

## Price and Warranty period decision in different channel structures for durable product

Xiaoyan Zhou<sup>1,a</sup>, Jincal Huang<sup>1,b</sup>, Qing Cheng<sup>1,c</sup>

Key Lab of Information System Engineering, NUDT, Changsha, 410073, China.  
<sup>a</sup> zxy7755755@163.com, <sup>b</sup> huangjincal@gmail.com, <sup>c</sup> sggggs@163.com

**Keywords:** supply-chain; Stackelberg model; Nash model

**Abstract.** A decision problem is investigated in a decentralized two-echelon supply chain with one manufacturer and one retailer during a fixed sales time. There are three possible power balance scenarios: Manufacturer-Stackelberg, retailer-Stackelberg, vertical Nash classified by the different influence on the market between the manufacturer and the retailer. Aiming at profit maximization, from the perspectives of the consumer, the manufacturer and the retailer, respectively, the results of the three scenarios are analyzed, in which the retailer price  $P$ , warranty period  $W$  and the whole price  $C_p$  are decision variables. Moreover, the sales over time can be characterized by a stochastic Bass model in the form of a non-homogeneous Poisson process (NHPP-Bass model) and the production system is a make-to-order type of system. Finally, A numerical example is provided to illustrate the effects of some key parameters, including the product reliability, price elasticity, and warranty period elasticity, on the optimal settings of the retail price and warranty period.

### Introduction

The paper provides a price and warranty period decision problem in a decentralized supply-chain. A decentralized supply-chain deriving from the rational man supposition emphasize the independent decision-making of supply chain members. Related works generally can be divided into two parts: one part is based on the inventory control, doing the research on the equilibrium inventory strategy under the relationship of upstream and downstream game; the other part considers the strategy variables, such as the retail price  $P$  determined by the retailer and the wholesale price  $C_p$  determined by the manufacturer based on the pricing strategy in the market economics, which is exactly the problem we considered in this paper. Blischke and Murthy [11] discuss several key problems in the warranty plan, while Djameludin and Murthy [9] conclude all the warranty strategy available. In the paper, we adopt the non-renewable free minimal warranty policy. When discussing the game problem between companies in supply chain, most research consider only the manufacturer leading the pack in a supply chain (Chen Fangruo et al. [12]) rather than the dominant retailer or manufacturer & retailer with equivalent size.

The production and consumption of durable product is chosen based on two meaningful reasons: 1) the service life of the durable product is usually more than a year, and is relatively more expensive than the consumable product, so the consumers pay more attention to the price and warranty period factors when choosing products. 2) For the manufacturer and the retailers, the durable products usually have high techniques and large profit space, which is worth discussing and doing research to make the optimal decision.

A make-to-order supply chain with one manufacturer and one retailer is considered in this paper. Here is how it works: the consumers book the durable product from the retailer who transfers the order to the manufacturer for production. Then the manufacturer sells the product to the retailer with the wholesale price and the consumers buy the product with retail price. Additionally, the manufacturer will provide a warranty period for the consumers and all the repair cost during this period loaded by the manufacturer.

There are three possible power balance scenarios: manufacturer-Stackelberg, retailer-Stackelberg, vertical Nash classified by the different influence on the market between the manufacturer and the retailer. Aiming at profit maximization, let the retailer price  $P$ , warranty period

$W$  and the whole price  $C_p$  be decision variables. We analyze the results of three scenarios, representing three types of markets, which has differently influence on the consumer, the manufacturer and the retailer, respectively and implies different groups' tendency and persuit for the future markets.

