[14] M.J. Egenhofer, E. Clementini, and Di Felicem P. Topological Relations Between Regions With Holes. *Int. J. Geographic Information Systems*, 8(2):129–142, 1994.

[15] M.J. Egenhofer and R.D. Franzosa. PointSet Topological Spatial Relations. *Int. J. Geographic Information Systems*, 5(2):161–174, 1991.

[16] M.J. Egenhofer and J.R. Herring. A Mathematical Framework for the Definition of Topological Relationships. In *Proceedings of the 4th international Symposium on Spatial Data Handling*, volume 2, pages 803–13, 1990.

[17] M.J. Egenhofer and J. Sharma. Topological consistency. In P. Bresnahan, E. Corwin, and D. Cowen, editors, *Proceedings of the 5th International Symposium on Spatial Data Handling*, volume 2, pages 335–343, Charleston, 1992. IGU Commission of GIS.

[18] K. D. Forbus, P. Nielsen, and B. Faltings. Qualitative Kinematics: A Framework. In D. S. Weld and J. De Kleer, editors, *Qualitative Reaoning About Physical Systems*, pages 562–574. Morgan Kaufman, 1990.

[19] K.D. Forbus, P. Nielsen, and B. Faltings. Qualitative Spatial Reasoning: the CLOCK Project . *Artificial Intelligence*, 51:417–471, 1991.

[20] Guesgen, H.W. Spatial reasoning based on allen's temporal logic. Technical Report TR-89-049, International Computer Science Institute, Berkeley, California, 1989.

[21] Th Hadzilacos and Tryfona N. An extended entity-relationship model for geographic applications. In *SIGMOD RECORD*. ACM, 1997.

[22] S-Y Lee and F-J Hsu. Picture Algebra for Spatial Reasoning of Iconic Images Represented in 2D C-string. *Pattern Recognition*, 12:425–435, 1991.

[23] A. Mukerjee and G. Joe. A Qualitative Model for Space. In *Proceeding of the 8th National Conference on Artificial Intelligence, AAAI, 1990*, pages 721–727, 1990.

[24] D. Papadias and T. Sellis. Spatial reasoning using symbolic arrays. In *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, LNCS 716, pages 153–161. Springer Verlag, 1992.

[25] D.A. Randell, A.G. Cohn, and Z. Cui. Computing Transitivity Tables: A Challenge for Automated Theorem Provers. In *CADE*, Lecture Notes In Computer Science, 1992.

the propagation of empty and non-empty intersections between object components. Finally, the method is applied in a finite known number of steps (equal to the number of components of the intermediate objects) which allows its implementation in spatial information systems.

# References


[1] A. I. Abdelmoty and El-Geresy B. A. A General Method for Spatial Reasoning in Spatial Databases. In *Proceedings of the Fourth International Conference on Information and Knowledge Management (CIKM'95)*. ACM, 1995.

[2] Alia I. Abdelmoty, Norman W. Paton, M. Howard Williams, Alvaro A.A. Fernandes, Maria L. Barja, and Andrew Dinn. Geographic Data Handling in an Deductive Object-Oriented Database. In D. Karagiannis, editor, *Proc. 5th International Conference on Database and Expert Systems Applications (DEXA)*, pages 445–454. Springer Verlag, Sep 1994.

[3] B. Bennett. Spatial Reasoning with Propositional Logics. In *Principles of Knowledge Representation and Reasoning (KR94)*, pages 51–62. Morgan Kaufmann, 1994.

[4] S.K. Chang and S.H. Liu. Picture Indexing and Abstraction Techniques for Pictorial Databases. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, PAMI-6(4):475–484, 1984.

[5] A. Clementini and P. Di Felice. A Model for Representing Topological Relationships Between Complex Gemetric Features in Spatial Databases. *Information Sciences*, xx, 1994.

[6] E. Clementini, P. Di Felice, and G. Califano. Composite Regions in Topological Queries. *Information Systems*, 20(7):579–594, 1995.

[7] F.P. Coenen and V. Pepijn. A generic ontology for spatial reasoning. In *Research and Development in Expert Systems XV, proc. of ES'98*, pages 44–57. Springer Verlag, 1998.

[8] A.G. Cohn, B. Bennett, J. Gooday, and N.M. Gotts. Qualitative Spatial Representation and REasoning with the Region Connection Calculus. *Geoinformatica*, 1(3):1–42, 1997.

[9] A.G. Cohn and N.M. Gotts. Expressing Spatial Vagueness in Terms of Connection. In *GISDATA Specialist Meeting: Spatial Conceptual Models for Geographic Objects with Undetermined Boundaries.* Taylor & Francis, 1994.

[10] A.G. Cohn and N.M. Gotts. The "Egg-Yolk" Representation of Regions with Indeterminate Boundaries. In P.A. Burrough and A.U. Frank, editors, *Geographic Objects with Indeterminate Boundaries, GISDATA*, pages 171–187. Taylor & Francis, 1996.

[11] A.G. Cohn, D.A. Randell, Z. Cui, and B. Bennet. Qualitative Spatial Reasoning and Representation. In P. Carrete and M.G. Singh, editors, *Qualitative Reasoning and Decision Technologies*, 1993.

