Key Engineering Materials
ISSN: 1662-9795, Vols. 488-489, pp 246-249
doi:10.4028/www.scientific.net/KEM.488-489.246
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Online: 2011-09-21

Study on gradient-dependent nonlocal damage constitutive models of concrete under freeze-thaw action

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Keywords: gradient-dependent, nonlocal, damage, freeze-thaw

Abstract. Concrete materials with a large number of micro-cracks were defined as inhomogeneous materials. And the mechanical properties of inhomogeneous materials were regarded as high-order deformation gradient dependent. Starting from the nonlocal thermodynamics theory and the hypothesis of total energy equivalence, the gradient-dependent nonlocal damage constitutive models of concrete under freeze-thaw action would be established in this paper.

Introduction

Concrete is a composite material which consists of coarse aggregate, fine aggregate and cement paste. The composition of concrete materials is often ignored. And concrete materials are regarded as homogeneous continuum. What's more, concrete materials are customarily assumed to be initially isotropic. But concrete materials contain a large number of microcracks, especially at the interface between aggregate and mortar, even before the application of external load. When microcracks produced, the mechanical properties of concrete materials are no longer isotropic. So Noll defined the materials as inhomogeneous materials [1].

Due to the inhomogeneities in the calculation, many scholars presented non-local models based on the concept of space average [2,3]. Under freeze-thaw action, area weighted average nonlocal constitutive models of concrete was presented in ref.[4]. But the nonlocal constitutive models could not model strain-softening in the calculation. For inhomogeneous materials, the mechanical properties were regarded as high-order deformation gradient dependent [1]. The gradient-dependent nonlocal damage constitutive models of concrete for elasto-plasticity were presented in refs.[5,6]. Starting from the nonlocal thermodynamics theory and the hypothesis of total energy equivalence [7], the gradient-dependent nonlocal damage constitutive models of concrete under freeze-thaw action would be established in this paper.

Nonlocal thermodynamics theory

The nonlocal first law of thermodynamics can be described as the time change rate of kinetic energy $\dot{T}$ and the internal energy $\dot{U}$ is equal to the sum of the rate-work $\dot{W}$ done upon the body by the external forces and heat energy rate $\dot{Q}$. Thus mathematically

$$\dot{T}+\dot{U}=\dot{W}+\dot{Q} \tag{1}$$

The variables $\dot{T}$, $\dot{U}$, $\dot{W}$ and $\dot{Q}$ can be written as follows respectively

$$\dot{T}=\int_V \rho \boldsymbol{v}\cdot\dot{\boldsymbol{v}}dV \tag{2}$$

$$\dot{U}=\int_V \rho\dot{e}dV \tag{3}$$

$$\dot{W}=\int_V \rho\boldsymbol{f}\cdot\boldsymbol{v}dV+\int_A \boldsymbol{\sigma}\cdot\boldsymbol{n}\cdot\boldsymbol{v}dA \tag{4}$$

$$\dot{Q}=\int_V \rho\dot{r}dV-\int_A \boldsymbol{q}\cdot\boldsymbol{n}dA \tag{5}$$

where, ρ is mass density, $\boldsymbol{v}$ is velocity vector, e is internal energy density per unit mass, $\boldsymbol{f}$ is external body forces vector per unit mass, $\boldsymbol{\sigma}$ is stress tensor, $\boldsymbol{n}$ is surface direction cosine vector, $\boldsymbol{q}$ is the heat flow vector, $\dot{r}$ is distributed internal heat sources.

Substituting Eqs.(2)~(5) into Eq.(1) , we get

$$\int_V(\rho\boldsymbol{v}\cdot\dot{\boldsymbol{v}}+\rho\dot{e})\ dV=\int_V(\rho\boldsymbol{f}\cdot\boldsymbol{v}+\rho\dot{r})dV+\int_A\boldsymbol{\sigma}\cdot\boldsymbol{n}\cdot\boldsymbol{v}dA-\int_A\boldsymbol{q}\cdot\boldsymbol{n}dA \tag{6}$$

Using Green theorem

$$\int_A\boldsymbol{\sigma}\cdot\boldsymbol{n}\cdot\boldsymbol{v}dA=\int_V(\nabla\cdot\boldsymbol{\sigma})\cdot\boldsymbol{v}dV+\int_V\boldsymbol{\sigma}\cdot(\nabla\cdot\boldsymbol{v})dV \tag{7}$$

$$-\int_A\boldsymbol{q}\cdot\boldsymbol{n}dA=-\int_V\nabla\cdot\boldsymbol{q}dV \tag{8}$$

