Simple Nonparametric Tests of Technological Change: Theory and Application to U.S. Agriculture

Ziv Bar-Shira and Israel Finkelshtain

Several nonparametric tests of technological change are proposed conditional on alternative maintained behavioral assumptions. The tests are simple as they require verification of axioms via binary comparisons that are analogous to those of the revealed preference theory. Our model allows a broad interpretation of technological changes, yet trivial rationalization of the data with either profit-maximization or cost-minimization behavior is excluded. The tests are applied to study the U.S. aggregate agricultural data, supporting the hypotheses of rankable and neutral Technological Variations.

Key words: neutral technological variations, nonparametric analysis, rankable technological variations, revealed technological superiority.

Nonparametric analyses of production data are aimed at testing the consistency of observed behavior with a particular optimization rule and certain technological restrictions, with no need for maintained assumptions about the form of the production function. These nonparametric methods have been developed by Afriat, Hanoch and Rothschild, and Varian (1984). They recently gained special popularity in analyzing technological changes and productivity in general, and in the agricultural sector in particular (e.g., Fawson and Shumway; Chavas and Cox 1988, 1990, 1994; Cox and Chavas; Tauer; Featherstone, Moghnieh, and Goodwin). Further contributions utilizing nonparametric methods include works by Silva and Stefanou, and Williams and Shumway. The former authors proposed a test for homothetic production by relaxing the linear homogeneity assumption and used the test to construct a range for the degree of homogeneity as well as a joint confidence region for the input demands while the latter proposed a series of aggregation tests.

Two main nonparametric approaches for testing technological changes have been proposed in the literature. The first is formulated by Chavas and Cox (1988, 1990) who derive conditions for testing technological changes jointly with profit maximization and cost minimization. The conditions account for both neutral and nonneutral as well as monotonic and nonmonotonic technological changes, and can be verified by linear programming. The tests are applied to investigations of technological change in the aggregate technology of U.S. agriculture (Chavas and Cox 1988), and technological progress in the American and Japanese industrial sectors (Chavas and Cox 1990). Recently, Chalfant and Zhang show that the associated quantitative indices measuring technological changes may not be invariant to changes in the measurements units.

Fawson and Shumway propose an alternative approach employing a variant of Varian's (1984) Weak Axiom of Profit Maximization (WAPM) to test the joint hypothesis of profit maximization and monotonic technological progress. Their test condition is intuitive and may be verified through simple binary comparisons. Fawson and Shumway study monotonic progress in the aggregate technologies of various subregions of U.S. agriculture. Their approach is also applied by Featherstone, Moghnieh, and Goodwin to investigate progress among microlevel farm technologies and by Williams and Shumway to test for aggregation subject to nonregres-

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The authors are lecturer and senior lecturer, respectively, in the Department of Agricultural Economics, the Hebrew University of Jerusalem, Rehovot, Israel.

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sive technological change.[1] However, their test does not account for nonmonotonic technical progress and does not distinguish between neutral and nonneutral technological changes.

This article suggests alternative nonparametric procedures for testing the specific structure of technological changes under profit maximization and cost minimization.[2] Drawing on Bar-Shira and Finkelshtain, who propose a nonparametric framework for analyzing pooled production data subject to technological variations, we extend and formalize the framework proposed by Fawson and Shumway and bridge it with the Chavas and Cox's methodology. The proposed procedures require only binary comparisons rather than solving linear programs as implied by the tests by Chavas and Cox. In addition, the proposed procedures bring descriptive insight and broaden the interpretation of technological changes. The generalization of Fawson and Shumway include (*a*) maintaining cost minimization as the maintained behavioral rule; (*b*) consideration of both neutral and nonneutral technological change; and (*c*) testing for both monotonic and nonmonotonic technological progress.

We apply our approach to the U.S. aggregate agriculture data set created by the U.S. Department of Agriculture (USDA) and reported by Ball et al. The analysis provides new empirical evidence regarding the nature and type of the technological changes experienced in American agriculture between 1948 to 1994. Our findings indicate that the technological changes over those years were neutral and rankable. Similar findings for the period ending in 1983 were reported by Chavas and Cox (1988), who used a different methodology. In addition we define a quantitative dual measure of the technological change, which is consistent with the ranking of the technologies associated with the different years. This measure is compared to the total factor productivity (TFP) index reported by the USDA.

After introducing notation, the concept of rankable technological changes developed by Bar-Shira and Finkelshtain is reviewed. This is followed by a discussion of monotone technological progress and neutral technological change. We then characterize the type of technological change via a series of tests, which are applied to the U.S. agricultural data. The article ends with final remarks summarizing the main findings.

Rankable Technological Variation

Technological change is a shift in the production set. Such a shift can take various forms, and the essence of nonparametric analysis of technological changes is to infer their specific structure from a given production data set. The data available to the researcher are T observations regarding the actual netput choices made by a competitive producer at different points in time. The netput vector at time t is denoted by $y_t \equiv (\mathbf{z}^t, -\mathbf{x}^t) \in \Re^n_+ \ \forall\ t \in \mathcal{T} \equiv \{1, \ldots, T\}$, where $\mathbf{z}^t$ and $\mathbf{x}^t$ are the output and input vectors. The corresponding market conditions are given by a vector of netput prices: $\mathbf{p}^t \equiv (q^t, w^t) \in \Re^n_+ \ \forall\ t \in \mathcal{T}$.

Each observation corresponds to a set of technological-environmental conditions given by a vector $\boldsymbol{\eta}^t \in \Re^m\ t \in \mathcal{T}$. It is assumed that $\forall\ t \in \mathcal{T}$, both $\mathbf{p}^t$ and $\boldsymbol{\eta}^t$ are known to the producer, but only $\mathbf{p}^t$ is observable by the researcher. The production technology is represented either by the Production Set (PS) $Y(\boldsymbol{\eta}^t)$ or by the Input Requirement Set (IRS) $V(\boldsymbol{\eta}^t, \mathbf{z}^t)$.

Since we allow for variations of the technological conditions from one observation to the next, each observation is characterized by its own PS or IRS. That is, the T observations correspond to a family of technologies: either $Y = \{Y^1, \ldots, Y^T\}$ or $V = \{V^1, \ldots, V^T\}$. It is worthwhile to note that each member V^t is itself a nested family of IRS corresponding to different levels of output.

It should be emphasized that if no structure is imposed on the technology shifts and thus on the technical change, then any data set can be rationalized as consistent with profit maximization or cost minimization and some family of technologies. For example, any data are consistent with profit maximization under the degenerate family of technologies $Y \equiv \{\{y^1\}, \ldots, \{y^T\}\}$ (all production sets are singletons with the actual netput as the unique production plan).

To eliminate such a trivial rationalization and still permit hypothesis testing, we follow Bar-Shira and Finkelshtain and impose a very general structure of technological change.

[1] Note that the concepts "nonregressive technical change" and "monotone technological progress" are equivalent.

[2] For consistency of data with a fairly general optimization hypothesis under any a finite number of constraints, see Chavas and Cox (1993).