**Notation**

- $C_W$ : the fixed minimal repair cost per unit in warranty period
- $C_0$ : the production cost of the first unit produced
- $C_p$ : the wholesale price per unit
- $P$ : the retail price per unit
- $W$ : the warranty period
- $\theta$ : the failure rate of the unit
- $\Lambda(t)$ : the expected cumulative sales quantity by time  $t$
- $N(t)$ : the number of sales by time  $t$
- $G(t)$ : the cumulative proportion of adopters
- $m$ : the size of the new product's potential market
- $\Pi_M$ : the total profit of the manufacturer
- $\Pi_R$ : the total profit of the retailer
- $\pi_M$ : the wholesale profit per unit
- $\pi_R$ : the sales profit per unit
- $L$ : the fixed period for selling the product

**The profit model**

**The profit model for the retailer**

According to [1], it is assumed that  $N(t)$  follows an NHPP(Non-Homogeneous Poisson Process ) with intensity function  $\lambda(t)$ . And the expected cumulative number of sales by time  $t$  is  $\Lambda(t) = E[N(t)] = \int_0^t \lambda(u)du$ . Besides, the Bass Model (Bass 1969) is adopted to postulate the trajectory of cumulative sales of a new product with two parameters, the innovator factor  $\alpha_1$  and the imitator factor  $\alpha_2$ , which can be shown as follows:

$$\Lambda(t) = m(P, W) \frac{1 - e^{-(\alpha_1 + \alpha_2)t}}{1 + \frac{\alpha_2}{\alpha_1} e^{-(\alpha_1 + \alpha_2)t}} \tag{1}$$

Let

$$G(t) = \frac{1 - e^{-(\alpha_1 + \alpha_2)t}}{1 + \frac{\alpha_2}{\alpha_1} e^{-(\alpha_1 + \alpha_2)t}} \tag{2}$$

represent the cumulative proportion of adopters by the time  $t$ , then  $\Lambda(t) = m(P, W)G(t)$ . Furthermore, we adopt the model proposed by Glickman and Berger (1976) to describe the potential market size  $m$  as a displaced log-linear function of retail price  $P$  and warranty period  $W$ , namely,  $m(P, W) = \kappa_1 P^{-\beta_1} (W + \kappa_2)^{\beta_2}$ , where  $\kappa_1, \kappa_2 \geq 0$ ,  $\beta_1 > 1$ , and  $0 < \beta_2 < 1$ . The constant  $\kappa_1$  is an amplitude factor, and  $\kappa_2$  is a constant for time displacement which allows for nonzero demand when  $W = 0$ . Parameters  $\beta_1$  and  $\beta_2$  could be interpreted as the price elasticity and displaced warranty period elasticity, respectively.

Then, the total profit function of the retailer can be obtained like this:

$$\begin{aligned} \Pi_R(P, W, C_p) &= \Lambda(L)(P - C_p) \\ &= m(P, W)G(L)(P - C_p) \end{aligned} \tag{2}$$

where  $0 < C_w < C_0 < \underline{C_p} \leq C_p \leq \overline{C_p} < P$ .

For the wholesale price  $C_p$ , we give the upper and lower bounds which is higher than the production cost and lower than the retail price, to restrain the decision-making behavior of the manufacturer.

### The profit model for the manufacturer

It is obvious that the profit of the manufacturer comes from the wholesale revenues removing the production cost and the repair cost during warranty period. In the paper, we adopt the Non-renewable Free Minimal Warranty policy which means the manufacturer only provides a free minimal repair during the warranty period (Elsayed[16]). Let  $C_w$  be a fixed minimal repair cost for handling each warranty claim. We have the expected warranty cost to the manufacturer expressed  $C_w \Lambda(L)\theta W$ .

Hence, the total profit function for the manufacturer can be shown as follows:

$$\begin{aligned} \Pi_M(P, W, C_p) &= \Lambda(L)(C_p - C_0 - C_w\theta W) \\ &= m(P, W)G(L)(C_p - C_0 - C_w\theta W) \end{aligned} \tag{3}$$

### Stackelberg game and Nash game

In the following, we discuss the decision problems in three channel structures for the durable products:

#### Manufacturer-Stackelberg

In this scenario, the market is made up of some large manufacturers and some smaller retailers, such as food processing industry, and the OTC market, etc. In other words, the market is controlled by the dominant manufacturer in Stackelberg game. After the manufacturer giving the wholesale price  $C_{p0}$  and the warranty period  $W_0$ , the retail price  $P^*(C_p, W)$  would be provided by the retailer.

