Review of Educational Research
September 2010, Vol. 80, No. 3, pp. 372–400
DOI: 10.3102/0034654310374880
© 2010 AERA. http://rer.aera.net

Methods of Instructional Improvement in Algebra: A Systematic Review and Meta-Analysis

Christopher R. Rakes, Jeffrey C. Valentine, Maggie B. McGatha, and Robert N. Ronau
University of Louisville

This systematic review of algebra instructional improvement strategies identified 82 relevant studies with 109 independent effect sizes representing a sample of 22,424 students. Five categories of improvement strategies emerged: technology curricula, nontechnology curricula, instructional strategies, manipulatives, and technology tools. All five of these strategies yielded positive, statistically significant results. Furthermore, the learning focus of these strategies moderated their effects on student achievement. Interventions focusing on the development of conceptual understanding produced an average effect size almost double that of interventions focusing on procedural understanding.

Keywords: achievement, hierarchical modeling, instructional practice, mathematics education, meta-analysis, conceptual understanding

In response to calls for higher standards in secondary mathematics, curriculum reforms have made algebra the backbone of secondary mathematics education in the United States (Chambers, 1994). Stronger skills in math, particularly in algebra, have been ascribed to college and career success (Vogel, 2008). The National Mathematics Advisory Panel (2008) found that completion of Algebra II doubles the probability of college graduation. Unfortunately, low pass rates (approximately 39%; Gates, 2008) and the sharp decline in mathematics achievement when students begin studying algebra raise concerns about the effectiveness of traditional algebra instruction (National Mathematics Advisory Panel, 2008). Furthermore, studies examining improvement in student mathematics achievement scores have obtained inconsistent results (Lee, Grigg, & Dion, 2007). Because algebra forms the core of the high school mathematics curriculum, improving the teaching and learning of algebra is critical to improving these long-term trends. Likewise, defining the fundamental concepts that students should learn in algebra is equally important.

Authors' Note: An earlier version of this article was presented at the 2009 annual meeting of the American Educational Research Association, San Diego, CA.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

In search of a definition for algebra. Individual researchers have attempted to clarify the composition of algebra for several decades with varying degrees of success (e.g., Leitzel, 1989; Thorpe, 1989; Usiskin, 1980). Several have repeatedly noted the importance of variability concepts to the structure of algebra (e.g., Briggs, Demana, & Osborne, 1986; Edwards, 2000; Graham & Thomas, 2000; Kalchman & Koedinger, 2005; Kieran, 2008). MacGregor and Stacey (1997) and Torigoe and Gladding (2006) found that these same variability concepts, when misunderstood, create a barrier to the deep learning of algebra: Students have difficulty assigning meaning to variables and fail to recognize the systemic consistency in the multiple uses of variables. Küchemann (1978) found that students interpret variables through six progressive (i.e., hierarchical) levels: (1) as a single value, through trial and error evaluation; (2) as irrelevant (i.e., students ignoring the variable in a contextual situation); (3) as an object or label; (4) as a specific unknown; (5) as a generalized number; and (6) as a functional relationship. While the first three of Küchemann's levels represent concrete variable interpretations, the three highest levels comprise the formal, abstract ways of interpreting variables. The National Mathematics Advisory Panel (2008) recommended dividing the study of algebra into six major topics: symbols and expressions, linear equations, quadratic equations, functions, polynomials, and combinatorics and finite probability.

The National Council of Teachers of Mathematics (NCTM) has refined its definition of algebra several times. In 1989, their definition emphasized equations, inequalities, and matrices. In 2000, they organized algebra by four overarching concepts and skills: functions, algebraic symbols, mathematical modeling, and analyzing change. Abstract variable meaning in algebra informed all four of the NCTM high school algebra standards. The first standard focused on developing students' understanding of variables as functionally related quantities. The second standard promoted an emphasis on understanding the symbols used to represent algebraic variables. The third standard emphasized the learning of mathematical modeling to represent quantitative (both functional and nonfunctional) relationships. The fourth standard supported an emphasis on teaching rates of change in algebra. All four of these standards focus on Küchemann's (1978) fourth, fifth, and sixth levels of variable interpretation. In a recent position statement, algebra was defined as "a way of thinking and a set of concepts and skills that enable students to generalize, model, and analyze mathematical situations" (NCTM, 2008).

The present study used the NCTM algebra standards and the National Advisory Panel topics to define algebra topics. So, interventions were considered to involve algebra if they targeted the learning of one or more of the topics or skills listed previously. The NCTM principles also recommended that the learning of algebra should occur in every grade, so the sample of studies was not limited to the middle and high school levels. Only one study (Suh & Moyer, 2007) examined the learning of algebra in elementary school, investigating the algebraic modeling of variables in a third-grade classroom.

From this body of literature, the definition of algebra remains elusive; nevertheless, there is agreement that algebra is critically important to the success of students throughout high school and college. To learn algebra means to navigate

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

through several complex topics with multiple sources of difficulty. These topics form the foundation for every advanced application of mathematics.

The unique challenges of algebra. Students beginning the study of algebra face learning challenges that form a general foundational set of understandings necessary to negotiate this topic. First, algebra is often the first course in which students are asked to engage in abstract reasoning and problem solving (Vogel, 2008). Researchers have demonstrated that the abstract nature of algebra increases its difficulty over arithmetic (Carraher & Schliemann, 2007; Howe, 2005; Kieran, 1989). Students who have experienced mathematics only at a concrete or procedural level, typical in many classrooms, must negotiate the difficult gap from concrete to abstract reasoning with no preparation (Freudenthal, 1983). This inexperience with abstractness for the construction of meaning directly affects the ability of students to manage multiple representations of algebraic objects (Kieran, 1992; Vogel, 2008).

Second, the learning of algebra requires students to learn a language of mathematical symbols that is also completely foreign to their previous experiences (Kilpatrick, Swafford, & Findell, 2001). The multiple ways in which this language is described and used during instruction often prevent students from connecting algebraic symbols to their intended meaning (Blanco & Garrote, 2007; Socas Robayna, 1997). In some cases, students are completely unaware that any meaning was intended for the symbols (Küchemann, 1978). In other cases, they may know that meaning exists, but limited understanding prevents them from ascribing meaning to the symbols, or they may assign erroneous meaning to the symbols (Küchemann, 1978). For example, as students study topics such as functions and graphs, they begin to understand and interpret one set of algebraic objects in terms of another (e.g., a function equation with its graph, a data set by its equation, a data set by its graph, as in Leinhardt, Zaslavsky, & Stein, 1990). McDermott, Rosenquist, and Van Zee (1987) found that students are generally able to plot points and equations; however, in spite of this procedural fluency, students still lack the ability to extract meaning from graphical representations. They concluded that the difficulty lies in the connection of a graph to the construct being represented. Specifically, students are readily capable of demonstrating procedural fluency, but memory and procedural understanding is unable to guide students through problems involving interpretation (Skemp, 1976/2006).

Kieran (1992), Howe (2005), and Carraher and Schliemann (2007) recognized that learning the structural characteristics of algebra creates a third challenge for students. For example, students often fail to recognize the differences between expressions and equations. They also have difficulty conceptualizing an equation as a single object rather than a collection of objects. The meaning of equality is often confused within algebra contexts as well. Taken together, these three example structural challenges often prevent students from recognizing the utility of algebra for generalizing numerical relationships.

These three foundational understandings, abstract reasoning, language acquisition, and mathematical structure, are often unique challenges for students. Although each in itself can serve as a unique obstacle to learning, the interaction of all three forms a much more formidable impediment to mastering algebra for many

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

students. As a result, these students have a poor initial experience with algebra and therefore fail to gain an adequate foundation for future learning. For example, consider the expression $a + b$: How students interpret the meaning of each variable depends on how well they can handle the abstract nature of the symbols. Furthermore, students must recognize that the expression $a + b$ represents the total number of items from a set of a and b items (Kieran, 1992).

The teaching methods used to convey content often exacerbate these algebra learning barriers, possibly becoming a unique barrier themselves (Leitzel, 1989; Thorpe, 1989). Sfard (1991) found that both students and teachers often expect immediate rewards for teaching and learning efforts. Instead, Sfard noted that relational understanding of abstract mathematical ideas often requires a lengthy, iterative process. Teaching methods that focus on skill or procedural levels on cognitive demand fail to address these foundational understandings and therefore fall short of providing students the tools necessary to find their way once they waver from a scripted path. Kieran (1992) extended Sfard's findings to algebra. She suggested that a great deal of time must be spent connecting algebra to arithmetic before proceeding to the structural ideas of algebra. Instead, teachers often spend a short period of time reviewing arithmetic and then proceed directly into a textbook sequence of instruction, which are often insufficient for helping students understand the abstract, structural concepts necessary for supporting the demonstrated procedural activities in algebra (Kieran, 1992).

The difficulties of achieving competence in abstract reasoning, language acquisition, and mathematical structure within the learning of algebra require teaching strategies that purposefully target the needs of learners. For example, in recognition of the difficulties some students have learning algebra in isolation, cooperative and collaborative learning (e.g., Slavin & Karweit, 1982) offers a relevant pedagogical option. For students struggling to connect abstract concepts with concrete examples, mastery learning and problem-based learning may be an appropriate strategy.