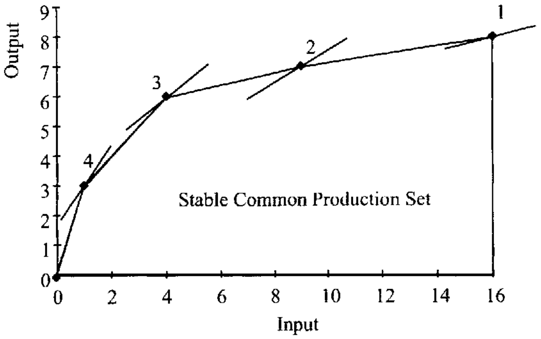

Figure 1. Common stable technology

This amounts to the assumption that the family Y is an ordered family. That is, the technologies associated with the various periods are different, but rankable, and therefore can be ordered from the least to the most advanced. Next, the formal definition is presented.

DEFINITION 1. *Rankable technological variation (RTV)*[3]*: A family of technologies, Y, exhibits RTV if and only if $\forall\ t, t' \in \mathcal{T}$, either $Y^t \subseteq Y^{t'}$, or $Y^{t'} \subseteq Y^t$.*

Thus, the relation $\subseteq$ is complete over the family Y; that is, any pair of technologies in Y are rankable. The completeness and the transitivity of the relation $\subseteq$ over the family Y imply that the technologies associated with the various years form an ordered family, so that $Y^{i_1} \subseteq Y^{i_2} \ldots \subseteq Y^{i_T}$, where i_1 and i_T denote the indices of the least and the most advanced technologies, respectively. The order does not necessarily correspond to the chronological order. While this structure accounts for neutral (Hicks) technological change, it also allows for many other types of nonneutral technological changes.

Consider a graphical example where the four netputs: (8, −16), (7, −9), (6, −4), (3, −1), and their respective prices: (1, 0.25), (1, 0.60), (1, 0.80), (1, 1.50) are observed. These data, the origin,[4] their convex monotonic hull, and the corresponding iso-profit lines are all presented in figure 1. Since the iso-profit lines in figure 1 cut through the convex monotonic hull, we conclude that these data are not consistent with profit maximization subject to a common stable technology. However, these data are consistent with profit maximization subject to an ordered family to technologies exhibiting RTV. An example of an ordered family of technologies rationalizing the data with profit maximization is as follows. Observation 1 is associated with the least advanced technology given by its convex monotonic hull (figure 2). Figure 3 shows the intermediate technology which could have produced the second observation and is given by the convex hull of observations 1 and 2. Observations 3 and 4 are associated with the most advanced technology, which is given by the convex monotonic hull of all the observations (figure 4).

To test for RTV jointly with profit maximization, we review a few concepts of revealed technological superiority, introduced by Bar-Shira and Finkelshtain. These concepts are based on the ideas presented by Varian (1983) in the context of a revealed preferences approach to consumer theory and they have

[3] Formally, we use the term "ordered" in conjunction with "family of technologies" and the term "rankable" in conjunction with "technological variation."

[4] The possibility of shut down is maintained.

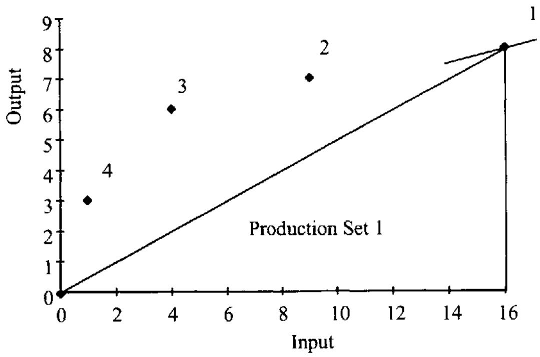

Figure 2. Ordered family—the least advanced technology

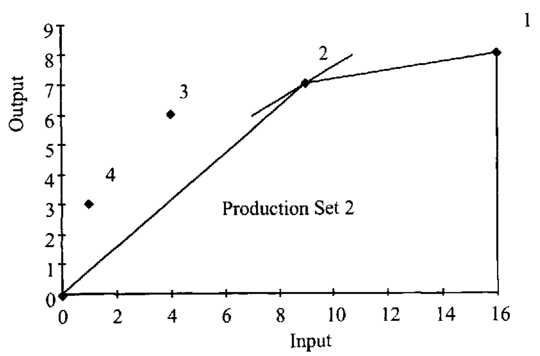

Figure 3. Ordered family—the intermediate technology

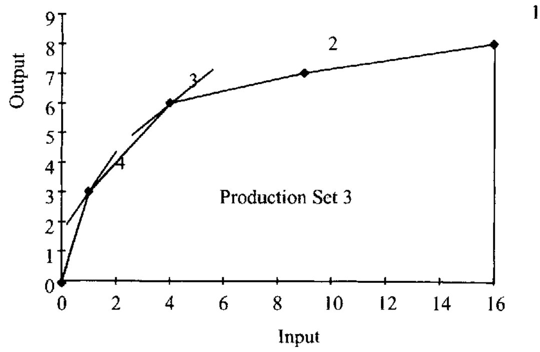

Figure 4. Ordered family—the most advanced technology

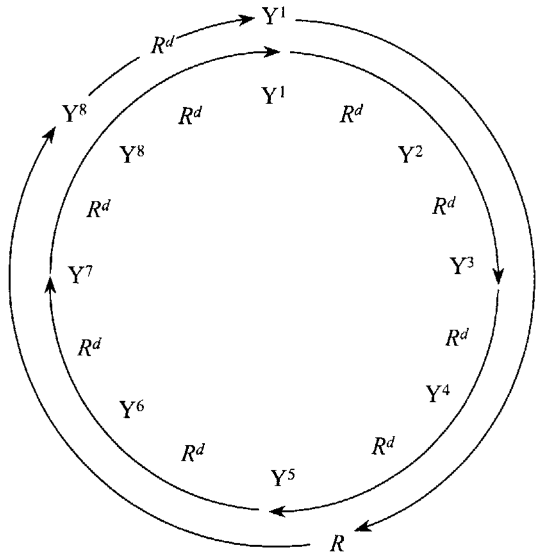

Figure 5. Violation of GARTS

different interpretations depending on the underlying optimization.

In analyses of consumer behavior, one does not observe the consumer's objective function—his or her utility. However, the budget constraint is observable which suffices to test whether a given data set is consistent with constrained utility maximization. For this purpose, Varian (1982) proposed the generalized axiom of revealed preferences (GARP). In analyses of producer behavior, the value of the producer's objective function—his or her profit or cost—is observable, but the technological constraint—the production set or input requirement set—is not. Moreover, technological variations over time mean that the production set varies between observations. Bar-Shira and Finkelshtain show that in such cases procedures that resemble GARP can be employed to nonparametrically test for consistency of production data with profit maximization and RTV.

