WP1.1
revision of partial pre-orders

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Attached documents:

"Revising Partially Ordered Beliefs".
"Reasoning with Partially Ordered Information in a Possibilistic Logic Framework".

1 introduction

One interesting recent advance in AI, supported by the REV!GIS Project, in the managent of uncertainty is the using of partial pre-orders [BLPb, BLPa].

In many applications, the reliability relation associated with available information is only partially defined, while most of existing uncertainty frameworks deal with totally ordered pieces of knowledge. Partial pre-orders offer more flexibility than total pre-orders to represent incomplete knowledge. Moreover, they avoid to compare unrelated pieces of information.

The need of extension of uncertainty frameworks to partial pre-orders is even crucial if we consider the dynamic of knowledge. Namely, even if the available information is totally ordered, then it may happen, when using some updating operators [KM91], that incorporating a new piece of information leads to a partially ordered knowledge.

Another situation is when we merge multiple sources information. Indeed the application of some merging technique, like the one based on preferred subtheories [Bre89] [BCD$^+$93], can also result in a set of partially ordered pieces of information.

In other respects, total pre-orders are not suitable to model the case where decision making is impossible or not arbitrary, as it is shown in the folowing example.

2 Example

Suppose an agent has to make a decision on the cultivation of a new plot according to three rules given by experts.

A first rule, $R1$ specifies that if some agronomical conditions hold (warm climate, deep soil, acidity, etc...), called *condition 1*, then *the cultivation of tobacco is feasible*. The second rule $R2$ specifies that if the agronomical conditions (*condition 1*) hold and the zone is mildewed (mildew parasite could ruin the plantation) called *condition 2*, then *the cultivation of tobacco is not feasible*. The third rule $R3$ specifies that according to the regulation of production of tobacco, if the area of the plot is not greater than the authorized area, called *condition 3*, then *the cultivation of tobacco is feasible*.

The question is to define a pre-order on the three rules in order to make a decision on the cultivation of tobacco. Since $R2$ is more specific than $R1$, it is natural to prefer R_2 over R_1 (namely $R2 < R1$ holds). However, *condition 3* is not related with *condition 1* nor with *condition 2*, then it seems reasonable to consider $R3$ incomparable with $R1$ and $R2$. On contrast, if we want to impose a total pre-oder on rules, we have to set $R3$ relatively to $R2$, therefore there are two intuitive choices, either $R1 \leq R3$ or $R3 \leq R2$. In the first case, the following total pre-order holds: $R2 < R1 \leq R3$ this means that if *condition 1*, *condition 2* and *condition 3* are both satisfied, according to the total pre-order we make the decision that *the cultivation of tobacco is not feasible*. In the second case, the following total pre-order holds: $R3 \leq R2 < R1$ this means that if *condition 1*, *condition 2* and *condition 3* are satisfied, according to the total pre-order we make the decision that *the cultivation of tobacco is feasible*.

These two total pre-orders lead to contradictory decisions, and there is no reason to choose the first one or the second one, the choice can only be arbitrary. Since we think that arbitrary choices have to be excluded, we think that a better solution is to consider $R3$ and $R2$ as incomparable (and hence all total pre-orders are considered), and thus to define a partial pre-order between rules. In such cases, an epistemic state has to be represented by a partial pre-order on interpretations and revision operations of partial pre-orders by formulas have to be defined.

3 Extension to partial pre-orders

The generalization of the representation and revision of an epistemic state to a partial pre-order leads to the following diagram:

$$\begin{array}{ccc} \preceq_\Sigma & \rightarrow & \preceq_\Psi \\ \downarrow & & \downarrow \\ \preceq_{\Sigma \circ^{sy} \mu} & \rightarrow & \preceq_{\Psi \circ^{se} \mu} \\ \downarrow & & \downarrow \\ Bel^{sy}(\Psi \circ^{sy} \mu) & \equiv & Bel^{se}(\Psi \circ^{se} \mu) \end{array}$$

We propose a generalization of two revision operators to the case of partial pre-orders.

We propose the generalization of the semantic revision operations to partial pre-orders and the generalization of the syntactic counterparts of these operations to partially ordered belief bases. These generalizations are rather direct. On contrast the generalization of the mapping from the syntactic level to the semantic level is more problematical. We need to define a pre-order on set of formulas from set. The definition of a partial pre-order between formulas leads to two possible partial pre-orders between subsets of formulas. We have finally presented the syntactic computation of the belief set corresponding to an epistemic state.

For more details, see the attached documents, which contain two accepted papers to the 9th International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU'2002) and to the 9th International Workshop on Non-Monotonic Reasoning (NMR'2002).

4 Prospects

In a future work, we have to investigate the properties of these revision operators and the presented approach could be generalized to the revision of a partial pre-order by a partial pre-order. Moreover, in order to provide reversibility the encoding by polynomials of partial pre-orders on interpretations and partially ordered belief base could be investigated. The restriction to lattices could be an interesting way to explore.

Another future work is to develop more efficient algorithms for computing the belief set at the syntactical case, and to apply them to landcover change problem (see WP1.3) where available information is often partially ordered.

References

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