 (experimental) group. Each group contained a total of 18 subjects.

The following procedure was used:

1. All groups received the same introduction. This introduction was a video-taped lesson which consisted of the definition of regular polyhedra, a proof of the existence of exactly five regular polyhedra, and a verbal definition of a semi-regular polyhedron which listed its characteristics ["convex polyhedra with faces in the shape of more than one kind of regular polyhedra and with all spatial vertices exactly alike" (p. 260)].
2. The C group was then presented eight examples of semi-regular polyhedra. The E_1 group received four examples followed by four non-examples. The E_2 group received four pairs of polyhedra -- each pair was an example and a closely related non-example. An example and a non-example were defined to be closely related if they differed on at most two attributes. In all three groups the subject's attention was focused on the critical attributes of the concept via a verbal justification.
3. Immediately following the treatments, each group received (on video-tape) a posttest of 20 polyhedra which were to be classified as semi-regular or not. The posttest had a split-half reliability of 0.82.

4. Findings

The mean posttest scores for the C, E_1, and E_2 groups were 15.39, 15.78, and 16.72, respectively. The standard deviations for C, E_1, and E_2 groups were 2.50, 3.23, and 2.59 respectively. A one-way ANOVA was performed on the data. No significant difference in concept acquisition was found among the three treatment groups.

The authors dropped one class from the analysis and reanalyzed the remaining data with a new n of 27, or 9 subjects per treatment. The class was dropped because the authors felt two of the three treatment groups within the class did not pay close attention to the video-tape presentation. The new means for the C, E_1, and E_2 groups were 14.76, 17.33, and 18.00, respectively.

The accompanying standard deviations were 1.87, 2.18, and 2.55, respectively.

A one-way ANOVA was performed on this reduced data. A significant $F = 5.69$ ($p < .009$) indicated concept acquisition differed among the three treatment groups of the reduced sample. A t-test for independent groups was performed to determine the differences. Results of these t-tests showed both the E_1 ($t = 2.785$, $p < 0.013$) and E_2 ($t = 3.162$, $p < 0.006$) groups outperformed the C group; however, no significant difference was found between the E_1 and E_2 groups ($t = .596$).

5. Interpretations

Using the reduced sample, the authors concluded: (1) examples and non-examples produced better concept acquisition than examples only; and (2) the order of presenting non-examples had no significant effect on acquisition of the semi-regular polyhedron concept.

Abstractor's Comments

1. As follow-up statistical tests, the authors state "...the t-test for independent samples was then used to determine which groups differed" (p. 262). Using three separate t-tests without planned, or a priori, orthogonal comparisons, the collective alpha risk (assuming the three t-tests were done at the .05 level) would be about .14 -- a rather high risk of a Type I error. Tukey's test seems to be more appropriate. However, with the high level of significance given for the t-tests (and assuming an $\alpha = .05$), the conclusions drawn by the authors are still valid.
2. A 20-item posttest was given but the authors do not indicate how many items were examples or non-examples. Is the distribution 10 examples and 10 non-examples or 5 examples and 15 non-examples reflecting the truth table for a conjunctive concept?
3. Was the discarding of data based on observations recorded prior to the no-differences analysis?
4. The authors' review and interpretation of the experimental psychology, mathematics education, and educational psychology research pertinent to their study lead smoothly into the hypotheses to be tested.

5. The authors indicated two important limitations: (1) a small, reduced sample (27 subjects) was used; and (2) the subjects were drawn from a rather specialized population.
6. Discussing the results and calling for studies of similar design, the authors write: "It is recommended, however, that the scope and sequence of the instructional phase reflect actual classroom behavior" (p. 263). Why? The authors leave us with this "recommendation" but fail to give any reasons. Does it not depend on whether one is trying to develop or validate a theory?
7. The authors indicate the consistency of their results with other researchers; however, they fail to give any classroom or research implications.
8. The author's research is valuable and most welcomed. They have appropriately and successfully linked the research of the experimental and educational psychologists with mathematics education research.

Karplus, Robert; Adi, Helen; and Lawson, Anton E. INTELLECTUAL DEVELOPMENT BEYOND ELEMENTARY SCHOOL VIII: PROPORTIONAL, PROBABILISTIC, AND CORRELATIONAL REASONING. School Science and Mathematics 80: 673-683; December 1980.

Abstract and commments prepared for I.M.E. by JAMES K. BIDWELL, Central Michigan University.

1. Purpose

The study investigated ways of measuring concrete, transitional, and formal reasoning among students from sixth grade to college level. The authors sought to answer these three questions:

1. What categories are required for classifying the subjects' responses on tasks requiring proportional, probabilistic, or correlational reasoning?
2. How effective are these tasks for assessing these aspects of formal reasoning?
3. What implications for teaching are suggested by the observed distributions of student responses among the categories required?

2. Rationale

This study is part of the larger research interests of AESOP (Advancing Education through Science-Oriented Programs). It also fits into the studies on development of formal reasoning conducted by many investigators over the last ten years. Extensive related literature is referenced. The current study is interlinked with the other research of AESOP.

3. Research Design and Procedures

The subjects were 505 students at grades 6, 8, 10, and 12, and at college freshman and sophomore age levels. The school-age subjects were from the "middle to upper-middle class suburban community." These subjects were tested in neutral classroom situations: eighth grade in English, tenth grade in biology, twelfth grade in social studies. The college students were enrolled in physical science courses for non-majors. The age groups were about equal in number, with more males (291) than females (214) in all groups but grade 6.

The study was concerned with written responses (with justification) to six group-administered tasks. The subjects were given a booklet with written questions and space for answers. The first four items were read aloud and the first three had demonstrations. The testing time was 40 minutes. The six tasks were:

1. (Proportion). The task involved water levels in two cylinders of different diameters but equally spaced graduations.

2. (Probability). The task was to state "whether there was a greater chance of pulling a white block from sack A or sack B", when both sacks contained white and black blocks.

3. and 5. (Probability Reasoning). The tasks were to give the chances of a simple event. Task 3 involved a known collection of colored blocks; Task 5 involved a sample of mice from a field.

4. and 6. (Correlational Thinking). The tasks were to state if "there was a relation between" the attributes of mice and fish, each classified by numbers into a 2 x 2 cell design. Subjects were shown pictures of the animals.

Based on results from previous studies and student responses to the current study, the three authors established categories of reasoning for each type of task. At least two of the authors evaluated all responses. Disagreements were resolved by discussion in about 10 percent of the responses.

4. Findings

The following categories were judged to be required to sort the explanations given for the tasks:

Proportion: I: none, illogical, guess; A: focus on difference in water level; Tr: additive procedure based on correspondence of amounts; R: using constant ratio.

