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THE UNITY AND SCOPE OF KNOWLEDGE

by

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A dissertation submitted to the
Graduate School-New Brunswick
Rutgers, The State University of New Jersey
in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Philosophy

written under the direction of

JASON STANLEY

and approved by

New Brunswick, New Jersey

October 2013

ABSTRACT OF THE DISSERTATION

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Orthodoxy has it that knowledge is bifurcated between different kinds of states and in particular that there are *species* of knowledge that cannot be reduced to knowledge of truths. Moreover, it is commonly alleged that knowledge of truths alone falls short of explaining a distinctive kind of human capacity: the human capacity for skillful actions. This dissertation challenges both these orthodoxies.

In the first chapter, “Know how and Gradability,” I defend the unity of knowledge against the single most powerful and thus far unanswered argument against it, what I call “The argument from gradability.” According to this argument from gradability, due to Gilbert Ryle (151) in *The Concept of Mind*, *know how* and propositional knowledge cannot be the same state, because the first comes in degrees, whereas the latter is absolute. In this chapter, I argue that the Rylean argument from gradability to dualism fails, as it moves too quickly from the surface form of ascriptions of *know how* to conclusions about the state that is ascribed by means of those ascriptions.

According to *Intellectualism about know how*, knowing how to do something is a matter of possessing a piece of propositional knowledge. Intellectualists about know how routinely appeal to *practical modes of presentation* in characterizing the relevant kind of propositional knowledge. But we are never told much about the nature of these modes of presentation. In Chapter II, I propose a positive view of practical

modes of presentation.

In the final chapter “Skills as knowledge,” I argue that propositional knowledge explains the human capacity for skillful and intelligent actions. In the first part of the chapter, by elaborating on the picture of content developed in Chapter II, I argue that having a skill to ϕ for a task ϕ is a matter of knowing a particular kind of answer to the question *How to ϕ* . In the second part of the chapter, I propose a direct argument for thinking of skills as propositional knowledge states.

Acknowledgments

My graduate career has been long and difficult. I was lucky to get a lot of help, from a lot of people, along the way.

As an undergraduate student in Italy, I studied lots of history and hermeneutics, but little analytic philosophy. I have to thank Diego Marconi for encouraging me to go get an analytic education abroad, an idea that otherwise would have never crossed my mind. Best career advice I have ever received.

But my English was poor. It was so poor that at the time I thought it was a good idea to go learn it in Scotland, of all places, while taking advantage of the philosophical scene at St Andrews University. During my times at St Andrews, I have not learned much philosophy—I was too busy honing my language skills. Besides, all the philosophical discussions I was exposed to were well far above my head. But I did acquire the sense of how difficult it would be to become an analytic philosopher. That convinced me to apply for graduate school in the States, where I was told students have more time to grow philosophically. I got helpful advice from Roy Cook, Richard Heck and Daniel Nolan on this matter. Special thanks to Aidan McGlynn, Marcus Rossberg, and Elia Zardini for their encouragement and support at those times.

I got into Pitt, and went there. Life at Pittsburgh was difficult. But I did have the great fortune of learning from Cian Dorr while there and of taking logic classes from Nuel Belnap. And I did make some very good friends: in particular, I am very grateful to Jonathan Surovell, and Shawn Standefer. I owe Cian also the encouragement to transferring to Rutgers—second best career advice.

The first few years at Rutgers were an amazing intellectual experience. My classmates were all brilliant and so enthusiastic about philosophy. I want to thank them all: Thomas Blanchard, Tom Donaldson, Mike Hicks, Erik Horvestein, Blake Roeber, Mary Salvaggio, Kurt Sylvan. I entered the program while the language graduate

community was flourishing. Among the language people I learned the most, there are Luvell Anderson, Josh Armstrong, Gabe Greenberg, Michael Johnson, Karen Lewis, Ricardo Mena, Will Starr. At Ruccs, philosophers, linguistics and computer scientists would meet regularly to discuss their ideas. I owe a great deal to the linguists at Rutgers, in particular to Mark Baker, to Jane Grimshaw, Maria Bittner, Roger Schwarchild, Veneeta Dayal. Finally, I have to thank the epistemologists among the graduate students, for letting me in in their community. In particular, I learned from Bob Beddor, Matthew Benton, Lisa Miracchi and Kurt Sylvan.

I owe an enormous debt to my committee members.

Although I never completed our independent studies on the *de se* and that project was not going anywhere, Andy Egan was kind enough to join my committee on a different project. His perspective on content have greatly influenced my thinking throughout graduate school (although that may not be clear from the final outcome). His marginalia to students' papers and the speed at which he writes them are legendary. The philosophical world is so much better because Andy is there.

Jonathan Schaffer has the rare ability of being extremely critical without ever being discouraging. The breadth of his thinking and the easiness with which he enters new debates has been of great help to me. Some of his objections and subsequent email exchanges with him were crucial to developing my views on the topic of skills and knowledge. I am also very grateful to him for how generous with his time he has been, even taking the time to help me with my interview spiel before the APA.

I have to thank Ernie Sosa for letting me attend his mythical discussion and reading groups, from which I learned most of what I know about epistemology. I benefited so much from Ernie's philosophical insight and from his comments to my papers. And it is so comforting to see that so much philosophical talent can go together with the kindest soul.

Since I started doing analytic philosophy, Timothy Williamson's philosophical

writings have always been for me the model of deep, far-reaching and at the same time lucid and rigorous thinking. Then Tim was so kind to serve as external committee member. I met a couple of times in person with Tim about the topics of my research, and each time had a huge impact on my work, leading me to reconsider big portions of it. His open-mindedness, kindness, intellectual honesty and the incisiveness of his comments have been truly humbling.

Ultimately, this dissertation was made possible only through the generous help and encouragements of my advisor. The influence of Jason Stanley's work on mine is obvious. I started thinking about this topic in my first year at Rutgers, during his graduate seminar on know how, and we have not stopped talking about the topic since. Jason's enthusiasm for philosophy is contagious, and his nose for interesting unexplored philosophical topics extraordinary. I am very grateful for his encouragement to work on what turned out to be an extremely rich project. I am also grateful to him for always treating me as an epistemic peer and for not thinking any less of me despite quite a few flops. Being both neurotics and hot-headed, it has not always been easy. But we did reasonably well. I am very proud of having him as Doctor Father. I also should thank Emile Thande Stanley, who cheered me up with a beautiful song just before a job talk and Njeri Thande for her kind support and understanding.

Other people besides my committee have deeply influenced me during these years. Matthew Stone has been an extremely helpful interlocutor for me on this project, by guiding me through readings in computer science and by showing me avenues for my arguments that I had not previously considered. Chapter II of this dissertation would not have been possible without his guidance. Matthew is a true blessing for graduate students at RUCCS and one of the finest philosophers I had the luck to work with (as well as one of the finest linguists, one of the finest computer scientists, etc.) I also learned a lot from Brian Weatherson during his time at Rutgers. Chapter I of this dissertation benefited a lot from his comments. More generally, his classes,

philosophical judgment and advice have been of great value to me. Discussions and email exchanges on skills with Jonathan Krakauer were also helpful. I want to also thank Brian McLaughlin, Frankie Egan, Jeff McMahan who have provided helpful suggestions during my practice interview and job talk. I should also thank Holly Smith, Branden Fitelson, Doug Husak and Martin Lin for their interest in my work.

During the job season, I had helpful discussions (in random order) with Brian Weatherson, Sarah Moss, David Manley, Gordon Belot, David Baker, Rich Thomason, Peter Railton, Alan Gibbard, Owen Flanagan, Michael Ferejohn, Walter Sinnott-Armstrong, Alex Rosenberg, Wayne Norman, Andrew Haniak, David Wong, Fred Dretske, David Sanford, Karen Neander, Sally Haslanger, Alex Byrne, Stephen Yablo, Roger White, Brad Skow, Vann McGee, Caspar Hare, Richard Holton, Rae Langton, Philip Bricker, Hilary Kornblith, Joe Levine, Scott Sturgeon, Jim Pryor and Susanna Siegel.

At Rutgers, I have made some of my very best friends. Alex Anthony, Josh Armstrong, Marco Dees, Lucy Jordan, Ben Levinstein, Ricardo Mena and Lisa Miracchi have provided the much needed distraction during stressful times. Special thanks go to my roommate Bob Beddor, for our endless philosophical discussions and uneven cocktail nights.

In all these years, I have been in a long distance relationship with Alessandro. His witty humor and penetrating intelligence, his healthy and balanced perspective on things, his deep insight into people's nature that makes him such a great writer and (last but not the least) his excellent cooking skills have provided me with so much comfort and happiness through the years as nothing else could have. The future looks brighter for us. I look forward to spending it on the same side of the pond.

Like Alessandro, the rest of my family, my parents Nanni and Ivana, my siblings Federico, Gaia and Jacopo, and my grandmothers Mafalda and Rosanna are all highly skillful people. My work vindicates how much they know and how important

their knowledge is. This dissertation is dedicated to the fond memory of my dear grandmother Mafalda, who just passed away.

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Chapter 1

KNOW HOW AND GRADABILITY

1.1 Introduction

Absolutism about knowledge is the view that knowledge is absolute—i.e., cannot come in different degrees nor hold to different extents. In epistemology, absolutism about knowledge is the orthodox view: Virtually every epistemologist nowadays believes that knowledge is an all or nothing matter.¹ In fact, philosophers do not usually even provide arguments for absolutism.² It just seems like an obvious fact about knowledge.³

Though *absolutism* is commonly assumed in epistemology, most would deny that *every* kind of knowledge is absolute. In particular, it is almost platitudinous that *know how* is a state in which a subject can be to different degrees or to different extents—i.e., a *gradable* state. After all, the gradability of know how seems to be reflected in our ordinary parlance. Know how is ordinarily ascribed in English by means of ascriptions of the form ‘S knows how to ϕ ’ as in “I know how to play my trumpet,” “Peter knows how to fix the sink,” and “Mary knows how to sing La Traviata.” And ascriptions of know how are gradable—i.e., they can be modified by adverbial modifiers such as “partially,” “for the most part,” “in part” and “well.” Moreover, ascriptions of know how can ordinarily enter into comparative constructions. For example, it is true of me that I know only in part how to write documents in L^AT_EX,

¹One exception is Hetherington(84), who argues against *absolutism*, which he considers one of the current dogmas of epistemology. Another notable, although rather peculiar, exception is Sosa(171), Sosa(172), Sosa(169) with his notion of *knowing full well*. As I understand his view, even for Sosa, the two main dimensions of knowledge, *animal knowledge* and *reflective knowledge*, are each absolute. What in his view qualifies as an instance of knowing *full well* is a belief that manifests both first-order competence and second-order competence—i.e., that is both apt and meta-apt. If this is correct, then also for Sosa, knowledge does not come in degrees. However, beliefs that qualify as cases of knowledge can in addition be more or less apt.

²Though see Stanley(175) for an explicit defense of *absolutism*.

³For instance, Dretske(44) p. 363 remarks: “Knowing that something is so, unlike being wealthy or reasonable, is not a matter of degree. . . . In this respect factual knowledge is absolute. It is like being pregnant: an all or nothing affair.”

that I do not know how to play the trumpet very well, and that, if I only practiced more, I would know how to play better than I do.

