Frequency Domain Equalization of DMT/OFDM Systems with Insufficient Guard Interval

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***Abstract*— In this paper, we propose a new, generalized approach for equalization of discrete multi-tone (DMT) systems, which not only uses redundancy in the time domain by adding a guard interval, but also takes advantage of existing or intentionally placed redundancy in the frequency domain. For this purpose, we replace the one tap frequency equalizer in the DMT receiver by a block equalizer and derive sufficient conditions for zero forcing equalization, i.e. perfect removal of intersymbol and intercarrier interference. We show that the equalizer matrix is sparse, thus resulting in a low computational complexity. Furthermore, the equalization approach allows to trade off the guard interval in the time domain for unused subcarriers in the frequency domain.**

I. Introduction

The restriction of Discrete Multi-Tone (DMT) or Orthogonal Frequency Division Multiplexing (OFDM) systems, namely the fact that the guard interval (GI) introduced in form of a cyclic prefix has to be at least as long as the channel impulse response (CIR), mainly affects the achievable bandwidth efficiency and latency time of practical DMT systems. To overcome this problem, it has been proposed to to shorten the effective CIR length by means of a short FIR filter at the input of the receiver, the so called time-domain equalizer (TEQ) [1], [2], [3], [4]. In per-tone equalization [5], the TEQ is transferred to the frequency domain, resulting in a complex frequency domain equalizer (FEQ) for each tone separately. The FEQ allows to optimize the SNR and hence the bit rate for each tone at a comparable computational cost as the TEQ. Furthermore, the equalization effort can be concentrated on the most affected tones by increasing the number of equalization filter coefficients for these tones. No effort is wasted to equalize unused subcarriers (due to hybrids in the signal path, finite transition of anti-aliasing and interpolation filters, frequency-selective zeros) when setting the number of taps for their equalizers to zero.

In this paper, we propose a frequency domain equalization (FEQ) approach where not only no effort is spent to equalize unused subcarriers but where we explicitly use the information contained in these subcarriers to obtain a better compensation of ISI and ICI in used subcarriers for the case of an insufficient GI. For this, we replace the DMT equalizer, consisting of a complex multiplication per subcarrier, by a block equalizer. Deriving sufficient conditions for zero-forcing equalization, we end up with a surprising discovery: With only a few linear combinations of unused subcarrier output samples with used ones ISI and ICI can totally be eliminated. The number of unused subcarriers needed depends on the number of samples by which the CIR exceeds the GI, i.e. we can achieve zero-forcing equalization even in receiver structures without a GI or combine the new equalization principle with the common GI insertion method. This forms a generalized DMT (GDMT) structure which allows for optimal distribution of redundancy, either to time or frequency domain, depending on the designated design criteria.

II. The Transmission System

The generalized DMT transmission scheme is depicted in Fig. 1 where we have replaced the original DMT equalizer by a block equalizer. The transmitter $\mathbf{G}$ transforms the input DMT symbol $\mathbf{u}(k)$ from the frequency domain into the time domain and adds the GI of length L. It can be expressed in matrix form as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{L\times M-L} & \mathbf{I}_L \\ & \mathbf{I}_M \end{bmatrix} \mathbf{W}_M^*/M, \quad [\mathbf{W}_M^*]_{k,l} = \exp(j\frac{2\pi kl}{M}) \qquad (1)$$
$$k,l = 0,\ldots,M-1$$

where $\mathbf{W}_M^*/M$ is the IDFT matrix. The resulting time domain symbol $\mathbf{v}(k)$ of length $M+L$ is then parallel to serial transformed and transmitted over the channel. We assume that the channel can be modeled as an LTI system with FIR impulse response $c(n)$ of length L_c and additive noise $r(n)$.