Probability: I: none, guess; misunderstanding; AV: comparison of absolute numbers of blocks; 1C: comparing blocks in one sack; 2C: comparing blocks in two sacks.

Probability Reasoning: I: none or illogical response; Ap: approximate description; Q: quantitative description.

Correlations: I: no or illogical explanation; NR: no relation mentioned between cells; TC: comparison of number in 2 cells; FC: comparison using 4 cells; Co: 2 quantitative comparisons using 4 cells.

The percentages of responses for each grade level are summarized in the following composite table. For probability reasoning, the possible paired category responses for the two tasks were used. For correlations, the higher category of the two responses given was used.

Table 1

Response Frequencies on Tasks (Percent)

	Category	Grade 6	Grade 8	Grade 10	Grade 12	College
Proportions	I	17	22	6	6	7
Task 1	A	73	59	28	20	13
	Tr	7	10	13	12	6
	R	3	10	55	63	74
Probability	I	7	10	8	4	1
Task 2	AV	14	10	0	2	2
	1C	24	14	20	13	4
	2C	55	66	73	81	93
Probability	(I,I)	19	14	9	7	2
Reasoning	(Ap,I),(Ap,Ap)	27	25	5	3	2
Tasks, 3,5	(Q,I),(Q,Ap)	31	25	29	18	16
	(Q,Q)	23	36	57	72	79
Correlation	I	65	45	32	21	14
Tasks 4,6	NR	26	12	27	17	15
	TC	4	13	8	6	1
	FC	5	23	14	19	20
	Co	0	8	18	37	50

5. Interpretations

Question 1: The various levels of reasoning can be distinguished as formal, transitional, and concrete. Adding to these inconclusive responses, the various categories determined for the study can be assigned as in Table 2,

Table 2

Assignment of Response Categories to Reasoning Levels

Task	Formula	Transitional	Concrete	Inconclusive
Proportions	R	Tr	A	I
Probability	--	2C	AV	I,1C
Probability	(Q,Q)	(I,Q);(Ap,Q)	(I,Ap);(Ap,Ap)	(I,I)
Correlation	Co	FC	NR,TC	I

Question 2: A major limitation of the study was the impossiblity of probing the subject's thoughts. Other limitations involved interaction between students' reading, writing, and concentration skills. Advantages of the procedure include reproducibility and the large number of subjects studied.

The percentage of inconclusive responses (I, 1C, (I,I) is the major problem of the group task method. These percentages were satisfactory except for the categories of 1C and I (Correlations), which were unacceptably high. A major confusion in the correlations tasks concerned the word "relation" which was interpreted as parental relationship rather than a characteristic one.

Question 3: The poor performance on "proportional reasoning even in high school suggests that the mathematics courses dealing with ratio and proportions do not provide sufficient instruction in applying these concepts." Probabilistic reasoning was "accomplished much more successfully than correlational reasoning." ". . .our findings suggest that science and social studies courses should pay more explicit attention to the analysis of data for correlations." These activities should be concentrated in the high school years.

New curriculum should tie together mathematical study of numerical and algebraic relations with the science applications of these relations.

Abstractor's Comments

It always seems comforting to see tables of percentages that total

100 percent and seem to tell us something about which we have always wondered. This illusory comfort is bad enough when we are dealing with directly measured skills which indicate that say 86 percent of x-graders can write the correct answer to 36 x 27. It is worse when we take the thinking machines of 505 individuals and process their written responses through three subjective authors' minds and report that 22 percent show formal correlational reasoning. This abstractor contends that such percentages have very little validity. He believes that reasoning involves such complex schema that any attempt to categorize thinking, through a group-administered test in particular, by age and level of formalism is in reality fruitless.

If we do study the percentages in this study, we find confirmation of what everyone hopes is the case: that formal reasoning levels increase without exception with increased age and formal education. Correspondingly the so-called concrete levels decrease. The reader should be made aware that the authors use the term "concrete reasoning" to indicate incomplete thought process based on concrete evidence that leads to incorrect answers. This contrasts with other interpretations which view concrete reasoning as valid but nonverbal reasoning obtained from using physical materials as aids.

The results of the study also show that proportionality is a difficult topic. (It would have been better in the abstractor's view to have made Task 2 also a proportion one and to have dropped the intuitive probability task.) This finding is consistent with many other testing results. The authors are correct in suggesting more adequate teaching of this concept. It is also the case that simple probability involves comparing 2 numbers, while proportion involves 4 numbers, and correlation involves - 4 numbers in two ways. Proportion is probably inherently more difficult than mathematics and science educators have previously wanted to believe.

It seems to the abstractor that a more valuable and naturally more difficult study would be a longitudinal one which records the transitions by individuals from concrete to formal reasoning on similar tasks. This would give us a better grasp of the evolution of the schemas that provide the formal responses we so frequently desire from school-based learning. Such information might well lead to improved sequences for learning that could be offered to avoid the large amounts of confusion and error now common in school learning of mathematics.

Leder, Gilah. BRIGHT GIRLS, MATHEMATICS AND FEAR OF SUCCESS. Educational Studies in Mathematics 11: 411-422; November 1980.

Abstract and comments prepared for I.M.E. by JOANNE ROSSI BECKER, Virginia Polytechnic Institute and State University.

1. Purpose

The two studies reported in this paper examined the relationship between the fear of success (FS) construct and sex differences in performance and participation in mathematics.

2. Rationale

The author reviews information on the mathematical performance of bright females and males in Australia, the USA, Great Britain, and Russia, concluding that males are consistently outperforming females. Also, data on the participation by sex in non-compulsory mathematics courses through grade twelve in Victoria, Australia show that equal numbers of males and females enrolled only in the general mathematics course; for the more intensive mathematics courses, males outnumbered females 2 or 3 to 1. In addition, the author claims no improvement has occured in the ratio of males to females who complete a degree in mathematics at Victoria universities. Thus the pattern of lower female participation in mathematics, and the consequences of that avoidance, are similar in Australia and this country.

The motive to avoid success postulated by Horner (1968) holds that, for high-ability females, success in areas generally considered masculine and inappropriate for females produces anxiety which has an adverse effect on performance. Leder draws a parallel between the development of sex differences in mathematics achievement and of FS, stating that both are more characteristic of older, high-ability students and both are associated with cultural pressures leading to sex stereotyping. Thus FS merits investigation as a possible factor related to sex differences in participation and performance.

