A Tuning Algorithm of PD-type Iterative Learning Control

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A Tuning Algorithm of PD-type Iterative Learning Control

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Abstract: Iterative Learning control (ILC) is a powerful control concept that iteratively improves the behaviors of processes that are repetitive in nature. A new and systematic frequency-domain approach of the PD iterative learning control has been presented based upon frequency domain of continuous model. For design specifications, including the convergence margin and cut-off frequency which is determined by useful input signal, a proper PD learning operator can be tuned practically and the obtained rate of convergence is quit good. It is also noted that, for a given plant, frequency band can not been increased much more with PD learning operator for guaranteeing convergence. Simulation results show that the proposed algorithms are practical and effective.

Key Words: Iterative Learning Control, PD Controller, Convergence Rate, Frequency Domain

1 INTRODUCTION

In many motion control tasks, it is usually required to follow a trajectory repeatedly. Conventional control algorithms do not take advantage of the repetitiveness. The basic idea of iterative learning control (ILC) is to construct a compensation signal based on the tracking error in each repetition so as to reduce the tracking error in the next repetition. In recent years, much attention has been devoted to the design of iterative learning controllers [1-8] that progressively improve the performance in repeatedly attempting to track pre-specified trajectories. It is a recursive control method that relies on less calculation and requires less knowledge about the system dynamics.

Iterative learning control is an open-loop control and can not suppress unanticipated, non-repeating disturbances. In real application, we often design the learning operator for the closed-loop systems that have taken a good performance by proper feedback controller. Although many popular ILC algorithms, such as inversion method, H_∞ method and quadratic optimal design, have been proposed [9], PD-type learning function and its various variations are still the most common design techniques and widely used in practical systems, and the integral rarely used because ILC has a natural integrator action from one trial to next.

This article presents analysis and design of the PD learning operators based upon frequency domain. A key reason that analysis and design in terms of frequency domain is so valuable is that the frequency respond can be determined experimentally with no prior knowledge of the system's model or transfer function, for example, by excited a sinusoid with varying frequency [10-12].

The major contributions of this paper are that a simple, practical and systematic design method for PD-type ILC in term of S-domain is proposed. The rest of this paper is organized as follows. In Sec. 2, general iterative learning algorithm and convergence conditions are given. In Sec. 3, basic P and D type learning operator is discussed briefly. New design methods of PD iterative learning algorithm are proposed in Sec. 4. An illustrative example is given in Sec.5. Finally, conclusions are presented in Sec. 6 with remarks on further investigation.

2 GENERAL LEARNING ALGORITHM

Consider the discrete-time, LTI, SISO system

$$Y_j(s) = P(s) \cdot U_j(s) . \tag{1}$$

where j is the iteration index, U_j and Y_j are the control and output signals, and $P(s)$ is the transfer function of closed-loop system with feedback controller and is asymptotically stable.

General learning algorithm by adding a learning filter has the form

$$U_{j+1}(s) = Q(s)[U_j(s) + L(s)E_j(s)] \ . \tag{2}$$

To apply the S-transformation to ILC, signals must be defined over an infinite time horizon. Since all practical applications of ILC have finite trial duration, the S-domain representation is an approximation of the ILC system.

Using (1) and (2), learning algorithm for S-domain are given by

$$U_{j+1}(s) = Q(s)[1 - L(s)P(s)]U_j(s) + L(s)Q(s)Y_d(s) . \tag{3}$$

Define

$$\|T(s)\|_\infty = \sup_{\forall\omega} |T(j\omega)|$$

The stability condition can be obtained by contraction mapping, that is

$$\|Q(s)[1 - L(s)P(s)]\|_\infty < \gamma \qquad \gamma < 1 \ . \tag{4}$$

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where γ is a chosen margin to guarantee fast convergence and safe operation, and $e_\infty(k)=0$ for all k, if and only if the system is AS and *Q(s)=1*.

