*Applied Mechanics and Materials*
*ISSN: 1662-7482, Vol. 626, pp 202-207*
*doi:10.4028/www.scientific.net/AMM.626.202*

*Submitted: 2014-06-25*
*Accepted: 2014-07-03*
*Online: 2014-08-28*

# Reactive power pricing by Power Flow tracing method

V.Mahesh[1,a*], N.Kamaraj [2,b]

[1,2] Thiagarajar College of Engineering, Madurai, India

[a] maheshv@tce.edu, [b] nkeee@tce.edu

**Keywords**: Reactive power cost, Power tracing, superposition principle, Quadratic cost function.

**Abstract**: Power flow tracing based reactive power pricing scheme is presented in this paper. A method is suggested to solve the problem of bidirectional flow of reactive power. The reactive power allocated by each generator to all the load buses is found and the corresponding cost to be paid by the load to generator is calculated. Super position principle based method is used to calculate the allocation of reactive power by the generators to the loads. Costs allocated for generators are found using quadratic cost function. The method is implemented on IEEE standard bus system and the results are found satisfactory.

**1 Introduction**: As power system is restructured, it is necessary to have separate pricing scheme for each component. Reactive power is essential for voltage control and system security. In the monopolistic structure, reactive power is not considered for pricing. In the case of a deregulated market, a direct pricing method for reactive power is necessary for competitive production. The existing reactive power pricing schemes are–spot pricing based on OPF formulation and power flow and graph theory based pricing [1]. Total cost cannot be recovered by OPF based method and the other methods generally use proportional sharing principle. In these types of methods, the amount of reactive power produced by each generator for each individual load is calculated and the generators are paid by suitable pricing scheme from the charges obtained from loads. In deregulated market, the responsibility of ISO is to monitor the balance between reactive power production and load. Also ISO has to fix the price to be paid by the loads to generators for reactive power production [2]. If any load takes more reactive power, it has to be penalized and the reactive power production of generator has to be considered as ancillary service for pricing purpose.

Reactive power cost has two different components; the capital cost and the production cost. The capital cost is very high and the production cost is generally very low and hence capital cost plays an important role in reactive power cost. In competitive market, opportunity cost should be considered for a large extent. Due to capacity limits, active power production of generators as to be reduced. Opportunity cost is the charge paid for generators for this loss in active power revenue [3].

In this paper, a quadratic reactive cost function is adopted for the generator which is derived from the active power quadratic cost function of the generators using the generator power factor. The problem of bidirectional flow in reactive power pricing is discussed in section 2. The proposed pricing structure is discussed in section 3. It is followed by the results obtained by applying the proposed method on IEEE 14 bus system in section 4.

## 2 Reactive power flow tracing:

In Power tracing, power flow result is obtained first. Based on it, power flow path is traced from either generator or load. The contribution of active power by any generator to any load can be calculated easily. But bidirectional flow of reactive power makes the calculation complex [4]. The discontinuation of reactive power flow path at such branches makes power tracing a difficult problem. The problem is discussed in 6-bus system shown in Fig. 1, which shows the active and reactive power flow directions through different branches. Unlike real power, reactive power flows either from both ends towards centre or towards one branch from two lines.

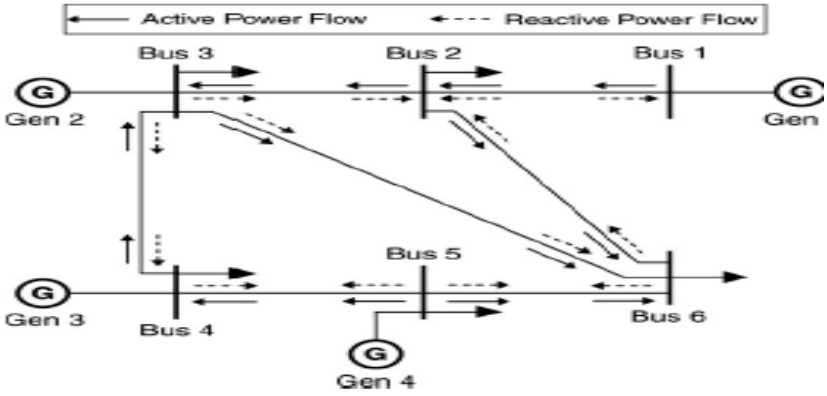


**Fig 1**. Active and reactive power flow of 6-bus test system.

Circuit theory based solution for this problem is proposed in this paper. Any transmission line is considered to have series inductive reactance and shunt line charging admittance [5]. Inductive reactance causes line losses and the line charging admittance acts as reactive power generator. The inward flow or outward flow of reactive power from any branch will be determined by the effect of reactive power loss and generation. Let $Q_{loss}$ represents the reactive power loss due to line reactance and $Q_C$ represents the power supplied by line charging capacitance. Let the reactive power flowing into any branch i-j is $Q_{in}$ and the amount flowing out of the branch is $Q_{out}$. $Q_C$ for any branch can be calculated as $\mathrm{Qci}-\mathrm{j}=V_i^2\,Yc/2_+V_i^2\,Yc/2$ and
$Qlossi-j=Im(I_{i-j}^2 Z_{i-j})$.