3. Research Design and Procedures

In the first study conducted in 1975, the author interviewed grade 11 students (11 females and 7 males) who had scored very well in a mathematics competition in Victoria, and their families. Students were asked about their long-term ambitions, and were asked to describe an imaginary student (of their sex) who was at the top of his or her mathematics class. Sample responses are provided in the report.

The second study, conducted in 1978, included 50 grade 8 students (20 females and 30 males) selected for special programs for able students in the Melbourne area. Students were selected for one program by means of the WISC IQ test, and for the other by a school-constructed mathematics test. Students were given five instruments:

1. An instrument to measure FS. Students were asked to write a brief story in response to three verbal cues depicting a figure of the same sex. Scores on this instrument range from -6 to 24. Stories were scored by the author and rescored six weeks later, with a self-consistency index of 0.92. Mean scores by school and sex were reported.
2. A mathematics test (Tests of Reasoning in Mathematics level 1 in on school and a teacher-constructed test in the other school). No actual data were reported.
3. A questionnaire to determine future course selection by asking about life plans, whether students hoped to pursue a career in a mathematical field, and whether they intended to take as much mathematics as possible in high school.
4. A questionnaire to determine future career intentions. Students were asked to rate five occupational categories. Percentages by sex of students who intended to pursue a career in mathematics/science were reported.
5. An instrument to determine to what students attribute the fact that fewer women than men study mathematics/science. Students were given four alternatives, two giving innate explanations and two giving social pressure explanations. Frequencies by sex for innate vs. social pressures were given.

In addition, females who reported plans to take as much mathematics as possible were compared on FS with the rest of the females. The author also compared females who indicated an intention to pursue a career in mathematics with those who did not. Only mean FS scores were reported in these comparisons.

4. Findings

From the first study, it was found that males' long-term ambitions centered around work and job satisfaction, while females' ambitions centered around happiness, emotional security, friends, or marriage.

From the second study, it was found that:

1. The mean FS of females was higher than that of males in each school (6.88 vs. 4.93; 7.00 vs. 4.75).
2. 53 percent of the males and 24 percent of the females indicated an intention to have a career in mathematics/science.
3. Females who intended to take as much mathematics as possible had a lower mean FS than the remaining females (5.90 vs. 8.00).
4. The mean FS of females who intended a career in mathematics differed little from the mean FS of the others (6.80 vs. 7.00).
5. Females were more likely than males to attribute sex differences in mathematics/science careers to external and social factors; also, the mean FS score of the four females who blamed innate reasons was less than that of the 15 females who blamed social pressures (4.74 vs. 7.53).
6. For males, in each school, FS and mathematics performance were correlated positively (r = 0.09 at each school); for females these two variables were negatively correlated (r = -0.14 and -0.50 at the two schools).

5. Interpretations

From the first study the author concluded that males were less ambivalent in their responses to a successful male figure than females were to an analogous female figure, providing support for the motive-to-avoid-success construct.

The author concluded that the second study provided further evidence of the FS construct being related to mathematics performance and participation. Leder suggested that teachers can play an important part in breaking down stereotypical views of mathematics and in alleviating the deep-seated concerns the FS construct measures. Intervention and remedial mathematics programs were recommended as the most promising directions for action.

Abstractor's Comments

This report is of special interest to us in the United States because it provides data from Australia in an area of research which has received a great deal of attention here. Of particular interest are the figures on female participation in non-compulsory mathematics.

Certainly investigation of the relationship of FS and mathematics participation and achievement is an important contribution to the growing literature aimed at isolating factors related to the study of mathematics by women. The FS construct has been questioned because of failure to replicate Horner's original results, although there is some evidence that it may be related to mathematics achievement and participation. Perhaps the author could have provided a more comprehensive review of the research related to FS to give the reader a better picture of the controversy surrounding Horner's theory, while still providing a rationale for its further study as an explanation of female underachievement in mathematics.

More important are concerns about the data presented from these two studies. So little information is given about the interview study (types of questions asked, information gleaned from the families, and type of analysis used, to mention just three items omitted) that it is difficult to judge the limited data presented.

The second study also suffers from some problems in reporting and methodology. Probably most mathematics educators are unfamiliar with an instrument to measure FS. An example of the verbal cues used, and more information about scoring and interpretation of scores would have been helpful. For example, are scores of 4-7 on this instrument considered high or low? The method for measuring self-consistency also was omitted.

The lack of a standard mathematics test given to all students makes interpretation difficult. At one school a teacher-constructed test was used; no information on the reliability or validity of that instrument is presented in the report.

The instrument used to determine to what students attribute the fact of fewer women in mathematics/science is insufficiently described. Were students asked to respond in writing to each of four alternatives or were they to choose the one explanation they preferred?

There are also some gaps in data presented in the report. No data on mathematics performance were presented; we do not know if these data were tested for sex differences. Certainly that is an important question to consider, even in a sample this small. A parenthetical comment mentions that school attended was not a statistically significant variable, but we are not told how the author tested this or on which of the five instruments this conclusion was based. Some of the data are presented by school, some are combined. Are we to assume significant school differences showed up on FS, thus accounting for separate school data being presented?

A summary of data provided by the two questionnaires to determine future course intentions and future career intentions would have provided more information to the reader. More males than females expressed a desire for a career in mathematics or science. I would have liked to know how the sexes compared on intention to select mathematics courses in the future. Were females or males uninterested in mathematics/science careers planning to take as much mathematics as possible anyway?

Are correlations reported between FS and mathematics performance Pearson product-moment correlations? Are any of these correlations significantly different from zero? No statistical tests for significance are reported anywhere in the paper. With the small sample size, especially for the data separated by school, the significance of the sex differences is questionable, and drawing conclusions from them is difficult.

Although the studies suffer from flaws, the limited data do indicate a need for further investigation of the variables of FS and attribution of sex differences to innate vs. external factors. The latter is particularly interesting in light of a recent study in the United States (Wolleat, Pedro, Fennema, and Becker, 1980) which found that females more than males attribute their personal failures in mathematics to ability, presumably an innate, stable factor; the Leder data showed females more likely than males to attribute sex differences in representation in careers in mathematics/science to external, social factors.

Whether FS is an important factor in explaining sex differences in participation and achievement in mathematics cannot be determined from this study, but the data hint that this variable merits further investigation.

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Mitchelmore, Michael. PREDICTION OF DEVELOPMENTAL STAGES IN THE REPRESENTATION OF REGULAR SPACE FIGURES. Journal for Research in Mathematics Education 11: 83-93; March 1980.

Abstract and comments prepared for I.M.E. by TERRY A. GOODMAN, Central Missouri State University.

1. Purpose

The purpose of this study was to test the generality of a scheme used to classify drawings of regular space figures into developmental stages. Further, data were obtained in an attempt to explain the relative difficulty of drawing different space figures.