As seen from (2), an extra freedom in design of *L(s)* is given to improve the convergence properties since we are also free to choose *Q(s)*. *Q(s)* is typically chosen as a low pass filter with a cut-off frequency ω_c, where unity gain for low frequencies $[0,\omega_c]$ and zero gain for higher frequencies. Clearly, the *Q*-filter can be employed to determine which useful frequencies band are emphasized. In Nyquist diagram, the effect of *Q(s)* is to move the base of the vector close to the origin as the frequency increased, and the phase error is of no importance as long as *L(s)P(s)* is small. So that, convergence and robust can be achieved.

As mentioned above, the learning operator *L(s)* should be tuned to maximize the range $[0,\omega_c]$, over which *L(s)P(s)* lies inside the unit circle centered at 1 in Nyquist diagram. Therefore, some ILC algorithm set *Q(s)=1* for perfect tracking. However *Q*-filter can improve transient behavior and robustness for high frequencies disturbance.

On the basis of the abovementioned arguments, the following frequency domain design procedure is proposed.

Step 1. Choose $L(s)\approx P^{-1}(s)\quad \forall\omega\in[0,\omega_c]$, i.e. choose $L(s)$ to be the approximate inverse of $P(s)$.

Step 2. Choose $Q(s)$ to be a low-pass filter with cut-off frequency near ω_c, with $|Q(s)|=1\quad\forall\omega\in[0,\omega_c]$, and $|Q(s)|=0\ \ \forall\omega>\omega_c$.

It is obvious that the error would converge to zero after one single cycle if the learning operator equal to the inverse transfer function of the system, i.e., $L(s)=P^{-1}(s)$. But this is often an impractical approach. Firstly, the reason we utilize the learning scheme is that it is difficult to get exact plant model and inverse transfer model due to unknown and nonlinear dynamics. In addition, inversion of non-linear dynamics system may be difficult. A mismatch between the nominal model and actual dynamics can make convergence worse and lead to poor transient behavior. Secondly, all practical system falls rapidly to zero as the frequency increases, implying that the learning operator should have high gain for frequency. This is undesirable property because of the influence of high-frequency noise. Thirdly, inversion of the non-minimum phase systems, which are common in practical application, will cause an unstable learning operator. Finally, complete inverse learning operator will force the system tracks all the high-frequency signals, and might put excessive stresses on the actuators.

Actually, frequency bandwidth of useful signal is normally limited and higher frequencies need not to be compensated. So that, a proper learning operator *L(s)* discussed below should be designed to satisfy

$$|1-L(s)P(s)|<\gamma\quad\forall\omega\in[0,\omega_c]\ . \tag{5}$$

3 P AND D-TYPE OPERATORS

(1) P-type learning algorithm is as

$$L(s)=\Gamma$$

Corresponding learning function can be written as

$$U_{j+1}(s)=Q(s)\left[U_j(s)+\Gamma\cdot E_j(s)\right]$$

and the stability condition from (5) is

$$|1-L(s)P(s)|=\sqrt{1-2\Gamma A(\omega)\cos\Phi(\omega)+\Gamma^2A^2(\omega)}<\gamma$$

where $A(\omega)$ and $\Phi(\omega)$ are respectively the magnitude and phase of $P(s)$.

To have a real solution of Γ, one requires that

$$4\cdot[A^2(\omega)\cos^2\Phi(\omega)]^2-4\cdot(1-\gamma^2)A^2(\omega)\geq0$$

which gives $\gamma^2\geq\sin^2\Phi(\omega)$. (6)

(2) D-type learning algorithm is generally as

$$L(s)=\Gamma\cdot s$$

$$U_{j+1}(s)=Q(s)\cdot[U_j(s)+\Gamma\cdot s\cdot E_j(s)]$$

The stability condition is similarly obtained from (5) as

$$|1-L(s)P(s)|=\sqrt{1+2\Gamma\omega A(\omega)\sin\Phi(\omega)+\Gamma^2\omega^2A^2(\omega)}<\gamma$$

To have a real solution of Γ, one requires that

$$\gamma^2>\cos^2\Phi(\omega)\ . \tag{7}$$

The inequality (6) and (7) means that, for given range ω of interest, the ILC convergence rate cannot be faster than the limit characterized by γ. As such, the way to achieve a faster ILC convergence process is to well design the learning operator to reduce the phase lag of the system designed.