The No Child Left Behind Act (2002) called for the use of research-based strategies to help districts, schools, and educators choose the most appropriate programs and materials for their particular settings. Easton (2010) advanced this call by proposing the development of better collaboration between researchers and practitioners. Unfortunately, educators have been largely left to synthesize a broad base of research on their own—a time-consuming task, for which most have limited training. The best tools available to educators are systematic reviews and meta-analyses that provide convenient summaries of current research on a particular topic. In the case of algebra, the core of the high school mathematics curriculum, little progress has been made by the mathematics education research community to compile research on effective ways of improving instruction and learning. In recognition of the major gap in focused literature on algebra instruction, the National Council of Teachers of Mathematics compiled its 70th yearbook around topics in algebra instruction (Greenes & Rubenstein, 2008). This comprehensive guide focused on offering practical advice to those wishing to improve algebra instruction at the classroom, school, and district levels. The yearbook addressed a portion of the gap in algebra research by building a pedagogical knowledge base for algebra. However, other important gaps still exist; specifically,

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

which instructional improvement strategies have been studied, how effective those strategies have been, and the consistency of the evidence for the efficacy of those strategies.

Existing analyses of mathematical interventions have focused on algebra only as a subcomponent within a larger framework. For example, one analysis examined interventions in elementary mathematics education (e.g., Slavin & Lake, 2008). Other studies examined specific improvement strategies in mathematics across all grade levels, such as technology use (e.g., Hartley, 1977), calculators (e.g., Ellington, 2003, 2006), peer tutoring (e.g., Hartley, 1977; Lou, Abrami, & d'Apollonia, 2001; Lou, Abrami, Spence, & Poulsen, 1996; Roscoe & Chi, 2007), proximal zones of influence (e.g., Seidel & Shavelson, 2007), and structured inquiry (e.g., Butler & Winne, 1995; Klauer & Phye, 2008; Pajares, 1996; Rosenshine, Meister, & Chapman, 1996). However, none of these studies focused on algebra instructional improvement and its effects on student achievement.

One meta-analysis did focus on interventions in algebra (Haas, 2005); however, several key features of a systematic review were missing, limiting the usefulness of that research to meet the needs of the mathematics research community (e.g., did not provide rationale for the inclusion criteria and did not take steps to estimate and maximize the reliability of data extraction). Haas (2005) identified six instructional intervention categories: direct instruction, cooperative learning, communication and study skills, multiple representations, problem-based learning, and technology. Basing recommendations solely on the point estimates of effect sizes, he concluded that educators should focus on direct instruction, problem-based learning, and multiple representations, noting that his categories served not so much as approaches to teaching but as tools to be incorporated into any lesson.

Several methodological issues within Haas's (2005) meta-analysis make the validity and interpretability of his results unclear (e.g., did not state how effect sizes were computed and weighted, did not account for nonindependent observations, and did not investigate the possible effects of publication bias). For educators interested in improving algebra instruction, perhaps the most problematic aspect of the review is that it is unclear what rules were used to place interventions into different instructional categories. For example, "Direct instruction is a teaching method type that may encompass all the others. . . . Like direct instruction, problem-based learning is a teaching method type that may encompass all the others" (Haas, 2005, p. 31). With nonindependent categories, the interpretation of effect sizes becomes problematic. Furthermore, the nature or quality of the interventions was not captured, so the reader is left to speculate on the meaning of each effect size. Additionally, Haas reported only the point estimates (i.e., average effect sizes) without considering their confidence intervals, which are critical to interpretation of the point estimate. Finally, he failed to test his conclusions using moderator tests. Based on these methodological and conceptual issues, we concluded that the Haas study has limited ability to meet the needs of educators interested in improving algebra instruction.

We therefore set out to conduct a review that would be more likely to meet these needs by reexamining the types of instructional strategies used to improve student achievement in algebra, providing a more transparent process for coding and measuring effect sizes, and synthesizing these results into findings that are

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

meaningful to both researchers and practitioners. We recognized that simply measuring the effects of treatment names would miss an important component of these interventions. Specifically, how a treatment was implemented has at least as much impact as the intervention type. We, therefore, examined previous research for a framework to use to code how each intervention sought to enhance the understanding of algebraic concepts.

Algebra learning foci: Teaching for understanding. "There may be debate about what mathematical content is most important to teach. But there is growing consensus that whatever students learn, they should learn with understanding" (Hiebert et al., 2000, p. 2). The framework described by Hiebert and Carpenter (1992) differentiated between conceptual and procedural understanding. In their framework, procedural knowledge isolated from conceptual meaning can result in misunderstandings similar to those described by Skemp (1976/2006) as resulting from instrumental understanding.

Skemp (1997/2006) described instrumental understanding of mathematics as a way of learning that forces students to rely on memorization and prescriptions. Kieran (2007) agreed with Skemp's viewpoint of the limiting nature of instrumental mathematics. Even the manipulation of symbols, once considered primarily an algorithmic process, has become recognized as fundamentally dependent on concept meaning and connections (Kieran, 2007). To better illustrate the meaning of instrumental understanding, Skemp gave the analogy of a person trying to navigate through a new city. A person with an instrumental understanding of the city may have a number of ways to get from Point A to Point B. The difficulty with this understanding arises when the person deviates from the original course. In such a case, the person gets lost. Instrumental understanding of algebra produces similar results. For instance, students may learn a set of prescriptions for solving equations of the form $ax + b = c$; when they encounter equations of the form $ax + b = cx + d$, their prescriptions are unable to accommodate the new form.

For example, an algebra class learning about quadratic functions might be asked purely procedural questions such as, "Use the quadratic formula to solve $x^2 + 6x + 8 = 0$." In such a problem, students can find a solution without any understanding of the meaning of quadratic functions simply by factoring or use of the quadratic formula. Alternatively, a conceptually focused question might ask students to graph $y = x^2 + 6x + 8$ and explain how the x intercepts of the graph are related to the factors of the equation. Such an approach requires students to make connections between the factors of the function (i.e., $[x + 2]$ and $[x + 4]$), the roots of the function (i.e., $x = –2$, $x = –4$), and the x intercepts of the function (i.e., [–2, 0] and [–4, 0]).

Students who acquire only procedural knowledge often "get lost" when subjected to unfamiliar situations and are unable to apply important mathematical concepts and structure in those situations. The key to avoiding this and other pitfalls, according to Hiebert and Carpenter (1992), Hiebert and Grouws (2007), and Skemp (1976/2006), is to focus first and primarily on the meaning of important mathematical ideas and the connections linking these ideas. Such a focus runs counter to the traditional intuitions of educators:

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

> Since the [conceptual] program uses a format that requires the student to do more than memorize the formula to successfully answer a series of computational problems on that concept, students' academic performance should have favored the [procedural] group. The data support the opposite, showing the [conceptual] group outperforming the [procedural] group on all four unit tests. (Peters, 1992, p. 94)

The NCTM principles, along with the National Research Council (Kilpatrick et al., 2001), have recognized the importance of conceptual understanding and called for an increased focus on central concepts and integrating disparate parts:

> Teachers should strive to organize the mathematics so that fundamental ideas form an integrated whole. Big ideas encountered in a variety of contexts should be established carefully, with important elements such as terminology, definitions, notation, concepts, and skills emerging in the process. (NCTM, 2000, p. 14)

Hiebert and Grouws (2007) described two observable features for a classroom focusing on conceptual understanding: (a) Teaching focuses explicitly to connections between facts, procedures, and ideas, and (b) students are allowed to struggle with important mathematical concepts. Procedural fluency is supported in this goal: In a conceptually based environment, procedures are learned as emergent from connecting concepts (Rittle-Johnson & Alibali, 1999). This conceptual understanding epistemology holds to a belief that focusing on conceptual knowledge and relational understanding carries several benefits for students: Knowledge becomes more adaptable to new tasks, learning becomes generative, and students begin developing their own knowledge (Hiebert & Carpenter, 1992; Skemp, 1976/2006). Memory is enhanced, while at the same time, students need to memorize less (Hiebert & Carpenter, 1992; Van De Walle, 2007). Furthermore, the building of relationships between concepts and procedures identifies the *identical elements* (Thorndike, 1913) necessary for preexisting knowledge to transfer to new knowledge (Hiebert & Carpenter, 1992).

Both conceptual and procedural epistemological viewpoints can dictate how a teaching method is implemented. For example, manipulatives, whether virtual or physical, can be used to build connections among ideas, as in Aburime (2007); Cavanaugh, Gillan, Bosnick, Hess, and Scott (2008); and Suh and Moyer (2007). Manipulatives can also be used to enhance skill proficiency, as in Goins (2001), Goldsby (1994), and McClung (1998). The same pattern is also true for every other category of instructional treatment.

The challenges faced by students learning algebra have led to steep declines in student mathematics achievement. Traditional instructional practices have led students to view mathematics as a set of disjointed algorithms. We therefore used the framework of Hiebert and Carpenter (1992) to capture the way algebra interventions have been used to address this critical component of mathematics learning. The descriptions of practice described by Hiebert and Grouws (2007) provided the criteria for this differentiation. By examining both the type of intervention and the epistemological focus of each intervention, the present study seeks to provide a valuable synthesis for both researchers and practitioners.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

Study Purpose

Although the 70th NCTM yearbook (Greenes & Rubenstein, 2008) advanced teaching strategies and theories of teaching algebra, and other meta-analyses (e.g., Ellington, 2003, 2006; Slavin & Lake, 2008) have touched on algebra while examining other mathematics topics, the impact of instructional improvement in algebra on student achievement remains largely unexplored. The purpose of the current study is to fill this gap by addressing three questions through a systematic review and meta-analysis of the literature on algebra instruction:

1. What methods for improving algebra instruction have been studied?
2. How effective have these methods been at improving student achievement scores?
3. Which characteristics of teaching interventions in algebra are the most important for determining the effectiveness of the intervention on student achievement?

To answer the first two questions, studies were organized into categories of instructional improvement methods. The effectiveness of these categories was measured using standardized mean difference effect sizes. Moderator tests were used to test for category differences in effect size variance. To answer the third question, we examined the epistemological learning focus of the study interventions, specifically whether the intervention sought to develop conceptual or procedural understanding. Hiebert and Grouws (2007) suggested that the degree to which learning is focused on developing conceptual understanding may determine the effectiveness of a teaching intervention. The synthesis of evidence addressing these three questions may offer important insight for practitioners and researchers on improving the learning of algebra.