2. Rationale

Various researchers have attempted to develop a classification scheme for drawings of regular space figures. Four developmental stages have been proposed by these researchers. The investigator summarized these stages as follows:

> Stage 1 (Plane schematic). The figure is represented by a single face face drawn orthoscopically (i. e., as if viewed orthogonally) or by a general outline.
>
> Stage 2 (Space schematic). Several faces are shown, but either the faces are drawn orthoscopically or hidden faces are included.
>
> Stage 3 (Prerealistic). The drawings attempt to represent the view from a single viewpoint and to depict depth.
>
> Stage 4 (Realistic). Parallel edges in space are represented by near-parallel lines on paper. (p. 84)

It was proposed that "the characteristics of drawings made in the four stages enable one to relate the sequence of stages to the development of children's concept of space from topological to projective and Euclidean (Piaget and Inhelder, 1967)" (p. 85).

It has been suggested that a child in Stage 1 focuses primarily on topological properties of a figure such as simple and closed. In Stage 2, a child begins to consider projective properties of space as evidenced by their representation of several faces of a figure. Children in this stage, however, do not appear to use a common frame of reference.

During Stages 3 and 4, children progress to projective space and eventually to well-established Euclidean concepts. In Stage 3 they show only visible faces of a figure and distort the figure to show depth. Finally, in Stage 4, the children represent parallel edges by parallel lines in their drawings.

Previous research (Lewis and Livson, 1967) and (Mitchelmore, 1978) has also indicated that cylinders are easier to draw than cubes and that cubes are easier than cuboids. No clear explanation of this result has been proposed.

3. Research Design and Procedures

Four male and four female students were selected from each of grades 3, 5, 7, and 9 from each of two schools in Columbus, Ohio. None of the subjects were from accelerated or remedial classes. This study was essentially a replication of a previous study (Mitchelmore, 1978) conducted with Jamaican students chosen from highly selective schools.

Subjects, tested individually, were asked to make drawings of five wooden models - cuboid, cylinder, pyramid, cube, and prism. Each student performed this task twice, once when each model was exposed for one second and once when each model was exposed for an indefinite period of time. This test was identical to one previously used by the investigator with the exception that the prism was not included in the earlier study. Each student was also given the Pacific Design Construction Test.

The drawings of the cuboid, cylinder, pyramid, and cube were classified independently by two researchers. Each drawing was assigned a score of 0, 1, 2, 3, or 4 and each student was given a total score for the eight drawings. These scores corresponded to Stages 1, 2, 3A, 3B, and 4. The reliability of the total drawing score was .92.

Drawing scores were then subjected to an analysis of variance with factors of Grade, Sex, Condition, and Figure, with repeated measures on the last two measures.

The researchers also attempted to predict the stages demonstrated in drawings of the fifth figure (prism). Typcial drawings predicted for each stage are shown in Figure 1.

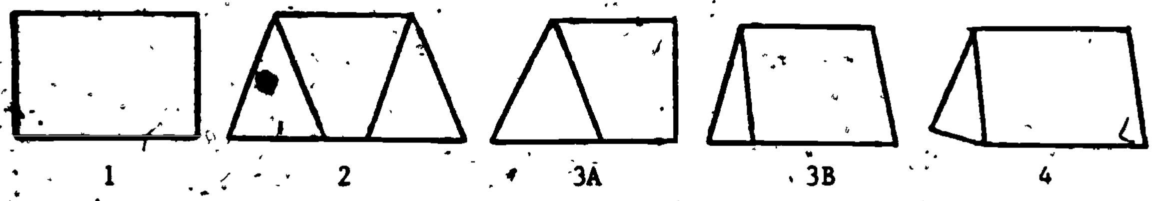

Figure 1. Typical drawings of a triangular prism predicted at each stage of representational development

4. Findings

Cross-cultural validity of the classification scheme was suggested by the fact that the two judges agreed on 88 percent of the drawings. Also, the reliability of the total drawing scores for the present study and the earlier study were similar. Finally, the correlation of the total score with the Pacific Design Test was .62 as compared to .59 in the Jamaican study.

The judges agreed on 89 percent of the classifications of the drawings of the prism. The correlation of the prism drawing score with the total score was .70 for the short term exposure and .73 for the indefinite exposure. This compared to item-test correlations of .65 to .85 for the other four solids. The results were suggested as support for the fact that the stages had been predicted validly.

The one significant interaction was grade x sex, with boys' scores being higher in Grades 5 and 9 but lower in Grades 3 and 7. The condition and figure effects were also significant.

Post hoc analysis revealed that the cuboid was more difficult than any of the other solids. There were no other significant differences between pairs of figures.

5. Interpretations

Based on the results of this study, the following conclusions and interpretations were offered:

1. Strong evidence was found to support the generality of the sequence of stages in the representation of regular space figures. This evidence included successful prediction of stages for a new figure as well as high test reliability and concurrent validity.

2. The variation between figures is stable. A child's stage of development for a given figure is dependent on both general level of perceptual development and the representational problems at each stage. For example, children

go to Stage 2 schema for the cylinder earlier because it is the most difficult to represent in Stage 1.

3. Representational ability is highly dependent upon spatial-perceptual development. It was proposed that the former will always lag behind the latter.

4. Children's schemata for representing space figures develops over time. Older children use a Euclidean space as opposed to a topological space and, hence, are able to represent more complex properties of the figures.

Abstractor's Comments

This study has several commendable aspects. The investigator made a careful effort to build upon previous research by replicating a prior study. By varying only two aspects of the previous study (a culturally different sample and including one new figure) the researcher was able to build upon the earlier results and generate more evidence regarding the stages of development. There is a need for more carefully coordinated research as represented by this study.

This study also is an example of well-done clinical research. The design and analyses were well-planned and should provide foundation for further studies. While this study has given us valuable information, there are several questions that should be considered.

While there is certainly evidence to support the proposed classification scheme, the generality of this scheme needs to be investigated further. The evidence used to support the generality of the stages included predicting the stages for a new figure. This gives a total, then, of five figures upon which the stages are based. The generality of the scheme would be much more conclusive if it were based on more than five model figures. What would happen if more figures were investigated? What would be the effects if less-familiar figures were used?

Because of the clinical nature of the study, relatively few students were tested (16 at each grade). Again, this may limit the generality of the results. Before these stages can be generalized, more students must be considered. It would also be useful to test different groups of students-gifted, remedial, etc. It was further the case that the results of the prior study were not completely replicated. For example, the pyramid was more difficult for the Jamaican students. Thus, more evidence may be needed to completely validate the stages of development.