4 PD-TYPE LEARNING OPERATOR

Typically, PD-type learning algorithm discussed below is given by

$$L(s)=K_p+K_d\cdot s\ . \tag{8}$$

here $K_p>0$, $K_d>0$.

Corresponding learning function can be written as

$$U_{j+1}(s)=Q(s)\cdot[U_j(s)+(K_p+K_d\cdot s)\cdot E_j(s)]$$

For design specifications γ and ω_c, the type and order of *Q*-filter, such as Butterworth, Chebyshev or FIR, is generally pre-specified and cutoff is ω_c. Then, two parameters including K_p, K_d should be tuned to satisfy (5) for getting good learning transients and small error.

The stability condition is as

$$|1-L(s)P(s)|=\left|1-(K_p+K_d\cdot s)\cdot A(\omega)e^{j\Phi(\omega)}\right|\leq\gamma$$

That is

$$N_1=A(\omega)^2\omega^2{K_d}^2+2\omega A(\omega)\sin\Phi(\omega)\cdot K_d +[1-\gamma^2-2A(\omega)K_p\cos\Phi(\omega)+A(\omega)^2{K_p}^2]\leq0\ . \tag{9}$$

For the quadratic equation (9) with unknown parameter K_d, in order to have a real solution of K_d, we require

$$N_2=-A(\omega)^2\cdot{K_p}^2+2A(\omega)\cos\Phi(\omega)\cdot K_p +[\gamma^2-\cos^2\Phi(\omega)]\geq0\ . \tag{10}$$

Due to $-A(\omega)^2 < 0$ and discriminant $\Delta = 4A(\omega)\gamma^2 > 0$, the equation $N_2 = 0$ with parameter K_p, for design specifications γ and ω_c, has two real solutions λ_1 and λ_2. Clearly, we always have the solutions $\lambda_1 \le K_p \le \lambda_2$ which satisfy (10), and the central value is

$$K_p = \frac{\cos\Phi(\omega_c)}{A(\omega_c)}$$

After K_p is selected, two solutions ξ_1 and ξ_2 can be got from $N_1 = 0$. $\xi_1 \le K_d \le \xi_2$ will make (9) and the central value is

$$K_d = -\frac{\sin\Phi(\omega_c)}{\omega_c \cdot A(\omega_c)}$$

So, we can design PD ILC which meets

$$K_p > 0,\ K_d > 0,\ \lambda_1 \le K_p \le \lambda_2,\ \xi_1 \le K_d \le \xi_2$$

This will guarantee

$$\left|1 - L(s)P(s)\right| \le \gamma \text{ at } \omega = \omega_c$$

But the design procedure above may not guarantee the stability condition at low frequency, for example at $\omega = 0$. In order to guarantee $\left|1 - L(s)P(s)\right| = 0$ at $\omega = 0$, it will be needed that $K_p = \dfrac{1}{A(0)}$ makes sure the error is zero in low frequency. So the design procedure can be modified further below.

For common unit-loop closed system with

$$A(0) = 1,\ K_p = 1.$$

(9) can be rewrite as

$$N_1 = a \cdot K_d^2 + b \cdot K_d + c \qquad 0 < N_1 \le \gamma^2$$

$$a = A(\omega)^2 \cdot \omega^2$$

$$b = 2\omega A(\omega)\sin\Phi(\omega)$$

$$c = 1 - 2A(\omega)\cos\Phi(\omega) + A(\omega)^2$$

Because of $a > 0$, for design specifications γ and ω_c, when

$$\frac{4 \cdot a \cdot c - b^2}{4 \cdot a} = [A(\omega_c) - \cos\Phi(\omega_c)]^2 < \gamma^2. \qquad (11)$$

That is

$$\cos\Phi(\omega_c) - \gamma \le A(\omega_c) \le \cos\Phi(\omega_c) + \gamma. \qquad (12)$$

K_d exists. Then solve the equation

$$M = a \cdot K_d^2 + b \cdot K_d + c = \gamma^2. \qquad (13)$$

two solutions ξ_1 and ξ_2 can be obtained from (13). If $\xi_1 > 0$ or $\xi_2 > 0$, we can choose one for the K_d that fulfill the requirement of the chosen margin to guarantee fast convergence and safe operation.