Formally, let $t, t' \in \mathcal{T}$. When we observe $p^t y^t < p^t y^{t'}$ we say that the technology $Y^{t'}$ is directly revealed superior to the technology Y^t and denote $Y^{t'} R^d Y^t$, since had $y^{t'}$ been feasible at year t, it would have been chosen under profit maximization. When we observe the opposite $p^t y^t \geq p^t y^{t'}$ we say that the technology $Y^{t'}$ is not directly revealed superior to the technology Y^t and denote $Y^{t'}$ not $\not R^d$ Y^t. When technological superiority is revealed through a third party, $Y^{t''}$, we omit the word "directly" and denote $Y^{t'} R Y^t$. Under RTV, if either $Y^{t'} R^d Y^t$ or $Y^{t'} R Y^t$ then the binary relation $Y^t \subseteq Y^{t'}$ holds.

In the context of production analyses, Bar-Shira and Finkelshtain suggest that a more meaningful name for GARP would be "Generalized Axiom of Revealed Technological Superiority" (GARTS). GARTS is an empirically verifiable condition, defined formally as:

Definition 2. *GARTS: If $Y^t R Y^{t'}$ then $Y^{t'}$ $\not R^d$ Y^t.*

Figure 5 illustrates a case in which GARTS is violated. The technology Y^1 is revealed superior to Y^8, and at the same time the technology Y^8 is directly revealed superior to Y^1, thereby closing a circle. When any such a circle can be closed, GARTS is violated, indicating that the observed behavior is not consistent with transitivity.

Rationalizing data, in the current context, simply means that there exists a family of technologies under which profit maximization would have generated the observed data.

Formally,

Definition 3. *Rationalizing data with profit maximization: A family of production sets Y p-rationalizes observed producer behavior if $p^t y^t \geq p^t y \ \forall \ y \in Y^t$.*

Theorem 1 sets the theoretical basis for the test for rankable technological changes under profit maximization. It shows that GARTS facilitates testing the joint hypotheses of profit maximization with rankable technological changes.

Theorem 1. *Consider a set of observations $y^1, \ldots, y^T$, corresponding to market conditions given by $p^1, \ldots, p^T$. The following state-*

ments are equivalent: (a) There exists an ordered family of production sets that p-rationalizes the data. (b) The data satisfy GARTS. (c) There exists an ordered family of closed, convex, negatively monotonic production sets that p-rationalizes the data.

Proof. See Bar-Shira and Finkelshtain.[5]

Condition 2 of theorem 1 facilitates the test. If the data satisfy GARTS (condition 2), then we are assured that there is an ordered family of technologies rationalizing the observed choice as consistent with profit maximization. Thus, when profit maximization is maintained, condition 2 makes a simple nonparametric test for RTV.

At this point, it is of interest to compare our test to others. If the data pass GARTS, then there exist h^t, γ^t which satisfy $h^t - h^s + \gamma^t p^t(y^t - y^s) \geq 0$, $\forall\ s, t \in \mathcal{T}$.[6] This is a test of weak separability of all netputs from the technological change presented by Varian (1984, p. 588) and Chavas and Cox [1988, equation (6), p. 305]. It is worth mentioning that if the production technology cannot be represented by some functional form, then GARTS generalizes equation (6) of Chavas and Cox to those cases. But when production technology is representable by some functional form, then both tests constitute necessary and sufficient conditions for profit maximization subject to a well-behaved production function exhibiting weak separation of all netputs from the technological change. The benefit of GARTS is twofold. First, it simplifies the computation, as it requires only pairwise comparisons rather linear programming.[7] The higher level of computational complexity associated with the linear programming may become crucial, especially in large problems. Second, it brings new economic interpretation to the notion of weak separability of all netputs from the technical change, implying a nested (ordered) family of production sets which can rationalize the data with profit maximization. It is important to note that this separability does not imply separability of all inputs from the technical change and, hence, does not imply Hicks neutrality.

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[5] In the proof, observations have two indices as the data are pooled. In the current context one index will do.

[6] The proof follows from Afriat's theorem as stated in Varian (1982).

[7] A computer code verifying GARTS (condition 2) is just four lines long (Varian 1982, p. 972).

Monotone Technological Progress

In many empirical studies of time series the prior hypothesis maintains monotone technological progress and implicitly assumes that adverse technological changes are usually rare. The proposed procedures for testing monotone technological progress rely on the theory reviewed in the previous section. Intuitively, after verifying GARTS, all that is left to verify is whether the ordered family Y is chronologically ordered or not. In the sequel, we formulate two alternative procedures for these tests, one for each of the behavioral conjectures: cost minimization and profit maximization. We begin with the formal definition.

DEFINITION 4. *Monotone technological progress: A family of technologies Y exhibits monotone technological progress if and only if its members form a chronologically ordered family such that* $Y^1 \subseteq Y^2 \subseteq Y^3 \ldots \subseteq Y^T$.

Monotone Technological Progress with Profit Maximization

To test for technological progress, one can strengthen condition 2 of theorem 1. That is, if GARTS holds and $\forall\ t, t' \in \mathcal{T}\ t > t'$ implies $Y^{t'} \subseteq Y^t$ ($Y^{t'} \not\supseteq Y^t$), then technological progress is accepted. Intuitively, the extension to condition 2 assures that the family rationalizing the data is not simply ordered but is chronologically ordered.

A modified version of Varian's Weak Axiom of Profit Maximization (WAPM) (Fawson and Shumway; Featherstone, Moghnieh, and Goodwin) constitutes a simpler nonparametric test for technological progress subject to profit-maximizing behavior. The Modified WAPM compares current realized profits with alternative profits associated only with past observed netputs rather than all observed netputs. Thus, the Modified WAPM accomodates technological progress by excusing violations associated with future netputs on the grounds that they simply were not currently available. Theorem 2 formalizes these ideas and presents a bridge between the work of Fawson and Shumway and this work by showing the equivalence of the Modified WAPM and the strengthened GARTS.

THEOREM 2. *Consider a set of observations* $y^1, \ldots, Y^T$, *corresponding to market condi-*

tions given by $p^1, \ldots, p^T$. *The following statements are equivalent: (a) There exists a chronologically ordered family of production sets that p-rationalizes the data. (b) The data satisfy* $\forall\ t, t' \in \mathcal{T}, t > t'$ *implies* $p^t y^t \geq p^t y^{t'}$ *(Modified WAPM). (c) The data satisfy GARTS and* $\forall\ t, t' \in \mathcal{T}, t > t'$ *implies* $Y^{t'} \subseteq Y^t$ $(Y^{t'} \not R\ Y^t)$. *(d) There exists a chronologically ordered family of closed, convex, negatively monotonic production sets that p-rationalizes the data.*

Proof. $(a) \Rightarrow (b)$: We will show that (b) is necessary for (a). Maintaining rationalization by an ordered family, $t' > t$ and $p^{t'}y^{t'} < p^{t'}y^t$ imply $Y^{t'} \subset Y^t$ which violates chronological order.

$(b) \Rightarrow (c)$: If $\forall\ t' > t \in \mathcal{T}\ p^{t'}y^{t'} \geq p^{t'}y^t$ then $\forall\ t' > t \in \mathcal{T}$ it is not the case that $Y^t R^d Y^{t'}$, which in turn implies both GARTS and $\forall\ t' > t \in \mathcal{T}$ it is not the case that $Y^t R Y^{t'}$.