Since the purpose of the study was to validate the proposed classification scheme and since the scheme was designed to describe an individual student's level of development, it would have been useful to include more information about individual student responses. Further discussion of individual differences would have given the reader more insight into the nature of the developmental stages being considered; perhaps more detailed description of two or three students' responses to all the figures would have been helpful.

More consideration needs to be given to the relationship between the proposed classification scheme and Piaget's theory. The investigator claimed that this relationship was quite strong. Previous research, however, raises some question here. Much of the previous research has focused on children's representations of plane figures and there has been some rather conflicting results with respect to Piaget's model. Martin (1976) found that geometric conceptualization was dominant at all ages. Geeslin and Shar (1979) obtained results that supported the notion that children relate figures by the amount of distortion needed to transform one to the other. How do these results relate to the results from the current study? Do children develop, for space figures, according to Piaget's model but follow different stages of development for plane figures?

Finally, there are several questions that could be investigated with further research:

1. What is the effect of school instruction on these stages of development?
2. How would tactile examination of the model figures affect the children's drawings?
3. Would more evidence for the generality of these stages be found if one group of students were studied over a period of time rather than several groups tested at one point in time?

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Abstract and comments prepared for I.M.E. by CHRISTIAN R. HIRSCH, Western Michigan University

1. Purpose

The general purpose of this study was to investigate possible aptitude-treatment interactions in the teaching of an introductory unit on Euclidean transformations in grade eight. In particular, the purpose was to search for interactions between Piagetian levels of cognitive development and instructional methods varied between classes along the dimensions (1) induction/deduction; (2) student activity/teacher demonstration; and (3) testing with or without the use of physical materials.

2. Rationale

This study was based on the premise that an adequate theory of instruction must rest on the "simultaneous consideration of both the situation dimensions and the person dimensions" (Snow, 1970). The "person dimensions" of interest in this study were the concrete and formal-operational stages of mental development as identified by Piaget and the sex of the learner. Studies by Collis (1973, 1978) are cited in support of potential interaction between developmental level and inductive/deductive instructional methods. It was hypothesized that concrete-operational students would show little achievement under deductive methods, whereas formal-operational pupils would achieve under either method. Support for the potential interaction between developmental level and student activity/teacher demonstration methodologies is found in Piaget (1974) and in Inhelder et al. (1974). For Piaget, teacher demonstrations alone may not promote learning in concrete-operational students. For Inhelder, either method may be effective for all students as long as the method emphasizes mental activity and corresponds to the students' levels of development. The inclusion of the testing dimension was based upon the belief that if concrete materials are necessary for the learning of concepts and skills by concrete-operational pupils, then the presence or absence of such materials during testing may differentially affect their achievement. Finally, based on a research review by Kelly (1976), it was hypothesized that females may profit more from deductive methods and teacher demonstrations of problem solving.

3. Research Design and Procedures

The subjects for this study were 245 students (108 females, 137 males) enrolled in eight grade 8 classes in a large metropolitan high school in South Australia. Four instructional treatments were identified: (I) deduction with individual student manipulation of materials; (II) induction with individual student manipulation of materials; (III) deduction with teacher demonstration; and (IV) induction with teacher demonstration. Each treatment was assigned at random to two intact classes and within each class students were randomly assigned to one of the two testing methods.

The instructional phase for each treatment consisted of seven 40-minute lessons incorporating the use of physical materials and taught by the same teacher. Each class followed the same sequence of content development; they differed only with respect to methodology. The deductive methods followed a rule-example paradigm. When a question or problem arose during lesson development, the appropriate rule was given and the solution was carried out by the student (treatment I) or demonstrated by the teacher (treatment III). The inductive methods followed an example-rule paradigm. Problems or questions that arose were "countered whenever possible with questions from the teacher designed to provide conflict situations" (p. 56) from which new results could evolve. In treatment II students were guided to their own solutions, whereas in treatment IV the teacher demonstrated the solutions.

The mathematical content consisted of an introduction to line reflections, rotations, and translations in a plane and to a coordinate analysis of preimage-image relationships. The concepts of line and rotational symmetry were also treated.

Subjects were identified as concrete or formal-operational by means of a group-administered version of Piaget's pendulum experiment (Rowell and Hoffman, 1975). This test, which was administered before and after the experimental treatments, had a test-retest reliability of 0.675. A 40-minute paper-and-pencil achievement test, with or without the aid of concrete materials, was used as both a pre- and posttest. The two halves of each class assigned to each form of this test were examined separately.

4. Findings

A t-test applied to the pretest means of the concrete and formal-operational groups indicated a significant difference ($p < .001$) favoring the formal-operational group. Hypotheses concerning aptitude-treatment interactions and factor main effects were tested via multiple linear regression analysis. No significant interactions were found. However, significant main effects were evident in the adjusted (by pretest) posttest scores for developmental level and for student activity vs. teacher demonstration. Subjects classified as formal-operational performed significantly better ($p < .001$) than their concrete-operational peers. Students who were instructed by teacher demonstration performed significantly better ($p < .03$) than their counterparts in the student activity classes.

5. Interpretations

The absence of any empirical confirmation for the theoretically predicted interaction effects may be due to the use of physical materials in all instructional treatments. It was hypothesized that the absence of such materials may lower the performance of concrete-operational students, thereby increasing the potential for aptitude-treatment interactions. The greater effectiveness of teacher demonstration in comparison with individual student manipulation of materials provides support for the position of Inhelder and suggests that the "necessity, if it exists, for 'hands-on' student manipulation of materials may be limited to the very young" (p. 59). Further investigation of the relative effectiveness of teacher demonstration vs. student activity is warranted. The fact that the posttest mean of the concrete-operational group was less than the pretest mean of the formal-operational group suggests, at least in the case of transformation geometry, that either new ways of minimizing the achievement gap between these groups must be found or some method for tracking students in terms of developmental level should be explored.