[12] Oracle Corporation. http://www.oracle.com/database/options/spatial/index.html. 1999.

[13] M.J. Egenhofer. Deriving the composition of Binary Topological Relations. *Journal of Visual Languages and Computing*, 5:133–149, 1994.

$c \rightarrow a < c$), and to the subset relations such as `contain` and `inside` (for example, $inside(A,B) \wedge inside(B,C) \rightarrow inside(A,C)$, $east(A,B) \wedge east(B,C) \rightarrow east(A,C)$).

Transitive property of the subset relations was employed by Egenhofer [13] for reasoning over topological relationships. Transitive property of the order relations has been utilised by Mukerjee & Joe [23], Guesgen [20], Chang & Lu [4], Lee & Hsu [22] and Papadias & Sellis [24].

Although order relations can be utilised in reasoning over point-shaped objects, they cannot be directly applied when the actual shapes and proximity of objects are considered.

- Theorem proving (elimination): Here, reasoning is carried out by checking every relation in the full set of *sound* relations in the domain to see whether it is a valid consequence of the composition considered (theorems to be proved) and eliminating the ones which are not consistent with the composition [11].

  Bennett [3] have proposed a propositional calculus for the derivation of the composition of topological relations between simple regions using this method. However, checking each relation in the composition table to prove or eliminate is not possible in general cases and is considered a challenge for theorem provers [25].

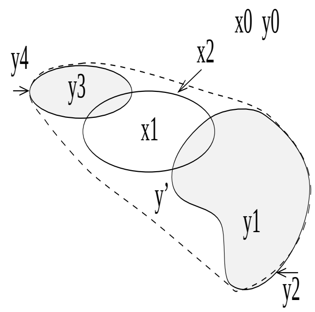
In general the limitation of all the methods in the above two approaches are as follows:

- Spatial reasoning is studied only between objects of similar types, e.g. between two lines or two simple areas. Spatial relations exist between objects of any type and it is limiting to consider the composition of only specific object shapes.
- Spatial reasoning was carried out only between objects with the same dimension as the space they are embedded in, e.g. between two lines in 1D, between two regions in 2D, etc.
- Spatial reasoning is studied mainly between simple object shapes or objects with controlled complexity, for example, regions with holes treated as concentric simple regions. No method has yet been presented for spatial reasoning between objects with arbitrary complexity.

The method proposed here is simple and general - only two rules are used to derive composition between objects of random complexity and is applicable to different types of spatial relations (topological and order).

# 5 Conclusions

A general approach is presented for the representation and reasoning over regions in space. The approach is general and can be used for regions with random complexity and for composite regions formed from multiple separate components. It is based on a uniform representation of the topology of the space as a connected set of components. A structure called adjacency matrix is proposed to capture the topology of regions with different complexity. It is shown how topological spatial relations can be uniquely defined between different types of regions. The reasoning method consists of a set of two general constraints to govern the spatial relationships between objects in space, and two general rules to propagate relationships between objects in space. The reasoning process is general and can be applied on any types of objects with random complexity. It is also simple and is based on the application of two rules for




(a)

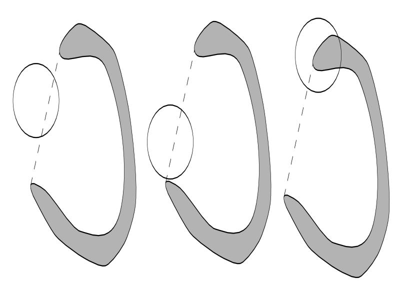
| $x_0$ | $x_2$ | $x_1$ | | $z_1$ | $z_2$ | $z_3$ | $z_0$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | $y_1$ | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | $y_2$ | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | $y_3$ | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | $y_4$ | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | $y'$ | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | $y_0$ | 1 | 1 | 1 | 1 |

(b)

Figure 11: (a) A different relationship between $x$ and $y$. (b) Corresponding intersection matrix.

| | $z_1$ | $z_2$ | $z_3$ | $z_0$ |
|---|---|---|---|---|
| $x_1$ | ? | ? | 1 | 0 |
| $x_2$ | ? | ? | 1 | 0 |
| $x_0$ | 1 | 1 | 1 | 1 |

(a)

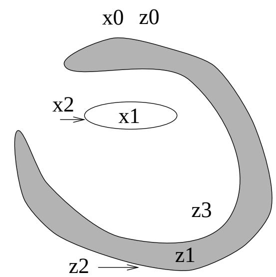


(b)

Figure 12: (a) Resulting intersection matrix for the composition in figure 11. (b) Matrix maps to one of three possible relations.

| | $z_1$ | $z_2$ | $z_3$ | $z_0$ |
|---|---|---|---|---|
| $x_1$ | 0 | 0 | 1 | 0 |
| $x_2$ | 0 | 0 | 1 | 0 |
| $x_0$ | 1 | 1 | 1 | 1 |

(a)




(b)

Figure 10: (a) Resulting intersection matrix for the composition in figure 9 (b) Its corresponding definite relation.