At the receiver the incoming data stream is serial to parallel transformed. The block receiver $\mathbf{F}$ discards the GI, performs a DFT to transform the received symbol from the time domain into the frequency domain, and then equalizes the values through a block equalizer $\mathbf{E}$. In matrix notation it writes:

$$\mathbf{F} = \mathbf{E}\,\mathbf{W}_M \begin{bmatrix} \mathbf{0}_{M\times L} & \mathbf{I}_M \end{bmatrix}, \qquad (2)$$
$$[\mathbf{W}_M]_{k,l} = \exp(-j\frac{2\pi kl}{M}), \quad k,l = 0,\ldots,M-1,$$

where $\mathbf{W}_M$ is the DFT matrix. The P/S conversion at the transmitter, the convolution with the CIR, and the S/P conversion at the receiver can be expressed in terms of a channel matrix $\mathbf{C}$. In general, we can assume that the length L_c of the CIR is shorter

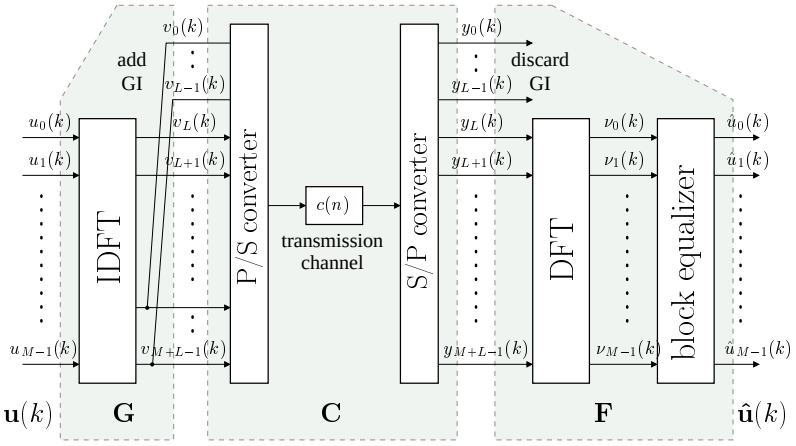

Fig. 1. Discrete Multi-Tone transmission scheme with block frequency equalizer

than the symbol length M. For this case, $\mathbf{C}$ is of size $P \times 2P$ and can be expressed as [6], [7]:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \end{bmatrix} \tag{3}$$

with

$$\mathbf{C}_0 = \begin{bmatrix} 0 & \cdots & c_{L_c-1} & \cdots & c_1 \\ \vdots & \ddots & & \ddots & \vdots \\ \vdots & & \ddots & \ddots & c_{L_c-1} \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{4}$$

$$\mathbf{C}_1 = \begin{bmatrix} c_0 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ c_{L_c-1} & \cdots & c_0 & \ddots & \vdots \\ & \ddots & & \ddots & 0 \\ 0 & \cdots & c_{L_c-1} & \cdots & c_0 \end{bmatrix}, \tag{5}$$

The input/output relationship of the transmission system for the noise-free case is thus given by:

$$\hat{\mathbf{u}}(k) = \mathbf{F} \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \end{bmatrix} (\mathbf{I}_2 \otimes \mathbf{G}) \begin{bmatrix} \mathbf{u}(k-1) \\ \mathbf{u}(k) \end{bmatrix} \tag{6}$$

where $\otimes$ symbolizes the Kronecker product. The output symbol $\hat{\mathbf{u}}(k)$ is identical to the input symbol $\mathbf{u}(k)$ if:

$$\mathbf{F} \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \end{bmatrix} (\mathbf{I}_2 \otimes \mathbf{G}) = \begin{bmatrix} \mathbf{0}_M & \mathbf{I}_M \end{bmatrix} \tag{7}$$