Abstractor's Comments

The investigators are to be commended for conducting an ATI study in a school-based setting and one in which the treatments focused on the learning of a standard, but significant, body of mathematical content over more than one or two instructional periods. Within this setting, the results of the study offer no support for the general ATI hypothesis first advanced by Cronbach or the specific hypotheses suggested by Piagetian theory. The failure to provide such support may be due in part to the research design and procedures used. In particular:

1. Rather than using two aptitude measures and three treatment dimensions leading to an examination of 20 possible interactions (four-way and higher interactions were ignored due to small subsample sizes), it may have been preferable to use cognitive level and a single treatment dimension (e.g., induction/deduction) carefully interpreted and "tuned" so as to take advantage of the potential interaction predicted by theory.
2. Similarly, more attention might have been devoted to the development and refinement of the achievement test. It would have been preferable to use parallel forms rather than the same test as both a pretest and posttest. Moreover, pretest and posttest means of 4.48 and 6.31 for the formal-operational group suggest the test itself was not sensitive enough or too difficult. Of course, another possible explanation is that very little learning occurred during the experiment. If this is the case, then again more care should have been taken in developing the instructional methods.
3. Test-retest reliability of the measure of developmental level was relatively low. In retrospect, perhaps more than a single task should have been used to classify subjects as concrete or formal-operational.
4. Since classes, not students, were assigned to instructional treatments, the unit of analysis should have been the class mean. Thus, the sample size of this study was n = 8 classes, not 245 subjects. Insufficient data are reported to conjecture as to whether a reanalysis would result in any significant interactions.

Several concerns arise in connection with the reporting of the study.

1. Although the rationale for the study was clear, the contextual framework was limited. No attempt was made to relate the present

study to previous ATI research efforts in mathematics education or to summaries thereof as found, for example, in Kilpatrick (1975) or Cronbach and Snow (1977).

2. The authors indicated that test details are available upon request. However, since the results hinge upon the single investigator-developed achievement test, as a minimum the report should have provided (a) sample test items; (b) the number of items and nature of scoring; and (c) measures of validity and reliability.
3. Although instructional methodology was carefully explicated, there was no indication that the intended treatments did in fact occur and were consistent across each of the two classes.
4. Basic to all instructional methods was the use of concrete materials to model line reflections, rotations, and translations. However, no indication was given of the specific materials used. Geoboards and tracing paper (transparencies) lend themselves to both teacher demonstration and individual student manipulation. However, MIRAS and cosmetic mirrors are most appropriate for student use. Information about the materials used is critical to any interpretation of the results of this study, particularly in light of the significant advantage reported in favor of teacher demonstration over individual student activity.
5. Notably absent were any descriptive statistics other than the pretest and posttest means in the two instances where significant differences were detected. There was even no information given about the numbers of students identified as concrete or formal-operational.

In summary, the authors chose interesting and in some cases heretofor unexamined ATIs to investigate. However, the low performance by subjects on the posttest suggests serious problems with instrumentation and/or instructional procedures and brings into question the validity of the results. The difficulty of interpretation is further compounded by the incompleteness of the report. The finding of greater effectiveness for teacher demonstration with concrete materials in comparison with individual student manipulation is consistent with an earlier study (Vance, 1969) conducted with seventh- and eighth-grade pupils. Given the increasing financial constraints on urban school systems together with the usual problems associated with multiple sets of apparatus, further research in this area with junior high school pupils is warranted.

Future ATI investigations involving geometric content might wish to consider the van Hiele levels of geometric thinking (Wirszup, 1976) as opposed to Piaget's levels of cognitive development.

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Abstract and comments prepared for I.M.E. by JANE O. SWAFFORD, Northern Michigan University.

1. Purpose

The purpose of the study was to assess the effect of weekly computer enhancement assignments on achievement and attitude in second-year algebra.

2. Rationale

Various reports have recommended that computers be incorporated into the school mathematics curriculum and have pointed out the need for research to determine how this might best be done. This study evaluates the effectiveness of a plan which uses out-of-class weekly computer assignments designed to enhance instruction without taking class time or replacing standard content to teach computer programming.

3. Research Design and Procedures

One hundred one (101) second-year algebra students were randomly assigned to two control and two experimental sections. One teacher taught both control classes and one experimental class. The other experimental class was taught by one of the investigators. The two teachers coordinated their instruction extensively. Assignments, tests, and other treatments were the same for all classes except that the experimental classes received weekly computer assignments from Computer Resource Book - Algebra (by Tom Dwyer and Margot Critchfield) which were graded and returned. The assignments related to the algebra topic being studied and required less than 30 minutes to complete. No programming was formally taught. No class time was spent discussing the assignments and very little teacher time was spent in helping students outside class. In all, the additional assignments added 15 to 30 minutes to the teacher's normal work load.

During September, the Lankton First Year Algebra Test was administered. It, with the Otis-Lennon Mental Ability Test, was used to determine the comparability of the two groups at the beginning of the year. The Otis was further used to partition the subjects into high ability and average ability groups for analysis. In January and May, the Cooperative Mathematics Test: Algebra II, the Suydam-Trueblood Attitude Toward Mathematics Scale, and the Suydam-Beardslee Attitude Toward the Instructional Setting Scale were administered to all students. Additionally, the experimental group completed an Attitude Toward Computer Use Scale. Two-way ANOVA (treatment x ability) were used to analyze the achievement and attitude data; t-tests were used to analyze initial differences between the experimental and control groups and differences from January to May on the experimental group's attitude toward computer use.

4. Findings

No significant differences were found between the two groups at the beginning of the year. No significant differences were found between the two groups on algebra achievement, attitude toward mathematics, and attitude toward the instructional setting. Differences in ability did have a significant effect on achievement but not on attitude. No significant interactions between treatment and ability were found. Both ability levels in the experimental group improved significantly in their attitude toward computer use during the second half of the year.

5. Interpretations

The investigators concluded that "computer related assignments can be given weekly throughout an entire course in second-year algebra with no effect on achievement and without taking time and topics from algebra content." The investigators observed that the initial reaction to the computer assignments was negative, particularly for the high ability group, being viewed as an extra, unnecessary requirement.

Abstractor's Comments

This well-done study focuses on an important issue in mathematics education, the appropriate use of computers in the curriculum. Although it found no significant differences, it tells us more than it says. In the study, a plan for the comprehensive use of computers was carefully implemented in a way so as not to distract from class time or course content. The process was carefully monitored. The end result was no improvement in achievement or attitude. Given the cost of outfitting a computer room, one would hope for more. If anything, the computer assignments were met with hostility, although September attitude data were not collected. With findings like this, one is tempted to ask what good is all of this technology. Unfortunately, only informal observations were available to testify to the fact that the students learned a good deal about computers. But if knowledge about computing is the goal of instruction, it can probably best be achieved by teaching computing, not algebra.

What this study tells us is that the effective integration of computers into the curriculum is more than an Apple on every teacher's desk. Like anything in the curriculum, to make a difference it must become an integral part of the classroom life whether it is problem solving, computational skills, or computers. Finding the most appropriate way to enhance the teaching and learning of mathematics with the new technology is not going to be easy. This study helps to underline the magnitude of the problem we face.

Wagner, Sigrid. CONSERVATION OF EQUATION AND FUNCTION UNDER TRANSFORMATIONS OF VARIABLE. Journal for Research in Mathematics Education 12: 107-118; March 1981.

