Issues on the Stability of Fuzzy Cognitive Maps and Rule-Based Fuzzy Cognitive Maps ♠

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Abstract

This paper focus several stability issues regarding the modeling of the dynamics of qualitative real world systems, and the ability of Fuzzy Cognitive Maps and Rule-Based Fuzzy Cognitive Maps to provide a faithful modeling in what concerns the stability properties of those systems. It also introduces the concept of Intrinsic Stability as a necessary property of qualitative system dynamics modeling tools.

1. Introduction

Decision makers, whether they are social scientists, politicians or economists, usually face serious difficulties when approaching significant, real-world dynamic systems. Such systems are composed of a number of dynamic qualitative concepts interrelated in complex ways, usually including feedback links that propagate influences in complicated chains. Axelrod [1] work on Cognitive Maps (CMs) introduced a way to represent real-world qualitative dynamic systems, and several methods and tools have been developed to analyze the structure of CMs. However, complete, efficient and practical mechanisms to analyze and predict the evolution of data in CMs were not available for years due to several reasons. System Dynamics tools like those developed by J.W.Forrester [2] could have provided the solution, but since in CMs numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [4][5][6], were developed as a qualitative alternative approach to system dynamics. However, FCM are Causal Maps (a subset of Cognitive Maps that only allow basic symmetric and monotonic causal relations)[7], and in most applications, a FCM is indeed a man-trained Neural Network that is not Fuzzy in a traditional sense and does not explore usual Fuzzy capabilities. They do not share the properties of other fuzzy systems and the causal maps end up being quantitative matrixes without any qualitative knowledge. To avoid the above-mentioned limitations of existing approaches, Rule Based Fuzzy Cognitive Maps (RB-FCM) were introduced in [7][8][9][10][11][12] and are being developed as a tool that models and simulates real world qualitative system dynamics. RB-FCM are essentially fuzzy rule based systems where we added fuzzy mechanisms to deal with feedback, and defined different kinds of timing mechanisms and different relations that can cope with the complexity and diversity of the dynamic qualitative systems we are trying to model. Among new contributions brought by RB-FCM, there is a new fuzzy operation - the Fuzzy Carry Accumulation -, which is essential to model the mechanisms of qualitative causal relations (FCR – Fuzzy Causal Relations) while maintaining the simplicity and versatility of FCM.

When one model the dynamics of a certain system, especially when the system involves lots of non-linear feedback links, the problem of system stability analysis becomes very important, and usually is the first thing one tries to find after finishing the model. Therefore, one

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cannot be surprised that the existence of an analytic method that can give a faster answer to the problem of system stability analysis in FCM – similar to the methods one can find in quantitative analytic models - has been one of the grails in FCM research.

However, there are several more important issues that rarely are mentioned in stability analysis. This is due to the fact, that most dynamic simulation modeling methods involve quantitative models of quantitative systems using analytic methods that can provide rather faithful models. But that's not that case of FCM or RB-FCM - one must not forget, that they were developed due to the inherent difficulties of analytic modeling of qualitative systems. Therefore, the problem of stability analysis must regress a few steps before we try to find a fast method to find stability in FCM or RB-FCM: more important than knowing if a model is or not stable, is the guarantee that the modeling process is faithful enough not to change the stability of the system being modeled (i.e. the model must capture the essence of the qualitative system without oversimplifying its complexity); and more important than the modeling process, is the necessary guarantee that the modeling tool is able to maintain the stability properties of the system we want model. Therefore, one can distinguish three important issues concerning the stability of qualitative dynamic systems:

- The stability of the real world system
- The stability of the model of the system
- Possible stability changes caused by the modeling tool

2. Stability of the Real-World System

As mentioned above, the problem of stability analysis of a dynamic system is always present when one models such a system. In quantitative systems, it is possible to find a precise model of that system, but as a result, we might end up with heavy differential equations that can be hard to work up with in terms of simulation. Therefore, analytic methods to speed up stability answers - like Liapounov function analysis – are incredibly useful and widespread. Kosko [5] approached the problem of finding an analytic method to provide a faster answer to the problem of system stability analysis in FCM using Liapounov functions, and concluded that despite of being adapted to the study of stability in Feedback Standard Additive Models (SAMs – which share some of FCM characteristics), one cannot extend such conclusions to FCM (according to Kosko, due to the large number of feedback links involved in FCM [5][6]).