Method

Study Inclusion Criteria

We applied four criteria to determine study inclusion in the sample. First, the intervention had to target the learning of algebraic concepts, regardless of the title of the classes being examined. For example, elementary and middle school mathematics classrooms sometimes examined how students learn algebraic concepts (e.g., functions, polynomials, variable expression simplification, solving linear/nonlinear equations/inequalities, graphing equations/inequalities) even though the courses were not labeled *algebra* (e.g., Reys, Reys, & Lapan, 2003; Suh & Moyer, 2007). Advanced high school mathematics courses such as precalculus were also used as a setting for the study of algebraic concepts (e.g., Whicker & Nunnery, 1997). This first criterion addressed the need for content validity within the sample (Urbina, 2004).

Second, the intervention had to involve a method for improving learning as measured by student achievement (e.g., standardized tests, teacher-made tests, researcher-made tests, grades, GPA). For example, some studies were excluded because the outcome of interest was motivation (e.g., Githua & Mwangi, 2003), success in school (e.g., Ellington, 2005), or teacher outcomes (e.g., Wenglinsky,

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

2000; Wiesner, 1989) rather than achievement. Third, the study had to employ an experimental design with a comparison group. We included quasi-experimental designs along with random experiments to maximize statistical power and external validity. We excluded, however, observational studies (e.g., Malloy & Malloy, 1998) and exploratory studies with no treatment (e.g., Berenson, Carter, & Norwood, 1992). Fourth, the comparison group had to have received the "usual instruction." We therefore excluded, for example, studies that compared the effectiveness of two novel treatments but did not include a usual instruction group (e.g., Woolner, 2004). Taken together, the second, third, and fourth criteria addressed the construct validity of the study (Shadish, Cook, & Campbell, 2001). Based on these inclusion criteria, we interpreted effect sizes as the magnitude of the impact of pedagogical strategies on student algebra achievement.

We chose to refrain from setting date limitations on our sample, which included studies from 1968 to 2008. The inclusion of studies more than 40 years old was both acceptable and desirable for the purposes of this analysis for two reasons. First, the differentiation between conceptual and procedural understanding can be traced back at least as far as Brownell's (1938) statement regarding the correction of "errors in understanding and computation" (p. 498). Second, the traditional methods of the early and middle 20th-century continue today. Welch (1978) described the typical mathematics classroom as following a rote procedure that focused solely on solving a high number of homework problems. Likewise, a decade later, Stodolsky (1988) claimed that in the United States, "most instruction is geared to algorithmic learning" (p. 7). Another decade later, the purpose of mathematics lessons had changed little (Stigler & Hiebert, 1997, p. 18), and evidence suggests that these patterns continue today (Hiebert & Grouws, 2007). Based on these reported trends, we concluded that an arbitrary date limitation would reduce the ability of our sample to represent teaching method interventions focusing on procedural understanding.

Electronic literature search strategy. To maximize the representativeness of our sample, we searched 20 electronic databases related to education and the psychological sciences. From EBSCOhost, we searched Academic Search Premier, Education Administration Abstracts, ERIC, Middle Search Plus, Primary Search, Professional Development Collection, Psychology and Behavioral Sciences Collection, PsycINFO, Sociological Collection, and Teacher Reference Center. From H.W. Wilson, we searched Education Full Text and Social Sciences Index. In JSTOR, we limited our search to the Mathematics and Education disciplines. In ProQuest, we searched the Research Library, Digital Dissertations, and the Career and Technical Education Database. In the ISI Web of Knowledge, we searched the Science Citation Index Expanded, Social Sciences Citation Index, and Arts and Humanities Citation Index. We also searched the IEEE Electronic Library. Additional resources included an online university library catalog, Google Scholar, and the What Works Clearinghouse website. Finally, bibliographies of related articles were searched to find relevant studies that were missed in the databases searches.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

To reduce the threat of publication bias, we included "gray literature" such as dissertations, conference proceedings, and reports (which are usually not approved on the basis of their results) and focused attention on conducting a thorough search of other unpublished literature not easily accessible through electronic databases as recommended by Cooper (1998) and Thornton and Lee (2000).

Search terms were chosen to identify studies meeting the first inclusion criteria (the intervention focused on the learning of algebra). We searched for the keywords *algebra, function, equation, expression, quadratic, polynomial, exponent*, and *rational*. To filter out studies of algebra not involving instruction, we also included the search terms *teach, learn, instruction*, and *education*. Finally, we contacted several well-established scholars in the area of algebra instruction to see if they were aware of additional studies that were relevant but not easily accessible.

Coding Studies

The coding of studies took place in four stages. All studies were coded by the first author with a second coder examining a random sample at each step to measure interrater agreement. First, the titles and abstracts of electronic search results were scanned; those that were clearly not related to mathematics (e.g., studies that examined chemistry or physics teaching) were excluded. We identified 594 potentially relevant studies. Second, upon completion of the electronic search, a judgment was made about the likely relevance of the studies based on a reading of titles and abstracts. Studies were considered not relevant for this review if they clearly did not meet the aforementioned inclusion criteria; if relevance could not be determined from their titles and abstracts, the studies were obtained and reviewed. Upon completion of the second round of coding, we retained 82 relevant studies as meeting our inclusion criteria. Third, the number of independent samples within each study was identified, followed by the recording of the appropriate student achievement means, standard deviations, and study characteristics such as study descriptors (e.g., author, date of publication), the sample (e.g., age or grade level, ethnicity), the intervention (e.g., the specific instructional improvement strategy used), the measure used (e.g., properties of tests), and the results (e.g., effect sizes). Fourth, instructional strategies were analyzed qualitatively to determine categories of instructional treatment.

Interrater reliability was measured for Stages 2, 3, and 4 of the coding process using a random sample of 65 studies from the original 594 identified studies. At Stage 2, we focused on initial inclusion or exclusion for each study. At this stage, we agreed initially on the inclusion status of 53 studies (88.3%). If either rater thought a study might be relevant, it was included in Stage 3. As a result, 36 studies were retained in the reliability subsample. At Stage 3, relevance of these studies was determined through an analysis of the full text, and we agreed initially on the continuing inclusion status of 34 studies (94%). Based on this analysis, 12 studies were retained for Stage 4. At this stage, we coded study characteristics (e.g., instructional strategy categories, number of independent samples within the study). We agreed initially on 95% of the study characteristics; however, the disagreements were not random. We found, instead, that most disagreement centered on coding the instructional treatment category. For this study characteristic, initial agreement measured approximately 45%. We deemed this level of reliability too

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

low to rely on any single judgment, so we proceeded to code the instructional treatment category for all 82 studies with a panel of three mathematics education researchers. This panel agreed that five categories represented the observed interventions independently: instructional strategies, manipulatives, technology tools, technology-based curriculum, and nontechnology curriculum.

Instructional strategies consisted of teaching methods such as cooperative learning, mastery learning, multiple representations, and assessment strategies. In Slavin and Karweit (1982), student teams and mastery learning were used to address limitations in group-paced algebra instruction. Ives (2007) examined the use of graphic organizers to clarify the meaning of algebra problems. Wineland and Stephens (1995) investigated the effects of a spiral testing strategy for improving student achievement.

Goins (2001) defined *manipulatives* as "concrete objects that are used to help students understand a concept" (p. 10). In her study, rectangular tiles were used to help students develop polynomial multiplication skills and to develop understanding of the meaning of polynomial multiplication. Aburime (2007) investigated the use of cardboard geometric cutouts to represent shapes such as triangles, quadrilaterals, pentagons, hexagons, circles, cubes, and cylinders.

Technology tools included calculators, graphing calculators, computer programs, and java applets. For example, Durmus (1999) investigated the use of graphing calculators as a method for carrying out computations and checking solutions. K. B. Smith and Shotsberger (1997) focused instead on the use of graphing calculators for changing the way students approach problem solving. Suh and Moyer (2007) and Cavanaugh et al. (2008) examined the use of java applets as a substitute for physical manipulatives to learn algebraic concepts such as balancing equations.

Technology-based curricula included computer-based curricula for use in onsite classes, online courses, and tutoring curricula. For example, Koedinger, Anderson, Hadley, and Mark (1997); Morgan and Ritter (2002); and Shneyderman (2001) examined the use of the Cognitive Tutor as a way of redesigning algebra instruction in on-site classes. Weems (2002) and O'Dwyer, Carey, and Kleiman (2007) compared online and on-site course effectiveness for algebra learning. Hannafin and Foshay (2008) examined the impact of the PLATO learning system as a way of teaching algebra.

Nontechnology curricula included reform-based curricula such as Math Thematics (Reys et al., 2003), Connected Mathematics (Reys et al., 2003), UCSMP Algebra 1 (Thompson & Senk, 2001), and a researcher-developed curriculum based on NCTM principles and standards (McCaffrey, Hamilton, & Stecher, 2001). This category also included traditional curricula such as Saxon (e.g., Johnson & Smith, 1987; Lawrence, 1992; McBee, 1984) and CORD Algebra 1 (Keif, 1995).

We referred to the theoretical framework of Hiebert and Grouws (2007) to differentiate between instructional strategies that focused on conceptual understanding or procedural understanding. In their review of research, they described conceptual understanding as beginning with the "engagement of students in struggling with important mathematics" (p. 391). Going into more detail, they described conceptual instruction as paying "explicit attention to connections among ideas,

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

facts, and procedures" and "posing problems that require making connections and then working out these problems in ways that make the connections visible for students" (p. 391). Both conceptual and procedural foci often appeared within each intervention category. For example, the concrete-representational-abstract method of instruction concentrated on skill development in one study (Konold, 2004), while in another study (Witzel, Mercer, & Miller, 2003), this same type of intervention was used to give explicit attention to developing meaning and connections between mathematical ideas.