Abstract and comments prepared for I.M.E. by JOHN G. HARVEY, University of Wisconsin-Madison.

1. Purpose

The study had two purposes:

1) to extend the use of Piagetian methods and techniques to the study of the conservation of relations and
2) to investigate the conservation by students of the equational relation "is the same equation as" and of the functional relation "is the same function as" when the variables appearing in equations and functions were changed to different letters of the alphabet.

2. Rationale

A relation between two sets is a subset of the Cartesian product of those two sets and is *conserved* by an individual when that person reliably identifies that the critical attributes of the relation are invariant under transformations of the relation by irrelevant attributes. Set relations are important to the study and use of mathematics; they are used, for example, to define equivalent fractions. In addition, *function*, a particular kind of set relation, is a predominant idea of modern mathematics. Thus it is important to discover how well students conserve set relations.

In many instances, once two sets have been identified, the critical attributes of a relation between those two sets are described using literal (letter) variables. Since these variables are arbitrarily chosen, and hence are irrelevant attributes of the relation, it is important that students conserve relations under alphabetic transformations of literal variables.

Two important relations taught in schools are (a) "is the same equation as" (i.e., the equations have the same solution set) and (b) "is the same function as." The investigator chose to study the conservation of these two relations using a linear equation and three functions with 6-point domains.

3. Research Design and Procedures

a) Subjects. There were 15 middle school subjects (8 M, 7 F); they were from either a Title I public elementary or junior high school or an academically selective private elementary school, were in grades 5-8, and ranged in age from 10 to 15 years (median age: 13 years). Five of these subjects had studied at least one semester of algebra.

There were 14 high school subjects (6 M, 8 F); an additional male student was initially selected but screened out by his performance on one of the warm-up tasks. The high school subjects were from a comprehensive high school, were in grades 9-12, and ranged in age from 15 to 18 years (median age: 16½ years). Ten of the high school students had studied at least one semester of algebra.

All four of the schools were in New York City. In each school teachers were asked to select two or three students of varying mathematical ability; the sample of subjects was selected from this pool of students.

b) The Instrument. The instrument consisted of six tasks, two warm-up tasks and four experimental tasks; one conservation-of-equation task and three conservation-of-function tasks. Each task was printed on a separate card; the numbers and operations symbols were printed in red and the variables in black.

The equation warm-up task consisted of a card on which the equation 2 x N + 3 = 11 was printed. The function warm-up task consisted of a card on which a table of ordered pairs for the function N = 2M was printed.

The conservation-of-equation task card had the equation 7 x W + 22 = 109 as its first line. The second and last line on this card was the same as the first line except that a hole was cut where the variable appeared on the top line. Under the hole was a sliding strip which permitted either the variable "W" or "N" to be displayed.

There were three conservation-of-function tasks: one free-response task and two furnished-response tasks (A & B). Each task card presented a table having six places for ordered pairs in two columns; on each card the complete set of domain values for the function used were displayed. Both the domain and image columns were given (different) variable names. On the free-response task card there was a hole cut in the card so that the image variable name could be changed; the last data entry in the image column was blank on this card. The function used to generate the image data values on this card was $C = -B^2 + 12B + 9$.

On furnished-response task card A holes were cut in the card so that the image variable name and the fifth image data entry could be changed. When the variable name "Z" was showing on this card, the fifth image data entry was "27"; when the variable name was "Y", the fifth image data entry was blank. The function used for task card A was $Y = 2X + 3$.

On furnished-response task card B the image column variable name could be changed as could the third data entry in that column; when the image variable name was "K" on this card, the third entry in the column was "8"; when the image variable name was "K + 3", the third data entry was blank. A function which generates the image data on this card is $K = J^2 - 1$.

c) Procedure. The investigator conducted a clinical interview with each subject; each interview lasted approximately 20 minutes and was audiotaped. The investigator made written notes of student responses during the interviews. Each subject was presented with all six tasks. In presenting the tasks to the subjects, the furnished response tasks A and B were always presented in order and in conjunction to each other. The conservation-of-equation task was always the first or second experimental task presented. All four of the possible permutations of experimental task orders were used and were assigned to subjects randomly.

When presented with the equation warm-up task card, a subject was asked to read the equation on the card and find the value of N which would make that sentence true. Immediately following completion of this task, the conservation-of-equation task was presented; when the conservation-of-equation task card was shown to a subject the first and second lines on the card were identical. After a subject acknowledged that the two lines were the same, the interviewer stated that she was going to change "W" to "N" and moved the strip on the back of the card so that the second line became 7 x N + 22 = 109. Then the interviewer asked the subject which value, W or N, when found would be larger. After the subject responded to this question, the interviewer asked, "Why?" or "How can you tell?"; she asked one of these two questions last in each experimental task with each subject.

The function warm-up task always immediately preceded the first conservation-of-function task presented to a subject. When presented with this warm-up task, subjects were asked to supply three missing values for the function. When given the free-response task, subjects first supplied a value for the blank data entry in the image column; then the variable name for that column was changed and the subject was asked if the value of the blank data entry would be the same or different from that originally given.

On both furnished-response tasks the image data values associated with domain data values were discussed with the subject while the variable "Z" or "K" was showing, respectively. Then, by sliding the strip on the back of the card, the variable names were changed and a data entry in that column became blank. Subjects were then asked what the missing value was and why.

d) Scoring. Each subject was scored on each task as being a conserver, a non-conserver, or transitional. A subject was judged as a conserver if he or she responded that the two variables names had the same value, as a non-conserver if he or she responded that the two values were different, and as transitional if he or she gave conflicting answers or justified equal values on computational grounds.

Using this scoring scheme the investigator and a high school teacher independently scored all of the student responses using as protocols the interviewer's written notes. Cramer's *V* coefficient of interrater agreement greater than 0.92 is reported for each task. Only the interviewer's data are reported and used in the data analyses.

4. Findings

a) Because of the small sample size (N = 29) the overall effect of task order could not be determined; however, "the order in which the two types of function tasks were presented had no statistically significant effect on responses to any (*sic*) task" (p. 113).

b) The responses to the conservation-of-equation, free-response, and furnished-response A tasks were bimodally distributed (Table 1, p. 113 is reproduced below). The responses to furnished-response task B were skewed toward non-conservation; as a result these data were deleted from subsequent statistical analyses.

c) The joint response frequencies for each pair of the three remaining tasks were analyzed; the coefficient of association was statistically significant for each pair ($p < .02$) and for all three tasks ($p < .05$).