Let

$$\frac{d\Pi_R(P, W_0, C_{p_0})}{dP} = m(P, W_0)G(L)(1 - \beta_1 + \frac{\beta_1 C_p}{P}) = 0 \tag{4}$$

we get  $P^* = \frac{\beta_1}{\beta_1 - 1} C_p$ . When the wholesale price is increased, the retailer would accordingly increase the retail price in order to guarantee his profit. However, the profit is still reduced due to the size of new product potential market is falling down.

Let

$$\frac{\partial \Pi_M(P^*, W, C_p)}{\partial W} = m(P^*, W)G(L)(\frac{\beta_2}{W + \kappa_2}(C_p - C_0 - C_w\theta W) - C_w\theta) = 0 \tag{5}$$

$$\frac{\partial \Pi_M(P^*, W, C_p)}{\partial C_p} = m(P^*, W)G(L)(1 - \frac{\beta_1}{C_p}(C_p - C_0 - C_w\theta W)) = 0 \tag{6}$$

We get

$$W^* = \frac{\beta_2 C_0 - (\beta_1 - 1)C_w\theta\kappa_2}{C_w\theta(\beta_1 - 1 - \beta_2)} \quad \text{and} \quad C_p^* = \frac{\beta_1(C_0 - C_w\theta\kappa_2)}{\beta_1 - 1 - \beta_2}$$

According to the constraints,  $W^* \geq 0$ , if  $W^* < 0$ , then  $W^* = 0$ , means the manufacturer provide no warranty service. Besides,  $\frac{C_P}{\beta_1} \leq C_P^* \leq \overline{C_P}$ , if  $C_P^* > \overline{C_P}$ , then  $C_P^* = \overline{C_P}$ ; if  $C_P^* < \frac{C_P}{\beta_1}$ , then  $C_P^* = \frac{C_P}{\beta_1}$ . Let  $P^* = \frac{\beta_1}{\beta_1 - 1} C_P$ , we get  $P^* = \frac{\beta_1^2 (C_0 - C_W \theta \kappa_2)}{(\beta_1 - 1)(\beta_1 - 1 - \beta_2)}$ .

### Retailer-Stackelberg

In this scenario, there are many relatively large retailers, such as some international large supermarket chains (Wal-Mart, Carrefour and Metro) and some other large sales companies in the clothing market. Under this circumstances, the production of the manufacturer is strongly influenced by the orders from the retailers. After given the retail price P by the retailer, the manufacturer accordingly provide the wholesale price  $C_P$  and the warranty period W. Given  $P_0$ , let

$$\frac{\partial \Pi_M(P_0, W, C_P)}{\partial W} = m(P_0, W)G(L) \left( \frac{\beta_2}{W + \kappa_2} (C_P - C_0 - C_W \theta W) - C_W \theta \right) = 0 \quad (7)$$

$$\frac{\partial \Pi_M(P_0, W, C_P)}{\partial C_P} = m(P_0, W)G(L) > 0 \quad (8)$$

then, we have

$$W(C_P) = \frac{\beta_2 (C_P - C_0) - C_W \theta \kappa_2}{C_W \beta_2 \theta + C_W \theta} \quad (9)$$

However, above formulations imply that, for the manufacturer, the higher wholesale price means more profit and in our model the potential market is only related to the retail price and the warranty period, so the manufacturer would increase the wholesale price as much as possible when not affecting the orders. Taking  $C_P^*$  to be  $\overline{C_P}$ , we have

$$W^* = W(C_P^*) = \frac{\beta_2 (\overline{C_P} - C_0) - C_W \theta \kappa_2}{C_W \beta_2 \theta + C_W \theta} \quad (10)$$

Then, for the retailer, the optimal retail price  $P^*$  fulfilled that

$$\frac{d \Pi_R(P, W^*, C_P^*)}{dP} = m(P, W^*)G(L) \left( 1 - \beta_1 + \frac{\beta_1 C_P^*}{P} \right) = 0 \quad (11)$$

we have  $P^* = \frac{\beta_1}{\beta_1 - 1} \overline{C_P}$ .