Dualism about knowledge is the view that know how and propositional knowledge are different kinds of states. Such a divided view of knowledge follows from absolutism about propositional knowledge and the gradability of know how. The argument—first hinted at by Gilbert Ryle in *The Concept of Mind*—proceeds as follows: know how can hold to different degrees or to different extents. Propositional knowledge, on the other hand, is absolute. Since know how and propositional knowledge have different properties, they must be different kinds of states.

The Rylean Argument stands out as the single most powerful and thus far unanswered challenge to a unitary view of knowledge. My purpose in this chapter is to provide a definitive response. The general response strategy is to show that the argument moves too quickly from the surface form of ascriptions of know how to conclusions about the state that is ascribed by means of those ascriptions. But the surface form of those ascriptions is not pellucid. What appear to be modifications of the state of knowledge ascribed are instead properly understood as modifications of other elements. Because of previously unrecognized complexity of the data, it will emerge that there are different *kinds* of graded ascriptions of know how. None of these different kinds of ascriptions are best understood as providing evidence that the knowledge state ascribed in graded ascriptions of know how is itself graded. As I show, the alleged evidence for the gradability of know how not only can be understood compatibly with a unified picture of knowledge and inquiry but is also best explained in those terms.

The state of know how is not gradable. We were misled by language into thinking that it is.

1.2 From Gradability to Dualism?

In *The Concept of Mind*, Ryle(151) first observed that we ordinarily speak *as if* know how could hold to different extents:

We never speak of a person having partial knowledge of a fact or truth, save in the special sense of his having knowledge of a part of a body of facts or truths. A boy can be said to have partial knowledge of the counties of England, if he knows some of them and does not know others. But he could not be said to have incomplete knowledge of Sussex being an English county. Either he knows this fact or he does not know it. On the other hand, it is proper and normal to speak of a person knowing in part how to do something, i.e. of his having a particular capacity in a limited degree. An ordinary chess-player knows the game pretty well, but a champion knows it better, and even the champion has still much to learn (Ryle(151), p.59).

In this passage, Ryle(151) has brought to our attention a genuine and quite robust *linguistic* phenomenon. In English, some knowledge ascriptions, such as ascriptions of the form ' S knows how to ϕ ', can be modified by so-called proportional modifiers such as “in part,” “largely” and “for the most part,” as in (1-a)-(1-c):

- (1)
 - a. Gianni knows in part how to fix the sink.
 - b. Francesco knows how to read Latin, for the most part.
 - c. My students largely knew how to solve their math problem.

By contrast, standard ascriptions of propositional knowledge—i.e., ascriptions of the form ' S knows that p '—cannot be sensibly modified by the same modifiers, as evidenced, for instance, by (2-a)-(2-d):

- (2)
 - a. ??The math teacher knows in part that his students cheated on the final.
 - b. ??The student knew that Paris is the capital of France, for the most part.
 - c. ??Pierre in part knows that his friends are arriving.
 - d. ??Pierre knows in part that Mark's recipe is a way to make lasagna.

The phenomenon is arguably not restricted to proportional modifiers. With some qualification, it extends to *degree modifiers* too, such as “well.” Ascriptions of know how can ordinarily be modified by “well,” as in (3-a)-(3-b):

- (3) a. Federer knows very well how to play tennis.
- b. Maurice Andre’ knew well how to play the trumpet.

In (3-a)-(3-b), the use of a degree modifier appears to increase the degree at which the state of knowing how to play tennis (or the trumpet) is claimed to hold of its subjects, much in the way in which the degree modifier in (4-a) affects the degree at which Marco is claimed to be acquainted with the facts:

- (4) a. Marco is well acquainted with the facts.

By contrast, for most know-*that* ascriptions degree modifiers are only marginally (if at all) acceptable, as, for instance, in (5-a)-(5-b):

- (5) a. ?Pierre knows well that Obama is president.
- b. ?Pierre knows well that Paris is the capital of France.

It is true, as observed by Dutant(46), that some know-*that* ascriptions tolerate degree modification, as in (6)-(8):

- (6) Tim knows well that we must have no illusions that somehow the danger has passed. (Dick Cheney)
- (7) But Chirac knows well that, while times change, an unchanging rule of politics is this: you say whatever you have to say to get elected. (International Herald Tribune)
- (8) And he knows very well that it requires work to get past old grievances. (GW

Bush)

However, it is doubtful that these ascriptions are instances of the same kind of phenomenon that is observable with know how ascriptions.

For one thing, the availability of know how ascriptions to modification by degree modifiers is much more *systematic*. Degree modifiers are not only perfectly felicitous into know how ascriptions, as illustrated by the above (3-a)-(3-b). Know how ascriptions, when modified by “well,” also naturally enter into *degree* constructions—i.e., complex syntactic constructions formed out of an adjective or an adverb and a degree morpheme, such as “-er,” “less,” “so,” etc. as in (9-a)-(9-c):⁴

- (9) a. Nobody knows how to play tennis better than Federer.
 b. My mother knows how to make lasagna at least as well as the best Italian chef.
 c. Only Maurice André knew how to play the trumpet better than Louis Armstrong.

By contrast, similar degree constructions with know *that* ascriptions are very marginal and by comparison much less felicitous than those involving know how ascriptions, as illustrated by the following (10-a)–(10-c):

- (10) a. Mary knows better than John how to make *bucatini all'amatriciana*.
 b. ??Mary knows better than John that their aunt's recipe is a way to make *bucatini all'amatriciana*.
 c. ??Mary knows better than John that *bucatini all'amatriciana* is a classic

⁴These ascriptions are ambiguous between two interpretations that depend on the scope of the degree construction in them. Only one reading is relevant for one purpose. I will return to this important observation in the section dedicated to this kind of adverbial modification into know how ascriptions, Section 2.5.

part of Cucina Romana.

Moreover, with know how ascriptions, it is possible to variously *embed* the degree constructions, as evidenced by (11-a)-(11-d):

- (11)
- a. Mark wishes he knew how to play his trumpet better.
 - b. Did Maurice André know how to play the trumpet better than Louis Armstrong?
 - c. Does Felipe know how to play the trumpet well or not so well?
 - d. Mario wishes he knew how to solve the economic crisis better than his predecessors.

In contrast, the availability of know *that* ascriptions to modification by means of degree modifiers is more *constrained*: ascriptions such as (12-a) cannot be freely embedded within other attitude reports or in the context of questions, as evidenced by (12-b)-(12-e):

- (12)
- a. Mark knows full well that speeding is dangerous.
 - b. ??Mary wishes her son knew full well that speeding is dangerous.
 - c. ??Mary wishes her son knew better that speeding is dangerous.
 - d. ??Does Mary know full well that speeding is dangerous?
 - e. ??Does Mary know very well or not so well that speeding is dangerous?

This lack of systematicity strongly suggests that, in the case of know-*that* ascriptions, their limited adverbial modifiability may well demand a different explanation: Perhaps, it is to be explained away on pragmatic grounds or—as suggested by

Schaffer&Szabó(155)—should be diagnosed as fringe non-literal parlance.⁵ Given the systematicity of the adverbial modification into know how ascriptions, by contrast, there is no room for dismissing the gradability of know how ascriptions on the same grounds.⁶

⁵There is a small literature on the marginality of the adverbial modification into ascriptions of the form ‘*S* knows that *p*’. Stanley(175) argues that in those constructions, the modifiers work as *pragmatic indicators* rather than as genuine degree modifiers. Against Stanley(175), Dutant(46) observes that pragmatic indicators such as “really” *can* be embedded, as in “Myers does not really know our military will have the will to win” or as in “Does Mary really know that our military will have the will to win?” By contrast, degree modifiers into know that ascriptions cannot be easily embedded. So perhaps that of “pragmatic indicators” is not the right diagnosis after all. Another option, suggested *en passant* by Schaffer&Szabó(155), is to treat those constructions as *idiomatic*. One reason to think that the marginal adverbial modification into knowing that ascriptions is idiomatic is that idioms are “inflexible”—i.e., they typically occur in a limited number of syntactic frames and constructions (see Nunberg&Others(126)). Another possible explanation, sketched by Kennedy(94), is that the degree modifier does not directly modify “know” but rather the process of *getting* to know. In this explanation, the availability of degree constructions would mirror the gradability of the past principle “known.” Another avenue to explain the contrast takes “well” to have two different meanings, one when it combines with know-that ascriptions and another when it combines with know-*wh* ascriptions. This ambiguity may explain why comparative constructions are systematically possible only in the latter case. There is some independent evidence for an ambiguity of “well.” See Kennedy&McNally(93). The ambiguity is between degree modifiers and manner adverbial. Accordingly, “well” occurs as manner adverbial into know that ascriptions and as degree modifiers into know how ascriptions. Ultimately, my explanation for this contrast is that knowledge how ascriptions ascribe knowledge of answers, and answers can be good or bad to different degrees. See Section 2.5 for a development of this analysis. What the best explanation is of the limited availability of know-that ascriptions to modification by means of degree modifiers is a very interesting question, but it is one with respect to which I would like to remain neutral. What matters for our purposes is the rather uncontroversial observation that the gradability of ascriptions of know how is not limited nor constrained in the way that the gradability of know-that ascriptions is. The gradability of ascriptions of know how is definitely a *systematic* phenomenon, and that for this reason, it cannot simply be explained away on pragmatic grounds.

⁶There is a second dimension of contrast between the gradability of know how ascriptions and the marginal gradability of know-that ascriptions. When one says that one knows well that driving is dangerous, this ascription is naturally interpreted as grading the degree of evidence or justification at which the subject knows, or believes, that driving is dangerous (See for instance Ludlow(117)). By contrast, it seems uncontroversial that adverbial modification by means of “well” is possible in know how ascriptions *even* when what is graded does not have anything to do with the subject’s evidence or justification. Thus, for instance, by means of

- (i) Mary knows how to hit a tennis ball better than John.

we do not need to be comparing the degree of justification or evidence that Mary has with respect to the question *How to hit a tennis ball*. So whatever explanation is needed for the marginal gradability of know that ascriptions, it is doubtful that the same explanation will satisfactorily extend to the phenomenon of adverbial modification into knowing how ascriptions.

In the following, I will call the “phenomenon of gradability” the linguistic phenomenon whereby ascriptions of the form ‘S knows how to ϕ ’ can be sensibly and systematically modified by proportional modifiers such as “in part” and “for the most part” and by degree modifiers such as “well,” while ascriptions of the form ‘S knows that p ’ cannot, or at least cannot be so systematically.⁷ The phenomenon is robust cross-linguistically and not specific to English.⁸ Moreover, it is too systematic

⁷It should be flagged that in calling such a phenomenon the “phenomenon of gradability,” I am using the label “gradability” more loosely than it is used in the linguistic literature. In the linguistic literature, an expression is taken to be *gradable* just in case it can *directly* compose with comparison constructions “-er/more... than,” “as... as,” “less... than.” Neither know-*that* ascriptions or know how/know-*wh* ascriptions are gradable in this sense, for neither can *directly* compose with comparative constructions:

- (i) ??Marco knows how to play the trumpet more than you do.
- (ii) ??Jonathan knows that the streets are wet more than you do.