- If $Q_{loss}= Q_C$ , then $Q_{in}= Q_{out}$; the branch will behave like a lossless line.
- If $Q_{loss}> Q_C$, $Q_{in}> Q_{out}$ and the branch will show a net reactive power loss.
- If reactive power flowing into the branch $Q_{in}$ satisfies the relation $Q_{in}<( Q_{loss}- Q_C)$, then reactive power will flow into the branch from both the ends.
- If $Q_{loss}< Q_C$, $Q_{in}< Q_{out}$ and the branch will show a net reactive power gain. It will lead to a case where reactive power will flow out of both the ends.

Thus, if these two reactive power components $Q_{loss}$ and $Q_C$ are removed from the branch, the problem of bidirectional flow is solved. This is obtained by removing reactive power due to line charging capacitance as generation to the end buses and the reactive power loss due to the line reactance as load to the terminal buses. Finally a lossless network is obtained.
For branch i–j if $Q_C$ and Qloss are the reactive power supplied by this line charging capacitance and the reactive power loss due to line reactance respectively then we can write

If $(Q_{gi}+\frac{Qc}{2}-\frac{Qloss}{2}-Q_{li})>0$ , then

$$Q_{Gi}=(Q_{gi}+\frac{Qc}{2}-\frac{Qloss}{2}-Q_{li});\ Q_{Li}=0 \quad (1)$$

where $Q_{gi}$ is the actual generation and $Q_{li}$ is the actual load connected to bus i; $Q_{Gi}$ and $Q_{Li}$ are the net generation and net load connected to bus i after transferring line reactive loss and line charging capacitance from the branch to the end buses. $Q_{Li}$ is made zero as it is assumed to be supplied by the local generations available. So, net reactive supply is reduced and net load becomes zero in this case.

Else , $Q_{Gi}$=0; $Q_{Li}$=$Q_{li}-\left(Q_{gj}+\frac{Qc}{2}-\frac{Qloss}{2}\right)$ (2)

Similarly, $Q_{Gi}$ and $Q_{Li}$ can be evaluated; where $Q_{Gi}$ and $Q_{Li}$ are the net reactive power generation and net reactive load connected to j-th bus. Once these net generations and loads are calculated for all the buses, the system becomes lossless. The branch reactive power flow will also get modified as

follows. Assuming Qi and Qj are the reactive power flows at the end buses i and j of branch i–j, modified Qi i.e. Qinew and modified Qj i.e. Qjnew can be calculated for both the above cases represented by Eqs. (1) and (2) as

$$Q_{inew} = Q_{i-}\frac{Qloss}{2} + \frac{Qc}{2} \quad \text{and}$$
$$Q_{jnew} = Q_{j-}\frac{Qloss}{2} - \frac{Qc}{2} \tag{3}$$

The problem of bidirectional flow of reactive power flow can be eliminated by the following procedure. New nodes are introduced in those branches, and its exact position as to be determined [6]. The point of the branch where the reactive power is zero is the exact location of the new node.
.
In Fig.2(a), Q1 and Q2 are the reactive powers flowing out of the branch from the two ends; then the generator will be located at a distance of l=(Q1/Q1+Q2)L fraction of the said branch from the end bus 1. Similarly, in Fig. 2c, reactive power Q1 andQ2 are flowing towards the middle of the line and a load is assumed there. The exact location of the load is calculated using the procedure explained above.