Determining the appropriate epistemological foundation of the intervention required more than a cursory reading of the full text because studies within both groups often used the same terminology to mean two different ideas. For example, most studies referred to standards or improving algebra instruction over traditional instruction. In some studies, *standards-based* meant an explicit focus on connecting the meaning of ideas while in other studies *standards-based* referred to adherence to topics listed within a state or national standards document. For example, the term *standards-based* was used to imply a conceptual focus for a computer program titled *The Learning Equation* (TLE; Walker & Senger, 2007). The TLE software, however, focuses on problem-solving heuristics. For example, in a lesson meant to differentiate between functions and relations, the lesson focused on developing a heuristic for parsing out relationships rather than the meaning of the ideas. In another case, Ives (2007) used the term *mathematical concepts* repeatedly, but the intervention focused specifically on a set of prescriptive heuristics for solving linear equations. In short, "There are two effectively different subjects being taught under the same name, 'mathematics'" (Skemp, 1976/2006, p. 6). In this sample, we determined that the interventions of 25 studies (approximately 30%) focused on the development of conceptual understanding.

Approximately 12% of the 82 sample studies were randomly selected to measure interrater reliability for the coding of the intervention epistemological focus. Initial agreement (80%) indicated a high degree of reliability. Furthermore, the resultant groups demonstrated a high degree of discriminant validity, measured by examining the correlations (r = .054, p = .378) between the groups, as recommended by Furr and Bacharach (2008) and Urbina (2004).

Independence of effect sizes. The unit of analysis for this review was the independent sample. In most of the studies that met our inclusion criteria, more than one effect size was obtainable for a sample of students due to multiple subscales on a single test or multiple tests. For example, some researchers measured the same construct multiple ways (e.g., two versions of an assessment) or at multiple times (e.g., at posttest and at follow-up). Other researchers employed multiple treatment groups (e.g., by comparing two different teaching strategies to instruction as usual) or multiple comparison groups (e.g., by comparing a treatment group to multiple control groups). Ignoring this dependence can result in a study having too much weight in an analysis. In those cases in which the effect sizes would not be independent, an average effect size was calculated in order to ensure independence of data in the final data set (Lipsey & Wilson, 2001). In some cases, the samples of various subscales and assessments overlapped but lacked or gained a few students so that the sample sizes of each dependent effect size varied slightly. In these cases,

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

a weighted average effect size was calculated, and the final sample size used was an average of the sample sizes. For example, Coppen (1976) implemented a treatment known as Individual Mastery Instruction (IMI) in a single class. The achievement scores for that class were then compared to three classes receiving instruction as usual. Averaging the observed effects resulted in a single average effect size for the sample.

In other studies, different samples were studied with no overlap (e.g., 2 years, two different samples); for these studies, each effect size was independent and thus both were included in the meta-analysis (Lipsey & Wilson, 2001). For example, Coppen's (1976) study continued into a second year in which the design was repeated with an entirely new sample of students. This repetition resulted in a second effect size in the meta-analysis.

Such handling of multiple effect sizes does not, however, preclude within-study clustering effects on the reported means and errors (Raudenbush & Bryk, 2002). On the contrary, this method for handling multiple effect sizes only assures that each effect size represents a distinct group of students; it does nothing to address the dependence of students within a group. Because every study within our sample was grouped by class rather than student, this within-group design effect (Kish, 1965) needed to be addressed through statistical adjustments to the computed effect size to avoid spuriously small standard errors.

Computation of effect sizes. Because interventions in this sample measured outcomes on a variety of scales, the standardized mean difference effect size, d, was chosen to represent study results. In addition, when both pretests and posttests were available, we corrected posttest effect sizes by computing the "difference in differences" in the means from posttest to pretest and standardizing this mean difference by the pooled posttest standard deviation. Finally, some studies in the sample provided statistics other than means and standard deviations, such as dichotomous proportions (e.g., the percentage of students mastering a skill), focused F tests (e.g., only two groups being compared), t tests, and correlation coefficients (between an outcome and treatment membership). In each of these cases, standard statistical formulas were used to convert these scores to the equivalent standardized mean difference effect size (Lipsey & Wilson, 2001).

Data Clustering

We adjusted for the within-study dependence through two methods to minimize potential Type I error. First, we computed a design effect (Kish, 1965) with intraclass correlations provided by Hedges and Hedberg (2007). The design effect was used to adjust the standard error for clustering, thereby reducing Type I error. Second, we computed an empirical Bayes (EB) estimate, adjusting both the effect size δ_j*; (Equation 1) and its standard error using estimates from hierarchical linear modeling (HLM) procedures (Raudenbush & Bryk, 2002; Raudenbush, Bryk, Cheong, Congdon, & du Toit, 2004).

$$\delta^{*}_{j} = \lambda_j d_j + (1\text{-}\lambda_j)(\hat{\gamma}_0). \quad (1)$$

Both adjustments yield slightly different results and offer valuable perspectives; therefore, we present both models in the interests of transparency and replicability.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

TABLE 1
Weighted average effect sizes for intervention categories

Category	Empirical Bayes adjusted fixed effects weighted averages, δ_j*	*SE*	Design effect adjusted random effects weighted averages, *d*	*SE*
Curricula	0.207	0.024*	0.404	0.115*
Instructional change	0.322	0.030*	0.349	0.070*
Manipulatives	0.318	0.089*	0.335	0.132*
Technology tools	0.304	0.046*	0.165	0.073*
Technology curricula	0.311	0.050*	0.151	0.305*

*$p < .05$.

Results

Literature Search

We obtained 594 articles that were identified as potentially relevant in our search. Of these, 413 articles contained research reports. Of these articles, 124 studies examined the effect of an instructional improvement strategy in algebra on student achievement, but only 82 of these studies included enough information to compute an effect size. These 82 studies contained 109 independent experiments. Due to the effort to locate gray literature (i.e., dissertations, theses, conference papers, and unpublished reports), we expected limited publication bias effects: A random-effects trim and fill analysis (Duval & Tweedie, 2000a, 2000b) and funnel plot analysis confirmed this expectation by indicating no need to include publication bias adjustments. Most of the studies (97 experiments; 89%) were conducted in either middle school, Grades 6 through 8 (26 experiments; 23.9%); high school, Grades 9 through 12 (62 experiments; 56.9%); or both middle and high school (9 experiments; 8.2%); 11 experiments examined algebra at the college level (10.1%); and 1 experiment studied the learning of algebraic concepts in Grade 3 (0.9%). Treatment durations varied widely, from one lesson to a full school year. All college-level experiments examined treatments lasting the full semester. The treatment duration was not a statistically significant predictor of the sample effect sizes ($b_1 = 0.000021$, $p > .5$).

Bias Due to Quasi-Experimental Study Inclusion

The weighted average effect size for randomized experiments was 0.280 and 0.325 for quasi-experimental designs. The moderator test revealed that the difference in observed effects was not statistically significant, $Q(1) = 0.633$, $p = .426$. We therefore concluded that the inclusion of quasi-experiments in our sample did not significantly bias the results.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

TABLE 2
Pairwise category moderator tests

Category 1	Category 2	$Q(1)$
Nontechnology curriculum	Technology curriculum	0.02
	Instruction	5.83*
	Manipulatives	1.12
	Technology tools	0.18
Technology curriculum	Instruction	91.52***
	Manipulatives	91.29***
	Technology tools	99.79***
Instruction	Manipulatives	90.90***
	Technology tools	7.53**
Manipulatives	Technology tools	99.43***

*$p < .05$. **$p < .01$. ***$p < .001$.

TABLE 3
Weighted Average Effect Sizes For Epistemological Emphases

Interventions focused on the development of:	Empirical Bayes adjusted fixed effects weighted averages, δ_j^*	SE	Design effect adjusted random effects weighted averages, d	SE
Conceptual understanding	0.232	0.023*	0.467	0.099*
Procedural understanding	0.301	0.023*	0.214	0.044*

*$p < .05$.

Weighted Average Effect Sizes

For each category, we computed the design-effect adjusted effect size and the empirical Bayes effect size estimate (Table 1).

We found positive, statistically signficant effect sizes for every category in at least one model. A multivariate moderator analysis revealed statistically significant variance between categories, $Q(1) = 10.369$, $p = .001$. We therefore conducted a pairwise post hoc moderator analysis (Table 2).

In addition to the intervention categories, we also coded the epistemological learning focus of the intervention using the characteristics described by Hiebert and Grouws (2007). We found that 25 of the 82 studies (31%) examined interventions intended to develop conceptual understanding. The remainder of the studies examined interventions intended to develop procedural understanding. Procedural study effect sizes ranged from –1.096 to 1.391 while conceptual study effect sizes extended from –0.286 to 2.590. Procedural study effect sizes were normally distributed, measured using the Shapiro-Wilk test of normality ($W = 0.977$, $p = .640$). Conceptual studies, on the other hand, were not normally distributed ($W = 0.856$, $p < .001$); rather, they were skewed to the right (see Figure 1).