However, there still is an important difference between know how ascriptions and know-*that* ascriptions. Only the former can *indirectly* and systematically compose with degree constructions, as in the aforementioned, *via* “well:”

- (iii) Mary knows better than John how to make *bucatini all’ amatriciana*.

And as in (iv), *via* the phrase “to a large extent:”

- (iv) Marco knew how to solve the math problem to a larger extent than anybody else in the class.

Compare (iv) with a “gradable” ascription (in this restricted sense of gradable) such as (v):

- (v) Ann regrets that she offended Ben less/just as much/more than she regrets that she insulted Claire.

Because ascriptions of know how cannot directly compose with degree morphology—but only *via* other modifiers such as “well” and “to some extent”—they are not gradable in this sense. See Kennedy and McNally(95), (93), (92) for this sense of gradability.

⁸To my knowledge, every language in which ascriptions of know how are expressed by means of ascriptions of the form ‘Know + interrogative’ exhibits the same phenomenon, although the licensed position of the degree modifier in the sentence may vary from language to language. For instance, in Italian, the correspondents of “in part” (= *in parte*) and “for the most part” (= *per lo piu’*) tend to occur at the end of the sentence. It is well known that in Romance languages, ascriptions of know how can also take the form ‘Know + infinitive’, which is also gradable. Rumfitt(149) and Glick(65) use this linguistic evidence to argue that there is a kind of know how that is not reducible to knowing

and pervasive to be set aside as nonliteral discourse or fringe parlance. Because of that, a comprehensive account of knowledge ascriptions cannot neglect the fact that ascriptions of know how can be graded.⁹

For our purposes, the relevant issue is whether, as Ryle thought, the phenomenon of gradability provides evidence against the view that the state ascribed by reports of the form ‘*S* knows how to ϕ ’ is the same state ascribed by reports of the form ‘*S* knows that p ’ and thus evidence for *dualism about knowledge*. Clearly, the linguistic phenomenon of gradability only provides support for dualism if the gradability of the reports is to be explained by the gradability of the state ascribed by those ascriptions. Only with this assumption in play does it follow from the phenomenon of gradability that know how and propositional knowledge are distinct states.

It is convenient to make this assumption completely explicit by fleshing out the entire reasoning. Recall that a state is *absolute* just in case it is not gradable. And a state is gradable just in case a subject can be in it to different degrees and to different extents. With these definitions in place, the argument from the gradability to dualism goes as follows:

an answer to a question. But this argument is too quick. A correct analysis of those constructions (Stanley(177)) shows that they differ *only* superficially from the interrogative constructions. At logical form, also those reports embed interrogatives. If this is correct, the explanation of their gradability must be the same as the explanation of the gradability of the main form of ascriptions.

⁹Ryle(151) was the first to observe the phenomenon. Quite independently of the philosophical debate on know how, some aspects of the gradability of reports embedding interrogatives have been discussed by a small linguistics literature. In particular, what I will soon call *quantitative gradability* has been studied by Berman(17), Lahiri(105), Groenendijk(71), Beck&Sharvit(13), Lahiri(107), Williams(184). But as far as I can see, such a literature ignores the qualitative aspect to the phenomenon of gradability, for which I will argue more extensively in §2.5. Moreover, such a literature fails to appreciate that the quantitative gradability of ascriptions of the form ‘*S* knows how to ϕ ’—and more generally, the gradability of reports embedding interrogative in their mention-some reading—raises special difficulties, difficulties that I raise explicitly in §2.4. As a consequence of that, none of the accounts put forward so far in the linguistic literature can be extended to deal successfully with the quantitative gradability of ascriptions of know how. In the last decade, the gradability of ascriptions of know how has been mentioned by many as primary evidence against *Intellectualism* about know how. See Roberts(147), Stanley(176), and Dutant(46) for a brief discussion of the phenomenon and an attempted solution and Pavese and Stanley(131) for a preliminary analysis.

The Argument from Gradability to Dualism about Knowledge

1. Propositional knowledge is absolute. (*Absolutism*)
2. Know how is not absolute.
3. Propositional knowledge and know how have different properties.
(*From (1) and (2)*)
4. Know how is not propositional knowledge. (*Leibniz's law*)

Premise (2) in this argument is in need of support and *The Rylean Argument* is the natural way to use the phenomenon of gradability to provide support for this premise:¹⁰

The Rylean Argument:

1. Reports of the form 'S knows how to ϕ ' are gradable. (*The Phenomenon of Gradability*)
2. The state ascribed by reports of the form 'S knows how to ϕ ' is gradable. (*From (1)*)
3. The state ascribed by reports of the form 'S knows how to ϕ ' is not absolute. (*From (2) and definition of an absolute state*)
4. The state ascribed by reports of the form 'S knows how to ϕ ' is know how. (*Definition of know how*)
5. Know how is not absolute. (*From (3) and (4)*)

The Rylean Argument crucially relies on the step from the phenomenon of gradability to the gradability of the underlying state (from (1) to (2))—i.e., from the fact that

¹⁰Nowhere does Ryle fully flesh out the following argument. For this reason, I will dub it "The Rylean Argument."

know how ascriptions can be systematically modified by proportional and degree modifiers (*The Phenomenon of Gradability*) to the claim that the state ascribed by those ascriptions is itself gradable—i.e., it is one in which a subject can be to different degrees and at different extents. However, such a step should be granted only if the phenomenon of gradability entailed the gradability of the state reported or were best explained by it. But the phenomenon neither entails nor is best explained by the gradability of the state reported.

Here is the general form of my argument. Let a *predicate* ‘ P ’ be gradable just in case ‘ P ’ can be satisfied by a subject to different extents and/or to different degrees. An ascription of the form ‘ S knows that p ’ says of a subject S the predicate ‘knows that p ’. An ascription of the form ‘ S knows how to ϕ ’ says of a subject S that it satisfies the predicate ‘knows how to ϕ ’. The phenomenon of gradability shows that the latter predicate is gradable—i.e., can be satisfied by a subject to different degrees and to different extents. The phenomenon of gradability does not show, however, that the *state* ascribed by knowing how ascriptions is itself gradable. For the predicate ‘knows how to ϕ ’ might hold of a subject to different degrees or to different extents *by virtue of* the subject being in an absolute state of knowledge. If so, the phenomenon of gradability would be compatible with the absoluteness of the state ascribed by means of that predicate.¹¹

In order to show that the phenomenon of gradability is compatible with the absoluteness of the state ascribed, what has to be shown is that the satisfaction conditions of the gradable predicate can be specified in terms of the satisfaction conditions of an

¹¹Let me just highlight that I have so far distinguished between three senses of “gradable:” (1) an ascription being gradable; (2) a predicate being gradable, and (3) a state being gradable. An ascription of knowledge is gradable just in case it can be systematically modified by proportional and degree modifiers. A predicate is gradable just in case it can be satisfied to different degrees and to different extents. Finally, a state is gradable just in case a subject can be in it to different degrees and to different extents.

absolute predicate. It is natural to assume that this goal will be achieved if the satisfaction conditions of the gradable predicate are specified in terms of metalinguistic clauses where the predicate “know” is assumed to express a propositional knowledge relation. That is what I primarily purport to show in this chapter.

Something stronger can also be showed to be true. The satisfaction conditions of the gradable predicate not only can be specified in terms of the satisfaction conditions of a non-gradable knowledge predicate. Such satisfaction conditions, as specified, can be derived from a *compositional assignment of semantic values* to the sub-expressions of the two predicates, *via* a semantic analysis that, I argue, is the most linguistically motivated, given the distribution of the phenomenon of gradability across different kinds of reports. The phenomenon of gradability is not just compatible with postulating the absoluteness of the state ascribed by means of know how ascriptions. The phenomenon is also better explained that way.

The details of my case will vary slightly, depending on the kind of gradability at issue. As with other graded constructions, the phenomenon of gradability comes in *two* different flavors. Compare the gradability of knowing how ascriptions and the gradability of sentences containing *color* terms. Two objects may be green to the same extent along a *quantitative* dimension and yet be green to different degrees along a *qualitative* dimension. The following sentence is perfectly acceptable:¹²

(13) These two leaves are both entirely green, but one is greener than the other.

Likewise for the gradability of knowing how ascriptions: Ale and I may both know entirely how to make Indian curry, and yet Ale may know how to make it much better than I do. The distinction between these different varieties of gradability is impor-

¹²Kennedy and McNally(95) first observed the quantitative/qualitative distinction for the case of color predicates.

tant, and they deserve separate consideration. Section 1.3 discusses the phenomenon of *quantitative gradability*; Section 1.4 discusses and analyzes the phenomenon of *qualitative gradability*.

1.3 Quantitative Gradability

The phenomenon of quantitative gradability—as I call it—is the *linguistic* phenomenon whereby ascriptions of the form ‘S knows how to ϕ ’ can be sensibly modified by so-called *proportional modifiers* such as “in part,” “for the most part,” “partly,” etc., while ascriptions of the form ‘S knows that ϕ ’ cannot be so sensibly modified. To reiterate just one more instance of this contrast, consider how natural it is to say things such as “Mark knows *in part* how to make lasagna,” and how strained it would be to say instead “Mark knows in part *that his aunt’s recipe for making lasagna is a way to make lasagna*.” The goal of this section is to show that this contrast between the two kinds of ascriptions is due to the different *semantics* of their complements.

The complement of ascriptions of know how, ‘How to ϕ ’, is an interrogative.¹³ There does not seem to be any verb embedding interrogatives that does not embed ‘How to ϕ ’.¹⁴ Nor does there seem to be a verb embedding ‘How to ϕ ’ that does

¹³I will adopt the convention of using corner quotes to refer to the interrogative—e.g., ‘How to ϕ ’—and *emphasis* to refer to the question it expresses—e.g., *How to ϕ* . It is helpful to distinguish between the *intension* and the *extension*—or denotation—of an interrogative. I will indicate the intension of the interrogative by angle quotes \langle How to ϕ \rangle and the denotation of the interrogative by double brackets \llbracket How to $\phi\rrbracket$. By “questions,” I mean the metaphysical entity that happens to be the intension of the interrogative—i.e., *How to ϕ* = \langle How to ϕ \rangle . Moreover, I will use ‘ Q ’ as a meta-variable over questions and ‘ Q ’ as a meta-variable on interrogatives. I will adopt a parallel convention for *that*-clauses and their denotation. I will refer to *that*-clauses using corner quotes—‘that p ’—and I will use *emphasis* to refer to the propositions that *that*-clauses denote—*that p* .

¹⁴An anonymous referee gave me “not realize” as a counterexample to this generalization. For instance whereas “I did not realize who was coming” is perfectly acceptable, “I didn’t realize how to do it” is not. Two observations about this. First, “not realize” *can*, in some environments, embed ‘How to ϕ ’, such as in the following:

- (i) I had not realized how to solve the math problem until I saw Paul solving it.

not also embed other interrogatives.¹⁵ Moreover, we do ordinarily say things such as “The question is how to ϕ ” which suggests that ‘How to ϕ ’ contributes a question to

(ii) Before I saw Paul doing it, I had not realized how to do the exercise without making mistakes.