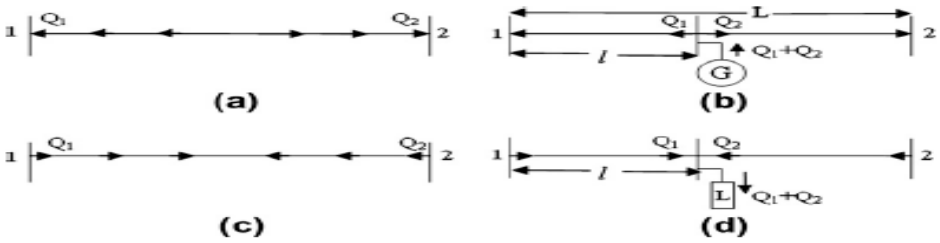


**Fig.2** Solutions for the problem of bidirectional flow of reactive power.

**3 Proposed method.**

Triangular method of generator reactive power cost given by is used in this paper. According to this pricing scheme reactive power cost also follows the same quadratic structure as active power and the cost coefficients are derived from the active power cost coefficients and the generator power factor. The reactive power cost function can be represented as

$$C(Q) = a_Q Q^2 + b_Q Q + c_Q \tag{4}$$

where $a_Q = a_p sin^2\theta$; $b_Q = b_p \sin\theta$ and $c_Q = c_p$.

$a_p, b_p, c_p$ are the active power cost coefficients and $a_Q, b_Q, c_Q$ are the reactive power cost coefficients.

The generators are assumed to have the following generation cost function

$C_1$=0.00533$P_1^2$+11.669$P_1$+213.1

$C_2$=0.00889$P_2^2$+10.333$P_2$+200

$C_3$=0.00741$P_3^2$+10.833$P_3$+240

where $P_1$,$P_2$,$P_3$ are the active power supplied by the three generators in MW at any particular instant and $C_1$ , $C_2$, $C_3$ are the cost of active power supplied by them in \$/h. The reactive power cost function for the generators determined using Eq(4) are

$C_{Q1}$=0.00004713$Q_1^2$+3.47$Q_1$+213.1

$C_{Q2}$=0.0000124$Q_2^2$+0.385$Q_2$+200

$C_{Q3}$=0.0000136$Q_3^2$+0.464$Q_3$+240 (5)

Where $Q_1$, $Q_2$, Q3 are the reactive power supplied by the three generators in MVAr and $C_{Q1}$, $C_{Q2}$, $C_{Q3}$ are the cost of reactive power in $/h [8].

Algorithm for the proposed method:

1. Power flow is executed.
2. Reactive power loss and reactive power generated due to line charging capacitance at each branch is calculated.
3. The net reactive load and net reactive generation at each bus are calculated using Eqs. (1) and (2).
4. New branch flows are evaluated using Eq.(3).
5. Determine cost coefficients for generators [9].
6. Apply superposition principle based power flow tracing method in [10] to find out the amount of reactive power supplied by a generator to a particular load
7. Calculate the amount of revenue to be paid to any source or to be charged to any load using step 5 and 6.

**4 Results and discussion**:

The proposed method has been tested in IEEE 9 bus system. Initially power flow is executed to find the line flows and losses. The result of power flow is shown in table 1. Then Qloss due to line inductance and Qc due to line charging capacitance are calculated. They are added to loads and generators respectively. The new line flows are evaluated using expression 3 and it is shown in table 2. To eliminate the presence of bi directional flow, four nodes-a, b, c, and d are introduced in the lines 4-5, 5-6, 8-2 and 8-9 respectively. In line 8-2, a factious generator is placed at a distance 5.2% of the actual branch length from the bus no.8. Similarly in all other three lines, loads are placed at distances calculated using the formula l=(Q1/Q1+Q2)L. The reactive losses allocated to loads by different buses are shown in table 4. Then superposition principle is applied to find the reactive power allocated by each generator to different loads. The results are shown in table 3. Then the cost to be paid by loads to generators for reactive power production is calculated using equation 5. The results are shown in table 5. Generator 3 acts as slack bus and it accounts for the losses in the system. The ISO has to pay the generators for the losses that occurred in the factious node introduced between the lines that have bidirectional reactive power flow.

**Table 1.** Power flow result of IEEE 9 bus system

| . Branch | From Bus | To Bus | From Bus Injection | | To Bus injection | | Loss | |
|---|---|---|---|---|---|---|---|---|
| | | | P (MW) | Q (MVar) | P (MW) | Q (MVar) | P (MW) | Q (MVar) |
| 1 | 1 | 4 | 71.95 | 24.07 | -71.95 | -20.75 | 0.000 | 3.32 |
| 2 | 4 | 5 | 30.73 | -0.59 | -30.55 | -13.69 | 0.174 | 0.94 |
| 3 | 5 | 6 | -59.45 | -16.31 | 60.89 | -12.43 | 1.449 | 6.31 |
| 4 | 3 | 6 | 85.00 | -3.65 | -85.00 | 7.89 | 0.000 | 4.24 |
| 5 | 6 | 7 | 24.11 | 4.54 | -24.01 | -24.40 | 0.095 | 0.81 |
| 6 | 7 | 8 | -75.99 | -10.60 | 76.50 | 0.26 | 0.506 | 4.29 |
| 7 | 8 | 2 | -163.00 | 2.28 | 163.00 | 14.46 | 0.000 | 16.74 |
| 8 | 8 | 9 | 86.50 | -2.53 | -84.04 | -14.28 | 2.465 | 12.40 |
| 9 | 9 | 4 | -40.96 | -35.72 | 41.23 | 21.34 | 0.266 | 2.26 |
| | | | | | | Total | 4.955 | 51.31 |