And equilibrium equation can be expressed as follow

$$\rho\dot{\boldsymbol{v}}=\nabla\cdot\boldsymbol{\sigma}+\rho\boldsymbol{f} \tag{9}$$

Substituting Eqs.(7)~(9) into Eq.(6) , we get

$$\int_V\rho\dot{e}dV=\int_V[\boldsymbol{\sigma}\cdot(\nabla\cdot\boldsymbol{v})+\rho\dot{r}-\nabla\cdot\boldsymbol{q}]dV \tag{10}$$

Eq.(10) is another expression of the nonlocal first law of thermodynamics.

Gradient-dependent nonlocal damage constitutive models under freeze-thaw action

Kachanov[8] considered the damage in a cross section to be measured by the relative area of voids. Most of the early damage theories followed this idea using a scalar as damage variable. And damage variables may be a scalar or a tensor. When selecting a scalar as damage variable, it implies that the damage is isotropic. However the mechanical properties of concrete with microcracks are no longer isotropic. A tensor should be selected as internal variable which can be described nonlocal damage properties of concrete. In ref.[5,6], strain gradient $\nabla\varepsilon$ and damage gradient ∇D were adopted respectively. It was verified that nonlocal effect of material could be described by a nonlocal internal variable[9]. So only damage gradient ∇D is adopted as internal variable of gradient-dependent nonlocal damage constitutive models of concrete in this paper.

Under freeze-thaw action, concrete is subjected to cyclic temperature field. Except for the outflow heat $\nabla\cdot\boldsymbol{q}$ and the inflow heat $\rho\dot{r}$, the dissipated energy with internal and external state change can all be transformed into heat. And the heat which is absorbed in the body raise entropy s. For volume V, one can obtain as follow

$$\int_V \rho\dot{s}dV = \int_V \frac{1}{T}[\boldsymbol{\sigma}_p : \dot{\boldsymbol{\varepsilon}}_p + Y\dot{D} + \boldsymbol{Y}^{(\nabla D)} \cdot (\nabla\dot{D}) + \rho\dot{r} - \nabla \cdot \boldsymbol{q}]dV \quad (11)$$

Where, $\boldsymbol{\sigma}_p$ is plastic stress tensor, $\boldsymbol{\varepsilon}_p$ is plastic strain tensor, T is temperature, D is damage variable, ∇D is damage gradient, Y , $\boldsymbol{Y}^{(\nabla D)}$ are the conjugate forces of thermodynamics corresponding to damage variable D and damage gradient ∇D respectively[6].

Substituting Eq.(11) into Eq.(10) , we get

$$\int_V \rho\dot{e}dV = \int_V [\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \boldsymbol{\sigma}_p : \dot{\boldsymbol{\varepsilon}}_p - Y\dot{D} - \boldsymbol{Y}^{(\nabla D)} \cdot (\nabla\dot{D}) + \rho T\dot{s}]dV \quad (12)$$

Where, $\boldsymbol{\varepsilon}$ is strain tensor. The potential function of thermodynamics is generally adopted in the expression of constitutive models, namely Helmholtz free energy function.

$$\psi = e - Ts \quad (13)$$

The time derivative of Eq.(13) is as follow

$$\dot{\psi} = \dot{e} - \dot{T}s - T\dot{s} \quad (14)$$

Substituting Eq.(14) into Eq.(12) , we get

$$\int_V \rho\dot{\psi}dV = \int_V [\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \rho s\dot{T} - \boldsymbol{\sigma}_p : \dot{\boldsymbol{\varepsilon}}_p - Y\dot{D} - \boldsymbol{Y}^{(\nabla D)} \cdot (\nabla\dot{D})]dV \quad (15)$$

The free energy is a function of all the state variables. The rate of the free energy ψ is expressed as follow

$$\dot{\psi}(\boldsymbol{\varepsilon}, T, \boldsymbol{\varepsilon}_p, D, \nabla D) = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}} : \dot{\boldsymbol{\varepsilon}} + \frac{\partial\psi}{\partial T}\dot{T} + \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}_p} : \dot{\boldsymbol{\varepsilon}}_p + \frac{\partial\psi}{\partial D}\dot{D} + \frac{\partial\psi}{\partial(\nabla D)} \cdot (\nabla\dot{D}) \quad (16)$$