Introducing the structures of the transmitter and receiver from (1) and (2), respectively, into (7), we obtain:

$$\mathbf{E}\,\mathbf{W}_M \begin{bmatrix} \mathbf{0}_{M\times L} & \mathbf{I}_M \end{bmatrix} \begin{bmatrix} \mathbf{C}_0 & \mathbf{C}_1 \end{bmatrix} (\mathbf{I}_2 \otimes (\begin{bmatrix} \mathbf{0}_{L\times M-L}\mathbf{I}_L \\ \mathbf{I}_M \end{bmatrix} \mathbf{W}_M^*/M)) = \begin{bmatrix} \mathbf{0}_M & \mathbf{I}_M \end{bmatrix} \tag{8}$$

Let us now denote by $\tilde{\mathbf{C}}_0$ and $\tilde{\mathbf{C}}_1$ the $M \times M$ matrices that describe the product of the GI introduction at the transmitter with the original channel $P \times P$ sub-matrices $\mathbf{C}_0$ and $\mathbf{C}_1$, respectively, and the removal of the GI at the receiver, i.e.:

$$\tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{0}_{M\times L} & \mathbf{I}_M \end{bmatrix} \mathbf{C}_i \begin{bmatrix} \mathbf{0}_{L\times M-L}\,\mathbf{I}_L \\ \mathbf{I}_M \end{bmatrix}, \quad i = 0,1 \tag{9}$$

Substituting this notation into (8) yields:

$$\mathbf{E}\,\mathbf{W}_M \begin{bmatrix} \tilde{\mathbf{C}}_0 & \tilde{\mathbf{C}}_1 \end{bmatrix} (\mathbf{I}_2 \otimes \mathbf{W}_M^*/M) = \begin{bmatrix} \mathbf{0}_M & \mathbf{I}_M \end{bmatrix} \tag{10}$$

which can be decomposed into:

$$\mathbf{E}\,\mathbf{W}_M\,\tilde{\mathbf{C}}_1\,\mathbf{W}_M^*/M = \mathbf{I}_M \tag{11}$$

$$\mathbf{E}\,\mathbf{W}_M\,\tilde{\mathbf{C}}_0\,\mathbf{W}_M^*/M = \mathbf{0}_M \tag{12}$$

It can be easily verified that $\tilde{\mathbf{C}}_0$ and $\tilde{\mathbf{C}}_1$ have the following form:

- If the GI is at least as long as the order of the CIR, i.e. $L \geq L_c - 1$:

$$\tilde{\mathbf{C}}_0 = \mathbf{0}_M \tag{13}$$

$$\tilde{\mathbf{C}}_1 = \begin{bmatrix} c_0 & & & c_{L_c-1} & \cdots & c_1 \\ \vdots & \ddots & & & \ddots & \vdots \\ \vdots & & \ddots & & & c_{L_c-1} \\ c_{L_c-1} & & & \ddots & & \\ & \ddots & & & \ddots & \\ & & c_{L_c-1} & \cdots & \cdots & c_0 \end{bmatrix} \tag{14}$$

Thus, in this case (12) is always satisfied and since $\tilde{\mathbf{C}}_1$ is cyclic, $\mathbf{W}_M\,\tilde{\mathbf{C}}_1\,\mathbf{W}_M^*/M$ becomes a diagonal matrix $\mathbf{D}$ whose entries are given by by the M-point DFT of the CIR. $\mathbf{E}$ is identical to the equalizer in the original DMT scheme, namely:

$$\mathbf{E} = \mathbf{D}^{-1}, \;\; [\mathbf{E}]_{k,k} = \frac{1}{C(e^{j2\pi k/M})}, \quad k = 0,1,\ldots,M-1 \tag{15}$$

where $C(e^{j2\pi k/M})$ denotes the channel frequency response at the normalized frequency $2\pi k/M$.