- $y_4$ intersections:

$$\begin{array}{rcl} \{x_0\} \sqsupseteq y_3 & \wedge & y_3 \sqsubseteq \{z_0\} \\ & \rightarrow & x_0 \cap z_0 \neq \phi \end{array}$$

- $y'$ intersections:

$$\begin{array}{rcl} \{x_0\} \sqsupseteq y_0 & \wedge & y_0 \sqsubseteq \{z_0, z_1, z_2, z_3\} \\ & \rightarrow & x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_1 \neq \phi \\ & \wedge & x_0 \cap z_2 \neq \phi \wedge x_0 \cap z_3 \neq \phi \end{array}$$

Applying rule 2 we get the following,

- $x_1 \sqsupseteq Y \wedge \sqsubseteq Z$ $x_1$ has no empty intersections with components in $Z$.
- $x_2 \sqsupseteq \{y_1, y_2\} \ \wedge \ \{y_1, y_2\} \sqsubseteq \{z_3\} \rightarrow x_2 \cap z_1 = \phi \wedge x_2 \cap z_2 = \phi$
- $z_1 \sqsupseteq \{y', y_0\} \ \wedge \ \{y', y_0\} \sqsubseteq \{x_0\} \rightarrow z_1 \cap x_1 = \phi \wedge z_1 \cap x_2 = \phi$
- $z_0 \sqsupseteq \{y_3, y_4, y', y_0\} \ \wedge \ \{y_3, y_4, y', y_0\} \sqsubseteq \{x_0\} \rightarrow z_0 \cap x_1 = \phi \wedge z_0 \cap x_2 = \phi$

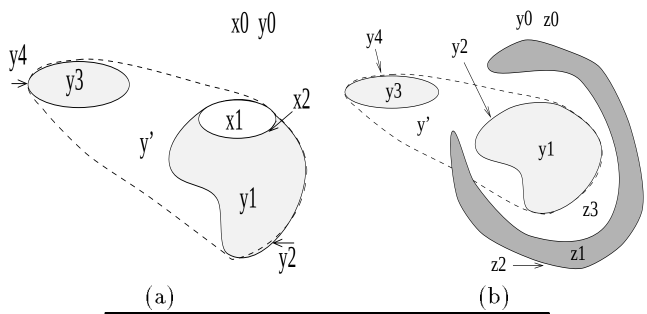
Refining the above constraints, we get the intersection matrix in figure 10(a) which maps to one definite relation 10(b).

A different conclusion is obtained if the relationship between objects $x$ and $y$ is as shown in figure 11(a). Their corresponding intersection matrix is in 11(b). The composition of the relationships between $x$, $y$ and $z$ in this case will result in an indefinite matrix in figure 12(a) which corresponds to one of the three relations in in 12(b).

## 4.4 Comparison with Related Approaches

Approaches to spatial reasoning in the literature can generally be classified into a) using *transitive propagation* and b) using *theorem proving*.

- Transitive propagation: In this approach the transitive property of some spatial relations is utilised to carry out the required reasoning. This applies to the *order relations*, such as `before`, `after` and $(<, =, >)$ (for example, $a < b \wedge b <$




| $x_0$ | $x_2$ | $x_1$ | | $z_1$ | $z_2$ | $z_3$ | $z_0$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | $y_1$ | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | $y_2$ | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | $y_3$ | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | $y_4$ | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | $y'$ | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | $y_0$ | 1 | 1 | 1 | 1 |

(c)

Figure 9: (a) and (b) Spatial relationships between different types of regions. (c) Corresponding intersection matrices.

and a simple region $x$ in (a) and a concave region $z$ in (b). The intersection matrices for both relations in shown in 9(c).

Given the relationships between the composite region $y$ and the simple region $x$ in 9(a) and its relationship with the concave region $z$ in 9(b), it is required to derive the possible relationships between $x$ and $z$.

The reasoning rules are used to propagate the intersections between the components of objects $x$ and $z$ as follows. From rule 1 we have,

- $y_0$ intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_0 \quad & \wedge \quad y_0 \sqsubseteq \{z_0, z_1, z_2, z_3\} \\ & \rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_0 \cap z_1 \neq \phi \\ & \wedge \quad x_0 \cap z_2 \neq \phi \wedge x_0 \cap z_3 \neq \phi \end{aligned}$$

- $y_1$ intersections:

$$\{x_0, x_1, x_2\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_3\} \rightarrow x_0 \cap z_3 \neq \phi \wedge x_1 \cap z_3 \neq \phi \wedge x_2 \cap z_3 \neq \phi$$

- $y_2$ intersections:

$$\begin{aligned} \{x_0, x_2\} \sqsupseteq y_2 \quad & \wedge \quad y_2 \sqsubseteq \{z_3\} \\ & \rightarrow \quad x_0 \cap z_3 \neq \phi \wedge x_2 \cap z_3 \neq \phi \end{aligned}$$

- $y_3$ intersections:

$$\begin{aligned} \{x_0\} \sqsupseteq y_3 \quad & \wedge \quad y_3 \sqsubseteq \{z_0\} \\ & \rightarrow \quad x_0 \cap z_0 \neq \phi \end{aligned}$$

*$Y$, the following is a governing rule of interaction for the three spaces $X$, $Y$ and $Z$.*

$$\begin{aligned}(x' \sqsupseteq y_j) \quad & \wedge \quad (y_j \sqsubseteq z') \\ & \rightarrow \quad (x' \cap z' \neq \phi) \\ & \equiv \quad (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \\ & \qquad \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\ & \qquad \wedge \cdots \\ & \qquad \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)\end{aligned}$$

The above rule states that if the component $y_j$ in space $Y$ has a positive intersection with every component from the sets $x'$ and $z'$, then each component of the set $x'$ must intersect with at least one component of the set $z'$.