So although it is an interesting question why a sentence “I didn’t realize how to do it” is not acceptable, the explanation for its unacceptability cannot be that “not realize” does not ever embed ‘How to ϕ ’. Secondly, it looks as if the correct generalization is that “not realize” does not always acceptably embed *infinitival* interrogatives:

(iii) ??I did not realize where to go for dinner.

(iv) ??I did not realize why to vote for Obama.

This suggests that there may be something about *infinitival* clauses that undermines the acceptability of those sentences. One speculation is that the source of the problem is the untensedness of the infinitival clauses. But I leave this issue open here.

¹⁵Against this generalization, somebody has suggested to me (p.c.) that “practice,” “demonstrate,” “perfect,” not embedding interrogatives, can embed ‘How to ϕ ’. If so, this would be evidence that ‘How to ϕ ’ can occur as something other than an interrogative in certain reports. But it strikes me as outright false that “practice,” “demonstrate”, and “perfect” cannot embed other interrogatives. For one thing, the following are perfectly acceptable:

(i) Mario practiced what to say to his parents.

(ii) Mario demonstrates what to do in case of an emergency.

(iii) Mario perfected what to say to his parents.

And it is commonly assumed in the linguistic literature than infinitival interrogatives such as “What to say to his parents” cannot be free relatives (See Schaffer(154), p. 489). Moreover, those same verbs can embed multiple *wh*-constructions, which cannot, for merely syntactic reasons, occur as free relatives (See Baker(9)):

(iv) Mario practiced at length what to say to which students.

(v) Mario will demonstrate to you what to do in which emergencies.

(vi) Mario must perfect what to say in which situation.

indent So I very much doubt that “practice”, “demonstrate”, “perfect” cannot embed interrogatives.

It has been argued by some that the complement ‘How to ϕ ’ occurs as a *free relative* in ascriptions of know how. But none of the arguments for this claim are at all persuasive. First of all, it is obvious that ‘how to ϕ ’ is not *always* a free relative. Free relatives have the same distributions as nominal phrases. But the interrogative ‘how to ϕ ’ can be embedded by verbs that do not tolerate nominal phrases as complements:

the sentences where it occurs:¹⁶

- (14) a. The question is not *if* but how and why to keep the shuttles flying.¹⁷
 b. The question is not how to survive but how to thrive with passion, compassion, humor, and style.¹⁸

(vii) I wonder how to address my father.

(viii) **I wonder the capital of France.

(ix) It is clear how to solve the problem.

(x) **It is clear the way to solve the problem.

So the only plausible claim is that in *some* occurrences and with *some* verbs, 'How to ϕ ' can occur as a free relative. What is the argument for the claim that *in ascriptions of know how* in particular, 'how to ϕ ' occurs as a free relative? The main argument for this claim is that the following ascriptions are roughly synonymous:

(xi) I know how to swim.

(xii) I know the way to swim.

But this argument is bad. The nominal phrase in (xii) is what linguists call a "concealed interrogative." Standardly, concealed interrogatives are given the same semantics as interrogatives (For semantic analyses of concealed interrogatives as denoting question-meanings, see Baker(11) , Grimshaw(68), Pesetsky(136)). The fact that the phrase "The way to swim" is an implicit question suffices to explain the synonymy between (xii) and "I know how to swim."

¹⁶Another competing view is Brogaard(25), Brogaard(24), Brogaard(23). Brogaard argues that, quite generally, embedded interrogatives have the semantic content of *predicates*. But as far as I can see, Brogaard(25)'s view is not really motivated. The main argument for it—the so-called argument from *iterated knowledge ascriptions*—is flawed. The argument goes as follows. If Mark knows that Judy knows that it is snowing, then Mark must also himself know that it is snowing (by factivity of knowledge (twice) and closure). By contrast, if Mark knows that Judy knows what the weather is like, it does not follow that Mark thereby knows himself whether it is raining or not. The argument concludes that embedded interrogatives do not contribute answers as their denotation. But there are several different ways of replying to this objection. One, the most straightforward, is to appeal to hyper-intentionality: *that*-clauses and embedded interrogatives denote the same kind of things (propositions or sets of propositions), but represent the same denotation in different ways. Another strategy is to take the embedded "Judy knows what the weather is like" to contribute an existentially generalized proposition, of the form "There is a proposition *p* that answers the question expressed by 'What the weather is like?' that Judy knows." Hence, there are at least two different strategies to resist Brogaard's conclusion.

¹⁷<http://nasawatch.com/archives/2010/04/the-question-is.html>

¹⁸<http://0theplaceswewillgo.blogspot.com/2011/12/question-is-not-how-to-survive-but-how.html>

The distribution of the phenomenon across reports embedding interrogatives is evidence that the source of the gradability of ascriptions of know how is their *interrogative* rather than the embedding verb. First, proportional modifiers are acceptable in most knowledge-*wh* ascriptions, as illustrated by (15-a)-(15-c):

- (15) a. For the most part, Mark knows what they serve for breakfast at Tiffany's.
 b. Mark mostly knows which students in his class cheated on the final exam.
 c. Mark knows in part what he will get for his birthday.

Second, the same modifiers can occur in several other kinds of reports embedding interrogatives, involving verbs other than “know,” as shown in (16-a)-(16-f):¹⁹

- (16) a. For the most part, Mark told me who cheated on the final exam.
 b. Mark told me in part what happened that night.
 c. For the most part, Mark has decided what to do after college.
 d. Mark has partly decided what to do after college.
 e. The school paper recorded in part who made the dean's list.
 f. For the most part, the school paper recorded what happened at the graduation ceremony.

Exactly like “know”, those verbs tolerate quantitative constructions *when* embedding interrogatives but do not allow those constructions whenever they embed *that*-clauses, as illustrated by the contrast between (16-a)-(16-f) and the following (17-a)-(17-d):

- (17) a. # For the most part, Mark told me that my students have cheated on

¹⁹In fact, it is a standard generalization in the linguistic literature that most *factive* verbs embedding interrogatives tolerate proportional constructions. See Lahiri(105) and Lahiri(107) for a detailed discussion of the distribution of quantitative constructions with verbs embedding interrogatives.

the final exam.

- b. # Mark told me only in part that he was robbed last night and the muggers got arrested.
- c. # For the most part, Mark has decided that he wants to become a plumber.
- d. # Mark has in part decided that he will not go to college.

The fact that the phenomenon of quantitative gradability systematically correlates with reports embedding interrogatives suggests that it is the interrogative that is responsible for the gradability of the whole report. This encourages an analysis of the phenomenon of quantitative gradability in which adverbial modification in such reports affects the semantic content contributed by interrogatives they embed and is allowed by the distinctive semantics of that complement.

In this analysis, quantitative gradability of reports embedding interrogatives is interpreted as an instance of a more *general* phenomenon of *adverbial modification into argument position*. Adverbial modification into argument position is rather common; and the availability of such adverbial modification is evidently compatible with the relation reported by the main verb being absolute. To illustrate, consider (18-a):

- (18) a. Mark's PhD students have for the most part found a job.
 b. Most of Mark's PhD students have found a job.

The most natural reading of (18-a) is not the one in which all of Mark's PhD students participate *to a large extent* in the property of having found a job. Rather, the most natural reading is the reading according to which *most* of Mark's PhD students have found a job: (18-a) seems to have the same truth conditions as (18-b). In (18-a) what is being graded is the argument "Mark's PhD students:" there, the expression "for the most part" appears to be quantifying over Mark's PhD students. Moreover, the

possibility of adverbial modification in (18-a) is surely compatible with the obvious fact that the property of finding a job is absolute—i.e., cannot hold to different extents. Similarly, *mutatis mutandis*, for (19-b). By using (19-b) in response to (19-a), we are saying that some of my colleagues have left the party before nine, not that all of my colleagues have left the party before nine to a *certain extent*. (19-b) seems to have the same truth conditions as (19-c):

- (19) a. What time did your colleagues leave the party?
 b. In part, they left before nine, but some remained until the end.
 c. Some of my colleagues have left the party before nine, but some remained until the end.

In (19-b), it is not the main verb that is affected by the adverbial modifier but its argument—i.e., “my colleagues.” And the possibility of adverbial modification appears here again to be fully compatible with the fact that leaving a party before nine is not something that can be done to different extents or in different degrees. By parity of structure, adverbial modification into reports embedding interrogatives may require the same kind of analysis: the phenomenon of quantitative gradability may just be an instance of a general pattern of adverbial modification into argument position. Since adverbial modification into argument position is quite generally compatible with the absoluteness of the relation expressed by the argument-taking verb, the phenomenon of quantitative gradability is *prima facie* compatible with the absoluteness of the state reported by ascriptions of know how.

The challenge is to provide an analysis of the phenomenon that is compatible with the absoluteness of the state reported by ascriptions of know how. If the phenomenon of quantitative gradability is to be analyzed as an instance of quantification into argument position, as suggested thus far, then what we want is an analysis of

(20) *S knows in part how to ϕ*

in terms of

(21) *For some — in the denotation of \ulcorner How to $\phi\urcorner$, *S knows* —.*

where the predicate “know” is understood as expressing propositional knowledge. The question is how to fill up the blanks in (21)—i.e., how to understand the quantification contributed by the proportional modifier “in part”—and how to think of the elements quantified over—i.e., how to think of the denotation of the interrogatives, which is the question I address next.

1.3.1 The Counting Approach

In the linguistics literature the currently most fashionable semantic theory of interrogatives—i.e., Hamblin and Karttunen’s semantics—identifies the denotation of embedded interrogatives such as “Who sang?” with *sets of true propositions*. The propositions in the set are those obtainable by first replacing the question word in the interrogative with a variable and by letting the assignment to that variable vary for every individual that actually satisfies the predicate of the interrogative. In other words, the set of true propositions denoted by “Who sang” is the set of propositions of the form *that x sang* for every assignment to x with an individual who actually sang—such as *that Mary sang*, *that John sang*, and so on, for any person who actually sang. Call each proposition so derived a true “Hamblin/Karttunen-answer”—or “HK-answer” for short—and call the set of true HK-answers a “Hamblin/Karttunen-true denotation”—“HKT-denotation” for short.²⁰

²⁰Karttunen(89) famously argues that only true HK-answers belong to the denotation of interrogatives. His main argument is that verbs that are not factive when embedding *that*-clauses such as “tell” are factive when embedding interrogatives. Against Karttunen, Egré&Spector(47), Beck&Rullman(12)

On this semantics, it is extremely natural to understand quantitative gradability in terms of *quantification* over HKT-denotations, as the following clauses make explicit (I indicate the denotation of an interrogative ‘ Q ’ at the actual world by putting it between denotation-brackets ‘ $\llbracket Q \rrbracket$ ’. Since ‘ Q ’ may contain some context-sensitive expression, its denotation is to be relativized to a context: $\llbracket Q \rrbracket^c$. I am ignoring other possible sources of context sensitivity):

THE COUNTING APPROACH

- ‘ S knows in part Q ’ is true relative to a context c if and only if for some HK-answers $p \in \llbracket Q \rrbracket^c$, S knows p .
- ‘ S knows for the most part Q ’ is true relative to a context c if and only if for most HK-answers $p \in \llbracket Q \rrbracket^c$, S knows p .
- ‘ S knows fully Q ’ is true relative to a context c if and only if for every HK-answers $p \in \llbracket Q \rrbracket^c$, S knows p .