- If the GI is of insufficient length, i.e. $L < L_c - 1$:

$$\tilde{\mathbf{C}}_0 = \begin{bmatrix} 0 & \cdots & 0 & c_{L_c-1} & \cdots & c_{L+1} \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & c_{L_c-1} \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{16}$$

$$\tilde{\mathbf{C}}_1 = \begin{bmatrix} c_0 & & & & c_L & \cdots & c_1 \\ \vdots & \ddots & & & \vdots & & \vdots \\ \vdots & & \ddots & & c_{L_c-1} & & \vdots \\ \vdots & & & \ddots & & \ddots & \vdots \\ \vdots & & & & \ddots & & c_{L_c-1} \\ c_{L_c-1} & & & & \ddots & & \\ & \ddots & & & & \ddots & \\ & & c_{L_c-1} & \cdots & \cdots & \cdots & c_0 \end{bmatrix} \tag{17}$$

For an arbitrary CIR of length L_c, it is no longer possible to solve (11) and (12) simultaneously.

III. Transmission with Insufficient GI and Unused Subcarriers

We now assume that K subcarriers are not used for data transmission, i.e. the value zero is transmitted in these subcarriers, and that the GI is of insufficient length. The block equalizer $\mathbf{E}$ then only needs to remove ISI and ICI from the $N = M - K$ subcarriers used for data transmission, since there is no need to equalize the unused subcarriers. This introduces new degrees of freedom that can be used to compensate ISI and ICI. The equalizer algorithm is derived through the following steps:

A. Removal of Unused Subcarriers:

In a first step, we explicitely set the unused subcarriers to zero through the introduction of a subcarrier selection matrix $\mathbf{S}_1$:

$$\mathbf{S}_1 = \mathrm{diag}(s_0, \ldots, s_{M-1}) \tag{18}$$

$$s_i = \begin{cases} 1 & \text{subcarrier is used} \\ 0 & \text{subcarrier is unused} \end{cases}$$

Eq. (10) then writes:

$$\begin{aligned} &\mathbf{S}_1^T \mathbf{E} \mathbf{W}_M \begin{bmatrix} \tilde{\mathbf{C}}_0 & \tilde{\mathbf{C}}_1 \end{bmatrix} (\mathbf{I}_2 \otimes (\mathbf{W}_M^*/M \cdot \mathbf{S}_1)) \\ &\quad = \mathbf{S}_1^T \begin{bmatrix} \mathbf{0}_M & \mathbf{I}_M \end{bmatrix} (\mathbf{I}_2 \otimes \mathbf{S}_1) \end{aligned} \tag{19}$$

We then remove all rows and columns from the upper equation that are zero on both hand sides of the equation sign, leading to:

$$\mathbf{S}_{1,\mathrm{red}}^T \mathbf{E} \mathbf{W}_M \begin{bmatrix} \tilde{\mathbf{C}}_0 & \tilde{\mathbf{C}}_1 \end{bmatrix} (\mathbf{I}_2 \otimes (\mathbf{W}_M^*/M \cdot \mathbf{S}_{1,\mathrm{red}}) = \begin{bmatrix} \mathbf{0}_N & \mathbf{I}_N \end{bmatrix} \tag{20}$$

where $\mathbf{S}_{1,\mathrm{red}}$ has been obtained by reducing $\mathbf{S}_1$ to its non-zero columns.

B. Decomposition of the Channel Matrix $\tilde{\mathbf{C}}$

Let us now decompose $\tilde{\mathbf{C}} = [\tilde{\mathbf{C}}_0 \quad \tilde{\mathbf{C}}_1]$ into the sum of a matrix $\tilde{\mathbf{C}}_{\mathrm{cycl}}$ describing cyclic convolution and an error matrix $\tilde{\mathbf{C}}_{\mathrm{err}}$:

$$\tilde{\mathbf{C}}_{\mathrm{cycl}} = [\mathbf{0}_M \quad \tilde{\mathbf{C}}_1 + \tilde{\mathbf{C}}_0 \mathbf{P}] \tag{21}$$

$$\tilde{\mathbf{C}}_{\mathrm{err}} = [\tilde{\mathbf{C}}_0 \quad -\tilde{\mathbf{C}}_0 \mathbf{P}] \tag{22}$$

where $\mathbf{P}$ is a permutation matrix that circularly shifts the columns of $\tilde{\mathbf{C}}_0$ to the left by L values, i.e.:

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{L\times(M-L)} & \mathbf{I}_L \\ \mathbf{I}_{M-L} & \mathbf{0}_{(M-L)\times L} \end{bmatrix}. \tag{23}$$

We have thus introduced the values in $\tilde{\mathbf{C}}_1$ in (17) that were missing to a cyclic matrix using the entries from $\tilde{\mathbf{C}}_0$ in (16). Substituting $\tilde{\mathbf{C}} = \tilde{\mathbf{C}}_{\mathrm{cycl}} + \tilde{\mathbf{C}}_{\mathrm{err}}$ into (20) and decomposing it into sub-equations, yields

$$\text{I.} \quad \mathbf{S}_{1,\mathrm{red}}^T \mathbf{E} \mathbf{W}_M (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{C}}_0 \mathbf{P}) \mathbf{W}_M^*/M \cdot \mathbf{S}_{1,\mathrm{red}} = \mathbf{I}_N \tag{24}$$

$$\text{II.} \quad -\mathbf{S}_{1,\mathrm{red}}^T \mathbf{E} \mathbf{W}_M \tilde{\mathbf{C}}_0 \mathbf{P} \mathbf{W}_M^*/M \cdot \mathbf{S}_{1,\mathrm{red}} = \mathbf{0}_N \tag{25}$$

$$\text{III.} \quad \mathbf{S}_{1,\mathrm{red}}^T \mathbf{E} \mathbf{W}_M \tilde{\mathbf{C}}_0 \mathbf{W}_M^*/M \cdot \mathbf{S}_{1,\mathrm{red}} = \mathbf{0}_N \tag{26}$$

Note that since $(\tilde{\mathbf{C}}_1 + \tilde{\mathbf{C}}_0 \mathbf{P})$ is a circular matrix,

$$\mathbf{W}_M (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{C}}_0 \mathbf{P}) \mathbf{W}_M^*/M = \mathbf{D} \tag{27}$$

in (24) is a diagonal matrix with

$$[\mathbf{D}]_{k,k} = C(e^{j2\pi k/M}), \qquad k = 0, \ldots, M-1. \tag{28}$$

C. Calculation of Equalizer Matrix $\mathbf{E}$

In a third step we decompose $\mathbf{E}$ into two independent column sets of equalizer coefficients

$$\mathbf{E} = \mathbf{E} \cdot \underbrace{(\mathbf{S}_0 + \mathbf{S}_1)}_{\mathbf{I}_M} = \underbrace{\mathbf{E} \cdot \mathbf{S}_0}_{\mathbf{E}_0} + \underbrace{\mathbf{E} \cdot \mathbf{S}_1}_{\mathbf{E}_1} \tag{29}$$

$$= \underbrace{\mathbf{S}_{1,\mathrm{red}} \mathbf{E}_{0,\mathrm{red}} \mathbf{S}_{0,\mathrm{red}}^T}_{\mathbf{E}_0} + \underbrace{\mathbf{S}_{1,\mathrm{red}} \mathbf{E}_{1,\mathrm{red}} \mathbf{S}_{1,\mathrm{red}}^T}_{\mathbf{E}_1} \tag{30}$$

with $\mathbf{S}_{0,\mathrm{red}}$ being obtained from $\mathbf{S}_0 = \mathbf{I}_M - \mathbf{S}_1$ by reduction to its nonzero columns. Fig. 2 illustrates the different sub-matrices of $\mathbf{E}$. Note that the number of relevant rows in $\mathbf{E}$ reduces to the number of used carriers N. $\mathbf{E}_0$ contains the linear combinations