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

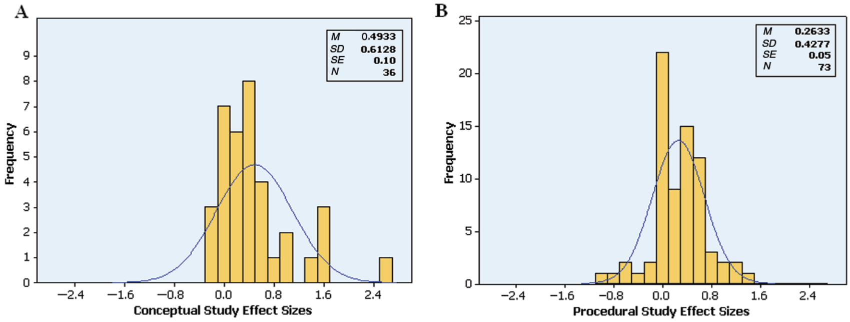

FIGURE 1. *Histogram of effect sizes for conceptual and procedural studies.*

Both analysis models showed that both epistemological emphases produced, on average, significant gains in student achievement (Table 3). The design effect model revealed that studies focusing on conceptual understanding produced an observed weighted average effect size more than twice the magnitude of the effects produced by interventions focusing on procedural understanding ($d_{conceptual} = 0.467$, $SE = 0.099$, $p < .05$; $d_{procedural} = 0.214$, $SE = 0.044$, $p < .05$). The moderator analysis revealed that these differences were statistically significant, $Q(1) = 7.069$.

The empirical Bayes model adjusted the observed effect sizes to closer approximate the unweighted grand mean, $\gamma_{10} = 0.325$, $p = .435$. The between-study variance was nonsignificant ($\tau = 0.002$, $p > .5$), while the within-study variances ranged from 0.004 to 0.557. The empirical Bayes estimates, therefore, reflect a high degree of shrinkage of the point estimates toward γ_{10}. Because the shift is additive (see Equation 1), negative effect sizes with small magnitudes for λ_j were shifted far enough to become positive. For example, the observed effect size for Abrams (1989) was –0.462, $\lambda_j = 0.106$. Using Equation 1, the empirical Bayes estimate for the effect size was $\delta_j^* = (0.106 \bullet -0.462) + (0.894 \bullet 0.325) = -0.049 + 0.291 = 0.242$. This computational outcome occurred for all 13 negative effect sizes in the procedural group, 4 of which were statistically significant in the original model. It also occurred for all 6 negative effect sizes in the conceptual group; however, none of these effects were statistically significant in the original model. The shrinkage effect, therefore, produced a weighted average empirical Bayes estimate of the effect size of 0.301 ($SE = 0.023$, $p < .05$) for the procedural studies. For the conceptual studies, the weighted average empirical Bayes estimate of the effect size was 0.214. For the conceptual studies, the weighted average empirical Bayes estimate of the effect size was 0.214 ($SE = 0.044$, $p < .05$). The difference between these estimates was statistically significant, $Q(1) = 4.614$ ($p < .05$).

Discussion

The present study began by seeking the most useful way to categorize research on instructional methods for improving student achievement in algebra. We found that five categories captured the breadth of interventions used to improve student achievement in algebra: implementation of new curricula, technology-based curricula, instructional strategies, manipulatives, and technology tools. The analyses of these categories resulted in five key findings:

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

1. While studies in all five categories (i.e., technology curriculum, nontechnology curriculum, instructional change, use of manipulatives, and use of technology) included both significant and nonsignificant effects, each category demonstrated positive weighted average effect sizes that were statistically significant in at least one model. This finding carries a direct implication for mathematics teachers. These strategies provide concrete methods for improving student achievement without relying on traditional drill and practice routines. This evidence suggests that these strategies should become ubiquitous in the algebra classroom.
2. Not only should these strategies be consistently implemented in algebra instruction, the focus of these strategies is also deeply important for student learning. This evidence suggests that a focus on the development of conceptual understanding will improve student achievement far better than the same strategy with a focus on procedural understanding. Teachers wishing to improve student achievement in their classrooms should therefore seek ways to explicitly target the meaning of important ideas in algebra and the connections between these ideas. Principals wishing to improve algebra achievement across an entire school should make these characteristics of instruction a target for teaching evaluations.
3. Duration of treatment did not account for differences in effectiveness on student achievement. Based on this finding, instruction occuring over small periods of time (e.g., at the end of a school year after state testing) may still have a statistically significant, positive effect on student achievement in algebra.
4. Pairwise moderator tests indicated that the grain size of the intervention did not account for significant differences in effect sizes. For example, there were no significant differences between whole-school studies and interventions involving only a single teacher. This finding suggests that both whole-department and individual-teacher efforts at reform have a positive impact on student achievement.
5. No significant differences were observed in effectiveness between quasi- and randomized experimental designs, $Q(1) = 0.633$, $p = .426$. This finding carries special import for researchers: Although the randomized, true experiment may provide the most compelling evidence (Whitehurst, 2002), quasi-experiments in algebra have produced statistically indistinguishable evidence. This finding does not suggest that randomized experiments are unnecessary; instead, it may provide reassurance that quasi-experiments may also be effective for studying student achievement in algebra.

Taken together, these findings illustrate effective ways to improve student achievement through the learning of algebra. For example, each category of algebra intervention yielded statistically significant positive effect size averages. This result indicates that reform efforts have consistently produced observable improvement in student achievement when compared to traditional algebra instruction. The weighted average effect sizes show that individual teachers can have a positive impact on student achievement in the algebra class. When whole departments coordinate their efforts by implementing a coherent curriculum, the benefits to

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

student achievement may be significantly greater. This outcome validates the NCTM (2000) curriculum principle: "A curriculum is more than a collection of activities: It must be coherent, focused on important mathematics, and well articulated across the grades." Furthermore, individual or whole department efforts have the greatest effect when they emphasize the meaning of important concepts and the connections between these concepts.

This study also examined the nature of the intervention within each study, focusing on whether the intervention emphasized conceptual or procedural understanding as a means to improving student achievement in algebra. Two important trends emerged as the coding of this characteristic proceeded. First, while procedural study dates ranged from 1968 to 2008, the earliest study focusing on conceptual understanding in algebra appeared in 1985. From this pattern of dates, we concluded that although traditional mathematics education has emphasized procedural understanding, reform efforts in the 1980s and 1990s succeeded in bringing attention to the need to focus on conceptual understanding. Traditional views for teaching mathematics, however, still persist in both practice and research.

Second, educators who stress both epistemological perspectives often use the same terms to mean something entirely different. Studies in our sample focusing on procedural understanding often described their goals as the development of concepts, yet the intervention consisted entirely of skill development (e.g., Chirwa, 1996; Ives, 2007). The term *standards-based* was also used to imply conceptual understanding. For example, in Walker and Senger (2007), the authors went to a great deal of effort to categorize their theoretical framework as emergent from the NCTM standards, yet the tool being used in the intervention focused, as far as we could determine, exclusively on skill development and the use of heuristic algorithms to solve problems. In this case, we concluded that *standards-based* was actually intended to mean that the mathematical topics being taught were included in the list of recommended topics for one of the NCTM content strands rather than that the method of instruction coincided with the NCTM principles (NCTM, 2000), which emphasize connections among ideas. Clarifying the language used to describe algebra interventions may be especially important for enhancing the usefulness of research to practitioners.

The moderator analyses between the conceptual and procedural studies in this sample presented a striking image of the nature of effective algebra instruction. Using the design effect model (i.e., observed effect sizes with adjusted standard errors), we found that the conceptual studies produced a statistically larger weighted average effect size on student achievement. This difference demonstrates that student achievement in algebra is not sacrificed by focusing on conceptual understanding. Quite the contrary, the data indicated that student achievement is actually enhanced by such an emphasis. Skemp (1976/2006) identified several benefits of building connections among ideas that may explain its effectiveness on improving student achievement: (a) improved ability to adapt to unfamiliar situations, (b) reduced need to memorize rules and heuristics, (c) enhanced student intrinsic motivation to learn mathematics, and (d) increased stimulation of student growth into independent, lifelong learners.

On the other hand, the empirical Bayes estimates of the effect sizes and standard errors presented a different picture, but we believe that this picture complements rather than contradicts the design effect model. In this analysis, the weighted

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

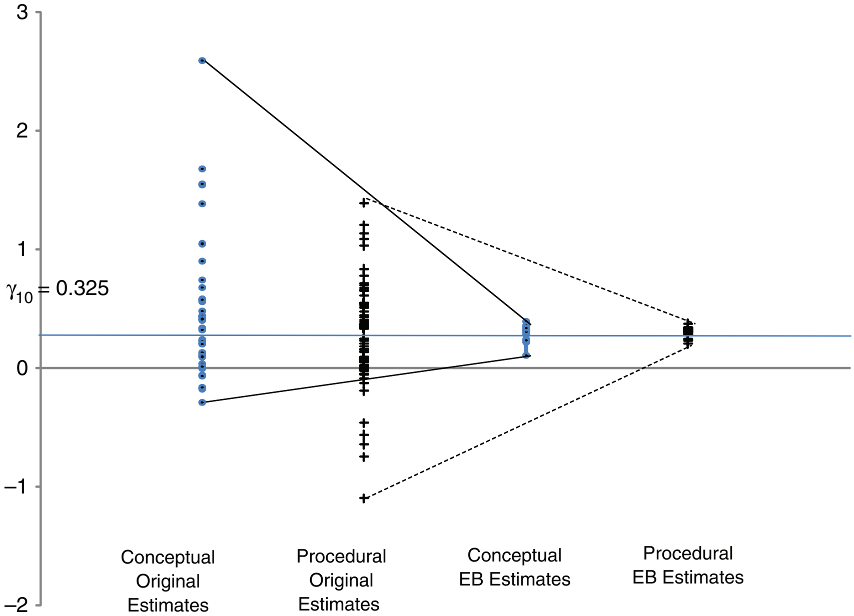

FIGURE 2. *Shrinkage of variability in empirical Bayes (EB) estimates.*

average effect estimates of the procedural studies was statistically significantly higher than that of the conceptual studies. We do not believe, however, that this result indicates that procedurally based instruction is more effective in algebra than conceptual instruction. Rather, this estimated outcome proceeded directly from the smoothing of the procedural study effect sizes up and conceptual studies down toward the overall mean effect size. This process, described by Raudenbush and Bryk (2002) as *shrinkage*, can be illustrated as a reduction of overall variability in the effect size estimates, as shown in Figure 2.