$(c) \Rightarrow (d)$. We prove by construction. Let the tth year technology be defined as

$$(1)\qquad Y^t \equiv com^-\{y^1, \ldots, y^t\}.$$

The above construction implies that $\forall\ t \in \mathcal{T}$ $y^t \in Y^t$, namely, y^t is feasible under Y^t. Furthermore, the family Y exhibits chronological order and each of its members is closed, convex, and negatively monotonic. It now remains to show that the above family p-rationalizes the data.

We must prove that $\forall\ y \in Y^t\ p^t y \leq p^t y^t$, $\forall\ t \in \mathcal{T}$. From the definition of Y^t it follows that any $y \in Y^t$ can be expressed as $y = \sum_{\tau=1}^{t} \omega^\tau(y^\tau - e^\tau)$, where $e^\tau > 0$, $\omega^\tau \geq 0$, and $\sum \omega^\tau = 1$; and from (c) it follows that $p^t y^\tau \leq p^t y^t\ \forall\ t, \tau \in \mathcal{T}$. The latter implies $\omega^\tau p^t(y^\tau - e^\tau) \leq \omega^\tau p^t y^t$. Now, sum over τ from 1 to t and use the former to get the result.

$(d) \Rightarrow (a)$: Trivial, since (d) is stronger.

If the data satisfy condition 2 of theorem 2, then one is assured that there is a family of technologies rationalizing the observed choices as consistent with profit maximization and technological progress. As profit maximization is a maintained hypothesis in the above tests, failure of the data to satisfy the conditions of theorems 1 and 2 may be the result of nonmaximization of profit. This could, mistakenly, lead researchers to reject monotone technological progress as well as RTV in favor of arbitrary technological changes. A partial solution to this problem is to formulate analogous tests based on a weaker maintained optimization assumption.

A clear candidate is cost minimization. For example, firms may choose an efficient input combination to minimize cost, even if they operate in an environment with price and/or multiplicative technological risks, or if they possess market power. In both situations, the maintained behavioral rule in theorems 1 and 2 based on profit maximization for given prices would not be fulfilled. Also, if the technology exhibits constant or increasing returns to scale, then the profit-maximization hypothesis is not relevant, while cost minimization remains a meaningful behavioral postulate. Therefore, we propose a variation of the above test, based on cost minimization rather than profit maximization.

Monotone Technological Progress with Cost Minimization

We employ a modified version of Varian's Weak Axiom of Cost Minimization (WACM) to test for monotone technological progress based on cost minimization. The WACM states that all input bundles capable of producing more output than the bundle x^t must be more expensive than x^t; otherwise cost minimization is violated. The Modified WACM is less restrictive: violation of cost minimization is declared only if there exists an earlier input bundle which had produced more output than the current bundle x^t and costs less at the current prices w^t. Violations associated with netputs chosen later than time t are forgiven on the grounds that their underlying technologies were not available at time t. The Modified WACM thus allows for monotone technological progress.

We turn to formal definitions and a corresponding theorem.

DEFINITION 5. *Rationalizing Data with Cost Minimization—A family of production sets: Y c-rationalizes observed producer behavior if* $w^t x^t \leq w^t x\ \forall\ x$, *such that* $(z^t, -x) \in Y^t$.

Theorem 3 sets the theoretical basis for the test of monotone technological progress based on cost minimization.

THEOREM 3. *Consider a set of observations* $(z^1, -x^1), \ldots, (z^T, -x^T)$, *corresponding to market conditions given by* $w^1, \ldots, w^T$. *The following statements are equivalent: (a) There exists a chronologically ordered family of production sets that c-rationalizes the data.*

(b) $\forall\, t, t' \in \mathcal{T}$, if $z^{t'} \geq z^t$ and $t > t'$, then $w^t x^t \leq w^t x^{t'}$ *(Modified WACM). (c) There exists a chronologically ordered family of closed, convex, negatively monotonic production sets that c-rationalizes the data.*

Proof. *(a)* ⇒ *(b)*: Suppose that $z^{t'} \geq z^t$ and $t > t'$. Since the production sets are chronologically ordered, $Y^{t'} \subseteq Y^t$, implying that $(z^{t'}, -x^{t'}) \in Y^t$. But, since the family of production sets rationalizes the data, it must be the case that $w^t x^t \leq w^t x^{t'}$.

(b) ⇒ *(c)*: We now have to show that there is a chronologically ordered family of closed, convex, negatively monotonic production sets that c-rationalize the data. The proof is constructive.

Define the *t*th year production set as

$$(2) \qquad Y^t \equiv \text{com}^-\{y^1, \ldots, y^t\}.$$

As required, the above construction implies that $\forall\, t \in \mathcal{T}\; y^t \in Y^t$. Furthermore, the family Y is chronologically ordered and each production set is closed, convex, and negatively monotonic. It is now left to show that the above family c-rationalizes the data.

We must demonstrate that $x \in V^t(z^t)$ implies $w^t x \geq w^t x^t$, $\forall\, t \in \mathcal{T}$. First, recall that the definition of $V^t(z^t)$ is given by

$$(3) \qquad V^t(z^t) \equiv \{x \,|\, (z^t, -x) \in Y^t\}$$

which indicates that the convexity and negative monotonicity of Y^t induces the convexity and positive monotonicity of V^t. The latter implies that $\forall\, x \in V^t(z^t)$ x can be expressed as

$$(4) \qquad x = \sum_{\tau=1}^{t} \delta^\tau \omega^\tau (x^\tau + e^\tau)$$

where $\omega^\tau > 0$, $e^\tau > 0$, $\Sigma\, \omega^\tau = 1$, and

$$(5) \qquad \delta^\tau = \begin{cases} 1, & \text{if } z^\tau \geq z^t \\ 0, & \text{if } z^\tau < z^t. \end{cases}$$

Condition 2 ensures that each x^τ for which $\delta^\tau = 1$, satisfies $w^t x^\tau \geq w^t x^t$, which after trivial manipulation yields

$$(6) \qquad \delta^\tau \omega^\tau w^t (x^\tau + e^\tau) \geq \delta^\tau \omega^\tau w^t x^t.$$

Now sum over τ from 1 to t and use equation (4) to obtain the result.

(c) ⇒ *(a)*: Trivial, since *(c)* is stronger.

Again, it is worth emphasizing that condition 2 of the theorem constitutes a practical test for monotone technological progress under cost minimization. By simple binary comparisons, we can examine whether a specific time series satisfies condition 2. If it does, then we are assured that there is a family of technologies rationalizing the observed choices as consistent with cost minimization and monotone technological progress.

