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Double Phase Differences Correction Method For Frequency and Phase in Spectrum Analysis

Luo Jiufei[1,a], Xie Zhijiang[1,b], Chen Ping[1,c]

[1]Department of Mechanical Engineering, Chongqing University, Chongqing 400044,People's Republic of China

[a]823761825@qq.com, [b]xie@cqu.edu.cn, [c]eptk@sina.com

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Abstract

This paper advances a new method, double phase differences correction method, which aims at correcting the errors of frequency, phase and amplitude of the harmonic signal. The employment of this method involves that two phase differences of two highest spectral lines in the main lob are used to get the correcting value of frequency and then phase and amplitude can be rectified by corresponding formula. It solves the problem that the traditional formula of translation of window center is only for rectangle window. This new formula can correct for the different window by the principle of translation of window center. Simulation shows that the maximum error of frequency of the signal with white noise is less than 1% of frequency resolution. The maximum error of phase is around 1.5º, and the maximum error of amplitude is within 0.01. The average errors in frequency, phase and amplitude are approximately 0.002Hz, 1º and 0.005, respectively.

1. Introduction

In spectrum analysis, as it is difficult to satisfy the integral period truncation in sampling, the leakage effect and fence effect in the discrete and limited transform are unavoidable, which leads to the errors in frequency, amplitude and phase[1,2].For analysis of a single harmonic with rectangular window and Hanning window, the maximum error of amplitude is up to 36.4% and 15.3%, respectively[1-4]. The maximum frequency error is about half of the frequency resolution for rectangular window[4]. The maximum error in phase is even up to 90 degree[5].

To solve the problems of the errors in the discrete spectrum, a lot of methods have been put forward. Phase difference correction method is one of them [1].This method is firstly proposed as phase interpolation estimator (PIE) to estimate the frequency of a single tone in 1986 by D.R.A. McMahon and R.F. Barrett[6].In 2000,I. Santamria , C. Pantaleon and J. Ibanez[7] improved it for frequency estimation as well. And in 1998, Y. Liu [8] presented window-length changing method to get phase difference to assess the frequency which changed the length of window in FFT. One year later, M. Xie and K. Ding [9] came up with a special case of the time-shifting correcting method of phase difference by two continuous time-domain signal segments. Later in 2002, K. Ding concluded the universal time-shifting correcting method of phase difference[10]. In 2003, Y.Z. Huang[11] proposed another new method called the method of translation of window center. At the same time,K. Ding and M. Xie[12] put forward synthesis method, including time-domain translation and the change of window width.

Under the condition of noises, phase difference correction methods display much higher resistance[13,14]. As a result, they are widely used in various aspects of signal processing and engineering application [15-23]. However, the traditional implements of phase difference correction methods are not as well as we have expected in engineering practice[24]. This paper proposed a new method to correct the frequency, phrase and amplitude of single frequency component.

2. Theory of new algorithms

2.1 The basic theory of traditional phase difference correction method

In order to explain the general theory of the phase difference correction methods of different kinds, we simply consider the case of a single cosine signal

$$x(t) = A\cos(2\pi f_0 t + \theta). \quad (1)$$

And A is the amplitude,with f_0 the accurate frequency and θ the phase. Its Fourier Transform is

$$X(f) = Ae^{j\theta}\delta(f - f_0)/2 + Ae^{-j\theta}\delta(f + f_0)/2. \quad (2)$$

The signal is truncated by a window $W_T(t)$ whose length is T. The window $W_T(t)$ is from the symmetrical window $W(t)$ by a time-shifting of $T/2$.We assume that the $W(f)$ is the Fourier Transform of $W(t)$,then the Fourier Transform of $W_T(t)$ is $W(f)e^{-j\pi fT}$ according to the time-shifting .The Fourier Transform of windowed signal is $X(f) * W(f)e^{-j\pi fT}$ according to the convolution theorem. To consider the positive frequency part of this spectrum, we can get

$$X(f) * W(f)e^{-j\pi fT} = \frac{A}{2}e^{j\theta}\delta(f - f_0) * W(f)e^{-j\pi fT} = \frac{A}{2}W(f)e^{-j[\pi fT(f-f_0)-\theta]}. \quad (3)$$

So, the phase now is $\varphi = \theta - \pi T(f - f_0) = \theta + \pi T\Delta k f_\Delta$ .We can further write the phase as

$$\varphi = \theta + \pi N\Delta T\Delta k f_S / N = \theta + \pi\Delta k. \quad (4)$$

Or it can be rewritten as $\theta = \varphi - \pi\Delta k$. (5)

In the equation (4) and (5), we can see the slope coefficient of the line is $\pi(-\pi)$.The slope coefficient depends up on how much the time window shifts.