So, we have design procedure of PD ILC. If

$$\frac{4 \cdot a \cdot c - b^2}{4 \cdot a} < \gamma^2$$

we can choose $K_d = \xi_1$ (if $\xi_1 > 0$) or $K_d = \xi_2$ (if $\xi_2 > 0$), and $K_p = 1$. Conditions below will be guaranteed.

$$\left|1 - L(s)P(s)\right| = \gamma \text{ when } \omega = \omega_c$$

$$\left|1 - L(s)P(s)\right| = 0 \text{ when } \omega = 0$$

5 AN ILLUSTRATIVE EXAMPLE

The closed-loop servo system model [12] for the joint movement of robot manipulation discussed below was obtained by measuring the phase and amplitude when different frequency sin signals were applied on the input of the servo system.

The estimated fourth-order AR model was given by the transfer function

$$P(z) = \frac{z^3}{333.3 \cdot z^4 - 1173 \cdot z^3 + 1596 \cdot z^2 - 991.8 \cdot z + 236.7}$$

with sampling time 0.01s. Note that $A(0)$ for the AR model is 1.

Corresponding transfer function of continuous model is

$$P(s) = \frac{0.082s^3 + 31.5s^2 + 5303s + 360400}{s^4 + 34.2s^3 + 2051s^2 + 41700s + 360400}$$

Desired convergence margin $\gamma = 0.8$ is selected to guarantee fast convergence and safe operation, and different cut-off frequency ω_c is adopted to design a proper PD iterative learning controller.

(1) For $\omega_c = 5(1/s)$

The magnitude and phase of $P(s)$

$$A(\omega_c) = 1; \quad \Phi(\omega_c) = -32^\circ$$

$$\cos\Phi(\omega_c) - \gamma = 0.048$$

$$\cos\Phi(\omega_c) + \gamma = 1.648$$

Clearly, (12) is satisfied. From (13), we can get the solution

$$K_d = 0.26,\ K_p = 1$$

The Nyquist diagram of $1 - L(s)P(s)$ is plotted in Figure 1 for the convergence criterion. The curve within ω_c should be in the unit circle with margin specified above to guarantee fast convergence and convergence margin. It is seen that $\left|1 - L(s)P(s)\right| \approx 0.8$ at $\omega_c = 5(1/s)$.

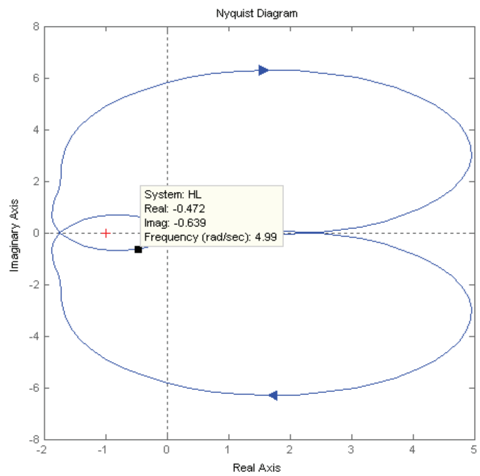

Fig. 1. Nyquist diagram when $\omega_c = 5(1/s)$ and $K_d = 0.26$

(2) For $\omega_c = 20(1/s)$

The magnitude and phase of $P(s)$

$A(\omega_c) = 0.575$; $\Phi(\omega_c) = -102^\circ$

$\cos\Phi(\omega_c) - \gamma = -1.0$

$\cos\Phi(\omega_c) + \gamma = 0.6$

Clearly, (12) is satisfied. Also, we can get the solution from (13)

$K_d = 0.07$ or $K_d = 0.1$, $K_p = 1$

The Nyquist diagram of $1 - L(s)P(s)$ is plotted in Figure 2 and 3. It is seen that $|1 - L(s)P(s)| \approx 0.8$ at $\omega_c = 20(1/s)$.