Teacher training may offer one reason for the even dispersal of positive and negative effects across procedural interventions while conceptual interventions witnessed only positive and/or nonsignficant negative effects. Procedural understanding interventions are far more similar to the traditional methods of teaching mathematics (Hiebert et al., 2005), meaning that these interventions may have required less innovation from the teacher. Focusing on the development of the meaning of mathematical ideas and the connections between those ideas, on the other hand, requires a unique, nonintuitive skill set (Hiebert, Morris, Berk, & Jansen, 2007) that necessitates specific professional development. As a result, teacher effects may have influenced the effectiveness of conceptual understanding interventions less than in the procedural. We concluded, therefore, that this collection of studies indicates that a focus on conceptual understanding may produce more consistently positive effects on student achievement in algebra. We further concluded that professional development for algebra teachers may impact student achievement more if it focuses on methods for developing conceptual understanding.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

The persistence of a procedural emphasis in traditional mathematics pedagogy (Hiebert, 2003; Stigler & Hiebert, 1997) suggests that although a great deal of evidence supports the importance of teaching mathematics conceptually, the information from that body of research has not yet influenced the teaching profession enough. Systematic reviews such as the present study provide an avenue for clarifying research results for the teaching community. The results of the present study indicate that a wide variety of reforms effectively improve student achievement in algebra. The degree to which these efforts focus on the development of conceptual understanding also influences the magnitude of effects. Put into consistent practice, the use of coherent curricula, teaching strategies, manipulatives, and technology to develop conceptual understanding may hold the key to the development of the three foundational understandings, abstract reasoning, language acquisition, and mathematical structure, which in turn may be critical to improving student achievement through the learning of algebra.

References

*References marked with an asterisk indicate studies included in the meta-analysis

*Abrams, B. J. (1989). A comparison study of the Saxon Algebra 1 text. *Dissertation Abstracts International-A*, *51*(07), 2251.

*Aburime, F. E. (2007). How manipulatives affect the mathematics achievement of students in Nigerian schools. *Educational Research Quarterly*, *31*, 3–15.

*Adams, J. L., & Stephens, L. J. (1991). A comparison of computer-assisted instructional methods. *International Journal of Mathematical Education in Science and Technology*, *22*, 889–893.

*Akinsola, M. K., & Animasahun, I. A. (2007). The effect of simulation-games environment on students achievement in and attitudes to mathematics in secondary schools. *Turkish Online Journal of Educational Technology*, *6*, 1–7.

Berenson, S. B., Carter, G. S., & Norwood, K. S. (1992). The at-risk student in college developmental algebra. *School Science and Mathematics*, *92*, 55–58.

*Berg, K. F. (1993, April). *Structured cooperative learning and achievement in a high school mathematics class*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

*Biesinger, K., & Crippen, K. (2008). The impact of a state-funded online remediation site on performance related to high school mathematics proficiency. *Journal of Computers in Mathematics and Science Teaching*, *27*, 5–17.

Blanco, L., & Garrote, M. (2007). Difficulties in learning inequalities in students of the first year of pre-university education in Spain. *Eurasia Journal of Mathematics, Science & Technology Education*, *3*, 221–229.

*Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning. *American Educational Research Journal*, *34*, 663–689.

Briggs, J., Demana, F., & Osborne, A. (1986). Moving into algebra: Developing the concepts of variable and function. *The Australian Mathematics Teacher, 42*, 5–8.

Brownell, W. A. (1938). A critique of the Committee of Seven's investigations on the grade placement of arithmetic topics. *Elementary School Journal, 38*, 495–508.

*Bunch, M. A. (1972). A study of the effects on retention and on the problem-solving ability of students when geometry is used as an aid in teaching factoring of second-degree polynomials. *Dissertation Abstracts International-A*, *34*(03), 1057.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

Butler, D. L., & Winne, P. H. (1995). Feedback and self-regulated learning: A theoretical synthesis. *Review of Educational Research*, *65*, 245–282.
Carraher, D. W., & Schliemann, A. D. (2007). Early algebra. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 669–706). Reston, VA: National Council of Teachers of Mathematics.
*Carroll, W. M. (1992, April). *The use of worked examples in teaching algebra*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.
*Carroll, W. M. (1994). Using worked examples as an instructional support in the algebra classroom. *Journal of Educational Psychology*, *86*, 360–367.
*Cavanaugh, C., Gillan, K. J., Bosnick, J., Hess, M., & Scott, H. (2008). Effectiveness of interactive online algebra learning tools. *Journal of Educational Computing Research*, *38*, 67–95.
Chambers, D. L. (1994). The right algebra for all. *Educational Leadership*, *53*, 85–86.
*Chirwa, A. S. (1996). Computers stimulate the generation of questions among students. *Dissertation Abstracts International-A*, *57*(08), 3464.
*Chung, G. K. W. K., Delacruz, G. C., Dionne, G. B., Baker, E. L., Lee, J., & Osmundson, E. (2007, April). *Towards individualized instruction with technology-enabled tools and methods*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.
*Clay, D. W. (1998). A study to determine the effects of a non-traditional approach to algebra instruction on student achievement. *Dissertation Abstracts International-A*, *31*(06), 3478.
Cooper, H. (1998). *Synthesizing research*. Thousand Oaks, CA: Sage.
*Coppen, M. J. (1976). The development and evaluation of an individualized mastery programme in mathematics for low-ability secondary school students. *New Zealand Journal of Educational Studies*, *11*, 132–142.
*Denson, P. S. (1989). A comparison of the effectiveness of the Saxon and Dolciani texts and theories about the teaching of high school algebra. *Dissertation Abstracts International-A*, *50*(10), 3173.
*Durmus, S. (1999). The effects of the use of the technology on college algebra students' achievements and attitudes toward mathematics: A constructivist approach. *Dissertation Abstracts International-A*, *60*(10), 3622.
Duval, S., & Tweedie, R. (2000a). A nonparametric "trim and fill" method of accounting for publication bias in meta-analysis. *Journal of the American Statistical Association*, *95*, 89–98.
Duval, S., & Tweedie, R. (2000b). Trim and fill: A simple funnel-plot-based method of testing and adjusting for publication bias in meta-analysis. *Biometrics*, *56*, 455–463.
Easton, J. Q. (2010, May). *Out of the tower, into the schools: How new IES goals will reshape researcher roles*. Presidential session presented at the annual meeting of the American Educational Researcher Association, Denver, CO.
Edwards, T. G. (2000). Some 'big ideas' of algebra in the middle grades. *Mathematics Teaching in the Middle School, 6*, 26–31.
Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education*, *34*, 433–463.
Ellington, A. J. (2005). A modeling-based college algebra course and its effect on student achievement. *Primus*, *15*, 193–214.
Ellington, A. J. (2006). The effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis. *International Journal of Instructional Media*, *106*, 16–26.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

*Elshafei, D. L. (1998). A comparison of problem-based and traditional learning in Algebra II. *Dissertation Abstracts International-A*, *60*(01), 85.

*Farrell, J. (1980). The effects of individualized instruction on the achievement of cognitive and affective objectives in selected first-year algebra students. *Dissertation Abstracts International-A*, *40*(12), 6087.

*Freeman, M. (1997). *Math and science on a personal level*. Lexington, KY: University of Kentucky, Appalachian Rural Systemic Initiative.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Boston, MA: Kluwer Academic.

Furr, R. M., & Bacharach, V. R. (2008). *Psychometrics: An introduction*. Los Angeles, CA: Sage.

Gates, J. (2008). *MIND Research Institute launches national readiness intervention program*. Retrieved from http://www.mindinstitute.net/pdf/08_0630_MI-AR-Ships%20Final.pdf.

*Gesshel-Green, H. (1987). The effect of interactive microcomputer graphics on student achievement and retention in second year algebra in an academic high school. *Dissertation Abstracts International-A*, *48*(02), 326.

Githua, B. N., & Mwangi, J. G. (2003). Students' mathematics self-concept and motivation to learn mathematics: Relationship and gender differences among Kenya's secondary-school students in Nairobi and Rift Valley provinces. *International Journal of Educational Development*, *23*, 487–499.

*Goins, K. B. (2001). Comparing the effects of visual and algebra tile manipulative methods on student skill and understanding of polynomial multiplication. *Dissertation Abstracts International-A*, *62*(12), 4098.

*Goldsby, D. S. (1994). The effect of algebra tile use on the polynomial factoring ability of Algebra I students. *Dissertation Abstracts International-A*, *57*(08), 3434.

Graham, A. T., & Thomas, M. O. J. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics, 41*, 265–282.

Greenes, C. E., & Rubenstein, R. (Eds.). (2008). *Algebra and algebraic thinking in school mathematics: Seventieth yearbook*. Reston, VA: National Council of Teachers of Mathematics.

Haas, M. S. (2005). Teaching methods for secondary algebra: A meta-analysis of findings. *National Association of Secondary School Principals Bulletin*, *89*, 24–46.

*Hannafin, R. D., & Foshay, W. R. (2008). Computer-based instruction's (CBI) rediscovered role in k-12: An evaluation case study of one high school's use of cbi to improve pass rates on high-stakes tests. *Educational Technology Research and Development*, *56*, 147–160.

Hartley, S. S. (1977). Meta-analysis of the effects of individually paced instruction in mathematics. *Dissertation Abstracts International*, *38*(07), 4003.