**Table 2.** Modified power flow result.

| Branch | From Bus Injection(MVar) | To Bus injection(MVar) |
|---|---|---|
| 1-4 | 24.07 | -20.75 |
| 4-a | -0.59 | 0 |
| a-5 | 0 | -13.69 |
| 5-b | -16.31 | 0 |
| b-6 | 0 | -12.43 |
| 3-6 | -3.65 | 7.89 |
| 6-7 | 4.54 | -24.40 |
| 7-8 | -10.60 | 0.26 |
| 8-c | 2.28 | 0 |
| c-2 | 0 | 14.46 |
| 8-d | -2.53 | 0 |
| d-9 | 0 | -14.28 |
| 9-4 | -35.72 | 21.34 |

**Table 3.** Reactive power allocated by each generator to different loads

| Branch No. | Loss allocated to load | | | | | Total Loss (MVar) |
|---|---|---|---|---|---|---|
| | Bus 5 | Bus 7 | Bus 9 | Bus c | Gen 3 | |
| 1-4 | 0 | 0 | 3.32 | 0 | 0 | 3.32 |
| 4-a | 0 | 0 | 0.04 | 0 | 0 | |
| a-5 | 0.94 | 0 | 0 | 0 | 0 | 0.94 |
| 5-b | 3.58 | 0 | 0 | 0 | 0 | |
| b-6 | 0 | 0.99 | 0 | 0 | 1.73 | 6.31 |
| 3-6 | 0 | 0 | 0 | 0 | 4.24 | 4.24 |
| 6-7 | 0 | 0.81 | 0 | 0 | 0 | 0.81 |
| 7-8 | 0 | 4.29 | 0 | 0 | 0 | 4.29 |
| 8-c | 0 | 0 | 0 | 2.27 | 0 | |
| c-2 | 0 | 0 | 0 | 14.47 | 0 | 16.74 |
| 8-d | 0 | 0.19 | 0 | 1.68 | 0 | |
| d-9 | 0 | 0 | 10.53 | 0 | 0 | 12.40 |
| 9-4 | 0 | 0 | 2.26 | 0 | 0 | 2.26 |
| Total | 4.52 | 9.69 | 16.15 | 16.73 | 4.24 | 51.31 |

**Table 4.** Reactive losses allocated to loads by different buses

| Load Bus | Gen.1 | Gen.2 | Gen.3 (slack) | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 4.4928 | 0 | 0 | 2.26 | 20.75 | 2.07 | 0 | 0 | 0 | 30.00 |
| 7 | 0 | 2.8019 | 0 | 0 | 0 | 14.65 | 14.28 | 2.69 | 0 | 35.00 |
| 9 | 18.48 | 3.0652 | 0 | 10.53 | 0 | 0 | 0 | 2.95 | 14.46 | 50.00 |
| Total | 22.91 | 5.86 | 0 | 13.69 | 20.75 | 20.38 | 14.28 | 5.64 | 14.46 | 115.00 |

**Table 5.** Cost to be paid by loads to generators for reactive power production.

| | Gen.1 ($/H) | Gen.2($/H) |
|---|---|---|
| Load 5 | 101 | 42 |
| Load 7 | 121 | 24 |
| Load 9 | 65 | 20 |
| Total | 287 | 86 |

**5 Conclusion**: A power flow tracing based reactive power pricing scheme is presented in this paper. Circuit theory based solution is proposed to eliminate the problem of bidirectional flow of reactive power. A lossles network is obtained by shifting the line loss and the reactive power due to line charging capacitance to the end buses as load and generation. The generators are provided with a quadratic reactive cost function based on active power cost function and the generator power factor and all other sources are charged with a fixed cost/MVar-h calculated from the capital investment cost. The total revenue offered to the sources is exactly recovered from the loads. So the proposed power tracing based reactive power cost allocation method can be considered to be fair and reasonable.

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