The FCM shown in Figure 1 can easily show this fact: Its Eigenvalues are 1.255, -0.084+1.408i, -0.084-1.408i, 0.543+0.325i and 0.543-0.325i [13]. Given these values, the system should be unstable, but for s_0 = (0,1,1,0,0), the system converges to a single equilibrium state s_4 = (1,0,1,1,1).

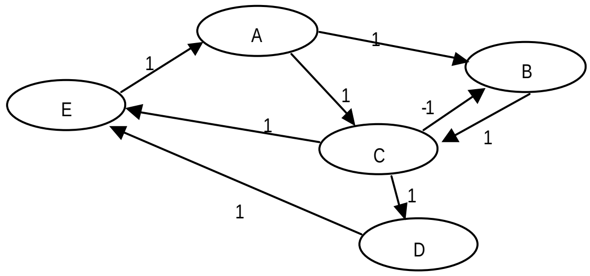

Figure 1 Theoretically unstable FCM

Up to date, to our knowledge, no other simple, fast and reliable method to predict the stability of a FCM, was found, and stability analysis of FCM must rely on the simulation of several scenarios. The same applies to RB-FCM: the fact of being qualitative semantic based and highly non-linear models prevents the application of variants of known stability analysis methods. However, the search for such a method should not be such an important issue in CM: since the primary goal of a RB-FCM is to model and simulate the evolution of real-world qualitative system scenarios, one only has to input the initial state of the scenarios we are analyzing and simulate their evolution to find whether the system converges or not (i.e., checking stability issues for those initial conditions we want to simulate). Of course it would be important to immediately know if that result was a particular case of stability, but one must not forget that Liapounov functions and similar methods are fast and easy but not that "informative", and their use is widespread because simulation of analytic dynamic models can be quite heavy and time consuming, which is not the case of FCM and RB-FCM. Therefore, the importance of the search for a fast method to check stability of FCM is essentially a theoretic and intriguing issue.

However, as it was said before, there are a few more important issues related with the stability of FCM and RB-FCM…

3. Model Stability

As we have seen, the main goal of stability (or other) analysis is the knowledge of the behavior of the real-world system. However, we always end up analyzing the stability of a model of that real-world system. Therefore, we should try to guarantee that the model is as faithful to reality as it could be. When one models a quantitative system using mathematical analytic methods, one has that guarantee, but since we are dealing with qualitative systems, the issue of model faithfulness should become our first priority.

3.1 Stability of an FCM Model

Regarding model faithfulness, FCM use a simple weight and a non-linear function to model complex relations between a pair of complex real world concepts [7]. These mechanisms are similar to those of a simple neural network (NN). One might think that, since a NN is a universal approximator, this should not be a problem. However, NN are indeed universal approximators, but a single connection between 2 neurons is not one (it cannot even simulate an exclusive-OR) – the universal approximator property comes from joining several neurons and several connections. Since an FCM simulates real world concepts as simple neurons, and relations as simple connections between 2 neurons, the modeling of relations in FCM is seriously injured. So when using an FCM representing a complex social, politic or economic system, we are always analyzing the stability of a model which has a high probability of being a very distant cousin of the real world system. This could not be a problem if we were trying to predict a short-term evolution of the system. But, since stability analysis usually involves lots of iterations and a long-term evolution, the feedback loops aggravate each minor difference between the model and the real world system as the number of iterations increases, and any long-term conclusion ends up being useless most of the time.