In the current linguistic literature THE COUNTING APPROACH is the standard approach to adverbial modifications into reports embedding interrogatives, particularly because of the work of Lahiri(107) and others.²¹ And, at least *prima facie*, it is an approach with many virtues.

First of all, THE COUNTING APPROACH defines the satisfaction conditions of a graded know how predicate in terms of an absolute knowledge state in accordance

and Lahiri (107) all argue that there are non-factive “tell”-reports that embed interrogatives. See also George(57) for discussion. In this essay, I will not attempt to resolve this open question. Because I am concerned here with knowledge ascriptions, which are factive both when embedding interrogatives and *that*-clauses, this issue is only tangential to the main aim of this paper. In providing a compositional semantics in §1.3.2.1 and Appendix §1.6.1.2, I will assume that the interrogative contributes a set of *true* answers.

²¹Lahiri(105), Williams(184), Lahiri(107), Lahiri(106) and Beck&Sharvit(13) have all defended some versions of THE COUNTING APPROACH to gradability. With only a few technical differences, Groenendijk and Stokhof(71) also implement the same kind of approach as THE COUNTING APPROACH. In general, none of the semantics of interrogatives thus far proposed can account for quantitative modification into mention-some interrogatives.

with the goal set at the outset: It analyzes quantitative gradability as quantification over HK-answers to a question of the form *How to ϕ* , and HK-answers are nothing else than propositions directly answering that kind of question:

- (22) a. *S knows in part how to ϕ* ;
 b. *For some true HK-answer p in the denotation of ‘How to ϕ ’ at c , S knows p .*

(22-b) specifies the truth conditions of (22-a) in terms of a propositional knowledge predicate, as desired.

One striking virtue of THE COUNTING APPROACH, moreover, is that it can elegantly explain the lack of gradability into know-*that* ascriptions. On the THE COUNTING APPROACH, know *wh*-ascriptions are *quantified* sentences, where the quantifier quantifies over the denotation of the interrogative—a set of true propositions. When ungraded, the quantifier is universal. When graded, however, the quantifier is either universal (“fully,” and “completely”), existential (“in part”) or a most quantifier (“for the most part”). Standardly, instead, *that*-clauses are not taken to denote *sets* of propositions. The denotation of *that*-clauses is a single proposition—i.e., not the right kind of entity to be quantified over. Hence, the non-gradability of know *that* ascriptions.²²

Despite all its virtues, THE COUNTING APPROACH cannot be the right analysis for quantitative gradability into know how ascriptions. Hence, it cannot be the whole story about the general phenomenon of quantitative gradability. To explain what the problem is, it is necessary to distinguish between two possible readings of embedded interrogatives: the mention-all reading and the mention-some reading. The mention-

²²Another attractive feature of THE COUNTING APPROACH is that it predicts that reports embedding whether-interrogatives are *not* quantitative gradable, a prediction that is borne out. See the final Appendix §1.6.1.1 for a discussion of the gradability into whether-interrogatives.

all reading requires, for every correct HK-answer to the embedded interrogative, that the subject bear the relevant attitude towards that answer; on their mention-some reading, by contrast, those reports can be true even if the subject bears the relevant attitude towards only *one* of the correct HK-answers to the embedded interrogative. To illustrate, (23) may be true even if Mark knows only of one location where he can find an open gas station:

(23) Mark knows where he can find an open gas station.

The mention-some reading is privileged for reports embedding *infinitival* interrogatives, including ascriptions of the form ‘ S knows how to ϕ ’.²³ For instance, for Mark to know where to find an Italian restaurant it suffices that Mark know the location of (or the directions to) *one* Italian restaurant, as opposed to having to know every location that hosts an Italian restaurant; and Mark can know how to make *tagliatelle al ragù* just by virtue of knowing *one* recipe for making *ragù*, as opposed to having to know all of *ragù*’s many possible recipes.²⁴

Now, the problem for THE COUNTING APPROACH is that, crucially, ascriptions of know how *can* be graded even in their mention-some reading. To illustrate, suppose Gianni knows only one recipe for cooking *tagliatelle al ragù*: a Bolognese recipe his grandmother Silvana, from Bologna, taught him. Gianni then knows only one HK-answer to the question *How to prepare tagliatelle al ragù*. Suppose now Gianni tries

²³As emphasized by Bhatt(18). Note that I am not taking a stance on how the mention-some reading is triggered and on whether it is a distinctively *semantic* reading. My only claim is that with infinitival interrogatives the mention-some reading is usually the most salient one (perhaps because of its more practical relevance). By no means am I claiming, moreover, that the mention-some reading is the *only* possible reading for infinitival embedded interrogatives. For example, in a sentence such as “Mark knows where to tread carefully in a minefield,” the mention-all reading seems more relevant and more salient.

²⁴There are three main different recipes for pasta *al ragù*, the Bolognese sauce, *ragù alla Napolitana* and *ragù alla Barese*. See <http://en.wikipedia.org/wiki/Rag%C3%B9> for more information. Besides these three main recipes, there exist many more local and regional versions.

to teach John, who only learns from Gianni how to knead the pasta for *tagliatelle* but not how to make the Bolognese sauce. It seems correct to say in this situation that John knows only in part how to prepare *tagliatelle al ragù*: What he has learned is part of one and only one HK-answer to the question *How to prepare tagliatelle al ragù*, that Gianni tried to teach him, without success.

On THE COUNTING APPROACH, John knows in part how to prepare *tagliatelle al ragù* if and only if, for some HK-answer p to the question *How to prepare tagliatelle al ragù*, John knows p . But, as the example just given illustrates, John may know in part how to prepare *tagliatelle al ragù* without knowing *any* full HK-answer to the question *How to prepare tagliatelle al ragù*, and only by virtue of his knowing part of any such full HK-answer. THE COUNTING APPROACH simply cannot account for this judgment.²⁵

1.3.2 Introducing The Part-Whole Approach

I propose to replace THE COUNTING APPROACH with a different kind of approach to the phenomenon of quantitative gradability. My approach correctly predicts the truth conditions of graded ascriptions of know how. Importantly, moreover, it can be generalized to an analysis of all quantitatively graded knowledge-*wh* ascriptions. I call it “THE PART-WHOLE APPROACH.”

There are two core ideas to THE PART-WHOLE APPROACH.

²⁵As I argue in Pavese(128), THE COUNTING APPROACH yields wrong predictions in a variety of other cases as well, such as reports embedding singular interrogatives—i.e., interrogatives expressing questions that happen to have only one true HK-answer. For instance, consider:

- (i) Mark knows in part but not completely what is the correct solution to the problem.

The question *What is the correct solution to the problem* has only one true HK-answer: The answer that x is the correct solution to the problem for an assignment of the correct solution to x . THE COUNTING APPROACH cannot account for quantitative gradability in (i), for the HKT-denotation of the embedded interrogative is just a singleton.

1. The first idea is that, *knowing in part* Q for any interrogative ' Q ', is to be analyzed as *knowing some part of a true complete answer to the question expressed by* ' Q ' (relative to a context c).

For instance, on this account, John knows in part how to prepare *tagliatelle al ragù* just in case John knows some part of a complete true answer to the question *How to prepare tagliatelle al ragù*; Mary knows in part who went to the party just in case Mary knows some part of a complete answer to the question *Who went to the party*.

2. The second idea is that, for the purpose of formal semantics, an answer to a question Q is not best thought of as a proposition. Rather, we should think of an answer to Q as an ordered pair that has as its first element a proposition answering Q and as its second element the question Q itself.

I discuss and develop these core ideas in turn.

According to THE PART-WHOLE APPROACH, the intuitive truth conditions of graded know *wh* ascriptions are as follows:

THE PART-WHOLE APPROACH²⁶

- ' S knows in part Q ' is true at a context c if and only if, for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, S knows some part of α .
- ' S knows for the most part Q ' is true at a context c if and only if, for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, one knows most part of α .
- ' S knows (fully) Q ' is true at a context c if and only if, for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, one knows every part of α .

²⁶As THE PART-WHOLE APPROACH makes explicit, the truth conditions of knowledge-*wh* ascriptions are given by an existential (count) quantification over complete true answers. The mass quantification is instead over parts of a complete true answer.

What is a *complete* answer to a question?

Whether an answer is complete depends on the kind of question it answers. There are three main kinds of questions:

1. *mention-some questions*—i.e., questions expressed (relative to a context) by a mention-some reading of an interrogative;
2. *mention-all questions*—i.e., questions expressed (relative to a context) by a mention-all reading of an interrogative;
3. *yes-no questions*—i.e., questions expressed (relative to a context) by a whether-interrogative.

A mention-some question is completely answered by an HK-answer. For instance, an HK-answer specifying one location where to find Italian newspapers is a complete answer to the question expressed by “Where can I find an Italian newspaper?” in its mention-some reading.

A yes-no question such as *Whether X is a location where I can find an Italian newspaper* for some assignment of a location to ‘X’ is instead completely and truly answered by either a positive answer such as *Yes, X is where you can buy an Italian newspaper*, or a negative answer such as *No, X is not where you can buy an Italian newspaper*.

Finally, a mention-all question is not necessarily completely answered by a single HK-answer. For instance, *Mary sang* does not completely answer *Who sang*, if Mary is not the only person who sang. A mention-all question such as *Who sang* is completely answered by an answer that specifies for all those who sang, that they sang. The conjunction of every of its true HK-answers is a complete answer to a mention-all question.²⁷

²⁷I want to leave it open in this essay whether a complete true answer to a mention-all question

So, THE PART-WHOLE APPROACH takes the denotation of *every* embedded interrogative to be a set of *complete* true answers in the sense clarified. Because a mention-some question can have more than one complete true answer—more than one HK-answer—the set corresponding to the denotation of a mention-some interrogative may contain more than one HK-answer. By contrast, the denotation of yes-no interrogatives and of interrogatives in their mention-all reading is always a *singleton*: In the first case, the singleton containing either a positive answer or a negative answer; In the second case—that of a mention-all interrogative—the singleton contains a unique exhaustive true answer.

So much for complete answers. The next question is: In what sense do complete answers have parts? The issue is pressing because answers are standardly taken to be propositions. And it is not clear that propositions have parts. If answers are not propositions, on the other hand, it is not clear that THE PART-WHOLE APPROACH succeeds at analyzing the satisfaction conditions of a gradable predicate in terms of an absolute propositional knowledge predicate. Moreover, if propositions had parts, we should expect quantitative gradability to extend into know *that* ascriptions, but it does not, and that is exactly the puzzle with which we started. So, why is not quantitative gradability possible into know *that* ascriptions?