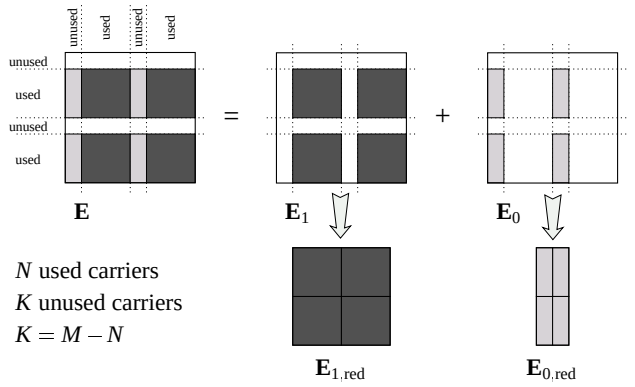

Fig. 2. Decomposition of equalizer matrix. White rows and columns denote zero entries.

of used subcarriers with unused carriers, whereas $\mathbf{E}_1$ includes all combinations with used carriers. $\mathbf{E}_{0,\text{red}}$ and $\mathbf{E}_{1,\text{red}}$ denote the nonzero parts of $\mathbf{E}_0$ and $\mathbf{E}_1$, respectively.

Substituting (30) and (27) into (24) and taking into consideration that $\mathbf{S}_{1,\text{red}}^T \mathbf{S}_{1,\text{red}} = \mathbf{I}_N$, yields

$$\mathbf{E}_{0,\text{red}} \underbrace{\mathbf{S}_{0,\text{red}}^T \mathbf{D} \mathbf{S}_{1,\text{red}}}_{\mathbf{0}} + \mathbf{E}_{1,\text{red}} \underbrace{\mathbf{S}_{1,\text{red}}^T \mathbf{D} \mathbf{S}_{1,\text{red}}}_{\mathbf{D}_{\text{red}}} = \mathbf{I}_N \tag{31}$$

The solution of the upper equation is independent of $\mathbf{E}_{0,\text{red}}$ and exists as long as the channel does not have any spectral nulls at the used subcarrier frequencies. It is given by:

$$\mathbf{E}_{1,\text{red}} = \mathbf{D}_{\text{red}}^{-1} \tag{32}$$

The remaining free parameters $\mathbf{E}_{0,\text{red}}$ in the equalizer matrix $\mathbf{E}$ will now be used for solving (25) and (26). A sufficient condition for solving both, (25) and (26), is that $\mathbf{S}_{1,\text{red}}^T \mathbf{E} \mathbf{W}_M$ lies in the left nullspace of $\tilde{\mathbf{C}}_0$, i.e.

$$\mathbf{S}_{1,\text{red}}^T \mathbf{E} \mathbf{W}_M \tilde{\mathbf{C}}_0 = \mathbf{0}_{N \times M} \tag{33}$$

Since $\tilde{\mathbf{C}}_0$ is an upper triangular matrix, see (16), its left nullspace is given by $[\mathbf{0}_{N \times (L_c-L-1)} \quad \mathbf{X}_{N \times M-(L_c-L-1)}]$ where $\mathbf{X}_{N \times M-(L_c-L-1)}$ denotes a matrix of don't care entries. Thus, in order to solve (25) and (26) we have to ensure that the first $L_c - L - 1$ columns of $\mathbf{S}_{1,\text{red}}^T \mathbf{E} \mathbf{W}_M$ are equal to zero, i.e.

$$\mathbf{S}_{1,\text{red}}^T \mathbf{E} \mathbf{W}_M \begin{bmatrix} \mathbf{I}_{L_c-L-1} \\ \mathbf{0}_{(M-(L_c-L-1)) \times (L_c-L-1)} \end{bmatrix} = \mathbf{0}_{N \times (L_c-L-1)} \tag{34}$$