Furthermore, most works using FCM tend to oversimplify FCM capabilities by reducing Concepts to bivalent or trivalent values, stretching even more the gap between reality and model, and aggravating the usefulness of any stability analysis. This can easily be seen with a simple FCM like the one pictured in Figure 2 (adapted from [13]). When one use bivalent, trivalent or continuous concepts, one obtain different long-term results that have entirely different meanings, therefore providing different conclusions (see Table 1). The bivalent and trivalent models converge to different limit cycles, and the continuous model does not converge at all – all one can identify is a chaotic cycle attractor [6], but it bears no resemblance to the other models. Notice that all solutions make sense, but they are so different that one cannot know which (or if any) will happen given the same initial state (and the same real world system would certainly not evolve to different states given the same conditions). One can also notice that different initial states lead to different limit cycles. These differences are due to the different model detail, and aggravated if one does not limit the weight values to 1 or –1. With such different results due to simplifications in the modeling process, it is obvious that one cannot pretend to conclude anything over such a model. The exceptions could only be those models where the real-world system is very simple (which excludes most systems involving social and economic issues) and does not show any traces of chaotic behavior (where the oversimplification would cause even worst results).

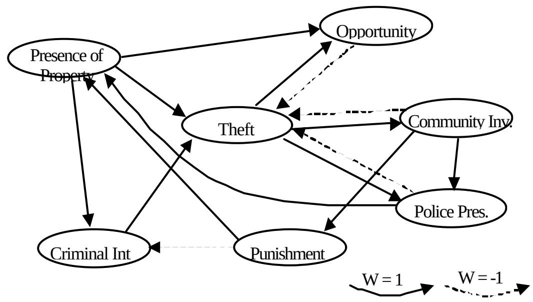

Figure 2 – Crime and Punishment Causal Map

One might ask why do people tend to reduce FCM capabilities if it naturally reduces the model faithfulness? And the answer seems to be that bivalent or trivalent FCM stability is easier to analyze since they usually converge to a single state or cycle of states instead of a chaotic attractor or no convergence at all. By limiting concept values, one reduces the number of possible states by a huge degree, and therefore stability becomes much more probable. However, what's the usefulness of a simpler analysis if one cannot trust its results?

3.2 Stability of an RB-FCM Model

When one uses RB-FCM to model a real-world system, one knows that since Fuzzy Rule based systems are universal approximators [14], any qualitative relation can be faithfully modeled as long as we are not limited by sheer model complexity (i.e., the number of linguistic variables in each Concept). However that's not always the case, and RB-FCM concepts with more than 11 or 13 membership functions can become a serious burden regarding knowledge modeling[1]. Therefore RB-FCM capability to model a system is not limited by its capability to model qualitative Relations, but by the number of membership functions (mbf) in each Concept, and we must check how the number of linguistic variables affect the stability of the model.

We modeled the system in Figure 2 using an RB-FCM (ignoring RB-FCM capabilities regarding relationship modeling to provide a fairer comparison): we tested the

[1] Although 11 mbf is a huge number for other rule-based fuzzy systems, is a perfectly acceptable number in RB-FCM due to the characteristics and simplicity of FCR.

system for concepts with 7, 11 and 15 linguistic variables, i.e., with 3, 5 and 7 linguistic terms for positive and negative variation and 1 indicating the absence of variation. All relations we used were FCR that simulate weights of 1 and –1 (i.e., the rule bases state that even a minor change in the antecedent will cause the largest possible change in the consequent).