Neutral Technological Change

An important question in the context of technological change is the degree of neutrality. That is, researchers are often interested in ascertaining whether the technical rate of substitution between inputs is preserved or modified by the technological change. Technological changes preserving the technical rate of substitution are regarded in the literature as the most general definition of Hicks-neutral changes. A general equivalent definition of Hicks neutrality suggests that the technologies associated with the various years rank any pair of input bundles equally. That is, if an input bundle x produces more output than another bundle x' under some technology, then x produces more output than x' under any other technology. We call this property input-rank-preserving technological variation (RPTV) and define it formally as follows:

DEFINITION 6. *RPTV: A family of technologies V exhibits RPTV if for all* z', z'', *and* t'' *either* $V^{t'}(z') \subseteq V^{t''}(z'')$ or $V^{t''}(z'') \subseteq V^{t'}(z')$

Thus, a family of technologies exhibits RPTV if, and only if, all the technologies have identical isoquant maps. However, a certain isoquant may correspond to different output levels under the various technologies. Moreover, RPTV does not necessarily imply RTV and vice versa. A family of technologies may exhibit RPTV, but the ranking of the technologies may vary with output levels, meaning that RTV does not hold. Similarly, a family may exhibit RTV, but isoquants associated with different technologies and output levels may cross, violating RPTV.

To test for Hicks-neutral technological changes, we only need to test for RPTV. The formulation of the test requires the introduction of the Generalized Axiom of Revealed Factor Superiority (GARFS).

We begin with the terminology. When we

observe $w^t x^t > w^t x^{t'}$, we say that the input bundle x^t was directly revealed superior to the input bundle $x^{t'}$. The only information from such an observation is that under technology t, the input bundle x^t produces more than $x^{t'}$; otherwise, cost minimization would have imposed the employment of the latter. Note that unless $x^{t'}$ is strictly greater than x^t the finding $w^t x^t > w^t x^{t'}$ is inconclusive regarding technological superiority. The word "directly" is omitted if factor superiority is inferred by invoking transitivity.

The formal definition of GARFS is as follows:

DEFINITION 7. *GARFS: If* $x^{t'} R x^t$ *then* $x^t \not R^d x^{t'}$.

Rationalization in this context is as in definition 5. Theorem 4 sets the basis for the neutrality test and shows that GARFS facilitates testing whether the observed data are consistent with cost-minimizing behavior and RPTV (definition 6).

THEOREM 4. *Consider a set of observations* $x^1, \ldots, x^T$, *corresponding to market conditions given by* $w^1, \ldots, w^T$. *The following statements are equivalent: (a) There exists a family of IRSs that exhibits RPTV and c-rationalizes the data. (b) The data satisfy GARFS. (c) There exists a family of closed, convex, positively monotonic IRSs that exhibits RPTV and rationalizes the data.*

Proof. $(a) \Rightarrow (b)$: Maintaining (a), showing GARFS is the same as showing that $w^t x^t > w^t x^{t'}$ implies $x^{t'} \notin V^t(z^t)$. Suppose this is not the case; then $w^t x^t > w^t x^{t'}$ and $x^{t'} \in V^t(z^t)$. This would mean that $x^{t'}$ was feasible at the year at which the choice x^t was made, contradicting cost minimization, and thus violating (a).

$(b) \Rightarrow (c)$: We now have to show that there is a family of closed, convex, positively monotonic IRSs that exhibits RPTV and c-rationalizes the data. The proof is constructive, the first step being to sort the data into a $1 \leq L \leq T$ ordered family of equivalence sets, $\phi^1, \ldots, \phi^L$ such that

$$(7) \qquad x^t, x^{t'} \in \phi^l \Rightarrow w^t x^t \leq w^t x^{t'} \quad \text{and} \quad w^{t'} x^{t'} \leq w^{t'} x^t, \; l \in \{1, \ldots, L\}$$

and

$$x^t \in \phi^{l+1} \Rightarrow \exists x^{t'} \in \phi^l, \quad \text{such that} \quad w^t x^{t'} < w^t x^t.$$

Since the data passes GARFS, this construction is well defined, and any input vector belongs to one and only one of the ϕ^l.

Now, let $\phi^s \ni x^t$ and define the tth observation IRS as

$$(8) \qquad V^t(z^t) \equiv com^+\{\phi^1 \cup \phi^2 \cup \ldots \phi^s\}.$$

As required, the above construction implies that $\forall\, t \in \mathcal{T}\; x^t \in V^t$, namely, x^t belongs to V^t. Furthermore, the family V exhibits RPTV, and each of the IRSs is closed, convex, and positively monotonic. It is now left to show that the above family c-rationalizes the data.

We must show that $\forall\, x \in V^t(z^t)\; w^t x \geq w^t x^t$, $\forall\, t \in \mathcal{T}$. Let n^t indicate the number of members in $V^t(z^t)$ and note that any $x \in V^t(z^t)$ can be expressed as $x = \sum_{\tau=1}^{n^t} \omega^\tau(x^\tau + e^\tau)$, where $x^t \in V^t(z^t)$, $e^\tau > 0$, $\omega^\tau > 0$, and $\sum \omega^\tau = 1$. By construction, $w^t x^\tau \geq w^t x^t\; \forall\, x^\tau \in V^t(z^t)$; hence, it follows that $\omega^\tau w^t(x^\tau + e^\tau) \geq \omega^\tau w^t x^t$. Now, sum over τ from 1 to n^t to obtain the result.

$(c) \Rightarrow (a)$: Trivial, since (c) is stronger.

As in the previous theorems, condition 2 is the practical one. This condition can be verified by a simple algorithm, and if it holds, one can conclude that the technological change preserves the technical rate of substitution between inputs. In the next section we discuss the possible implementation of various combinations of the above tests. To each combination of failure-success in passing the four axioms associated with the four theorems we propose an interpretation regarding the technological and behavioral consequences.

Integrated Analylsis

The four nonparametric tests, corresponding to the four theorems presented above, are combined such that clear conclusions are reached regarding the nature of the technological changes. In other words, the data are tested for the four axioms—GARTS, GARFS, Modified WAPM, and Modified WACM—simultaneously. The fail-pass pattern then identifies whether the technological change is wild, rankable, neutral, neutral rankable, monotonically progressive, or neutral progressive (not all are identifiable under both behavioral rules).

Before we explain the identification procedure, note that the verification order is not

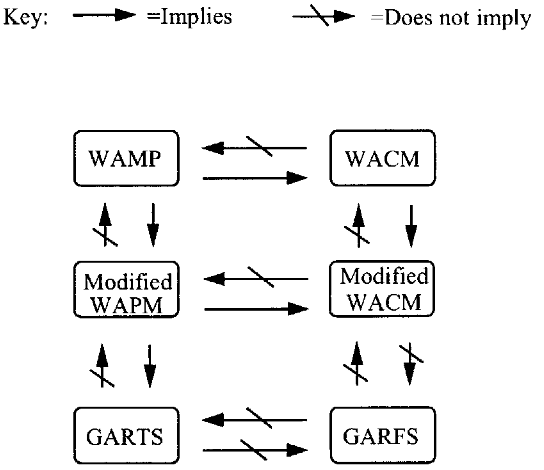

Figure 6. Relations between the axioms

significant and that not all possible fail-pass sequences are feasible, since some of the axioms are necessary for the others. For example, each of the modified axioms is necessary for its respective weak axiom, and the Modified WACM as well as GARTS are necessary for the Modified WAPM, but neither GARTS not GARFS is necessary for the other. These relationships are summarized in figure 6. Figure 7 outlines all feasible sequences of fail-pass and the implied nature of the technological variation. We illustrate with three examples.