### vertical Nash

In this scenarios, the retailer and the manufacturer are equivalent in size without any following or leading relationship, which is a Nash game. Given the strategy of one side, the other side would make the best choice for himself. In other word, given the retail price  $P_0$ , the manufacturer provides the warranty period  $W^*(P_0)$  and the wholesale price  $C_P^*(P_0)$  to maximize the profit  $\Pi_M(P_0, W, C_P)$ ; given the strategy of  $C_{P0}$  and  $W_0$ , the retailer makes the choice to maximize the profit  $\Pi_R(P, W_0, C_{P0})$ .

$$\frac{d \Pi_R(P, W_0, C_{P0})}{dP} = m(P, W_0)G(L) \left( 1 - \beta_1 + \frac{\beta_1 C_{P0}}{P} \right) = 0$$

$$\frac{\partial \Pi_M(P_0, W, C_P)}{\partial W} = m(P_0, W)G(L) \left( \frac{\beta_2}{W + \kappa_2} (C_P - C_0 - C_W \theta W) - C_W \theta \right) = 0$$

$$\frac{\partial \Pi_M(P_0, W, C_P)}{\partial C_P} = m(P_0, W)G(L) > 0 \quad (12)$$

we have  $C_p^* = \overline{C_p}$ ,  $P^* = P(C_p^*) = \frac{\beta_1}{\beta_1 - 1} \overline{C_p}$ ,  $W^* = W(C_p^*) = \frac{\beta_2(\overline{C_p} - C_0) - C_w \theta \kappa_2}{C_w \beta_2 \theta + C_w \theta}$ . (13)

**The comparison and analysis of results**

Let  $(P_1^*, W_1^*, C_{P1}^*)$ ,  $(P_2^*, W_2^*, C_{P2}^*)$  and  $(P_3^*, W_3^*, C_{P3}^*)$  denote the equilibrium points of three scenarios, respectively.

Table 1 The equilibrium points of three scenarios

	$P^*$	$W^*$	$C_p^*$
Manufacturer-Stackelberg	$\frac{\beta_1^2(C_0 - C_w \theta \kappa_2)}{(\beta_1 - 1)(\beta_1 - 1 - \beta_2)}$	$\frac{\beta_2 C_0 - (\beta_1 - 1)C_w \theta \kappa_2}{C_w \theta (\beta_1 - 1 - \beta_2)}$	$\frac{\beta_1(C_0 - C_w \theta \kappa_2)}{\beta_1 - 1 - \beta_2}$
Retailer-Stackelberg	$\frac{\beta_1}{\beta_1 - 1} \overline{C_p}$	$\frac{\beta_2(\overline{C_p} - C_0) - C_w \theta \kappa_2}{C_w \theta (1 + \beta_2)}$	$\overline{C_p}$
Vertical-Nash	$\frac{\beta_1}{\beta_1 - 1} \overline{C_p}$	$\frac{\beta_2(\overline{C_p} - C_0) - C_w \theta \kappa_2}{C_w \theta (1 + \beta_2)}$	$\overline{C_p}$

Theorem 1: The equilibrium points of three scenarios  $(P_1^*, W_1^*, C_{P1}^*)$ ,  $(P_2^*, W_2^*, C_{P2}^*)$  and  $(P_3^*, W_3^*, C_{P3}^*)$  have the following relationship of size:

$$P_1^* \leq P_2^* = P_3^*, W_1^* \leq W_2^* = W_3^*, C_{P1}^* \leq C_{P2}^* = C_{P3}^*$$