The obtained results are rather interesting (see table 2): first, although the concepts can assume a continuous value, the system converges to a cycle of states instead of the chaotic attractor we obtain in the FCM (in the present example, the cycle consists of 4 states); second, one notice that increasing the number of mbf above 11 does not change the conclusions one obtains from the system, since the limit cycles are similar in terms of semantics; third, one notice that a diminution of the mbf number affects the limit cycle (as one would expect), but the change is less dramatic than expected, since of the 28 values defining the convergence cycle, only 2 were not expected when compared to the other experiences (in the case of FCM, bivalent, trivalent and continuous cases are completely unrelated). One can conclude that the diminution of the number of linguistic variables affects modeling capabilities of the RB-FCM, but it's not that important in stability and convergence properties except in what concerns the "capture" of a chaotic attractor.

Several applications to different cases and systems [11], have shown that whenever a continuous FCM would converge to a chaotic attractor, a RB-FCM turns it into a limit cycle. This is due to the nature and restrictions of the membership functions used in the Fuzzy Causal Relations and the FCA [9][11], especially due to the fact that each linguistic variable must have a non-null interval where μ=1, and that $\mu(c)$=1, where c is the linguistic variable centroid, providing a small attractor for every linguistic variable. Experience also shows that systems that converge to a single state or cycle of states in continuous FCM also converge in RB-FCM, and chaotic systems remain chaotic while modeled by an RB-FCM, i.e., RB-FCM models do not limit or impose the stability of the real world systems in those cases, but in the case of pseudo-chaotic systems, stability analysis using RB-FCM is made easy because although being allowed to assume every value in its universe of discourse, each concept behaves like having n-levels, where n is the number of its linguistic variables.

One also noticed that in the shown example, the limit cycle is always the same regardless of the initial state (which does not happen with the FCM), but this is a particular property of this model. Other RB-FCM can be highly dependent of the initial state, converging to a single state, cycle of states or not converging at all like one can see on a qualitative model of macroeconomics in [11].

Therefore, with RB-FCM it is possible to easily conclude over the stability of a certain model, without the need to overly-manipulate the stability of the real world system, and providing a better way to simulate long-term system evolution and stability.

Notice that this section basically compares RB-FCM and FCM regarding model stability issues and not modeling issues, which have been previously addressed in [7] and [11].

4. Modeling Tool Stability

In the previous sections we introduced the issues of system stability and model stability. However, one even more important issue remains: Is the modeling tool able to simulate the model without affecting its stability properties? Is it possible that FCM or RB-FCM architecture can be responsible to change long-term model behavior, turning a theoretically unstable model into a stable one, or vice-versa?

As we have seen, both FCM and RB-FCM mechanisms can change long-term behavior of a model: theoretically unstable FCM often converge, and chaotic attractors can turn into limit cycles in RB-FCM. However, to properly answer these questions, and their implications, one can introduce the property of **Intrinsic Stability**:

A qualitative dynamic systems modeling tool is intrinsically stable if and only if the following system maintains its initial state for an infinite number of iterations

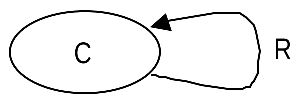

Figure 3 – Intrinsic Stability

In this system, C is a given concept, and R is a relation that causes a concept to follow the changes in its predecessor. For instance, given concepts A and B, if A "decreases much", B "decreases much", if A "increase just a few", B "increase just a few", if A rises 10ºC, B rises 10ºC, and so on. This relation is chosen because it maintains the dynamics imposed on the system. A relation that would describe an amplification would cause C to reach its maximum variation after a given number of iterations; a relation that would describe an attenuation would cause C to stop moving after a given number of iterations, therefore changing the initial dynamics. When we consider a CM, <u>if a simulating tool cannot provide such a relationship</u>, then it is not possible to guarantee that the simulation of the model maintains the stability of the system, because even if the original system maintains its dynamics, the modeling tool will always change it and eventually C will converge to its minimum or maximum value.

In a RB-FCM this relation can be easily described by a Macro FCR+ [10][11], which is a Fuzzy rule base where in each rule the antecedent and the consequent address the same linguistic variable. As long as we define the linguistic terms as membership functions abiding the restrictions imposed in [11] and referred in 3.2, such a system maintains its initial value. So RB-FCM have Intrinsic Stability.