1. The left branch indicates failure of all axioms. If profit maximization or cost minimization is maintained, this branch implies wild technological variations of which the degenerate technology mentioned earlier is a special case.
2. The right branch represents the case in which all axioms pass. This would imply consistency of the data with Hicks-neutral, monotonic technological progress joint with profit-maximizing behavior.
3. Consider the case of passing GARTS, failing Modified WAPM, failing GARFS, and passing Modified WACM. Since GARFS failed, Hicks neutrality is excluded. Now look at the pair passing GARTS and failing Modified WAPM, which supports the conjecture of profit-maximizing behavior sub-

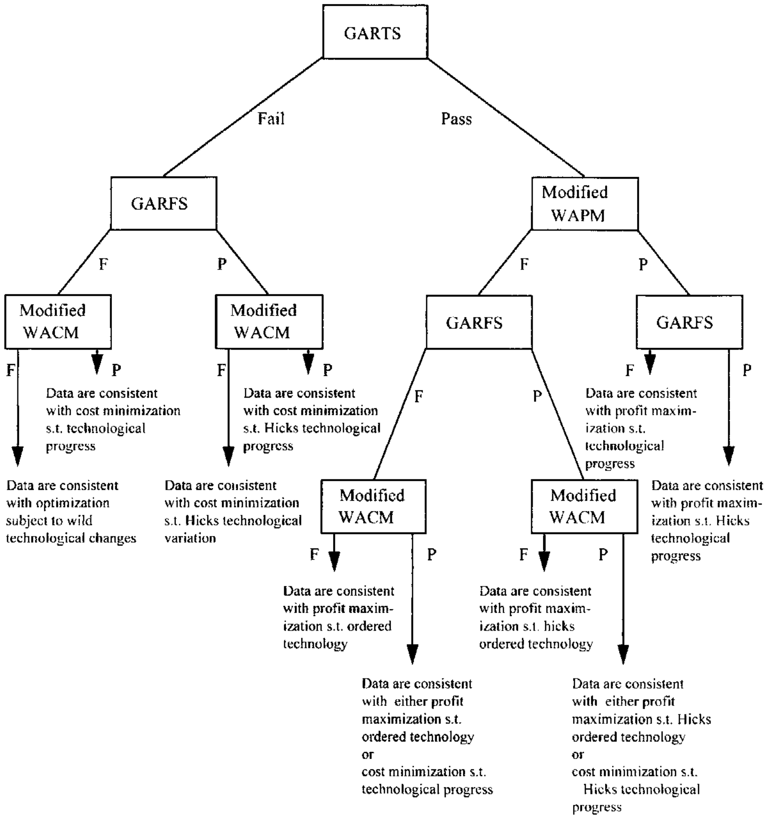

Figure 7. Test-conclusion tree

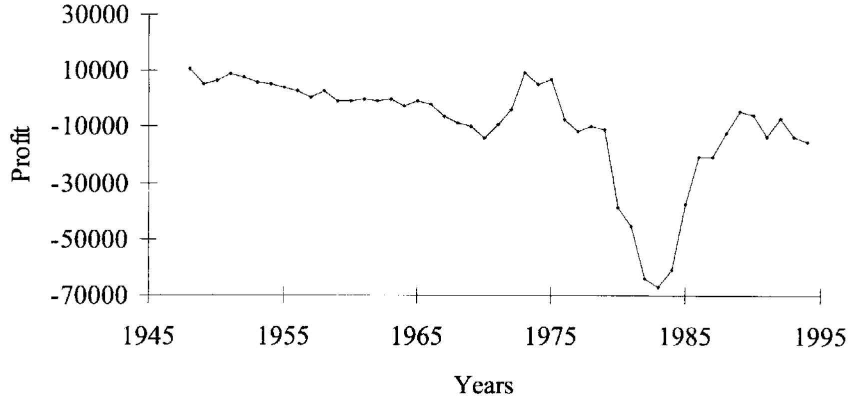

Figure 8. Economic profit (million $)

ject to rankable technological change. However, the pair failing Modified WAPM and passing Modified WACM supports cost-minimizing behavior subject to monotone technological progress. This is not a contradiction since models other than profit-maximizing behavior subject to rankable technological changes can generate data which pass GARTS, and models other than cost minimization subject to technological progress can generate data which pass Modified WACM. Both profit maximization subject to rankable technological changes and cost minimization subject to technological progress are possible models.

In general, passing GARTS is necessary for RTV joint with profit maximization. If the data also pass the Modified WAPM, then the technological variations are monotonically progressive. If, in addition, the data pass GARFS, then we know that the monotone technological progress is neutral (Hicks). Keeping in mind that the order of the tests is of no importance, the above logic applies to all branches of the tree.

Empirical Analysis

The current empirical analysis extends previous studies dealing with productivity and technological changes (Chavas and Cox 1988, Cox and Chavas 1990, and Chalfant and Zhang) which used aggregate agricultural data on output and ten inputs over the period 1948–83. The USDA recently made available an updated U.S. aggregate agricultural data set, which contains quantity indices and implicit prices of nine aggregate inputs and twelve aggregate outputs over the years 1948 to 1994. The aggregate output are Meat Animals, Dairy Products, Poultry and Eggs, Other Livestock, Food Grains, Feed Crops, Oil Crops, Sugar Crops, Cotton and Cotton-Seed, Vegetables and Melons, Fruit and Tree Nuts, and Other Crops; and the aggregate inputs are Agricultural Chemicals, Fuels and Electricity, Feed, Seed and Livestock Purchases, Other Purchased Inputs, Hired Labor, Self-Employed Labor, Durable Equipment, Real Estate, and Inventories. A complete description of the data set and its construction methodology is detailed in Ball et al. We only note that multiplication of the implicit price and the respective quantity index yields the actual value, which is a necessary condition for valid nonparametric analysis (Chalfant and Zhang).

The results of many studies, including Ball et al., indicate a significant productivity rise over time in the agricultural sector. It is interesting to look at the reflection of technological progress on profitability in the agricultural sector. Figure 8 presents the evolution of economic profit in the U.S. agricultural production sector. Economic profit declines as productivity rises. The yield of the technological progress in the agricultural sector is harvested elsewhere in the economy. Furthermore, economic profit becomes negative in later years, meaning that farmers do not recover the opportunity costs of their assets and labor. In-depth investigation of this observation is beyond the current scope, but possible explanations include measurement errors in the data, temporary disequilibrium, and unobserved benefits in agriculture as a way of life.