Proof: According to  $\underline{C_p} \leq C_p^* \leq \overline{C_p}$ , then  $C_{P1}^* = \frac{\beta_1(C_0 - C_w \theta \kappa_2)}{\beta_1 - 1 - \beta_2} \leq \overline{C_p} = C_{P2}^* = C_{P3}^*$

$$P_1^* - P_2^* = \frac{\beta_1^2(C_0 - C_w \theta \kappa_2)}{(\beta_1 - 1)(\beta_1 - 1 - \beta_2)} - \frac{\beta_1}{\beta_1 - 1} \overline{C_p} = \frac{\beta_1}{\beta_1 - 1} (C_{P1}^* - \overline{C_p}) \leq 0$$

$$\begin{aligned} W_1^* - W_2^* &= \frac{\beta_2 C_0 - (\beta_1 - 1)C_w \theta \kappa_2}{C_w \theta (\beta_1 - 1 - \beta_2)} - \frac{\beta_2(\overline{C_p} - C_0) - C_w \theta \kappa_2}{C_w \beta_2 \theta + C_w \theta} \\ &= \frac{\beta_2}{C_w \theta (\beta_1 - 1 - \beta_2)(1 + \beta_2)} [\beta_1(C_0 - C_w \theta \kappa_2) - (\beta_1 - 1 - \beta_2)\overline{C_p}] \# \\ &= \frac{\beta_2}{C_w \theta (1 + \beta_2)} (C_{P1}^* - \overline{C_p}) \\ &\leq 0 \end{aligned}$$

With reference to theorem 1, we can find that in Retailer- Stackelberg and vertical Nash the retail price, as well as the warranty period is much higher than in Manufacturer-Stackelberg. In fact, whether the consumers decide to buy the product or not and how many products to purchase, is based on the two main decision variables—price and warranty period. Which type of market is more beneficial to consumers depends on the consumers’ preference for the variables. If the consumers pursuit of low price rather than the length of the warranty period, the manufacturer dominant market is more beneficial for them which is why the price has always been the competition points for the OTC industry and food processing industry. If the consumers prefer to the warranty period, then the retailer dominant market is more suitable for them.

Theorem 2: The optimal sale profit per unit of three scenarios  $\pi_{R1}^*, \pi_{R2}^*, \pi_{R3}^*$  and the optimal wholesale profit per unit  $\pi_{M1}^*, \pi_{M2}^*, \pi_{M3}^*$  have the following relationship of size:

$$\pi_{R1}^* \leq \pi_{R2}^* = \pi_{R3}^*, \pi_{M1}^* \leq \pi_{M2}^* = \pi_{M3}^*$$

Proof:  $\pi_{R1}^* = P_1^* - C_{P1}^* = \frac{\beta_1(C_0 - C_W\theta\kappa_2)}{\beta_1 - 1 - \beta_2} (\frac{\beta_1}{\beta_1 - 1} - 1) = \frac{\beta_1(C_0 - C_W\theta\kappa_2)}{(\beta_1 - 1 - \beta_2)(\beta_1 - 1)}$

$$\pi_{R2}^* = P_2^* - C_{P2}^* = \frac{\beta_1}{\beta_1 - 1} \overline{C_P} - \overline{C_P} = \frac{1}{\beta_1 - 1} \overline{C_P}$$

then  $\pi_{R1}^* = \frac{1}{\beta_1 - 1} C_{P1}^* \leq \frac{1}{\beta_1 - 1} \overline{C_P} = \pi_{R2}^* = \pi_{R3}^*$

$$\begin{aligned} \pi_{M1}^* &= C_{P1}^* - C_0 - C_W\theta W_1^* \\ &= \frac{\beta_1(C_0 - C_W\theta\kappa_2)}{\beta_1 - 1 - \beta_2} - C_0 - C_W\theta \frac{\beta_2 C_0 - (\beta_1 - 1)C_W\theta\kappa_2}{C_W\theta(\beta_1 - 1 - \beta_2)} \\ &= \frac{C_0 - C_W\theta\kappa_2}{\beta_1 - 1 - \beta_2} \end{aligned}$$

then  $\pi_{M3}^* = \pi_{M2}^* = \frac{\overline{C_P} - C_0 + C_W\theta\kappa_2}{1 + \beta_2} \geq \frac{C_{P1}^* - C_0 + C_W\theta\kappa_2}{1 + \beta_2} = \frac{C_0 - C_W\theta\kappa_2}{\beta_1 - 1 - \beta_2} = \pi_{M1}^* \quad \#.$

According to theorem 2, compared to the manufacturer dominant market, the retailer earns more unit sales profit, as well as the manufacturer makes more wholesale unit profit in retailer dominant market or in equivalent market which is naturally more preferential for the retailer and the manufacturer. So that is the reason why many of the major home appliance retailers such as Gome, Suning, etc. occupy a large market share in appliances sales industry.