The constraint $x_i \cap z_1 \neq \phi \vee x_i \cap z_2 \neq \phi \cdots \vee x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label $a$ in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ ($x_1$ has a positive intersection with $z_2$, or with $z_4$ or with both). A $-$ in the matrix indicates that the intersection is either positive or negative.

| | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $\cdots$ | $z_n$ |
|---|---|---|---|---|---|---|
| $x_1$ | $-$ | $a$ | $-$ | $a$ | $-$ | $-$ |

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

**Rule 2: Propagation of Empty Intersections**

*Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space $Z$ whose total number of components is $n$ and $n' < n$; $z' \subset Z$. Let $y' = \{y_1, y_2, \cdots, y_{p'}\}$ be a subset of the set of components of space $Y$ whose total number of components is $p$ and $p' < p$; $y' \subset Y$. Let $x_i$ be a component of the space $X$.* Then the following is a governing rule for the spaces $X$, $Y$ and $Z$.

$$\begin{aligned}(x_i \sqsubseteq y') \quad & \wedge \quad (y' \sqsubseteq z') \\ & \rightarrow \quad (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi)\end{aligned}$$

**Remark:** if $n' = n$, i.e. $x_i$ may intersect with every element in $Z$, then no empty intersections can be propagated. Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces.

Note that in both rules the intermediate object ($y$) and its space components plays the main role in the propagation of intersections. Indeed, it shall be shown in the next example how the above two rules are applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations using this method becomes a tractable problem which can be performed in a defined limited number of steps.

## 4.3 Reasoning with Composite Regions

The example in figure 9 demonstrates the composition of relations using composite regions. Figure 9 shows the relationship between a composite region $y$

2. Every component from one space must intersect with at least one component from the other space.

   If one component of one space does not intersect with any component of the other space, either the two spaces are not equal or the spaces are not *connected*. Both conditions are excluded by the initial assumptions. This implies that there cannot exist a row or a column in the intersection matrix whose elements are all empty intersections, hence the combinatorial cases in the matrix where this case exists can be ignored.

## 4.2 General Reasoning Rules

Composition of spatial relations is the process through which the possible relationship(s) between two object $x$ and $z$ is derived given two relationships: $R_1$ between $x$ and $y$ and $R_2$ between $y$ and $z$. Two general reasoning rules for the propagation of intersection constraints are presented. The rules are characterised by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity) tables between any spatial shapes [1]- a task considered to be a challenge to automatic theorem provers [25].

### Reasoning Rules

Composition of spatial relations using the *intersection-based* representation approach is based on the transitive property of the subset relations. In what follows the following subset notation is used. If $x'$ is a set of components (set of point-sets) $\{x_1, \cdots, x_n\}$ in a space $X$, and $y_j$ is a component in space $Y$, then $\sqsubseteq$ denotes the following subset relationship.

- $y_j \sqsubseteq x'$ denotes the subset relationship such that: $\forall x_i \in x'(y_j \cap x_i \neq \phi) \ \wedge \ y_j \cap (X - x_1 - x_2 \cdots - x_n) = \phi \ where \ i = 1, \cdots n$. Intuitively, this symbol indicates that the component $y_j$ intersects with every set in the set $x'$ and does not intersect with any set out-with $x'$.

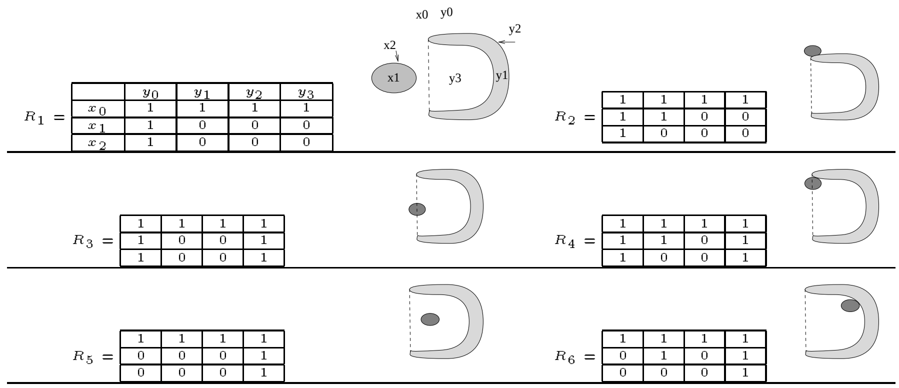
If $x_i$, $y_j$ and $z_k$ are components of objects $x$, $y$ and $z$ respectively, then if there is a non-empty intersection between $x_i$ and $y_j$, and $y_j$ is a subset of $z_k$, then it can be concluded that there is also a non-empty intersection between $x_u$ and $z_k$.

$$(x_i \cap y_j \neq \phi) \ \wedge \ (y_j \subseteq z_k) \rightarrow (x_i \cap z_k \neq \phi)$$

This relation can be generalised in the following two rules. The rules describe the propagation of intersections between the components of objects and their related spaces involved in the spatial composition.