We now turn to the primary objective of this article—to characterize the nature of the

Table 1. Axiom Violations

Aggregation	WACM	Modified WACM	GARFS	WAPM	Modified WAPM	GARTS
			2 Outputs–3 Inputs			
Counts	483	7	2	1,077	27	6
Percentage	44.68%	0.65%	0.19%	50%	2.50%	0.56%
Goodness of Fit	1.66%	0.011%	0.00026%	142%	3.13%	0.0046%
			12 Outputs–9 Inputs			
Counts	451	7	1	1,078	32	10
Percentage	41.72%	0.65%	0.09%	50%	2.96%	0.93%
Goodness of Fit	1.9%	0.011%	0.00013%	133%	22.7%	0.0049%
Maximum Violations	1,081	1,081	1,081	2,162	1,081	1,081

technological change. Table 1 presents the results of testing for six axioms—GARTS, Modified WAPM, WAPM, GARFS, Modified WACM, and WACM[8]—at two aggregation levels. The first, relatively disaggregated, level includes the twelve outputs and nine inputs mentioned above, and the second includes two outputs, livestock and crops, and three inputs, purchased inputs, labor, and capital. Both WAPM and WACM failed at the two aggregation levels with violation rates of more than 40%, indicating inconsistency with optimizing behavior subject to a stable technology.

Next we tested for GARTS. At the higher aggregation level we found six violations; two of them were revealed directly and four were found through transitivity. The direct violations were between 1980 and 1973 and between 1980 and 1975, indicating that the production set associated with 1980 technology crosses those of 1973 and 1975. Eliminating the 1980 observation and retesting for GARTS yields no violation. Alternatively, a 5% increase in 1980 profit also eliminates all violations yielding a measure of goodness of fit of 0.0046%.[9] At the second aggregation level we found ten violations, four of which were revealed directly while the other six were revealed through transitivity. The direct violations were between 1980 and 1971, 1980 and 1972, 1980 and 1973, and 1971 and 1972. In this case, increases of 5% in 1980 profit and 0.3% in 1972 profit eliminate all violations. The resulting goodness-of-fit measure is 0.0049%. We interpret these results as evidence in favor of the maintained profit-maximizing hypothesis and a family of technologies exhibiting rankable technological variations.

Comparing the violation rates of GARTS with those of the Modified WAPM reveals economically significant differences. The violation rates of the latter are 6.75 and 3.2 times higher under the two aggregation levels, respectively. However, comparing the violation rates of GARTS with those of the Modified WACM does not reveal economically significant differences. Comparing the respective goodness-of-fit measures shows the same. Hence, the data are consistent with both cost-minimizing behavior subject to monotone technological progress and profit-maximizing behavior subject to an ordered family of technologies. The hypothesis of monotone technological progress can be accepted only under cost minimization.

The tests for GARFS result in two violations under the higher aggregation level and only one under the lower. The violations occurred between 1986 and 1991 and between 1987 and 1990 under the higher aggregation level, and between 1987 and 1990 under the lower aggregation level. Decreases of 0.15% in 1990 total factor costs and 0.14% in 1991 total factor costs remove all violations. The resulting goodness-of-fit measures are 0.00026% and 0.00013% under the two aggregation levels, respectively. Thus, the hypothesis of neutral technological changes is accepted independently of the optimizing behavior.

The analysis thus far indicates that, maintaining profit-maximizing behavior, the technology associated with the various years forms an ordered family; however, the order

[8] In the empirical application, WACM and Modified WACM test for cost minimization subject to a given level of revenue rather than to a given level of output.

[9] The goodness-of-fit measure is calculated by dividing the sum of total percentage deviations from the optimizing hypothesis by the total number of possible deviations. For more details, see Varian (1990).

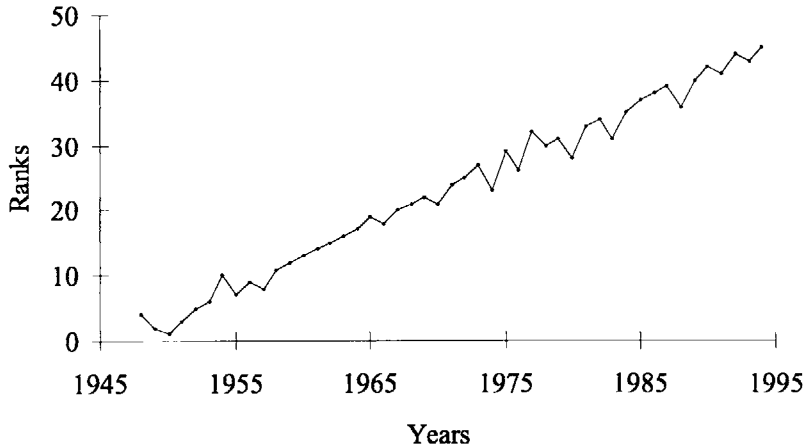

Figure 9. Ranking of the technologies

is obviously not chronological. This raises the question as to how the technologies rank. A simple sorting procedure based on the revealed superiority analysis presented above provides an easy approach to the answer. A technology is efficient as long as no other technology has been revealed superior to it. The sorting procedure is implemented as follows. All the efficient technologies are assigned the highest rank and put aside; then all the newly efficient technologies (among those remaining) are assigned the second highest rank and put aside, and so on.

Figure 9 presents the rank of forty-seven technologies. The graph, as expected, is not monotonic but shows a significant positive trend. This, in turn, constitutes evidence in favor of nonmonotone technological progress. To formally test for technological progress, we made use of the Spearman correlation coefficient between the rank and year vectors. We find that this correlation is 0.98826 with a p-value of 0.01% indicating a significant positive correlation between rank and time. Hence we conclude that the hypothesis of technological progress cannot be rejected at any reasonably significant level.[10]

It is interesting to compare the findings of the current article to the results of previous studies of technological change in U.S. agriculture. The comparison is focused on three characteristics of the technological change. The first is the ranking of the technologies in various years. The second is the magnitude of the change, and the third is its nature—biased versus neutral change.

We commence with the first issue by comparing the rank vector (figure 9) to the index of total factor productivity reported by Ball et al. (figure 10). Figures 9 and 10 show the same trend but imply different ranking in three time periods. The revealed superiority analysis ranks the 1973 technology ahead of the 1976 technology, that of 1975 ahead of 1980, and that of 1977 ahead of 1979, while the TFP measure ranks these three couples in reverse order. The study presented in this article shows that there does not exist a family of technologies which is ranked according to the order implied by the TFP measure, such that profit maximization with respect to it would have generated the observed data.

Comparison of the magnitude of the technological changes as measured by our approach and others requires the introduction of quantitative measure. For this purpose we suggest a quantitative dual measure of the technological differences. This allows us to bridge our methodology with other popular approaches including the TFP one. We measure the technological difference between adjoining technologies as the proportional increase in revenue per dollar expenses, when moving from the less advanced technology to the more advanced one, while keeping the prices unchanged. The calculation itself is conducted with respect to a family of lower-bound technologies. At each year, the technology is represented by a convex-monotonic hull of all equally and less-efficient observations relative to current observation, as de-

[10] This and the following results were obtained under the lower aggregation level.