*Haver, W. E. (1978). Developing skills in college algebra—A mastery approach. *The Two-Year College Mathematics Journal*, *9*, 282–287.

*Hawkins, V. J. (1982). A comparison of two methods of instruction, a saturated learning environment and traditional learning environment: Its effects on achievement and retention among female adolescents in first-year algebra. *Dissertation Abstracts International-A*, *43*(02), 416.

*Heath, R. D. (1987). The effect of calculators and computers on problem-solving ability and attitude toward mathematics. *Dissertation Abstracts International-A*, *48*(05), 1102.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

*Hecht, J. B., Roberts, N. K., Schoon, P. L., & Fansler, G. (1995, October). *Teacher teams and computer technology*. Paper presented at the annual meeting of the Midwestern Educational Research Association, Chicago, IL.

Hedges, L. V., & Hedberg, E. C. (2007). Intraclass correlation values for planning group-randomized trials in education. *Educational Evaluation and Policy Analysis*, *29*, 60–87.

*Henderson, R. W., & Landesman, E. M. (1995). Effects of thematically integrated mathematics instruction on students of mexican descent. *Journal of Educational Research*, *88*, 290–300.

Hiebert, J. (2003). What research says about the NCTM standards. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to "Principles and standards for school mathematics"* (pp. 5–23). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–100). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding. Portsmouth,* NH: Heinemann.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education*, *58*, 47–61.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., & . . . Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis*, *27*, 111–132.

*Hodo, L. B. (1989). The effects of study skills instruction on achievement and usage of selected study strategies in Algebra II classes. *Dissertation Abstracts International-A*, *50*(04), 862.

Howe, R. (2005). *Comments on NAEP algebra problems*. Retrieved from http://www.brookings.edu/brown/~/media/Files/Centers/bcep/AlgebraicReasoningConference Howe.pdf.

*Hutchinson, N., & Hemingway, P. (1987, April). *Teaching representation and solution of algebraic word problems to learning disabled adolescents*. Paper presented at the annual meeting of the American Educational Research Association, Washington, DC.

*Ives, B. (2007). Graphic organizers applied to secondary algebra instruction for students with learning disorders. *Learning Disabilities Research & Practice*, *22*, 110–118.

*Johnson, D. M., & Smith, B. (1987). An evaluation of Saxon's algebra text. *Journal of Educational Research*, *81*, 97–102.

*Kalchman, M. S. (2001). Using a neo-Piagetian framework for learning and teaching mathematical functions. *Dissertation Abstracts International-A*, *62*(11), 3719.

Kalchman, M., & Koedinger, K. (2005). Teaching and learning functions. In S. Donovan & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 351–393). Washington, D.C.: National Academy Press.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

*Kalyuga, S., & Sweller, J. (2005). Rapid dynamic assessment of expertise to improve the efficiency of adaptive e-learning. *Educational Technology Research and Development*, *53*, 83–93.

*Keif, M. G. (1995). Performance of students completing courses using the CORD Applied Mathematics curriculum in four Missouri school districts. *Dissertation Abstracts International-A*, *57*(09), 3863.

Kieran, C. (1989). The early learning of algebra: A structural perspective. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 33–56). Hillsdale, NJ: Lawrence Erlbaum.

Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (2007). Learning and teaching of algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707–762). Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (2008). *What do students struggle with when first introduced to algebra symbols?* Retrieved from http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Brief%20-%20What%20Can%20We%20Learn.pdf.

*Kieren, T. E. (1968). The computer as a teaching aid for eleventh grade mathematics: A comparison study. *Dissertation Abstracts International-A*, *29*(10), 3526.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

*Kim, T. (2006). Impact of inquiry-based teaching on student mathematics achievement and attitude. *Dissertation Abstracts International-A*, *67*(04).

Kish, L. (1965). *Survey sampling*. New York, NY: John Wiley.

Klauer, K. J., & Phye, G. D. (2008). Inductive reasoning: A training approach. *Review of Educational Research*, *78*, 85–123.

*Koedinger, K. R., Anderson, J. R., Hadley, W. H., & Mark, M. A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education*, *8*, 30–43.

*Konold, K. B. (2004). Using the concrete-representational-abstract teaching sequence to increase algebra problem-solving skills. *Dissertation Abstracts International-A*, *65*(08), 2949.

Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in School*, *9*, 23–26.

*Lawrence, L. K. (1992). The long-term effects of an incremental development model of instruction upon student achievement and student attitude toward mathematics. *Dissertation Abstracts International-A*, *53*(03), 747.

Lee, J., Grigg, W., & Dion, G. (2007). *The nation's report card: Mathematics 2007–National Assessment of Educational Progress at grades 4 and 8 [NCES 2007-494]*. Washington, DC: National Center for Education Statistics.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, *60*, 1–64.

Leitzel, J. R. (1989). Critical considerations for the future of algebra instruction or a reaction to: "Algebra: What should we teach and how should we teach it?" In S. Wagner, & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 25–27). Hillsdale, NJ: Lawrence Erlbaum.

Lipsey, M. W., & Wilson, D. B. (2001). *Practical meta-analysis*. Thousand Oaks, CA: Sage.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

Lou, Y., Abrami, P. C., & d'Apollonia, S. (2001). Small group and individual learning with technology: A meta-analysis. *Review of Educational Research*, *71*, 449–521.
Lou, Y., Abrami, P. C., Spence, J. C., & Poulsen, C. (1996). Within-class grouping: A meta-analysis. *Review of Educational Research*, *66*, 423–458.
*Lwo, L. S. (1992). Effects of individualized examples and personalized contexts in computer-based adaptive teaching of algebra word problems. *Dissertation Abstracts International-A*, *53*(06), 1832.
*Makanong, A. (2000). The effects of constructivist approaches on ninth grade algebra achievement in Thailand secondary school students. *Dissertation Abstracts International-A*, *61*(03), 923.
Malloy, C. E., & Malloy, W. W. (1998). Resiliency and Algebra 1: A promising non-traditional approach to teaching low-achieving students. *Clearing House*, *71*, 314–317.
*Mathews, S. M. (1997). The effect of using two variables when there are two unknowns in solving algebraic word problems. *Mathematics Education Research Journal*, *9*, 122–135.
*McBee, M. (1984). *Dolciani vs. Saxon: A comparison of two Algebra I textbooks with high school students*. Oklahoma City, OK: Oklahoma City Public Schools, Oklahoma Department of Planning, Research, and Evaluation. (ERIC Document Reproduction Service No. ED241348)
*McCaffrey, D. F., Hamilton, L. S., & Stecher, B. M. (2001). Interactions among instructional practices, curriculum, and student achievement: The case of standards-based high school mathematics. *Journal for Research in Mathematics Education*, *32*, 483–517.
*McClung, L. W. (1998). *A study on the use of manipulatives and their effect on student achievement in a high school Algebra I class*. Unpublished master's thesis, Salem-Teiko University, Salem, WV. (ERIC Document Reproduction Service No. ED425077)
McDermott, L. C., Rosenquist, M. L., & Van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics*, *55*, 503–513.
*McKenzie, S. Y. (1999). Achievement and affective domains of Algebra I students in traditional or self-paced computer programs. *Dissertation Abstracts International-A*, *60*(09), 3297.
*Morgan, P., & Ritter, S. (2002). *An experimental study of the effects of Cognitive Tutor Algebra I on student knowledge and attitude*. Retrieved from http://www.carnegielearning.com/web_docs/morgan_ritter_2002.pdf.
National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
National Council of Teachers of Mathematics. (2008). *Algebra: What, when, and for whom*. Retrieved from http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/Algebra%20final%2092908.pdf.
National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Retrieved from http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf.
*Nichols, J. D., & Miller, R. B. (1994). Cooperative learning and student motivation. *Contemporary Educational Psychology*, *19*, 167–178.
No Child Left Behind Act of 2002, 20 U.S.C. § 6301 et seq. (West, 2002).

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

*O'Dwyer, L. M., Carey, R., & Kleiman, G. (2007). A study of the effectiveness of the Louisiana Algebra I Online Course. *Journal of Research on Technology in Education*, *39*, 289–306.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research*, *66*, 543–578.

*Parham, J. W. (1993). An analysis of the effects of tutoring on seventh-grade students engaged in the mastery of pre-algebra concepts. *Dissertation Abstracts International-A*, *54*(11), 4021.

*Peters, K. G. (1992). Skill performance comparability of two algebra programs on an eighth-grade population. *Dissertation Abstracts International-A*, *54*(01), 1993.

*Pierce, R. (1984). A quasi-experimental study of Saxon's incremental development model and its effects on student achievement in first-year algebra. *Dissertation Abstracts International-A*, *45*(02), 443.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks, CA: Sage.

Raudenbush, S. W., Bryk, A. S., Cheong, Y. F., Congdon, R., & du Toit, M. (2004). *HLM 6: SSI Scientific Software International: Hierarchical linear & non-linear modeling*. Lincolnwood, IL: Scientific Software International, Inc.

*Rech, J. F., Juhler, S. M., & Johnson, H. L. (1995). The effectiveness of videotapes in the individualized instruction of intermediate algebra and college algebra. *International Journal of Mathematical Education in Science and Technology*, *26*, 463–472.

*Reys, R. E., Reys, B., & Lapan, R. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education*, *34*, 74–95.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? J*ournal of Educational Psychology, 91*, 175–189.

*Rodgers, C. E. (1995). An investigation of two instructional methods for teaching selected pre-algebra concepts to minority at-risk seventh-grade mathematics students. *Dissertation Abstracts International-A*, *56*(08), 3042.

Roscoe, R. D., & Chi, M. T. H. (2007). Understanding tutor learning: Knowledge-building and knowledge-telling in peer tutors' explanations and questions. *Review of Educational Research*, *77*, 534–574.