**Rule 1: Propagation of Non-Empty Intersections**

*Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ be a subset of the set of components of space $X$ whose total number of components is $m$ and $m' \leq m$; $x' \subseteq X$. Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space $Z$ whose total number of components is $n$ and $n' \leq n$; $z' \subseteq Z$. If $y_j$ is a component of space*




$$R_1 = \begin{array}{c|cccc} & y_0 & y_1 & y_2 & y_3 \\ \hline x_0 & 1 & 1 & 1 & 1 \\ x_1 & 1 & 0 & 0 & 0 \\ x_2 & 1 & 0 & 0 & 0 \end{array} \qquad R_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad R_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 8: A set of 6 spatial relationships between two solid bodies.

# 4 Reasoning over Regions in Space

The reasoning approach consists of: a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between the objects.

## 4.1 General Constraints

The intersection matrix is in fact a set of constraints whose values identifies specific spatial relationships. For example, the constraints used to represent the relationship in figure 5(a) are $x_1 \cap y_1 = 0, x_1 \cap y_2 = 0, x_1 \cap y_3 = 0, x_1 \cap y_4 = 1, \cdots$

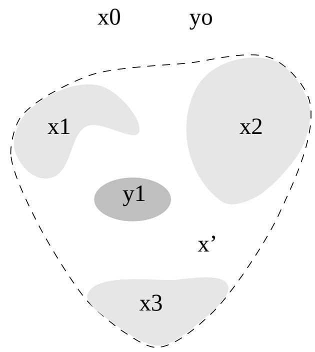
The process of spatial reasoning can be defined as the process of propagating the constraints of two spatial relations (for example, $R_1(A, B)$ and $R_2(B, C)$), to derive a new set of constraints between objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A, C)$).

A subset of the set of constraints defining all spatial relations are general and are applicable to any relationship between any objects. These general constraints are a consequence of the initial assumptions used in the definition of the object and space topology. The identification of these constraints is useful and can be used in checking the correctness of the relations and shall be used later in the paper to give some insight in the propagation of spatial relations.

The two general constraints are:

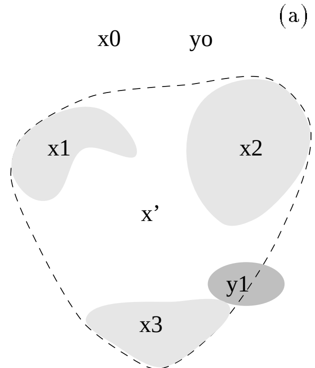
1. Every unbounded (infinite) component of one space must intersect with at least one unbounded (infinite) component of the other space.

   Intuitively this rule says that it is impossible for an infinite component in the space to only have an intersection with finite component(s). In this case the infinite component becomes a subset of the finite component(s) which is not possible. In figure 8, $x_0$ and $y_0$ always have a non-empty intersection.



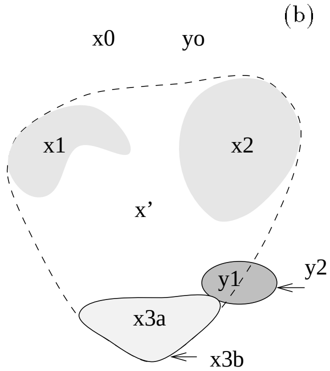

| | $y_0$ | $y_1$ |
|---|---|---|
| $x_0$ | 1 | 0 |
| $x'$ | 1 | 1 |
| $x_1$ | 1 | 0 |
| $x_2$ | 1 | 0 |
| $x_3$ | 1 | 0 |

(a)




| | $y_0$ | $y_1$ |
|---|---|---|
| $x_0$ | 1 | 1 |
| $x'$ | 1 | 1 |
| $x_1$ | 1 | 0 |
| $x_2$ | 1 | 0 |
| $x_3$ | 1 | 1 |

(b)




| | $y_0$ | $y_1$ | $y_2$ |
|---|---|---|---|
| $x_0$ | 1 | 1 | 1 |
| $x'$ | 1 | 1 | 1 |
| $x_1$ | 1 | 0 | 0 |
| $x_2$ | 1 | 0 | 0 |
| $x_{3a}$ | 0 | 0 | 0 |
| $x_{3b}$ | 1 | 0 | 1 |

(c)

Figure 7: (a) Possible relationship between a composite and simple region. (b) A different relationship, distinguished by $y$ connecting to $x_3$. (c) Exact nature of relationship is revealed by considering finer object detail.

| | $y_0$ | $y_1$ | $y_2$ |
|---|---|---|---|
| $x_0$ | 1 | 0 | 0 |
| $x_1$ | 1 | 0 | 0 |
| $x_2$ | 1 | 0 | 0 |
| $x_3$ | 1 | 1 | 1 |

(a)

| | $y_0$ | $y_1$ | $y_2$ |
|---|---|---|---|
| $x_0$ | 1 | 1 | 1 |
| $x_1$ | 1 | 0 | 0 |
| $x_2$ | 1 | 0 | 0 |
| $x_3$ | 1 | 0 | 0 |

(b)