Substituting (30) into (34), we obtain

$$\mathbf{E}_{0,\text{red}} \underbrace{\mathbf{S}_{0,\text{red}}^T \mathbf{W}_M \begin{bmatrix} \mathbf{I}_{L_c-L-1} \\ \mathbf{0}_{(M-(L_c-L-1)) \times (L_c-L-1)} \end{bmatrix}}_{\mathbf{W}_0} =$$

$$-\mathbf{E}_{1,\text{red}} \underbrace{\mathbf{S}_{1,\text{red}}^T \mathbf{W}_M \begin{bmatrix} \mathbf{I}_{L_c-L-1} \\ \mathbf{0}_{(M-(L_c-L-1)) \times (L_c-L-1)} \end{bmatrix}}_{\mathbf{W}_1} \tag{35}$$

This equation can be solved if the number of columns in $\mathbf{E}_{0,\text{red}}$, which equals the number K of unused carriers, corresponds to the column rank of $\mathbf{W}_1$. The latter can at most be equal to its number of columns $L_c - L - 1$. Thus, we obtain $\mathbf{E}_{0,\text{red}}$ as

$$\mathbf{E}_{0,\text{red}} = -\mathbf{E}_{1,\text{red}} \mathbf{W}_1 \mathbf{W}_0^{\dagger}, \quad K \geq L_c - L - 1 \tag{36}$$

where $\mathbf{W}_0^{\dagger}$ denotes the pseudo inverse of $\mathbf{W}_0$. The nonzero entries of the resulting equalization matrix $\mathbf{E}$ are illustrated in Fig. 3. Thanks to its sparse structure, the implementation cost of the ZF block equalizer is low. The entries on the diagonal are identical to the original DMT equalizer with one tap per subband. All that needs to be added are linear combinations of each used carrier with each unused carrier.

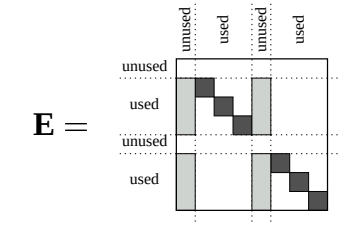

Fig. 3. Entries of equalizer matrix

IV. Simulation Results

We compared the proposed new equalization scheme with conventional DMT using a flexible, modular Matlab-simulated multi-tone system. Previous measurements based on a proprietary DMT solution [8] provided a pool of realistic channel data from twisted-pair loops with lengths from 2 to 30 km and a diameter of 0.9 mm. However, in order to apply these data for simulation, sampling rate and bandwidth needed to be restricted to the requirements of the mentioned DMT system with $f_S = 48$ kHz and $B = 21$ kHz ranging from 300 Hz to 21.3 kHz. We assumed AWGN with a noise floor of -30 dB, normalized to the signal power. The effective length of the channels was shortened by a TEQ filter using standard methods for TEQ adaptation. The typical length for the TEQ was $L_{\text{TEQ}} = 9$ coefficients. We restricted our simulations to GDMT with exclusively frequency domain redundancy as the opposite to conventional DMT.

Fig. 4 shows the performance of conventional DMT with GI vs. GDMT without GI depending on the introduced redundancy either with, or without TEQ. The CIR of the used 10km channel has a length of 244 coefficients, shortened by the TEQ to effectively 13 coefficients. When using a TEQ we achieve optimum performance as expected for about $K, L = 10 \ldots 12$. Without TEQ the situation changes completely: While conventional DMT performance collapses due to severe ISI/ICI distortions, GDMT outperforms the TEQ-aided result with even less needed redundancy. We repeated this simulation for various twisted pair loops with always the same observation: Obviously, the FEQ has a 'built-in' capability for shortening the

CIR. Although $L_c \gg K$, ISI/ICI distortion is kept below the level of typical background noise. Thus, the 'real-life' performance of GDMT for $L = 0$ is much better than the perfect reconstruction property $K \geq L_c - 1$ indicates. A separately TEQ filter, adapted after standard methods, is at least suboptimal if not obsolete at all.