Table 1

Total Response Frequencies for Each Conservation Task

Task	Type of response		
	Nonconserving	Transitional	Conserving
Equation	13	5	11
Free-Response	13	3	13
Furnished-Response, A	19	1	9
Furnished-Response, B	26	1	2

d) No significant association between age group and the ability to conserve equation or function was found.

e) There was a statistically significant association ($p < .05$) between the sex of the subjects and the conservation of equation. Males more frequently conserved or were transitional on this task.

f) There was a statistically significant association ($p < .05$) between having taken one semester of algebra and conservation on both equation and function.

5. Interpretations

"The results obtained in this study suggest that the ability to conserve equation or function varies among students of different ages, of either sex, and of various mathematical backgrounds. Less than half of the students interviewed gave conserving responses to any one of the four tasks used in this study" (p. 115). "This study documents two common misconceptions about variables: (a) that changing a variable symbol implies changing the referent and (b) that the linear ordering of the alphabet corresponds to the linear ordering of the number system" (p. 116).

Abstractor's Comments

As the investigator points out, students' understandings of relations have been extensively investigated but none of that research has employed the methods and techniques developed by Piaget. Thus the present study is worthwhile in that it demonstrates that these methods and techniques can be applied to further our knowledge of students' understandings of relations. In addition, while the questions investigated are small ones, those findings, too, add to our knowledge and may help us to improve instruction on equations and functions. The study is well conceived, instrumented, and executed; the investigator's conclusions are appropriate.

This study has several limitations; the investigator identified three: sample, number of items in the instrument, and wording (presentation) on the items. In her discussion she indicates awareness of the microscopic size of the problem and that her study did not reveal why one-third of the students who had studied algebra did not conserve. Without discussing other interesting problems which the investigator might have investigated, there are two other matters which should be mentioned:

1) The investigator does not give a definition of *variable*; the implied definition of "a letter which 'stands for' the elements in the solution set of the equation or the elements in the domain of the function" would appear to be incomplete (see Tonnessen, 1980) and thus may have led to measurement errors when assessing conservation of equation or function.

2) It is acknowledged that the equation and function warm-up tasks attempted to screen out subjects who did not know enough about solving equations, solution sets, and functions to conserve equation and function. However, it appears that these tasks are weakly discriminating. Thus it seems likely that a more extensive pretest of prerequisite knowledge should have been administered which was designed to indicate better how well subjects (a) knew what it meant to solve an equation, (b) recognized the phrase "the solution set of an equation", and (c) could identify examples and non-examples of functions.

This would appear to be a fruitful methodology for investigating students' understandings of relations and in particular, equational and functional relations. Thus it is good to report that the investigator and others are continuing research along these lines.

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Webb, Noreen M. AN ANALYSIS OF GROUP INTERACTION AND MATHEMATICAL ERRORS IN HETEROGENEOUS ABILITY GROUPS. British Journal of Educational Psychology 50: 266-276; November 1980.

Abstract and comments prepared for I.M.E. by F. RICHARD KIDDER, Longwood College, Farmville, Virginia.

1. Purpose

This study investigates the influence of the learning setting (small heterogeneous ability group or individual) on achievement of new and previously learned mathematical material.

2. Rationale

Citing several studies comparing heterogeneous ability grouping and individual conditions as justification, Webb selected a small subsample of her prior 1977 investigation to analyze further group interaction and achievement. In so doing, she assumed that the conditions, the format (very sketchily reported herein), and a small selected subsample of the original study were adequate for drawing further conclusions.

3. Research Design and Procedures

In the original study, 64 eleventh-grade students were selected from a pool of 181 students from three high schools in a suburban area of Northern California. From this 1977 study, Webb selected five groups of four students each. Each group contained one student of high-ability level, two of medium-ability level, and one of low-ability level. The mean IQs of these levels were 124, 117, and 105, respectively.

The study consisted of a training session, a problem-solving practice session, and testing. Two specific tasks were involved in all three: task I based on mathematical probability and task II dealing with positive and negative number bases. The three sessions comprising each task was spread over a time span of one week.

The training was identical for all subjects. The experimenter explained the nature of the study and procedures, then students worked alone on prepared training booklets. Problem practice sessions were conducted differently on each task. On task I, all students worked individually; on task II, all worked in groups. During group practice, each group was instructed to work as a group; that is, to ask questions of teammates and to explain the problems to each other. This verbal interaction was recorded. Testing consisted of one problem per task. The test item was of the same format as the task problem-solving practice.

4. Findings

Webb summarized her findings thusly:

> Averaging over all types of errors, high-ability students did best in the individual condition, and medium-ability students did equally well on both conditions. When types of errors were distinguished, for high-ability students, the group condition was detrimental to learning new material (the mathematical algorithm) but was advantageous for performance on previously learned material (computational and algebraic manipulations); for medium-ability students, the group condition was good for learning the algorithms but not for performance on computational and algebraic manipulations; for low-ability students, group work was beneficial for performance of new and previously learned material. Group interaction was related to achievement: students who described or received explanations about the mathematical algorithm did well on algorithm on the test, and students who explained how to perform computational or algebraic manipulations did well on these manipulations on the test.

5. Interpretations

In analyzing her findings, Webb reasoned that describing an algorithm may have helped students understand and memorize how to do the algorithm. Also, being the recipient of explanations about algorithims is better than reading about the algorithm. She attributed the ineffectiveness (for the explainer) of giving explanations on calculations to the fact that students tended to do calculations by rote, and suggested that the frequency of careless errors indicated that students were attending more to the algorithm than to the calculations. Total test performance masked the results by error types. It is interesting that the above findings are somewhat modified when error types are taken into consideration. Whether working in groups is considered as beneficial or detrimental compared to individual work may depend upon the type of error which is emphasized.

Abstractor's Comments

Webb's findings appear to substantiate the intuitive belief of many teachers: heterogeneous group work does indeed help lower-ability students and does, in some instance, penalize students of high-ability. Webb's error analysis is of particular interest, highlighting that "...whether working in groups is interpreted as beneficial or detrimental compared to individual work depends upon the type of error that is emphasized." If so, we might well examine more closely the material to be learned prior to grouping.

Two facets of the study are somewhat disturbing to this reviewer. How was the subsample selected? Was it random? It appears that one could eyeball a data bank of 64 subjects and from these select 20 which would support a desired thesis. Further explanation is also needed as to the ability level classification and labeling. Three-fourths of the subjects appear to have an average IQ of approximately 120. If one grouped the normal heterogeneous high school mathematics class into three levels, would the high-, medium-, and low-ability groups have mean IQ's of 124, 117, and 105 respectively? I suspect not. It appears that all subjects were above average in ability.

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