Figure 6: The corresponding **intersection matrices** for the relationships in figure 5 respectively.

the number of components in $X$ and $m$ is the number of components in $Y$, then a spatial relation $R(x, y)$ can be represented by one state of the following equation:

$$\begin{aligned} R(x,y) &= X \cap Y \\ &= \left( \bigcup_{i=1}^{n} x_i \right) \cap \left( \bigcup_{j=1}^{m} y_j \right) \\ &= (x_1 \cap y_1, \cdots, x_1 \cap y_m, x_2 \cap y_1, \cdots, x_n \cap y_m) \end{aligned}$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix as follows.

$$R(x,y) = \begin{array}{|c|c|c|c|c} \hline & y_0 & y_1 & y_2 & \cdots \\ \hline x_0 & & & & \\ \hline x_1 & & & & \\ \hline x_2 & & & & \\ \hline \vdots & & & & \end{array}$$

For example, the intersection matrices corresponding to the spatial relationships in figure 5 are shown in figure 6. The components $y_1$ and $y_2$ have a non-empty intersection with $x_0$ in 6(a) and with $x_3$ in 6(b).

Different combinations in the intersection matrix can represent different qualitative relations. The example in figure 7 shows an example of spatial relationships between composite regions and between composite and simple regions. The set of sound, or physically possible, spatial relationships between objects is dependent on the particular domain studied. Also, properties of the objects would affect the set of possible spatial relationships that can exist between them. For example, if one object is solid and the other is permeable, there cannot be any intersection of the inside of the solid object with any other component of the other object.

The example in figure 8 demonstrates the six possible spatial relations that can exist between two *solid* objects, one simple region and the other a concave one along with their intersection matrices. The example can be used to represent many situations, for example, a solid object falling into a container full of liquid, a ball thrown into a net, or a ship entering a bay of an island, etc.

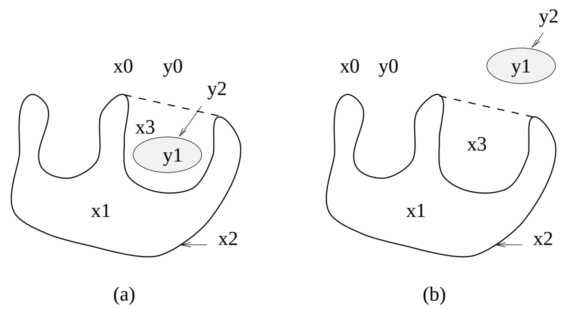



Figure 5: Different qualitative spatial relationships can be distinguished by identifying the appropriate components of the objects and the space.

interiors and exteriors. His method carries the same limitations as those of [16]. In another work [6], Clementini et al addressed the issue of defining composite regions for use in spatial query languages, by defining explicit relationships between all the components in the object, in the same way, regions with holes were defined in [14].

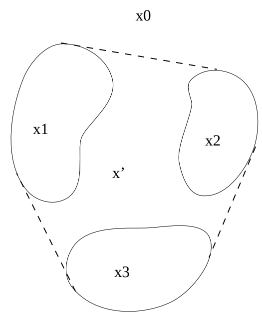
Coenen and Pepijin [7] proposed an ontology for objects and relationships in spatio-temporal domains. They assumed the space to be consisting of sets of points and used set-theoretic notions to define objects in that space. Their approach is distinctive from the above where space is considered to be discrete, not continuous. The method was used to define a general "object" and quantitative identifiers are used to qualify the object properties. Extending the method for distinguishing between different types of regions was not proposed.

# 3 Representing Topological Relations between Regions

Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces. For example, in figure 5 different relationships between two objects representing a ship ($y$) and an island ($x$) are shown, where in 5(a) the ship is inside the bay and in 5(b) the ship is outside the bay. The concave region representing the island ($x$) is decomposed into two components $x_1$ and $x_2$ and the rest of the space associated with $x$ is decomposed into two components ($x_3$ representing the bay and $x_0$ representing the rest of the ocean). The component $x_3$ is a virtual component, i.e. with no physical boundary to delineate its spatial extension. It is the identification of this component that makes the distinction between the two relationships in the figure.

The complete set of spatial relationships are represented by combinatorial intersection of the components of one space with those of the other space.

If $R(x, y)$ is a relation of interest between object $x$ and object $y$, and $X$ and $Y$ are the spaces associated with the objects respectively such that $n$ is




(a)

| | | | | |
|---|---|---|---|---|
| $x_0$ | | | | |
| 1 | $x'$ | | | |
| 1 | 1 | $x_1$ | | |
| 1 | 1 | 0 | $x_2$ | |
| 1 | 1 | 0 | 0 | $x_3$ |

(b)

Figure 4: (a) A composite region formed of multiple, separate, parts, $x_1$, $x_2$ and $x_3$, defined using its convex hull. (b) Its corresponding adjacency matrix.

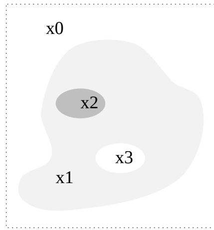
## 2.4 Comparison with Other Related Approaches

The main advantage of the representation method above is its uniformity. The same methodology is used for the definition of simple, complex and composite regions. The method is also adaptable, where different levels of representation can be devised by hiding or revealing the details of objects as required. The method is therefore well adapted for use as a basis for a spatial reasoning formalism.