With this knowledge, we compared the efficiency of time domain redundancy vs. frequency domain redundancy. Now, no TEQ was used for GDMT. For the variety of test channels, TEQ-aided DMT showed its maximum performance for an average GI length $L_g = 8$, while GDMT typically needed $K = 8$. As an example, Fig. 5 compares data rate of DMT and GDMT depending on the introduced redundancy for two different channels. It can be seen that GDMT outperforms DMT for shorter loops. For longer loops, maximum data rate is about equal. In both cases, GDMT offers a broader range of close-to-peak performance and is therefore less sensitive to mismatch of the introduced redundancy. Note that FEQ-only GDMT, for any amount of introduced redundancy K, always operates with the shortest possible system latency.

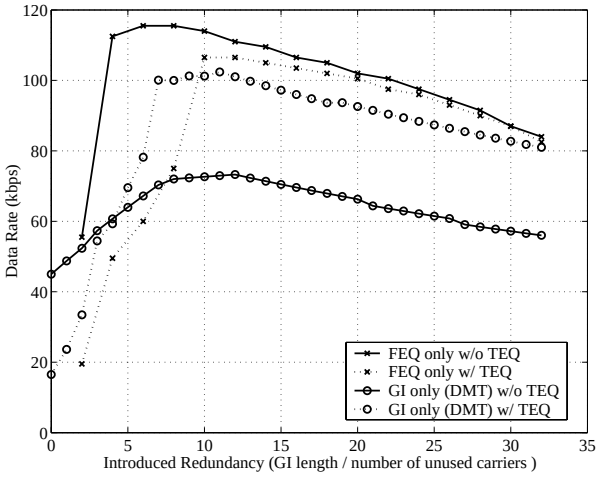

Fig. 4. Performance depending on TEQ usage for FEQ-only GDMT vs. DMT (10km twisted-pair loop)

V. Conclusion

In this paper we presented a new, general approach to equalization of multi-tone systems which includes conventional DMT as a special case. In addition to GI insertion, the GDMT algorithm incorporates frequency domain redundancy by extending the DMT single-tap equalizer to a block equalizer. We have given sufficient conditions for zero-forcing equalization and shown that the only non-zero entries in the block equalizer are the ones that are already present in conventional DMT plus additional branches that combine used subcarriers with unused subcarriers. GDMT thus utilizes the ISI/ICI leakage caused by the FFT operation at the receiver side. This leakage effect is especially strong and wide-spread for DMT 'thanks' to the poor

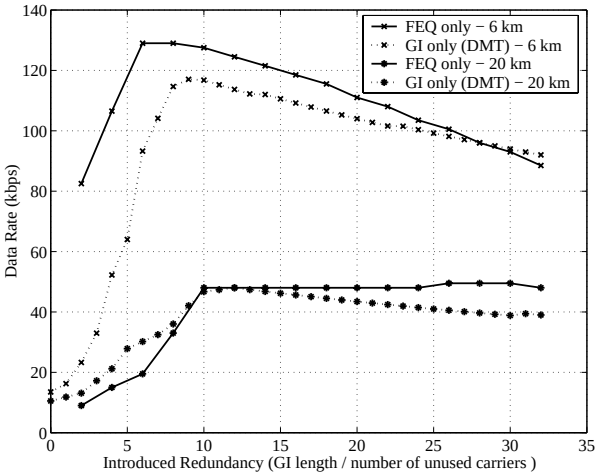

Fig. 5. Performance of FEQ-only GDMT (fixed latency) vs. DMT (increasing latency) for twisted-pair wires of different lengths

spectral selectivity of the DFT basis filters. While it is generally considered as one of the disadvantages of DMT, it here presents its positive aspects.

The application of GDMT can be seen in two different ways: First, it allows to utilize existing frequency domain redundancy in practical DMT systems. This is an essential advantage to previous methods, because performance is increased without reducing the bandwidth efficiency. Second, and even more interesting, it is possible to intentionally transfer redundancy from time domain to frequency domain. Perfect reconstruction is maintained even if no GI is used and all redundancy is placed in the frequency domain.

With its simplicity and close relation to existing DMT systems GDMT opens the door to a new field of short-latency multi-tone applications.

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