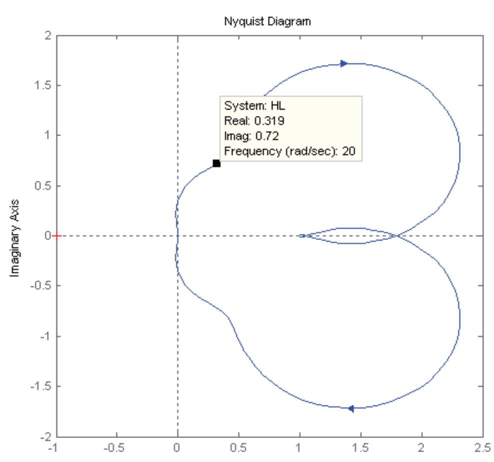

Fig. 2. Nyquist diagram when $\omega_c = 20(1/s)$ and $K_d = 0.07$

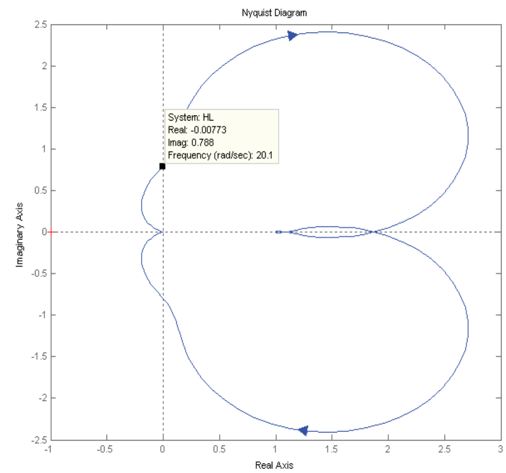

Fig. 3. Nyquist diagram when $\omega_c = 20(1/s)$ and $K_d = 0.1$

(3) For $\omega_c = 22(1/s)$

The magnitude and phase of $P(s)$

$A(\omega_c) = 0.538$; $\Phi(\omega) = -107^\circ$

$\cos\Phi(\omega_c) - \gamma = -1.09$

$\cos\Phi(\omega_c) + \gamma = 0.507$

Here, no solution exists for (13) because (12) is not satisfied. Note that no proper PD operator can be obtained when the higher cut-off frequency is specified.

Clearly, parameters of PD operator can be easily gotten from (13) for different design specifications. For the convergence margin mentioned above, maximum cut-off frequency is about 20(1/s).

Desired trajectory for illustration is shown in Figure 4 with

$$y_d(t) = 1 - \cos(4 * t) \qquad t \in (0, 1.57)$$

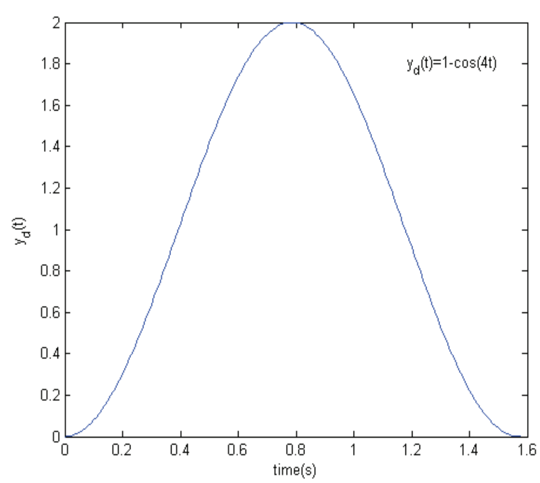

Fig. 4. Desired trajectory

In order to guarantee $|1 - L(s)P(s)| = 0$ at $\omega = 0$, we always make $K_p = 1$. For different values of K_d, IAE value of trajectory error are shown in Figure 5 by numerical simulations.

Corresponding trajectories of $y(t)$ are show in Figure 6, 7, 8 for $K_d = 0.26$, $K_d = 0.03$, $K_d = 0.1$ respectively.