The representation of complex regions has been addressed in many works. In [8], Cohn et al extended the RCC formalism to handle concave regions, and regions with holes (doughnut shapes). New axioms and theories had to be devised to define the new shapes. The main drawback of this approach is its complexity, as new, possibly considerable, extensions of the formalism have to be devised with every new shape considered.

In [16, 15], Egenhofer et al used point-set topology to define simple regions, using three components, boundary, interior and exterior. The method proposed here deviates from their work in one important respect which has far-reaching implications. We relaxed the constraint on the object components to be any possible set of components which satisfies the main assumptions behind the formalism. The notions of boundaries, interiors and exteriors were dropped and the notion of object and space components is used instead. Egenhofer's method is limiting and could not be extended to handle complex objects. Other methods were devised in [14] to define regions with holes, through the definition of spatial relationships between simple regions and no extension for the method was proposed for the definition of irregular, or concave, regions.

The work of Clementini and De Felice [5] follows closely the method of Egenhofer, and provides a definition for regions with holes using boundaries,




(a)

| | | | |
|---|---|---|---|
| $x_0$ | | | |
| 1 | $x_1$ | | |
| 0 | 1 | $x_2$ | |
| 0 | 1 | 0 | $x_3$ |

(b)

Figure 3: (a) A region with several holes. (c) Its corresponding adjacency matrix.

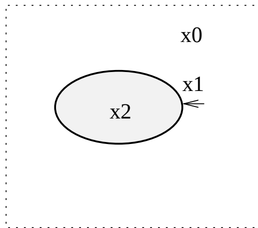
regions [8], regions with holes [14] and regions with indeterminate boundaries [9, 10]. The examples shown in figure 2 and 3 demonstrate how the approach proposed above is used for representing the first two cases.

As can be seen from the figures, the same methodology of object and space decomposition is used uniformly in the different cases. In the case of the concave region, the cavities in the region are represented as separate (virtual) components. They are initially identified using the convex hull of the object (the minimum convex polygon that can enclose the object), as demonstrated in 2 (b). Any number of cavities can be distinguished depending on the degree of precision required, Also, more precise relations will be distinguished if the boundary of the cavity is distinguished.

## 2.3 Representing Composite Regions

Composite regions are used here to denote regions with multiple separate components. This type of objects is needed in many application contexts. For example, a university may be built on different sites in a city; a country may consist of separate islands, etc. There is a need in spatial databases to model those aggregate objects as wholes, and hence enabling the representation of their relationships in space. This need has been identified in many works [2, 21]. However, so far, few works addressed this representation problem [6].

One possible method for representing a composite region, proposed here, is by using its convex hull, as shown in figure 4(a). The region is defined by the union of its separate component regions as well as their complement which lies within the convex hull, $x'$. A coarse level of representation is used initially. Further refinement of the object details may be used later for exact determination of relationships, as shall be shown later on in the paper.



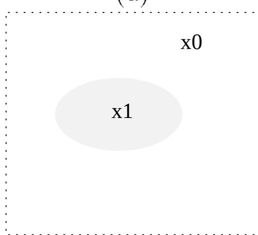

(a)

| | $x_0$ | $x_1$ | $x_2$ |
|---|---|---|---|
| $x_0$ | - | 1 | 0 |
| $x_1$ | 1 | - | 1 |
| $x_2$ | 0 | 1 | - |

(b)

| | | |
|---|---|---|
| $x_0$ | | |
| 1 | $x_1$ | |
| 0 | 1 | $x_2$ |

(c)




(d)

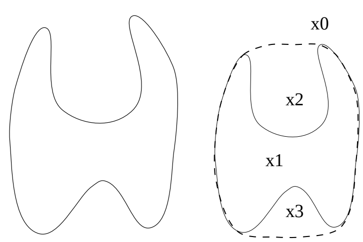
| | $x_0$ | $x_1$ |
|---|---|---|
| $x_0$ | - | 1 |
| $x_1$ | 1 | - |

(e)

| | |
|---|---|
| $x_0$ | |
| 1 | $x_1$ |

(f)

Figure 1: (a) and (d) Possible decompositions of a simple convex region and its embedding space. (b) and (e) Adjacency matrix of the two shapes in (a) and (d). (c) and (f) Half the symmetric adjacency matrix is sufficient to capture the object representation.




(a) (b)

| | | | |
|---|---|---|---|
| $x_0$ | | | |
| 1 | $x_1$ | | |
| 1 | 1 | $x_2$ | |
| 1 | 1 | 0 | $x_3$ |

(c)

Figure 2: (a) Irregular (concave) region. (b) Representation of virtual components using convex hull. (c) Its corresponding adjacency matrix.

space are divided into components according to a required resolution. The connectivity of those components is explicitly represented.

Let $S$ be the space in which the object is embedded. The object and its embedding space are assumed to be *dense* and *connected*. The embedding space is also assumed to be infinite. The object and its embedding space are decomposed into components which reflects the objects and space topology such that,

1. No overlap exist between any of the representative components.
2. The union of the components is equal to the embedding space.