However, in a FCM is not possible to model a relation where $C_{i+1} = C_i = S(w_C \cdot C_i)$, because there is not a single weight w that allows C to maintain its value for every possible initial value. On a FCM relation, if w were larger than 0.5, the relation should indicate that a variation in the antecedent should cause a larger variation in the consequent, and a smaller one should indicate a smaller variation. Therefore, w=0.5 should indicate a relation similar to the above mentioned a "following" relation. However, due to the non-linear function S used to compute the value of a concept this does not happen except for a single value v (which is usually 0 if $C\in[-1,1]$, and 0.5 if $C\in[0,1]$). For every other values, $|C_{i+1}|=|C_i+\partial|$, where ∂ can never be 0 (except if S is a linear function). It's easy to prove this issue for the most common S functions, since S' can never be constant except for k=0 (in which case C would always be zero):

$$S(y) = \frac{2}{1+e^{-ky}} - 1 \;,\; S'(y) = 2ke^{-ky}\left(1+e^{-ky}\right)^2$$

One must notice that if S becomes a linear function, FCM become completely linear and therefore unable to model non-linear systems.

Due to this structural limitation, FCM are not intrinsically stable, and therefore concepts in FCM tend to saturate easily than they should when as soon as they assume a value different from v. This is a sufficient motive to cause theoretically unstable FCM to converge, since saturation of Concepts limits the possible state space (like when one use bivalent concepts) and the result is often a limit cycle consisting of saturated concepts instead of a range of free-values.

5. Conclusions

This paper analyses FCM and RB-FCM in what concerns their capability to model qualitative systems without altering the stability of the original real-world system. In order to provide better models of real-world qualitative systems, it is necessary to guarantee that not only must the model provide mechanisms to model qualitative relations, but also that the model must not sacrifice quality to provide stability.

The paper also introduces the concept of "Intrinsic Stability", which regards the ability of a modeling method to maintain the original system stability, and shows that FCM cannot provide this guarantee in certain systems, causing theoretically unstable FCM to converge.

Despite the lack of analytic methods to predict and analyze stability, RB-FCM do not alter the stability properties of the real-world systems it models (since they have the intrinsic stability property), and are much more adequate to produce a model that is closer and faithful to those qualitative real world systems.

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Table 1

In each column:

Iteration	Opportunity	Community	Police Presence	Punishment	Criminal Intent	Presence of Property	Theft

Bivalent FCM	Trivalent FCM	Continuous FCM
t_{00}=(0,0,1,0,0,1,0)	t_{0}=(0,0,1,0,0,1,0)	t_{00}=(0.00,0.00,1.00,0.00,0.00,1.00,0.00)
t_{01}=(1,0,0,0,1,1,0)	…	t_{01}=(1.00,0.50,0.50,0.50,1.00,1.00,0.50)
t_{02}=(1,0,0,0,1,0,1)	…	t_{02}=(0.92,0.92,1.00,0.92,0.92,1.00,1.00)
t_{03}=(0,1,1,0,0,0,1)	…	t_{03}=(0.49,1.00,1.00,1.00,0.56,1.00,1.00)
t_{04}=(0,1,1,1,0,1,0)	t_{4}=(-1,0,1,0,0,1, -1)	t_{04}=(0.51,1.00,1.00,1.00,0.51,1.00,0.60)
t_{05}=(1,0,1,1,0,1,0)	t_{5}=(1,-1,-1,0,1,1,-1)	t_{05}=(0.87,0.95,1.00,1.00,0.51,1.00,0.54)
t_{06}**=(1,0,0,0,0,0,1)**	t_{6}**=(1,-1,-1,-1,1,-1,1)**	t_{06}=(0.90,0.94,1.00,1.00,0.51,1.00,0.90)
t_{07}=(0,1,1,0,1,0,1)	t_{7}**=(-1,0,1,-1,0,-1,1)**	t_{07}=(0.63,1.00,1.00,1.00,0.51,1.00,0.92)
t_{08}=(0,1,1,1,0,1,0)	t_{8}**=(-1,0,1,0,0,0, -1)**	t_{08}=(0.60,1.00,1.00,1.00,0.51,1.00,0.67)
t_{09}=(1,0,1,1,0,1,0)	t_{9}=(1,-1,1,0,0,1,-1)	t_{09}=(0.84,0.97,1.00,1.00,0.51,1.00,0.65)
t_{10}=(1,0,0,0,0,0,1)		t_{10}=(0.85,0.96,1.00,1.00,0.51,1.00,0.87)
t_{11}=(0,1,1,0,1,0,1)	$t_{0'}$=(-1,-1,1,-1,-1,1,-1)	t_{11}=(0.66,0.99,1.00,1.00,0.51,1.00,0.88)
	…	t_{12}=(0.65,0.99,1.00,1.00,0.51,1.00,0.71)
	$t_{3'}$=(-1,0,1,-1,0,-1,1)	t_{13}=(0.81,0.97,1.00,0.99,0.51,1.00,0.70)
	$t_{4'}$=(-1,0,1,0,0,0,-1)	t_{14}=(0.82,0.97,1.00,1.00,0.51,1.00,0.84)
	$t_{5'}$=(1,-1,-1,0,0,1,-1)	t