A method to change the slope coefficient between two segments is proposed by Y.Z. Huang. The first segment is N points adding a window with length of N, and the second segment is the same as the first segment but directly setting the former L points to zero. To the first segment, according the translation of window center, we can get the phase equation $\varphi_1 = \theta + \alpha\Delta k$ .To the second segment, we can also get the phase equation $\varphi_2 = \theta + \beta\Delta k$ .The the normalized frequency correction value is got by the equation

$$\Delta k = (\varphi_1 - \varphi_2)/(\alpha - \beta). \quad (6)$$

where α and β stands for their slope coefficient. The key step of the method of translation of window center is to set the former L points of the first segment to zero to constitute the second segment, which is the same as adding another shorter rectangle window on the windowed first segment. When the rectangle window is truncated , it is still a shorter rectangle window and symmetrical. But when the others are done by this, it becomes asymmetrical. In other word, it is difficult to work out the frequency correction value through formula (6) because it is hard to get the value of β. Moreover, the phase-shifting of this asymmetrical window is not strictly linear, either. Fortunately, in main lobe, the phase is still very close to a straight line.

2.2 The double phase differences correction method

Double phase differences correction method need choose the two highest spectral lines in the main lobe of each spectrum to get their phase information. For the first segment, it is assumed that φ_1 and ϕ_1 stands for the phase information of the first and second highest spectral line. For the second segment, it is similar that φ_2 and ϕ_2 stands for the two phase information. Then according to the linear phase-shifting theorem we can get four equations below.

$$\varphi_1 = \theta + \alpha\Delta k. \quad (7)$$

$$\phi_1 = \theta - \alpha(1 - \Delta k). \quad (8)$$

$$\varphi_2 = \theta + \beta\Delta k. \quad (9)$$

$$\phi_2 = \theta - \beta(1 - \Delta k). \quad (10)$$

According formula (7) and (9), it is easy to get the equation

$$\Delta k = (\varphi_1 - \varphi_2)/(\alpha - \beta) \quad (11)$$

According to formula (8) and (10), we can get the equation (12)

$$1 - \Delta k = -(\phi_1 - \phi_2)/(\alpha - \beta) \quad (12)$$

After canceling the factor $\alpha - \beta$, formula (13) is obtained

$$\Delta k/(1 - \Delta k) = -(\varphi_1 - \varphi_2)/(\phi_1 - \phi_2) = -\Delta\varphi/\Delta\phi . \quad (13)$$

In this new formula, $\Delta\varphi$ and $\Delta\phi$ is the phase difference of the two highest spectral line. Setting σ equal to $-\Delta\varphi/\Delta\phi$, the relationship between the normalized frequency correction value Δk and the ratio σ can be simply described as formula (14)

$$\Delta k = \sigma/(1 + \sigma) \quad (14)$$

Then the correcting value of frequency is

$$\Delta f = \Delta k f_{\Delta} \quad (15)$$

The formula (14) called the double phase differences correction method offers us a new way to obtain the frequency correcting value, which does not have to know the slope coefficient of phase-shifting of the window. Therefore, it avoids calculating the quantum of translation of window center which is a hard problem in the asymmetrical window.

By combining the formula (1) and (2) together, we can also get the formula (16)

$$\alpha = \varphi_1 - \phi_1 . \quad (16)$$

Substituting equation (9) into equation (1), it yields

$$\theta = \varphi_1 - (\varphi_1 - \phi_1)\Delta k = \varphi_1(1 - \Delta k) + \phi_1 \Delta k . \quad (17)$$

The value of corrected phase is obtained after the normalized frequency correction value is worked out through equation (10).The corrected amplitude value can be worked out as well by DFT in which the integral k is replaced by $k + \Delta k$ where k is the number of the spectrum lines[25].