The topology of the object and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix*. In the decomposition strategy, the complement of the object in question shall be considered to be infinite, and the suffix 0, e.g. ($x_0$) is used to represent this component.

Hence, the topology of a space $S$ containing an object $x$ is defined using the following equation.

$$x = \bigcup_{i=1}^{n} x_i \tag{1}$$

$$S_x = x \cup x_0 \tag{2}$$

where $S_l$ is used to denote the space associated with object $l$.

In figure 1(a) different possible decompositions of a simple convex region and its embedding space is shown along with their adjacency matrices. In 1 (a), the object is represented by two components, a linear component $x_1$ and an areal component $x_2$ and the rest of the space is represented by an infinite areal component $x_0$ representing the surrounding area. In 1(b), only one areal component is used to represent the region. Both representations are valid and may be used in different contexts. Different decomposition strategies for the objects and their embedding spaces can be used according to the precision of the relations required and the specific application considered. The higher the resolution used (or the finer the components of the space and the objects), the higher the precision of the resulting set of relations in the domain considered.

The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be collapsed to the structure in figure 1(c) and (d).

In the rest of this section, the method is applied for representing complex and composite regions.

## 2.2 Representing Complex Regions

Different types of regions may be of interest in different application domains. Several works have addressed the problem of representing irregular or concave `intersect in point(x,y)`. Queries involving qualitative spatial relationships are common in spatial decision support systems when no precise geometric information is needed. Spatial reasoning has also been proposed to maintain the consistency of the database [17].

In this paper, a representation and a reasoning approach is proposed for handling complex and composite regions in space. Complex regions are non-simple, irregular shaped areas, and composite regions are those defined by multiple, possibly, disconnected regions. Several works have been proposed for representing those object types. However, they are mostly limited to handling very specific, well-defined cases, or quite complex to form the basis for reasoning mechanisms. The method proposed in this paper is simple, based on a straightforward decomposition strategy for representing object and space topologies. It is also general and can be applied uniformally to regions with random complexity, and flexible, where different levels of representations can be readily devised. It is shown how topological relationships can be uniquely defined between the different region types. Two general rules and constraints are used for reasoning over those objects. A major advantage of the method is that reasoning between objects of any complexity can be achieved in a definite limited number of steps. Hence, the incorporation of spatial reasoning mechanisms in spatial information systems becomes plausible.

The rest of this paper is structured as follows. The representation formalism is presented in section 2. Examples are given for the representation of regions with variable complexity. The representation of spatial relationships is presented in section 3 and section 4 describe the general reasoning formalism. Comparison with other related approaches are given throughout the different sections and conclusions are given in section 5.

# 2 Representing Object and Space Topologies

In general, spatial representation formalisms aim to satisfy two main goals, namely, generality and flexibility.

**Generality:** The method should be general enough to be applied uniformly in spaces with different object complexities.

**Flexibility:** Different levels of representations may be needed for the same objects in different application contexts. Hence, the method should be flexible and adaptable to different levels of representation.

The representation method proposed here strives to satisfy both requirements. The method is first presented using simple regions and then applied on several cases of complex and composite regions.

## 2.1 The Underlying Representation

The method used for representing the space and the objects is similar to the space vocabulary described in [18] where objects of interest and their embedding

# Qualitative Reasoning with Complex and Composite Regions in 2D Space

Alia I. Abdelmoty and Baher A. El-Geresy
School of Computing, University of Glamorgan
Treforest, Wales, UK

**Abstract**

A new approach is proposed in this paper for the representation and reasoning over complex and composite regions in space. Complex regions are non-simple, irregular shaped regions with possible holes. Composite regions are regions defined through multiple separate regions. Both types exhibit different behaviour in space which is also distinct from simple regions. Spatial databases are currently limited to treating both types as simple, and approaches to qualitative reasoning are also limited to handling simple regions only. The approach proposed in this paper is general, where any type of region is treated uniformly, and flexible, where different levels of representation could be devised. A general reasoning approach is presented based on the representation formalism. It is shown how the composition of spatial relations is achieved between different types of regions. This is a significant contribution, compared to current approaches, where reasoning is only done using simple regions or regions with controlled complexity. The method proposed here is a step towards developing spatial reasoning engines for spatial databases.

## 1 Introduction

A need is emerging in many application domains for handling spatial and spatio-temporal data. This is evidenced by the spatial extensions developed in large database management systems [12]. Geographic information systems are the main driving force behind these developments, but other significant applications can also be found in many domains, including, medical and multimedia systems. Complex decision making tasks typically required in such systems depend to a large extent on data modelling and representation. The set of basic spatial data types used are between, vector; points, lines, and regions and raster; point-sets. However, it is well recognised that more complex data types need to be handled, typically made from specialisations or aggregations of the basic types.

Queries in these systems involve the derivation of different types of spatial relationships which are not explicitly stored. It is generally agreed that it is neither practical nor efficient to store the substantial amount of different types of spatial relationships that can exist in space [19]. Qualitative spatial reasoning has been proposed as a complementary mechanism for the automatic derivation of spatial relations which are not explicitly stored. It is based on the manipulation of qualitative spatial relationships such as, `near` and `touch`, as opposed to quantitative information such as, `at a distance of 10 m.` and