		$_{15}$=(0.68,1.00,1.00,1.00,0.51,1.00,0.86)
	t $_{6'}$=(-1,0,1,-1,0,-1,1)	t $_{16}$=(0.67,1.00,1.00,1.00,0.51,1.00,0.74)

Table 2

RB-FCM (7 mbf)	**RB-FCM (11 mbf)**	**RB-FCM (15 mbf)**
t0 -99 99 -99 56 23 12 0	t0 -99 99 -99 56 23 12 0	t0 -99 99 -99 56 23 12 0
t1 79 0 79 79 0 0 20	t1 83 0 83 83 0 0 20	t1 90 0 90 90 0 0 19
t2 -79 79 79 0 -79 96 0	t2 -83 83 83 0 -83 97 0	t2 -90 90 90 0 -90 98 0
t3 79 0 79 79 79 79 -64	t3 83 0 83 83 83 83 -63	t3 90 0 90 90 90 90 -61
t4 96 -79 -79 0 0 96 54	t4 97 -83 -83 0 0 97 53	t4 98 -90 -90 0 0 98 51
t5 0 79 7 -79 79 -79 99	t5 0 83 0 -83 83 -83 99	t5 0 90 0 -90 90 -90 99
t6 -96 79 96 79 0 0 -55	t6 -97 83 97 83 0 -83 -37	t6 -98 90 98 90 0 -90 -35
t7 79 -79 0 79 -79 96 -99	t7 0 -83 0 83 -97 97 -99	t7 0 -90 0 90 -98 98 -99
t8 96 -79 -96 -79 0 79 54	t8 97 -83 -97 -83 0 83 36	t8 98 -90 -98 -90 0 90 35
t9 0 79 7 -79 96 -96 99	**t9 0 83 0 -83 97 -97 99**	**t9 0 90 0 -90 98 -98 99**
t10 -96 79 96 79 0 0 -55	**t10 -97 83 97 83 0 -83 -37**	**t10 -98 90 98 90 0 -90 -35**
t11 79 -79 0 79 -79 96 -99	**t11 0 -83 0 83 -97 97 -99**	**t11 0 -90 0 90 -98 98 -99**
t12 96 -79 -96 -79 0 79 54	**t12 97 -83 -97 -83 0 83 36**	**t12 98 -90 -98 -90 0 90 35**
t13 0 79 7 -79 96 -96 99	t13 0 83 0 -83 97 -97 99	t13 0 90 0 -90 98 -98 99