**Numerical Example**

We further study the effects of some key parameters on the optimal settings of the retail price P, warranty period W.

Table 2 Parameters used in the numerical example

NHPP-Bass model	Manufacturer	Warranty	Potential market
$\alpha_1 + \alpha_2 = 0.4$	$C_P = \$100, C_0 = \$50$	$C_W = \$30$	$\beta_1 = 1.1, \beta_2 = 0.3$
$\frac{\alpha_2}{\alpha_1} = 100$	$\theta_{max} = 0.4, \theta_{min} = 0.1$		$\kappa_1 = 5e8, \kappa_2 = 9 \text{ years}$
$L = 5 \text{ years}$	$\theta = 0.2, \mu = 0.2$		

As shown in Table 3, when the failure rate  $\theta$  increases (i.e., the overall reliability becomes lower), the manufacture need to increase the warranty period to attract the customers to guarantee its profit not reduce. However, the retailer is losing some of its profit as the increasing of  $\theta$  leading to the decreasing of the expected total sales.

Table 3 Effect of failure rate  $\theta$  on the optimal total profit

$\Theta$	$P^*$	$W^*$	$\Lambda^*(L)$	$\Pi_M^*$	$\Pi_R^*$
0.15	\$1100	2.14 years	23927	\$2423600	\$23927000
0.16	\$1100	2.44 years	23608	\$2440000	\$23608000
0.17	\$1100	2.70 years	23318	\$2458000	\$23318000

0.18	\$1100	2.94 years	23054	\$2477700	\$23054000
0.19	\$1100	3.15 years	22811	\$2498600	\$22811000
0.20	\$1100	3.34 years	22587	\$2520700	\$22587000
0.21	\$1100	3.51 years	22380	\$2543800	\$22380000
0.22	\$1100	3.66 years	22187	\$2567700	\$22187000

Table 4 shows the effects of increasing the price elasticity  $\beta_1$ . One can see that as  $\beta_1$  is increased from 1.05 to 1.25, the retail price is decreased by the retailer to promote the sales (the sales is first increased by 48.43% and then decreased as the retail price continues to go down). Moreover, the warranty period and the manufacturer's profit also appear like a single peak curve with the  $\beta_1$  as the variable. On the other hand, we can find the price elasticity has a great effect on the retailer's profit as the profit sharply goes down when  $\beta_1$  increases (the profit is reduced by 73.94%).

Table 4 Effect of price elasticity parameter  $\beta_1$  on the optimal total profit

$\beta_1$	$P^*$	$W^*$	$\Lambda^*(L)$	$\Pi_M^*$	$\Pi_R^*$
1.05	\$2100	3.3535 years	16242	\$1805100	\$32485000
1.10	\$1100	3.3359 years	22587	\$2520700	\$22587000
1.15	\$767	3.3325 years	24108	\$2692600	\$16072000
1.20	\$600	3.3345 years	23208	\$2590900	\$11604000
1.25	\$500	3.3393 years	21164	\$2360000	\$8465600

As the warranty elasticity becomes more efficient, the total sales and the total profit for both the manufacturer and the retailer increase (by 13.42%, 9.27% and 13.42%, respectively). However, the warranty period goes down slowly.