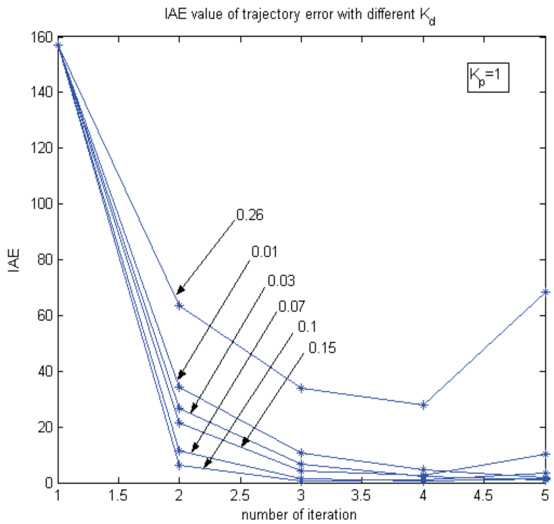

Fig. 5. IAE value of trajectory error with different k_d

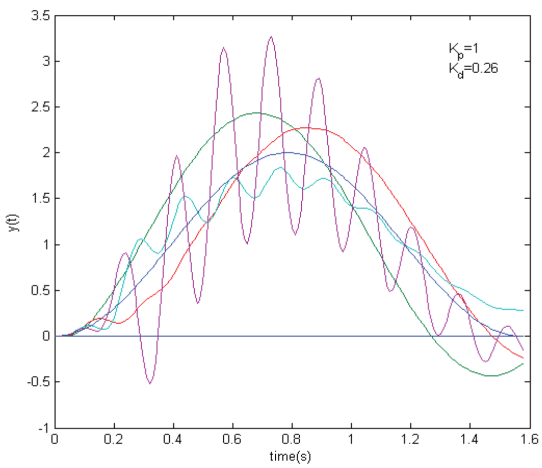

Fig. 6. Trajectory from trial 1 to 5 at k_d=*0.26*

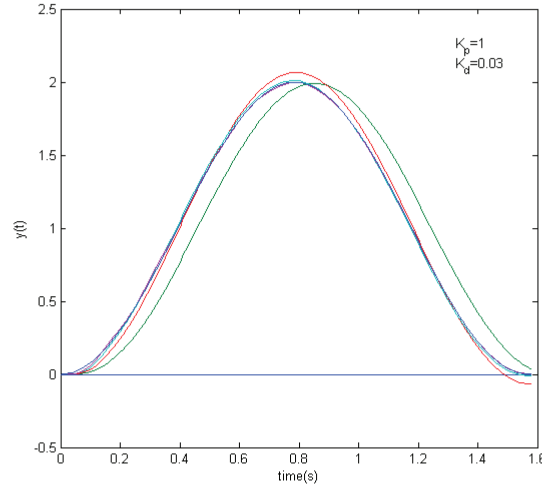

Fig. 7. Trajectory from trial 1 to 5 at k_d=*0.03*

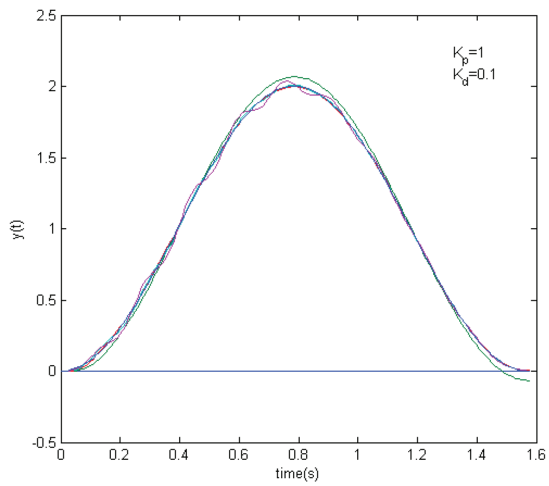

Fig. 8. Trajectory from trial 1 to 5 at k_d=*0.1*

It can be seen that K_d between 0.07 to 0.1, which means maximum cut-off frequency about 20(1/s) is achieved, will make relatively desired performance. Smaller or higher K_d can't satisfy the required specifications or make convergence worse.

6 CONCLUSIONS

A new and systematic design of the PD iterative learning control has been presented based upon frequency domain. For design specifications, including the convergence margin and cut-off frequency which is determined by useful input signal, a proper PD learning operator can be tuned practically and the obtained rate of convergence is quit good. Although it is well-known that increasing frequency band may enhance the learning performance, we noted that, for a given plant, frequency band can not been increased much more with PD learning operator for guaranteeing convergence.

Further, it would be of interest to apply fractional order PD learning operator because of more flexible tuning in the frequency domain [13][14].

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