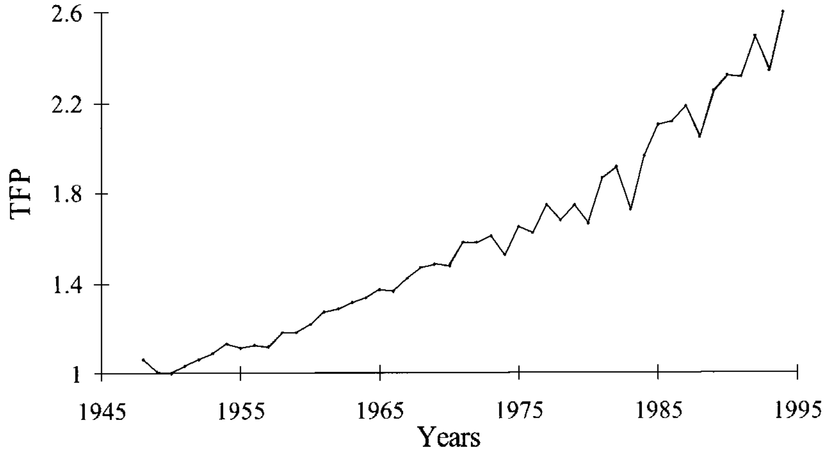

Figure 10. Total factor productivity

termined by the sorting procedure. Calculating the suggested measure involves no complicated optimization. Commencing with the least-efficient observation, one only needs to calculate the costs and the revenue of the next upper netput in current prices.

Figure 11 presents our dual measure and the TFP measure reported by Ball et al. Although there is a principal qualitative difference between the two as revealed by the different ranking, the resulting qualitative differences are minor. The progression paths indicated by the dual and TFP measures are almost parallel, resulting in average annual progression rates of 1.88% and 1.96% respectively. Our quantitative findings agree also with Capalbo and Vo who report an average annual progression rates of 1.57% for the years 1950–82.

Finally, we turn to the question of biasness. Our results supporting the hypothesis of neutral technological changes are consistent with the findings in previous nonparametric studies of U.S. agricultural data (e.g., Chavas and Cox 1988). In the parametric literature the findings are diverged. Capalbo and Denny accept the neutrality hypothesis while many other researchers (e.g., Antle, Binswanger) reject it. The difference between the parametric and nonparametric literatures with regard to this point may be the result of the maintained hypothesis regarding the form of the production function in the parametric literature or the lack of power in the nonparametric tests. The fact

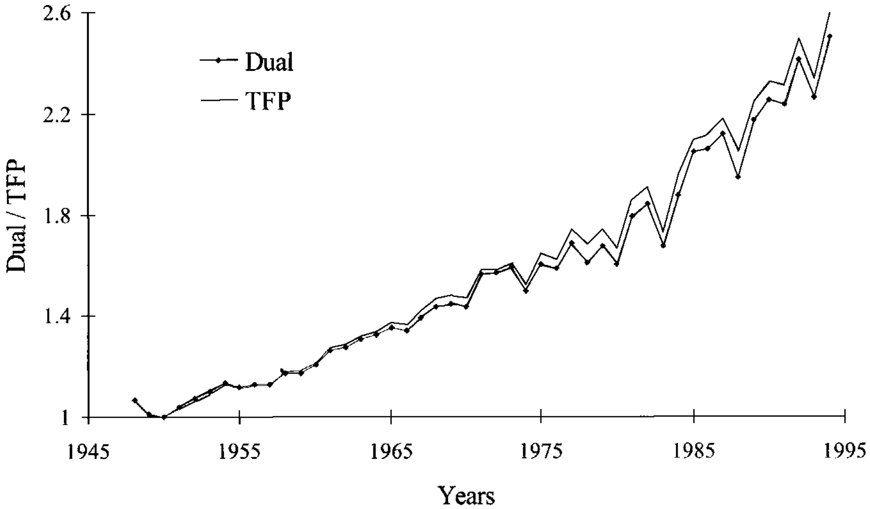

Figure 11. The TFP and the dual measures

that some parametric studies accept the neutrality hypothesis supports the argument that the parametric production function specification is the reason for the difference.

Final Remarks

Nonparametric tests of the type utilized in this study are usually promoted and motivated as being free of bias, which may be caused by an incorrect parametric specification of the objective function. But compared to standard statistical tests, they are not free of disadvantages. The critical regions of nonparametric tests, in contrast to statistical tests, are independent of the significance level. Nonparametric tests, therefore, may reject a hypothesis because of a small deviation which standard statistical tests would consider insignificant.

The economic literature has addressed the "significant" weakness of nonparametric tests. Hanoch and Rothschild, for example, proposed to declare violation of the WAPM only when there exists an alternative netput that could generate, say, 10% more than the actual profit, rather than just greater profit. When Varian (1985), Tsur, and Lim and Shumway incorporated random measurement error in the data-generating mechanism, they suggested that violation be declared only when the minimal perturbation of the data required to pass the test is large relative to the measurement error thought to be present in the data.

Varian (1990) suggested the goodness-of-fit measure to assess the fit of the data to the model. In this 1990 paper, Varian claims that the key determinant of the fit of the data to the model is the size of the deviation of the actual value of the objective function from its optimal value. This approach is different from the random measurement error approach that considers the size of the deviations of the decision variables from their respective optimal values. Varian shows that large deviations in the decision variables may result in only small deviations in the value of the objective function. In such cases, the deviations in the decision variables, although large, are economically insignificant. In this study, we adopt the goodness-of-fit measure to assess the economic significance of deviations of data from the model.

The analysis conducted here may be subject to other limitations, such as reliance on optimizing behavior, possible aggregation problems, dependence on price data, and measurement errors in data. These limitations, however, are not unique to the nonparametric methodology employed here. Measurement errors in the data and aggregation problems, for example, affect both parametric and nonparametric analyses; and both primal parametric and primal nonparametric analyses (Chavas and Cox 1994) do not rely on optimizing behavior or price data.

Several new procedures for testing for specific types of technological change under alternative behavioral postulates are presented. Specifically, the four following results are derived: (*i*) GARTS is a necessary and sufficient condition for rankable technological variations under profit maximization; (*ii*) GARFS is a necessary and sufficient condition for Hicks-neutral technological variations under cost minimization; (*iii*) the Modified WAPM is a necessary and sufficient condition for monotone technological progress under profit maximization;[11] and (*iv*) the Modified WACM is a necessary and sufficient condition for monotone technological progress under cost minimization. These new procedures require only pair-wise comparisons, whereas existing procedures for testing for Hicks-neutral and/or nonmonotonic technological variation require verifications of linear programs, and are therefore computationally simpler. All computations were performed via Microsoft Excel and took only a few seconds to complete. Furthermore, the procedures suggest a new "revealed preference" interpretation.

The empirical application to U.S. aggregate agricultural data yields the following results: (*i*) the data are consistent with profit maximization subject to RTV; (*ii*) the data are consistent with neutral technological changes independent of the behavioral postulate; (*iii*) the data are consistent with cost minimization and monotone technological progress; (*iv*) the data are not consistent with profit maximization and monotone technological progress, but show a significant trend.

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[11] Note that the strengthened GARTS is equivalent to the Modified WAPM (theorem 2) but the latter is simpler and thus preferred from an empirical point of view.

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