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**This document concerns the 2nd edition, Corr. 2nd printing, 2004**

**For the 1st printing of the 2nd ed., 2003:**

**<http://ams.jrc.it/soille/book2nd/book2nd.html>**

### Abstract

Following the success of the first edition (Soille, 1999), recent developments in the field of morphological image analysis called for an extended second edition (Soille, 2003), reprinted with corrections in 2004 (Soille, 2004). The text has been fully revised with the goal of improving its clarity while introducing new concepts of interest to real image analysis applications.

The book has grown by 25 percent leading to a total of 391 pages (including 39 additional figures, 3 new tables, 93 extra bibliographical references, as well as 1 supplementary index). This increase has been distributed more or less evenly across all chapters. More precisely, beyond a new chapter devoted to texture analysis, main additions include: processing of multichannel images, ordering relations on image partitions, connected operators and levellings, homotopy for grey tone images, translation-invariant implementations of erosions and dilations by line segments, reinforced emphasis on rank-based morphological operators, grey tone hit-or-miss, order independent homotopic thinnings and anchored skeletons, self-dual geodesic transformation and reconstruction, area based self-dual filters, anti-centre, and new scientific and industrial applications.

This self-contained book will be of value to engineers, scientists, and other practitioners interested in the analysis and processing of digital images.

Readers more familiar with the German language may still consider the original German edition (Soille, 1998).

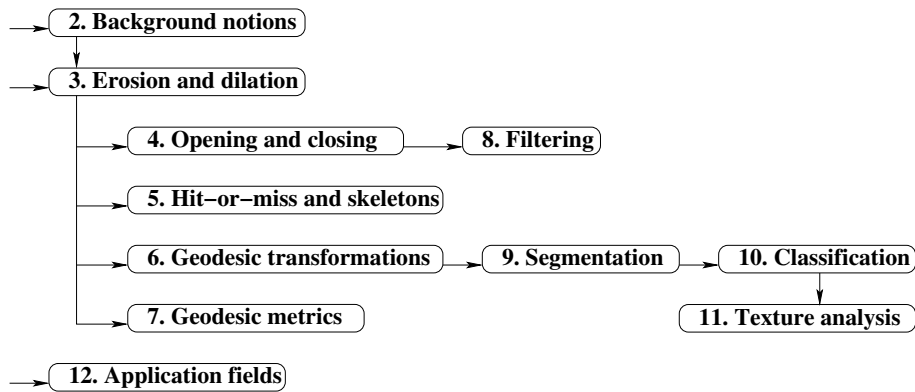


Figure 1: Some possible reading paths referring to the book chapters.

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## 1 Scope

*Mathematical morphology* (MM) or simply *morphology* can be defined as a theory for the analysis of spatial structures. It is called morphology because it aims at analysing the shape and form of objects. MM is not only a *theory*, but also a powerful image analysis *technique*.

The purpose of this book is to provide the reader with a detailed presentation of the principles and applications of morphological image analysis. This is achieved through a step by step process starting from the basic morphological operators and pursued until the most recent advances which have proven their practical usefulness. All concepts are illustrated with real applications to help the reader acquiring the expert knowledge necessary for building the chain of operators to resolve his/her own image analysis problem. The emphasis is therefore put on the technique rather than the theory underlying MM.

This volume will be valuable to all engineers, scientists, students, and practitioners dealing with the analysis/processing of digital images.

## 2 Book organisation

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A PDF file containing the table of contents can be downloaded [here](#).

### 3 Bibliography file for all cited publications

The BibTeX file containing all entries of work cited in the book is searchable at the *Collection of Computer Science Bibliographies*: Bibliography on Morphological Image Analysis. A HTML version with numerous hyperlinks is available [here](#).

### 4 Additional references

An on-line demonstration of the automatic recognition of plants in a crop field (see p. 357) is available at: <http://ams.jrc.it/soille/ivc2000/>.

### 5 Errata list

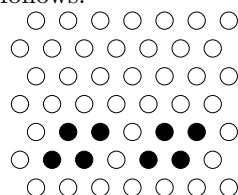
[For the errata list regarding the 1st printing, see <http://ams.jrc.it/soille/book2nd/node6.html>]

- Page 148, table 5.1, the two first configurations for the connectivity number in the 4-connected case can be combined into a single configuration since the lower right pixel is off in the first and on in the second:

$$\mathcal{N}\left(\begin{array}{cc} \blacksquare & \blacksquare \\ \blacksquare & \square \end{array}\right) + \mathcal{N}\left(\begin{array}{cc} \blacksquare & \blacksquare \\ \square & \blacksquare \end{array}\right) = \mathcal{N}\left(\begin{array}{cc} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{array}\right)$$

Simplification reported by students of Renato Keshet, Thu Jul 29 2004.

- Page 187, figure 6.5c, the 4th pixel on the 4th row should be displayed as a white circle since it does not belong to the geodesic erosion. That is, the correct figure is as follows:





## References

- P. Soille. *Morphologische Bildverarbeitung*. Springer-Verlag, Berlin, Heidelberg, 1998. ISBN 3-540-64323-0. URL <http://ams.jrc.it/soille/buch1st>.
- P. Soille. *Morphological Image Analysis*. Springer-Verlag, Berlin New York, 1999. URL <http://ams.jrc.it/soille/book1st>.
- P. Soille. *Morphological Image Analysis: Principles and Applications*. Springer-Verlag, Berlin Heidelberg New York, 2nd edition, 2003. See also <http://ams.jrc.it/soille/book2ndprint>.
- P. Soille. *Morphological Image Analysis: Principles and Applications*. Springer-Verlag, Berlin Heidelberg New York, corrected 2nd printing of the 2nd edition, 2004. See also <http://ams.jrc.it/soille/book2ndprint>.