Rosenshine, B., Meister, C., & Chapman, S. (1996). Teaching students to generate questions: A review of the intervention studies. *Review of Educational Research*, *66*, 181–221.

*Schumacker, R. E., Young, J. I., & Bembry, K. L. (1995). Math attitudes and achievement of Algebra I students: A comparative study of computer-assisted and traditional lecture methods of instruction. *Computers in the Schools*, *11*, 27–33.

*Seals, G. J. (2001). The effects of portfolio use as a learning tool on Algebra II students' achievement and their attitudes toward mathematics. *Dissertation Abstracts International-A*, *63*(01), 72.

Seidel, T., & Shavelson, R. J. (2007). Teaching effectiveness research in the past decade: The role of theory and research design in disentangling meta-analysis results. *Review of Educational Research*, *77*, 454–499.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*, 1–36.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2001). *Experimental and quasi-experimental design*. Boston, MA: Houghton Mifflin.

*Shneyderman, A. (2001). *Evaluation of the cognitive tutor algebra 1 program*. Retrieved from http://oer.dadeschools.net/algebra.pdf.

*Shoecraft, P. J. (1971). The effects of provisions for imagery through materials and drawings on translating algebra word problems, grades seven and nine. *Dissertation Abstracts International-A*, *32*(07), 3874.

*Silvis, K. T. (2007). A comparison of hands-on and traditional approaches for teaching eighth grade pre-algebra. *Dissertation Abstracts International-A*, *68*(03), 921.

*Siskind, T. G. (1994). The effect of calculator use on mathematics achievement for rural high school students. *The Rural Educator*, *16*, 1–4.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*, *12*, 88–95. (Reprinted from Mathematics Teaching, 77, 20–26, 1976)

*Slavin, R. E., & Karweit, N. L. (1982). *Student teams and mastery learning: A factorial experiment in urban math nine classes*. Baltimore, MD: Johns Hopkins University, Center for Social Organization of Schools. (ERIC Document Reproduction Service No. ED215904)

Slavin, R. E., & Lake, C. (2008). Effective programs in elementary mathematics: A best-evidence synthesis. *Review of Educational Research*, *78*, 427–515.

*Smith, K. B. (1994). Studying different methods of technology integration for teaching problem solving with systems of equations and inequalities and linear programming. *Journal of Computers in Mathematics and Science Teaching*, *13*, 465–479.

*Smith, K. B., & Shotsberger, P. G. (1997). Assessing the use of graphing calculators in college algebra: Reflecting on dimensions of teaching and learning. *International Journal of Instructional Media*, *97*, 368–376.

*Smith, L. R. (1985). Presentational behaviors and student achievement in mathematics. *Journal of Educational Research*, *78*, 292–298.

Socas Robayna, M. M. (1997). Dificultades, obstáculos y errores en el aprendizaje de las matemáticas en la educación secundaria. [Difficulties, obstacles, and errors in learning mathematics in secondary education]. In L. R. Romero (Ed.), *La educación matemática en la enseñanza secundaria* (pp. 125–154). Barcelona: Horsori y Instituto de Ciencias de la Educación.

*St. John, D. J. (1992). A comparison of two methods of teaching eighth-grade pre-algebra students how to construct tables for use in solving selected mathematical problems. *Dissertation Abstracts International-A*, *54*(02), 453.

*Stephens, L. J., & Konvalina, J. (1999). The use of computer algebra software in teaching intermediate and college algebra. *International Journal of Mathematical Education in Science and Technology*, *30*, 483–488.

Stigler, J. W., & Hiebert, J. (1997). Understanding and improving classroom mathematics instructions: An overview of the TIMSS video study. *Phi Delta Kappan*, *79*, 14–21.

Stodolsky, S. S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago, IL: University of Chicago Press.

*Suh, J., & Moyer, P. S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching*, *26*, 155–173.

*Tenenbaum, G. (1986). The effect of quality of instruction on higher and lower mental processes and on the prediction of summative achievement. *Journal of Educational Research*, *80*, 105–114.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

*Thompson, D. R., & Senk, S. L. (2001). The effects of curriculum on achievement in second-year algebra: The example of the University of Chicago School Mathematics Project. *Journal for Research in Mathematics Education*, *32*, 58–84.

Thorndike, E. L. (1913). *Educational psychology* (Vols. 1-2). New York, NY: Columbia University Press.

Thorpe, J. A. (1989). Algebra: What should we teach and how should we teach it? In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 11–24). Hillsdale, NJ: Lawrence Erlbaum.

Thornton, A., & Lee, P. (2000). Publication bias in meta-analysis: Its causes and consequences. *Journal of Clinical Epidemiology, 53*, 207–216.

Torigoe, E., & Gladding, G. (2006). Same to us, different to them: Numeric computation versus symbolic representation. *AIP Conference Proceedings*, *883*, 153–156.

Urbina, S. (2004). *Essentials of psychological testing*. New York, NY: John Wiley.

Usiskin, Z. (1980). What should not be in the algebra and geometry curricula of average college-bound students? *Mathematics Teacher*, *73*, 413–424.

*Uy, C. (2005). Critical trial use of TIMSS as an alternative instructional delivery tool. *Asia Pacific Education Review*, *6*, 28–40.

Van De Walle, J. A. (2007). *Elementary and middle school mathematics* (6th ed.). Boston, MA: Pearson.

Vogel, C. (2008). Algebra: Changing the equation. *District Administration*, *44*, 34–40.

*Walker, S., & Senger, E. (2007). Using technology to teach developmental African-American algebra students. *Journal of Computers in Mathematics and Science Teaching*, *26*, 217–231.

*Weems, G. H. (2002). Comparison of beginning algebra taught onsite versus online. *Journal of Developmental Education*, *26*, 10–18.

Welch, W. (1978). Science education in Urbanville. In R. Stake & J. Easley (Eds.), *Case studies in science education* (pp. 5-1–5-33). Urbana, IL: University of Illinois.

Wenglinsky, H. (2000). *How teaching matters: Bringing the classroom back into discussions of teacher quality*. Princeton, NJ: Educational Testing Service, Policy Information Center. (ERIC Document Reproduction Service No. ED447128)

*Whicker, K. M., Bol, L., & Nunnery, J. A. (1997). Cooperative learning in the secondary mathematics classroom. *Journal of Educational Research*, *91*, 42–48.

Whitehurst, G. J. (2002, October). *Evidence-based education*. Paper presented at the annual meeting of the School Accountability Conference, Orlando, FL. Retrieved from http://www.ed.gov/offices/OERI/presentations/evidencebase.html.

Wiesner, M. B. W. (1989). Concept learning and the learning of strategies for teaching graphing, probability, and statistics among inservice elementary and secondary mathematics teachers. *Dissertation Abstracts International-A*, *50*(06), 1588.

*Wilkins, C. W. (1993). *Effects of using a problem solving approach to Algebra I instruction*. Mississippi State, MS: Mississippi State University. (ERIC Document Reproduction Service No. ED372934)

*Wineland, J. N., & Stephens, L. (1995). Effects of spiral testing and review on retention and mathematical achievement for below-average eighth- and ninth-grade students. *International Journal of Mathematical Education in Science and Technology*, *26*, 227–232.

*Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, *18*, 121–131.

*Wood, J. B. (1992, October). *The application of computer technology and cooperative learning in developmental algebra at the community college*. Paper presented at the

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016

ninth annual computer conference of the League for Innovation in the Community College, Orlando, FL. (ERIC Document Reproduction Service No. ED352099)
Woolner, P. (2004). A comparison of a visual-spatial approach and a verbal approach to teaching mathematics. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2004* (pp. 449–456). Bergen, Norway: International Group for the Psychology of Mathematics Education.

Authors

CHRISTOPHER R. RAKES has recently completed his doctoral studies at the University of Louisville, Curriculum and Instruction in Secondary Mathematics Education. Over the past 10 years, he has taught mathematics in both urban and rural settings at the secondary and postsecondary levels. His research is framed by two overarching issues: (a) effective strategies to teach mathematics and (b) issues and approaches in the preparation of teachers of mathematics. His scholarly work involves multiple methods such as systematic review, meta-analysis, structural equation modeling (SEM), hierarchical linear modeling (HLM), and mixed methodology.

JEFFREY C. VALENTINE is an associate professor in the College of Education & Human Development at the University of Louisville. His research interests include how non-school activities affect students, both psychologically and academically. He is also interested in improving the methods used in systematic reviews and meta-analyses and in improving assessments of study quality.

MAGGIE B. MCGATHA is an assistant professor in the College of Education & Human Development at the University of Louisville. Her research interests include how mentoring and coaching affects teacher practice and in turn, student learning. She is also interested in improving mathematics teacher education for preservice and inservice teachers.

ROBERT N. RONAU is a professor of mathematics education at the University of Louisville whose research interests and publications include the implementation of instructional technology, teacher knowledge, teacher preparation, and assessment. Over the past 15 years, he has played a critical role in numerous statewide and local grant efforts including development of State Wide Mathematics Core-Content and Assessments, LATTICE (Learning Algebra Through Technology, Investigation, and Cooperative Experience), the Secondary Mathematics Initiative (SMI) of PRISM (Partnership for Reform Initiatives in Science and Mathematics), Kentucky's statewide systemic reform initiative, Technology Alliance, Teaching K-4 Mathematics in Kentucky, the Park City/IAS Geometry Project, and U²MAST. He currently serves as a Co-PI on the NSF-funded project, Geometry Assessments for Secondary Teachers (GAST), an initiative to determine critical geometry knowledge for secondary teachers and to design assessments to measure levels of teacher knowledge.

Downloaded from http://rer.aera.net at PENNSYLVANIA STATE UNIV on September 13, 2016