The core idea that I shall defend is that answers have parts only *qua* answers to a certain question. In other words, the part-whole structure of an answer is inherited by the part-whole structure of the question it answers. Questions are naturally thought

is a *weakly exhaustive answer* or a *strongly exhaustive answer*. A *weakly exhaustive answer* to a question is an answer that specifies the positive extension of that question's predicate. For instance, a *weakly exhaustive answer* to the question *Who came to the party* is an answer which specifies for all people who went to the party that they went. Thus an HKT-denotation is a *weakly exhaustive answer*. A *strongly exhaustive answer* is an answer which specifies both the negative and the positive extension. For instance, a *strongly exhaustive answer* to the question *Who came to the party* is one which specifies for all people who went to the party, that they went and for all people who did not go to the party that they did not go. Groenendijk&Stokhof's(69) semantics uses strongly exhaustive answers.

of as having *parts*: intuitively, parts of a question \mathcal{Q} are those questions that need to be answered to reach a complete answer to \mathcal{Q} . Parts of a question \mathcal{Q} are \mathcal{Q} 's *sub-questions*. As showed in Appendix 1.6.1.1, the notion of a sub-question can be given a formal and rigorous definition. The notion of part of an answer can in turn be defined *via* the notion of sub-question. To a rough approximation, an answer to a question \mathcal{Q} has as its parts the propositions that answer \mathcal{Q} 's sub-questions. Thus, answers to a question have as many parts as there are sub-questions to that question.

If answers have parts, do propositions have parts too? They do not need to, because as anticipated answers to a question are not better thought of simply as propositions answering it. Here is an argument for this conclusion. Relative to different questions, the same proposition may have different part-whole structures. Assuming that answers are individuated by the parts they have, that means that, relative to different questions, there may correspond more than one answer to the same proposition. This point can be illustrated vividly by considering the contrast in acceptability between the following two reports:

- (24) a. ??Mark knows in part whether this recipe is how his aunt makes lasagna
 b. ??Mark knows in part whether a certain recipe is how his aunt makes lasagna.
 c. ??Mark knows in part whether his aunt makes lasagna by following this recipe.
- (25) Mark knows in part how his aunt makes lasagna.

The contrast in acceptability between the sentences in (24-a)–(24-c) and (25) is striking, but hard to explain. After all, only one and the same proposition truly and completely answers the questions expressed by their embedded interrogatives. Consider (24-a). The proposition *that recipe r is how Mark's aunt makes lasagna*, for

a suitable assignment of a recipe to *r*, answers both the question expressed by its embedded interrogative and the question expressed by the embedded interrogative in (25). If the complete answers to the two questions *whether that recipe is how Mark's aunt makes lasagna* and *how Mark's aunt makes lasagna* are identical with the unique proposition that completely and truly answers both, it is not clear how to account for the contrast.²⁸ As another instance of the contrast, compare the following two sets of sentences:

- (26)
- a. ??Mark knows in part whether 106 Somerset Street, 5th Floor New Brunswick NJ is Rutgers philosophy's full new address.
 - b. ??Mark knows in part whether Rutgers philosophy's full new address is 106 Somerset Street, 5th Floor New Brunswick NJ.
 - c. ??Mark has come to know whether Rutgers philosophy's full new address is 106 Somerset Street, 5th Floor New Brunswick NJ
 - d. ??Mark has come to know in part whether 106 Somerset Street, 5th Floor New Brunswick NJ is Rutgers philosophy's full new address.
 - e. ??Mark has learned in part whether 106 Somerset Street, 5th Floor New Brunswick NJ is Rutgers philosophy's full new address.
 - f. ??For the most part, Mark has learned whether 106 Somerset Street, 5th Floor New Brunswick NJ is Rutgers philosophy's full new address.
- (27)
- a. Mark knows in part what is Rutgers philosophy's full new address.
 - b. Mark knows in part what Rutgers philosophy's full new address is.

²⁸One may wonder whether the contrast has to be explained semantically rather than otherwise, for instance, syntactically. It is hard to see what syntactic explanation could work in this particular case. After all, whether-interrogatives and wh-interrogatives are supposed to have the same syntax: They both are interrogative CPs. Moreover, a syntactic explanation should be the last resort. In Pavese(128) I give a more articulate argument for thinking that the contrast has to be explained semantically.

- c. Mark has come to know in part/for the most part what Rutgers philosophy's full new address is.
- d. Mark has come to know in part what Rutgers philosophy's full new address is.
- e. Mark has learned in part what Rutgers philosophy's full new address is.
- f. For the most part, Mark has learned what Rutgers philosophy's full new address is.

The two sets of sentences seem to widely differ in their acceptability. The explanation for the contrast *cannot* simply be that the interrogative construction in the first set of sentences is “heavy,” because the same contrast is observable also between (28-a)-(28-b) (for a suitable assignment to the demonstratives in them) and (28-c)-(28-d) and their interrogatives are about the same length:

- (28)
- a. ??Mark knows in part whether that is Rutgers philosophy's full, new address.
 - b. ??Mark learned in part whether Rutgers philosophy's full, new address is that one.
 - c. Mark knows in part what Rutgers philosophy's full, new address is.
 - d. Mark knows in part what is Rutgers philosophy's full, new address.

If the contrast is to be explained semantically, then the answers to the two questions expressed by the two interrogatives have to have a *different part-whole structure*. If answers are individuated by their part-whole structure, the true answer to the question *whether 106 Somerset Street, 5th Floor New Brunswick NJ is Rutgers philosophy's full new address* ought to be different from the true answer to the question *what is Rutgers philosophy's full new address*, even though the same proposition answers both questions. But if so, then an answer to a question—the semantic unity of

interrogatives—cannot simply a proposition that answers that question.

This kind of contrast can be explained if we take propositions to have parts only relative to a question—only *qua* answers to that question—and possibly different parts relative to different questions—*qua* answers to different questions. So, for instance, the proposition *that recipe r is how Mark’s aunt makes lasagna* has parts relative to the question *How Mark’s aunt makes lasagna* but no parts relative to the question *Whether recipe r is how Mark’s aunt makes lasagna*. Relative to different questions, the same proposition may have different part-whole structures. If answers are individuated by the parts that they have, the answers to those two questions must be different. So, the contrast in acceptability between (24-a) and (25) can be explained semantically, if we take the *answers* to the two questions to be different—i.e., only the latter having parts that Mark can know. To get an answer with a definite part-whole structure out of a proposition, one needs to specify what question it is an answer to. Once a question is specified, so is the answer’s part-whole structure.

This brings us to the second core idea of my THE PART-WHOLE APPROACH. For the purpose of the formal semantics, it is helpful to distinguish between a proposition—a *bare proposition*—and a proposition *qua* answer to a particular question—an *answer* properly said—as I will call it. The members of an interrogative’s denotation should not be thought of as bare propositions. Rather, the members of an interrogative’s denotations should be modeled as *ordered pairs*, having as their first element a proposition and as their second element the question it answers. For instance, the denotation of a mention-all question should be thought of as the singleton containing the following ordered pair:

(29) $\langle p, \textit{Who came to the party} \rangle;$

where *p* is a proposition completely and truly answering the mention-all question *Who came to the party* and where the question *Who came to the party* is a set of

propositions that completely (but not necessarily truly) answer it. The denotation of a mention-some question such as *How to ϕ* is the set containing ordered pairs such as:

$$(30) \quad \langle p, \textit{How to } \phi \rangle; \langle q, \textit{How to } \phi \rangle; \dots$$

where p, q, \dots , are propositions completely and truly answering the question *How to ϕ* , and the question *How to ϕ* itself is a set of propositions that completely, but not necessarily truly, answer it. As *How to ϕ* is a mention-some question, it will correspond to a set of possible HK-answers.

By thinking of members of an interrogative's denotation as ordered pairs in this fashion, we can formally capture some aspect of our intuitive notion of answers, such as their relativity—i.e., the fact that answers are answers *to* particular questions. Moreover, we can capture the fact that two distinct answers may be *propositionally equivalent*—i.e., may agree in their first members—and yet be distinct, because they have different part-whole structures, as in the example given above. (In the following, I will adopt the convention of calling “*bare proposition*” the first element of the ordered pair; I will reserve the label of “*answer*” for the ordered pair.)

Once the members of the denotation of interrogatives are modeled in this way, adverbial modification into embedded interrogatives can be given a general analysis. Embedded interrogatives can be thought of on the model of *mass terms*. Adverbial modification into *mass terms* is ubiquitous in English:

- (31) a. Coffee is for the most part grown in Latin America.²⁹
 b. Plasma in the blood is mostly made of water.³⁰

²⁹<http://www.coffeepartsplus.com/CoffeeBeans.htm>

³⁰http://wiki.answers.com/Q/What_is_plasma_mostly_made_of

The denotation of mass terms is standardly associated with *algebras* (Moravcsik(122), Roeper(148) and Lønning(116)). Parthood can then be specified *via* a *measure* defined over the elements of the algebra. A measure is a function from elements of the algebra to the number of those elements’ parts—a function that satisfies certain further constraints, such as additivity. Mass quantifiers quantify over algebras *via* measures responsible for counting the algebras’ parts.

On the same model, we can think of each complete answer as associated with an algebra of propositions (As showed in detail in the formal Appendix 2.7). The propositions in the algebra are parts of the answer. The top of the algebra is the first element of the answer. Just like standard mass quantifiers, adverbial modifiers into embedded interrogatives quantify over the elements of those algebras *via* a measure. Such a measure (associated with the meaning of the mass quantifiers “in part” and “for the most part”) is responsible for counting the *parts* of an answer. On this proposal, expressions such as “in part” and “for the most part” would be *mass* quantifiers: they quantify over *parts* of a complete true answer.³¹ The resulting analysis is one on which the truth conditions of (32) at a context *c* are then given by (32-b):

- (32) a. *S knows in part how to ϕ .*
 b. *For some true complete answer in the denotation of ‘How to ϕ ’ at a*

³¹The behavior and the logical properties of mass quantifiers are now well-studied (Lønning(116), Higginbotham(85)). Hence, their appearance here is neither surprising nor unmotivated. In fact, analyzing adverbial modification into embedded interrogatives quite generally as an instance of mass quantification into argument position has the advantage of generality: Since Link’s(114), (115) work on plurals, it is customary to look for structural analogies between plural terms and mass terms. If I am right, we observe a similar structural analogy in the domain of embedded interrogatives: “Plural” interrogatives (interrogatives that have as their HKT-denotations sets of HK-answers with cardinality higher than 1) and what I call “singular” interrogatives—i.e., interrogatives that express questions that happen to have only one HK-answer, are both quantified into by the same kind of quantifiers. Higginbotham(85) teaches us to think of mass quantifiers as functions from ordered elements of Boolean algebras, together with a measure responsible to count parts of those elements, to true values. He shows that, so-conceived, mass quantifiers behave like count quantifiers, in that they satisfy the Mostowski’s condition. In my analysis and in the Final Appendix 1.6.1.2, I will rely on this characterization of mass quantifiers.

context c , S knows part of that answer.

A complete answer to a mention-some question *How to ϕ* is just an ordered pair $\langle p, \textit{How to } \phi \rangle$, where p is what we previously called a “HK-answer,” a proposition of the form *w is how to ϕ* , for some way w to ϕ . Now, we have reserved the label of “HK-answer” for the ordered pair rather than for its first element. The ordered pair is true just in case its first element is. Then, (32-b) is equivalent to (33):

(33) *For some true HK-answer to the question how to ϕ , S knows part of that HK-answer.*

This analysis is exactly what we want.