Substituting Eq.(16) into Eq.(15) , we get

$$\begin{aligned}&\int_V [(\boldsymbol{\sigma} - \rho\frac{\partial\psi}{\partial\boldsymbol{\varepsilon}}) : \dot{\boldsymbol{\varepsilon}} - \rho(s + \frac{\partial\psi}{\partial T})\dot{T} - (\boldsymbol{\sigma}_p + \rho\frac{\partial\psi}{\partial\boldsymbol{\varepsilon}_p}) : \dot{\boldsymbol{\varepsilon}}_p - \\ &(Y + \rho\frac{\partial\psi}{\partial D})\dot{D} - (\boldsymbol{Y}^{(\nabla D)} + \rho\frac{\partial\psi}{\partial(\nabla D)}) \cdot (\nabla\dot{D})]dV = 0\end{aligned} \quad (17)$$

Because of the arbitrariness of $\dot{\boldsymbol{\varepsilon}}$, $\dot{T}$, $\dot{\boldsymbol{\varepsilon}}_p$, $\dot{D}$ and $\nabla\dot{D}$, we get

$$\boldsymbol{\sigma} = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}} \quad (18)$$

$$s = -\frac{\partial\psi}{\partial T} \quad (19)$$

$$\boldsymbol{\sigma}_p = -\frac{\partial\psi}{\partial\boldsymbol{\varepsilon}_p} \quad (20)$$

$$Y = -\frac{\partial\psi}{\partial D} \quad (21)$$

$$\boldsymbol{Y}^{(\nabla D)} = -\frac{\partial\psi}{\partial(\nabla D)} \quad (22)$$

The strain energy density expression of coupled thermo-elasto-plastic anisotropic damage is expressed as follow

$$\begin{aligned}\psi &= \frac{1}{2}\boldsymbol{\varepsilon}:\tilde{\boldsymbol{E}}^{(e)}(D):\boldsymbol{\varepsilon}+\frac{1}{2}\boldsymbol{p}:\tilde{\boldsymbol{E}}^{(p)}(D):\boldsymbol{p}+\frac{1}{2}(\nabla D)\cdot\tilde{\boldsymbol{E}}^{(\nabla D)}(D)\cdot(\nabla D)+\\ &\quad \theta\tilde{\boldsymbol{H}}^{(e)}(D):\boldsymbol{\varepsilon}+\theta\tilde{\boldsymbol{H}}^{(p)}(D):\boldsymbol{p}+\theta\tilde{\boldsymbol{H}}^{(\nabla D)}(D)\cdot\nabla D+\frac{1}{2}\tilde{\xi}(D)\theta^2\end{aligned} \quad (23)$$

Where, $\tilde{\boldsymbol{E}}^{(e)}(D)$ is the effective elastic modulus tensor with damage, $\tilde{\boldsymbol{E}}^{(p)}$ is the plastic hardening modulus tensor with damage, $\boldsymbol{p}$ is the accumulated plastic strain tensor, $\tilde{\boldsymbol{E}}^{(\nabla D)}(D)$ is the effective damage gradient modulus tensor with damage, θ is the temperature difference, $\tilde{\boldsymbol{H}}^{(e)}(D)$ is the effective thermal modulus tensor coupled with thermo-elasto-damage, $\tilde{\boldsymbol{H}}^{(p)}(D)$ is the effective thermal modulus tensor coupled with thermo-plasto-damage, $\tilde{\boldsymbol{H}}^{(\nabla D)}(D)$ is the effective thermal modulus vector coupled with thermo-damage and damage gradient, $\tilde{\xi}(D)$ is the effective specific heat coefficient with damage. The specific forms of parameters above are related to material properties and microscopic properties of damage. Due to the limit of space, they are not described in more details.

Summary

The mechanical properties of concrete materials with a large number of micro-cracks were regarded as high-order deformation gradient dependent. In the paper, damage gradient ∇D is adopted to describe nonlocal damage properties of concrete. Subjected to elasto-plastic field and cyclic temperature field, the gradient-dependent nonlocal damage constitutive models of concrete under freeze-thaw action are established.

Acknowledgement

This research is supported by the National Natural Science Foundation of P. R. China through Grant number 50908059.

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