Approximation Algorithms Minicourse

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Introduction

Goal

Approximation algorithms for optimization problems find solutions that have value provably near the optimal value in polynomial time. Usually such algorithms are designed for NP-hard problems, for which polynomial-time algorithms seem unlikely to exist. Although the area of approximation algorithms has been around since the 1960s (predating the concept of NP-completeness), it has grown significantly in just the past few years.

One of the reasons the field has grown is that several new techniques in the design of approximation algorithms have been discovered and applied. Consider the quotes below from the 1970s:

> One of our hopes for this thesis has failed to be realized. This was that the proofs of our results, and the ideas involved in them, might be of use to researchers investigating other problems. Unfortunately, the best proofs we could find have turned out to be quite domain dependent...
>
> D.S. Johnson (1973), *Near Optimal Bin Packing Algorithms*

> The methods used for designing such [approximation] algorithms tend to be rather problem specific, although a few guiding principles have been identified and can provide a useful starting point.
>
> M.R. Garey and D.S. Johnson (1979), *Computers and Intractability*

As the quotes attest, many years ago the design of approximation algorithms tended to be ad hoc, and each algorithm relied heavily on the structure of the particular problem at hand. In the past few years, however, there have emerged certain methods for designing approximation algorithms that have been successfully applied to a large number of problems. The goal of this course is to describe some of these methods, and to show recent results applying them to a wide variety of problems.

Another reason the field has grown recently is because of exciting new proofs showing that certain kinds of approximation algorithms cannot exist unless $P = NP$. A second goal of this course is to understand some of these results and the techniques used to derive them.

Syllabus

The general syllabus of the course will be as follows:

Monday	Introduction to Approximation Algorithms
	Metric Methods (minimum multicuts, balanced cuts, minimum flux cuts)
Tuesday	Randomized Methods (maximum satisfiability, maximum cut in dense graphs)
Wednesday	Semidefinite Methods (maximum cut, coloring 3-colorable graphs)
Thursday	Primal-Dual Methods (network design problems)
Friday	Hardness of Approximation (maximum satisfiability, maximum cut, maximum cliques)
	Wrap-up

The first four days will concentrate on methods for designing approximation algorithms, while the last day will describe some of the recent work proving that certain approximation algorithms cannot exist unless $P = NP$.

Schedule and Format

The format of the course will be a mixture of lecturing and problem solving. Each day I will hand out a problem set based on that day's material. We will discuss the solution to the problems the afternoon of the following day; I will invite the students to present the solutions that they found. Obviously there are some boundary problems with this system. For this reason I will lecture for an extra hour Monday afternoon, and there will be no problem set for Friday's material. Students are encouraged to collaborate with each other to solve the problem sets.

The daily schedule of the course is as below:

09.45-10.45	lecture
10.45-11.00	coffee break
11.00-12.00	lecture
12.00-13.30	lunch break
13.30-15.30	problem session

The lectures may on occasion run into the problem sessions in the afternoons.

Bibliography

Below is a bibliography of the papers discussed during the minicourse. Some general resources on approximation algorithms include an excellent survey by Shmoys [30], notes by Motwani [25], and a forthcoming book edited by Hochbaum [17].

Monday: Metric Methods

The approach to presenting the Garg, Vazirani, and Yannakakis result was drawn from Shmoys [31]. Thanks also to Seffi Naor and Baruch Schieber for showing me their simplified balanced cut result.

The main results covered are by Garg, Vazirani, and Yannakakis [11], Even, Naor, Rao, and Schieber [9], Aumann & Rabani [7], and Linial, London, and Rabinovich [23].

Tuesday: Randomized Methods

The main results covered are by Goemans & Williamson [13], and Arora, Karger, and Karpinski [4].

Other papers mentioned include those by Sahni & Gonzales [29], Johnson [18], Lieberherr and Specker [22], Raghavan & Thompson [28], Spencer (in Alon and Spencer [2]), and Yannakakis [33].

Good additional resources include the book by Motwani & Raghavan [27] and the forthcoming chapter by Motwani, Naor, and Raghavan [26].

Wednesday: Semidefinite Methods

The main results covered are by Goemans & Williamson [14], and Karger, Motwani, and Sudan [19].

Other papers mentioned include those by Lovász [24], Karloff [20], Frieze & Jerrum [10], Alon & Kahale [1], and Goemans & Kleinberg [12].

Good additional resources about the Theta function include the book by Grötschel, Lovász, and Schrijver [16], and notes by Knuth [21].

Thursday: Primal-Dual Methods

The results presented here are drawn almost entirely from the survey of Goemans & Williamson [15]. The interested reader can refer to the survey for the other papers mentioned.

Friday: Hardness of Approximation

The results in this area come from a long string of papers. I will mostly be discussing results from Arora, Lund, Motwani, Sudan, and Szegedy [6] and Bellare, Goldreich, and Sudan [8]. Some of the presentation is due to the survey chapter of Arora and Lund [5] and discussions with Madhu Sudan.

In addition to the Arora-Lund survey [5], good additional resources include the prize-winning theses of Arora [3] and Sudan [32].

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