3. Simulation calculation and result

The example signal is produced by computer according to the formula (18). It has three harmonic components, in which 200.6 Hz,209.2 Hz and 280.3Hz . There is also white noise $e(t)$ mixed with the harmonic components whose amplitude is uniformly distributed. The maximum amplitude of the noise is 0.25(a quarter of the unifrequent amplitude). There are some assumptions that the sampling frequency is 1024 Hz, and the number of carrying out FFT is 1024 points, so the frequency interval is 1 Hz. And the theoretical value of the amplitude of each harmonic component in the one-side spectrum is 1. In the experiment, each frequency component will be corrected based on rectangle window and Hanning window.

$$\begin{aligned} Y = {} & \sqrt{2}\cos(2\pi 200.6t + 45\pi/180) + \sqrt{2}\cos(2\pi 209.2t + 18\pi/180) \\ & + \sqrt{2}\cos(2\pi 280.3t + 90\pi/180) + e(t) \end{aligned} \quad (18)$$

Since the white noise is stochastic, the result of corrected frequency, phase and amplitude differs slightly every time. We take the maximum value, minimum value and the value on an average of 100 times into consideration to assess the two methods.

The result with rectangle window is shown in Table 1.The maximum error of the corrected frequency of the two adjacent frequency components by the old method is over 0.02Hz. The maximum error of the corrected phase and amplitude is about 7º and 0.3V. The maximum errors of 280Hz are much less than the two adjacent frequency components. The maximum error of the corrected frequency of all components by the new method with rectangle window is below 0.01Hz. And the maximum errors of the corrected phase and amplitude do not reach 1.5º and 0.01v. The average errors are under 0.003Hz for the frequency, 0.7º for the phase and 0.003v for the amplitude.

Compared with rectangle window, the performance of the old method with Hanning window worsens in frequency and phase. From table 2, we can see that the maximum error of frequency is beyond 0.05Hz.The maximum error of phase is even above 10º.And the maximum error of amplitude is surpasses 0.25Hz closed to the rectangle window. However, new method can correct the errors satisfactorily with the maximum value of errors about 0.01Hz, 1º and 0.01v, respectively.And the mean errors are less than 0.002Hz, 0.4º and 0.005v for frequency, phase and amplitude, respectively. The mean values of the new method in all aspects are much better than the old method.

Table 1 Results of correction with Rectangle window

		traditional method			New method		
CF [Hz]	Max	200.621243	209.228040	280.305510	200.602128	209.206439	280.304673
	Min	200.620794	209.221110	280.296383	200.594265	209.199309	280.296197
	Avg	200.621052	209.224490	280.300692	200.597735	209.202459	280.300162
CP [Deg]	Max	051.480720	012.115980	090.761760	046.140120	018.192780	090.779040
	Min	049.671900	010.809540	089.043300	044.678700	016.572240	089.235360
	Avg	050.560200	011.481120	090.053640	045.465660	017.396460	089.992620
CA [V]	Max	000.984267	001.008513	001.001873	001.008085	001.009810	001.005763
	Min	000.970708	000.995243	000.985779	000.995336	000.996608	000.990474
	Avg	000.977774	001.000761	000.995534	001.002212	001.002172	000.999043

CF Corrected frequency;CP Corrected phase;CA Corrected amplitude;

Table 2 Results of correction with Hanning window

		traditional method			New method		
CF [Hz]	Max	200.678820	209.239720	280.358815	200.605893	209.205012	280.306170
	Min	200.668730	209.232903	280.349534	200.592832	209.195852	280.295305
	Avg	200.674070	209.237100	280.353937	200.600247	209.199679	280.300236
CP [Deg]	Max	032.571680	025.230240	100.723500	046.253176	018.699480	090.919980
	Min	030.680100	023.917140	098.796780	043.828920	017.042580	088.909560
	Avg	031.673880	024.663780	099.789840	044.967780	018.068760	089.991900
CA [V]	Max	000.987206	001.009582	001.003162	001.011745	001.010853	001.007202
	Min	000.973325	000.996132	000.986914	000.997198	000.997659	000.990774
	Avg	000.980257	001.003089	000.996450	001.004646	001.004458	001.000064

4. Discussion and conclusion

The results of the simulation demonstrate that the traditional formula is only suitable for rectangle window. The new method does a good performance not only for rectangle window but also for Hanning window. This method can correct the errors of harmonic signal without caring the type of window added. The result of corrected frequency, phase and amplitude of the harmonic signal are closed to the theoretical values although the white noise is involved. The algorithm is simple and easy to achieve.

The advantage of this new method compared with the other kinds of phase difference methods is that it only needs the result of FFT of windowed data. This is of great significance because of its possibility to achieve a new, fast and simple algorithm of correction method by phase difference. As the method of phase difference of any other kind does, the double phase differences correction method is not suitable for the situation of densely populated peaks spectrum or the continuous spectrum.

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