Table 5 Effect of warranty elasticity parameter  $\beta_2$  on the optimal total profit

$\beta_2$	$P^*$	$W^*$	$\Lambda^*(L)$	$\Pi_M^*$	$\Pi_R^*$
0.28	\$1100	3.6333 years	21490	\$2434800	\$21490000
0.29	\$1100	3.4835 years	22029	\$2477000	\$22029000
0.30	\$1100	3.3359 years	22587	\$2520700	\$22587000
0.31	\$1100	3.1904 years	23163	\$2565800	\$23163000
0.32	\$1100	3.0471 years	23759	\$2612300	\$23759000
0.33	\$1100	2.9058 years	24375	\$2660400	\$24375000

## Conclusion

In the paper, a price and warranty period decision problem in a decentralized supply-chain is provided. Aiming at profit maximization, from the perspectives of the consumer, the manufacturer and the retailer, respectively, we analyze three scenarios: Manufacturer-Stackelberg, retailer-Stackelberg, vertical Nash classified by the different influence on the market between the manufacturer and the retailer. In section 2, the sales over time can be characterized by a stochastic Bass model in the form of a non-homogeneous Poisson process (NHPP-Bass model) and the

production system is a make-to-order type of system. After the comparison and analysis in section 3, we can conclude that: 1) If the consumers consider more about the price, then the manufacturer dominant market is more beneficial for them; 2) If the consumers care more about the warranty period, it means that the retailer dominant market and the equivalent market are more suitable for them; 3) From the perspective of the retailer and the manufacturer, the equivalent market is more profitable. Besides, we find that the wholesale price  $C_P$  builds a bridge between the manufacturer and the retailer, as it affects the strategy of the retail price  $P$  and the warranty period  $W$ . However, as we assume in our model that the wholesale price is not directly affecting the orders; it is hard to define appropriate constraints for the decision which becomes a limitation of our model.

In the numerical examples section, we use the optimization toolbox in MATLAB to get the optimal solutions to our problem. Our analysis of the key parameters affecting the total profit leads to the following insight. First, when the product reliability becomes lower, the optimal total sales and the profit of retailer decrease, but the optimal warranty period and the profit of manufacturer may not necessarily decrease. Second, the optimal retail price is mainly determined by the price elasticity and the fixed wholesale price according to theorem 1. So the retail price and the profit of retailer sharply go down when  $\beta_1$  increases, besides the warranty period remains slightly fluctuating. Lastly, as the warranty period elasticity increases, the optimal warranty period decreases while the total sales and the profit increase.

### Acknowledgement

This work was supported by the Hunan Provincial Innovation Foundation For Postgraduate under Grant no. CX2013B024.

### Reference

- [1] Wei Xie , Haitao Liao & Xiaoyan Zhu. IIE Transactions, 46(2014), 87-105.
- [2] Choi S C. Marketing Science, 10(1991) 271-296.
- [3] Adler P S, Clark K B. Management Science, 37 (1991) 267-281.
- [4] Bass F M, Krishnan T V, Jain D C. Marketing science, 13(1994) 203-223.
- [5] Huang H Z, Liu Z J, Murthy D N P. Iie Transactions, 39 (2007) 819-827.
- [6] Kamakura W A, Mittal V, De Rosa F, et al. Marketing science, 21(2002) 294-317.
- [7] Loerch A G. Incorporation of Learning Curve Costs in Acquisition Strategy Optimization[R]. army concepts analysis agency bethesda MD, 1995.
- [8] Huang H Z, Liu Z J, Murthy D N P. Optimal reliability, warranty and price for new products[J]. Iie Transactions, 39(2007) 819-827.
- [9] Murthy D N P, Djamaludin I. New product warranty: A literature review[J]. International Journal of Production Economics, 79(2002) 231-260.
- [10] Niu S C. A piecewise-diffusion model of new-product demands[J]. Operations research, 54(2006) 678-695.
- [11] Blischke W R, Murthy D.N.P. Warranty cost analysis. 1994.
- [12] Chen F, Federgruen A, Zheng Y S. Management Science, 47(2001) 693-708.
- [13] Bass, F.M. A new product growth model for consumer durables. Management Science, 15(1969) 215-227.
- [14] Mettas, A. Reliability allocation and optimization for complex systems, in Proceedings of the Annual Reliability and Maintainability Symposium, IEEE Press, Piscataway, NJ, (2000) 216-221.
- [15] Murthy, D.N.P. Optimal reliability choice in product design. Engineering Optimization, 15(1990) 281-294.
- [16] Elsayed, E.A.. Reliability Engineering, Addison Wesley, New York, NY, 1996.