First of all, it affords us the desired reduction of the satisfaction conditions of a gradable predicate ‘knowing how to ϕ ’ into an absolute propositional knowledge state. According to it, knowing how to ϕ is a matter of knowing *every part* of a complete true answer to the question *How to ϕ* ; knowing in part how to ϕ is a matter of knowing *some part* of that answer. Because parts of an answers are themselves elements of an algebra of *propositions*—i.e., are, in other words, themselves propositions—THE PART-WHOLE APPROACH succeeds at specifying the satisfaction conditions of the gradable predicate ‘know how to ϕ ’ in terms of an absolute knowledge state.

This approach, moreover, explains why adverbial modification is infelicitous into *know-that* ascriptions. I argued that propositions have parts only relative to questions. Only relative to a question does a proposition have a definite part-whole structure. This diagnosis suggests the following explanation of the lack of quantitative gradability into *know-that* ascriptions: *that*-clauses cannot be adverbially modified into because they do not specify questions relative to which their denotation—the proposition they denote—can be ground into parts. The denotation of *that*-clauses is thus a bare proposition, not an answer, in the special sense in which I have been

using the term. As such, the denotations of *that*-clauses are not associated with algebras. But because the meaning of mass quantifiers is associated with measures defined over algebras of propositions, the truth conditions of adverbially modified know-*that* ascriptions are simply undefined. Thus, as developed, THE PART-WHOLE APPROACH nicely explains why the denotation of *that*-clauses cannot be adverbially modified into.³²

A full development of THE PART-WHOLE APPROACH would require showing, for any kind of report embedding interrogatives, how to derive the truth conditions predicted by it from a compositional semantics. Moreover, a full development of the approach would require me to tell much more about the part-whole structure of different kinds of answers—i.e., to explain, for instance, why certain answers, such as answers to whether questions, do not have parts (On these issues, see Appendix 1.6.1.1 and Pavese(128)).³³

³²Do not *that*-clauses specify whether-questions? This is a controversial issue. Schaffer(153), for instance, believes that know-*that* ascriptions ascribe knowledge of an answer to a whether-question. It is important to observe, however, that whether or not *quantitative*-clauses specify whether-questions is irrelevant to whether or not my diagnosis is correct. According to my definition of sub-questions, which the reader can see in the final appendix, whether-questions do *not* have sub-questions, because no answer can merely partially answer a whether-question. According to my general account of parts of answers, not every answer has parts. An answer has parts only if it answers a question that has sub-questions. Because whether-questions do not have sub-questions, answers to whether-questions do not have parts. Only answers to questions that have sub-questions have a part-whole structure. See Appendix 1.6.1.1 for discussion.

³³Here is an objection worth considering. On my analysis, answers have parts because questions do: The part-whole structure of answers is—so to say—inherited from the part-whole structure of questions. So my analysis is committed to questions having parts. Now, intensional (or “rogative”) verbs, such as “wonder” and “ask,” take the intension of interrogatives as their arguments. And the intension of interrogatives is standardly assumed to be the question they express. However, reports with intensional verbs do not seem to allow for adverbial modification:

- (i) # Mark wonders in part how to cook the lasagna.
- (ii) # Mary asked in part how to cook the lasagna.

(i) does not have a reading on which Mary asked part of the question *How to cook lasagna*. On the assumption that the intension of interrogatives are questions and on the further assumption that “wonder” and “ask” are intensional verbs, we would expect the possibility of adverbial modification

Given the goal of this chapter, I will not take on this challenge here. Rather, here I will only endeavor to defend THE PART-WHOLE APPROACH as an analysis of *quantitative gradability into know how ascriptions*. In the next section, I show how to implement the intuitive truth conditions specified by THE PART-WHOLE APPROACH into a fully compositional semantics. In the section following next, I illustrate in rather intuitive terms how we should think of practical answers and their parts.

1.3.2.1 The Compositional Semantics

The denotation of an interrogative ‘How to ϕ ’—the set of true complete answers to the question *How to ϕ* —can be obtained compositionally from an abstract β given

into those reports.

It is true that intensional reports embedding interrogatives do not *always* allow for quantitative gradability and that this fact calls for an explanation. However, as also noticed by Beck&Sharvit(13), in *some context*, we do get quantitative gradability into intensional reports. Suppose I am unsure whether Mark has finally learned how to cook lasagna, and I asked my friend, who was present at Mark’s attempts at cooking it. Then the following make total sense:

- (iii) Did Mark manage to make lasagna on his own?
- (iv)
 - a. Yes, and for the most part he did not even ask me how to do it.
 - b. Yes, and for the most part he did not even wonder how to do it.

If we sometime get adverbial modification into intensional reports, then the unacceptability of (i) and (ii) does not show (even on the assumptions previously listed) that questions do not have parts. Moreover, observe that questions can explicitly be adverbially modified into, as in:

- (v) Mary asked her question only in part.
- (vi) Your question was partly answered by Mary.

So the unacceptability of (i) is hardly an objection to the analysis proposed here. An explanation of the unacceptability of those sentences is not really relevant to an assessment of the main proposal in this essay, which is about extensional (or responsive) reports and the denotation of embedded interrogatives. Let me just say that I suspect, following Beck&Sharvit(13), that the explanation of the oddity of (i) and (ii) has to do *not* with the impossibility of quantifying over parts of the intensions of interrogatives but rather with the specific presuppositional structure of the embedding verbs. The interested reader may find a more extensive discussion of these issues in Pavese(128).

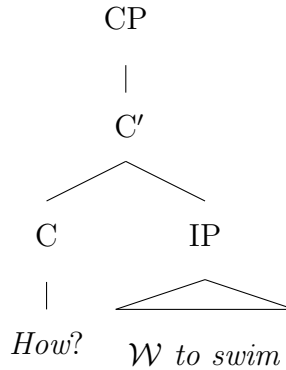
by (34):

$$(34) \quad \lambda \mathcal{W}_e (\mathcal{W} \text{ is a way to } \phi).$$

(34) is a property of ways to ϕ . We want to apply the meaning of a question word *How?* to the abstract (34) and get as a result the set of complete true answers to the question *How to ϕ* —i.e., the set of *ordered pairs* that have as their first element a proposition truly answering the practical question *How to ϕ* and as their second element the question itself—i.e., a set of true practical answers. For this purpose, the meaning of the question word is as follows:

$$(35) \quad \llbracket \text{How?} \rrbracket = \lambda \alpha_{\langle p, \mathcal{Q} \rangle} [\alpha = \langle p_{\langle s, t \rangle}, \mathcal{Q}_{\langle p, t \rangle} \rangle: p^T \ \& \ \exists \mathcal{W} (p = \lambda w'_s (\mathcal{W} \in \beta \text{ at } w')) \ \& \ \mathcal{Q} = \lambda p (\exists \mathcal{W}^*: p = \lambda w^*_s (\mathcal{W}^* \in \beta \text{ at } w*))].$$

By applying the $\llbracket \text{How?} \rrbracket$ to the abstract, we get the interrogative's denotation:



$$(36) \quad \begin{array}{l} \text{a. } \llbracket \text{How to } \phi \rrbracket = \llbracket \text{C} \rrbracket (\llbracket \text{IP} \rrbracket) = \\ \text{b. } \lambda \alpha_{\langle p, \mathcal{Q} \rangle} [\alpha = \langle p_{\langle s, t \rangle}, \mathcal{Q}_{\langle p, t \rangle} \rangle: p^T \ \& \ \exists \mathcal{W} (p = \lambda w'_s (\mathcal{W} \text{ is a way to } \phi \text{ at } w')) \\ \ \& \ \mathcal{Q} = \lambda p (\exists \mathcal{W}^*: p = \lambda w^*_s (\mathcal{W}^* \text{ is a way to } \phi \text{ at } w*))]. \end{array}$$

The interrogative 'How to ϕ ' semantically denotes a set of true answers α —i.e., the set of ordered pairs that have as a first element a proposition truly answering the question *How to ϕ* and as a second element the practical question *How to ϕ* itself. I indicate

such ordered pairs by $\ulcorner \alpha_{\langle \langle s,t \rangle, \langle \langle s,t \rangle, t \rangle} \urcorner$ (for short $\ulcorner \alpha_{\langle p, \mathcal{Q} \rangle} \urcorner$)—i.e., $= \langle p_{\langle s,t \rangle}, \mathcal{Q}_{\langle p,t \rangle} \rangle$. An answer $\alpha_{\langle p, \mathcal{Q} \rangle}$ is true (for short $\ulcorner \alpha^T \urcorner$) just in case its first element truly answers its second element—i.e., if and only if p truly answers \mathcal{Q} . (37) represents the denotation of the embedded interrogative:

$$(37) \quad \lambda \alpha (\alpha^T \in \text{How to } \phi).$$

The modifiers “in part”, “for the most part” take an answer α into those parts of α that are known by a subject. Recall that an answer is associated with an algebra of propositions $\mathcal{A}^*(\alpha)$ that has the first element of that answer as its top. μ is a measure defined over $\mathcal{A}^*(\alpha)$. On this analysis, S knows part of a practical answer if and only if the value of μ for the intersection between the top of $\mathcal{A}^*(\alpha)$ and the propositions known by S is different from zero:

$$(38) \quad \text{In part } [\alpha; \lambda p_{\langle s,t \rangle} (S \text{ knows } p)] = 1 \text{ iff } \mu (\mathcal{A}^*(\alpha) \cap \lambda p_{\langle s,t \rangle} (S \text{ knows } p)) \neq 0.$$

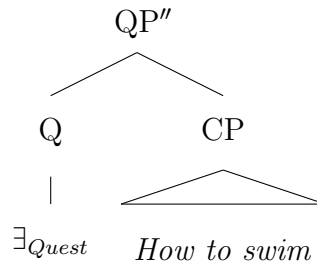
Less formally, according to (91), S knows part of a practical answer if and only if the number of propositions of the algebra associated with that answer that are known by S is different from zero. Compositionally, “in part” takes an answer into the property *being known by S* such that part of that answer falls under the propositions known by S . Equivalently, given the meaning of “in part,” a part of an answer falls under the property of being known by S just in case the measure of the intersection between the top of the algebra associated with that answer and the propositions known by S is not empty. So for simplicity, in the composition, I abbreviate (39) as (39-b):

$$(39) \quad \begin{array}{l} \text{a. } \llbracket \text{in part} \rrbracket = \lambda \alpha \lambda P_{\langle p,t \rangle} (\mu (\mathcal{A}^*(\alpha) \cap P) \neq 0). \\ \text{b. } \llbracket \text{in part} \rrbracket = \lambda \alpha \lambda P_{\langle p,t \rangle} (\text{PART}(\alpha) \in P). \end{array}$$

Finally, the predicate “knows” contributes the set of propositions known by a subject. The composition goes as follows:

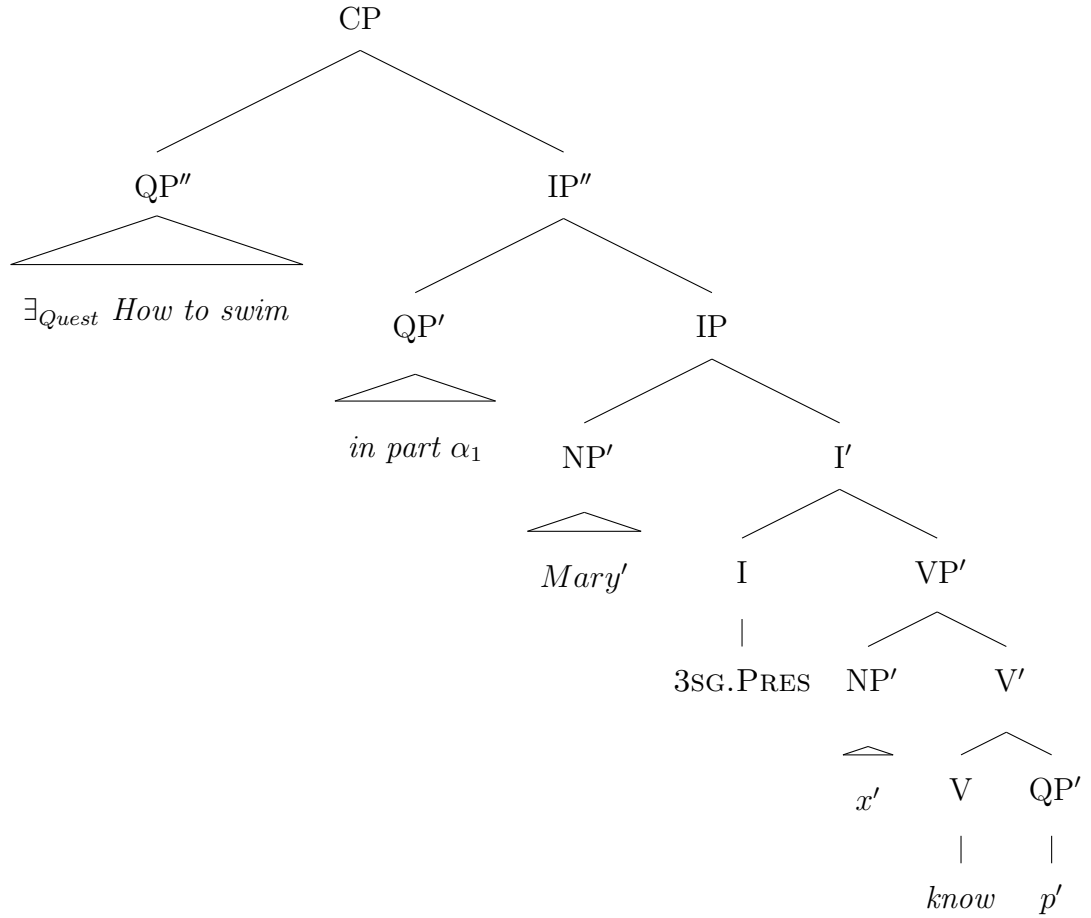
- (40) a. $\llbracket IP'' \rrbracket = \llbracket QP'' \rrbracket(\llbracket IP \rrbracket) =$
 b. $\llbracket in\ part\ \alpha_1 \rrbracket(\llbracket Mary\ knows\ p^*\ at\ w^* \rrbracket)$
 c. $\lambda P_{\langle p,t \rangle}(\text{PART}(\alpha) \in P)(knows(w^*)(p)(Mary))$
 d. $\text{PART}(\alpha) \in (knows(w^*)(p)(Mary))$ (by Functional Application).

$\lceil \exists_{Quest} \rceil$ is a lifted existential quantifier as in Heim & Kratzer(82). In my implementation, \exists_{Quest} takes as arguments two sets of answers and gives the truth just in case the intersection of the two sets is not empty. It first applies to the denotation of the interrogative, as in:³⁴



- (41) a. $\llbracket QP'' \rrbracket = \llbracket Q \rrbracket(\llbracket CP \rrbracket) =$
 b. $\llbracket \exists_{Quest} \rrbracket(\llbracket How\ to\ swim \rrbracket) =$
 c. $\lambda P_{\langle \alpha,t \rangle} \lambda P'_{\langle \alpha,t \rangle} \exists \alpha (P(\alpha) \ \& \ P'(\alpha)) (\lambda \alpha (\alpha^T \in How\ to\ \phi))$
 d. $\lambda P'_{\langle \alpha,t \rangle} \exists \alpha (\lambda \alpha (\alpha^T \in How\ to\ \phi)(\alpha) \ \& \ P'(\alpha))$
 e. $\lambda P'_{\langle \alpha,t \rangle} \exists \alpha (\alpha^T \in How\ to\ \phi \ \& \ P(\alpha))$ (by Functional Application).

³⁴The use I am making of $\lceil \exists_{Quest} \rceil$ is analogous to George’s (57). The main difference is that George takes it to be a quantifier over *propositions*, whereas I take it to be a quantifier over *answers*, which—on my analysis—are ordered pairs of propositions and questions. So, in positing an \exists_{Quest} operator, my analysis does not at all depart from orthodoxy.



(For the derivation of this LF, see the final appendix). The result then applies to the whole IP'' , as follows:

- (42)
- a. $\llbracket CP \rrbracket = \llbracket QP'' \rrbracket (\llbracket IP'' \rrbracket)$
 - b. $\llbracket \exists_Q \text{ How to swim} \rrbracket (\llbracket \text{in part } \alpha_1 \text{ Mary knows } p_1 \rrbracket) =$
 - c. $\lambda P'_{\langle \alpha, t \rangle} \exists \alpha (\alpha^T \in \text{How to } \phi \ \& \ P(\alpha)) (\text{PART}(\alpha) \in (\text{knows}(w^*)(p)(\text{Mary})))$
 - d. $\exists \alpha (\alpha^T \in \text{How to } \phi \ \& \ (\text{PART}(\alpha) \in (\text{knows}(w^*)(p)(\text{Mary})))) (\alpha)$
 - e. $\exists \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{PART}(\alpha) \in (\text{knows}(w^*)(p)(\text{Mary})))$ (by Functional Application).
 - f. For some practical answer to the question *How to* ϕ , S knows part of that practical answer.

This concludes the implementation of the intuitive truth conditions specified by the THE PART-WHOLE APPROACH into a fully compositional semantics.

1.3.2.2 Practical Answers

I showed how to derive the intuitive truth conditions of graded know how ascriptions from a compositional assignment of values to the sub-sentential expressions of the ascriptions. This section is devoted to explaining in much greater detail what practical answers and their parts are.

We observed that examples such as (43-a)-(43-b):

- (43) a. The question is not *if* but how and why to keep the shuttles flying.³⁵
 b. The question is not how to survive but how to thrive with passion, compassion, humor and style.³⁶

suggest that interrogatives of the form 'How to ϕ ' express a *single* question, the practical question *How to ϕ* . A proposition completely answering a practical question is a PRACTICAL PROPOSITION. To a first approximation, a practical proposition is of the form *that w is how to ϕ* for some substitution of w with a way to ϕ . Different practical propositions may truly and completely answer a practical question, for there is usually more than one possible way to ϕ .

The denotation of an interrogative of the form 'How to ϕ ' is in turn a set of ordered pairs—or PRACTICAL ANSWERS—having as a first element a practical proposition and as a second element the practical question expressed by the interrogative at a context. Knowing how to ϕ is a matter of knowing every part of a practical answer to the question *How to ϕ* . Knowing every part of a practical answer at least entails

³⁵<http://nasawatch.com/archives/2010/04/the-question-is.html>

³⁶<http://0theplaceswewillgo.blogspot.com/2011/12/question-is-not-how-to-survive-but-how.html>

that one knows a practical proposition—i.e., one knows that a certain way to ϕ is how to ϕ . Hence, knowing how to ϕ at least entails that one knows that a certain way to ϕ is how to ϕ .

The reverse does not seem true, however.³⁷ For it seems like one may know that a certain way is how to ϕ , without knowing how to ϕ . For instance, suppose I watch Phelps's swimming, and I come to know of his way of swimming that it is a way to swim. But I do not thereby come to know how to swim in the relevant sense, let alone come to know how to swim as Phelps does. The standard response to this challenge is to appeal to *practical modes of presentation*.³⁸ According to this move, knowing how to ϕ is matter of knowing that a certain way to ϕ is how to ϕ *under a practical mode of presentation*. But what is a practical mode of presentation?

My preferred way to think of practical modes of presentation is to think of them on the model of computer programs, as *representations of algorithms*.³⁹⁴⁰ It is indeed rather standard for computer scientists to think of algorithms and programs as Fregean senses (Girard (63)). On this construal, procedures stand to ways of ϕ -ing as senses stand to their referents. Accordingly, on my view, for one to know how to ϕ , one needs to know a way of ϕ -ing through some procedure. Though one may know that a certain way of swimming is how to swim just by looking at how Phelps swims, one may not know that way *through* a certain procedure to swim. That is why, in the original scenario, one may lack know how. Practical propositions have as their components senses that stand for ways of ϕ -ing. More specifically, a practical proposition

³⁷Trusting that the reader is clear by now on the difference between a practical proposition—a proposition answering a practical question—and a practical answer—a member of the denotation of an interrogative 'How to ϕ ', I will omit this qualification in what follows.

³⁸Stanley and Williamson (178).

³⁹In Chapter II, I develop this view of practical modes of presentation in detail. On the model of programs, a practical sense is a representation of an algorithm in some representational system or other.

⁴⁰This is not how Stanley and Williamson (178) and Stanley (176) seem to think of practical modes of presentation.

‘ $\alpha_{\mathcal{P}}$ ’ to the question *How to ϕ* has as its component a procedure \mathcal{P} *through which* it represents a way to ϕ .

Now, as I am thinking of them, procedures are naturally construed as having *parts*. In analogy with computer programs, a procedure can be thought of as *a maximal way of breaking a complex task into smaller and smaller sub-tasks* that are easier to carry out.⁴¹ Some sub-tasks cannot be further broken down into smaller tasks: They are MINIMAL—i.e., in so far as their internal structure does *not* matter for the execution of the complex task. Some sub-tasks are not minimal in this sense.

To illustrate, suppose I want to prepare *pasta and fagioli*.⁴² This task can be broken down into two simpler tasks, that of preparing the *fagioli*—i.e., the beans—and that of preparing the pasta. But the sub-task of preparing the beans is itself complex. Among other things, it requires unshelling the beans. And unshelling the beans itself requires obtaining a basket of unshelled beans and an empty bowl where to put them; it requires taking a bean from the basket, breaking open the bean pod, dumping the bean from the pod into the bowl and discarding the pod. Finally, it requires performing these operations in this order as long as there are beans in the basket. Only by addressing each sub-task, will one be in position to prepare the beans for one’s *pasta and fagioli*.

Another—non-culinary—example can be used to illustrate the same general idea of parts of a procedure and of minimal parts. Suppose one wants to sort a list of names into alphabetical order. This task can be easily accomplished by sorting every *subset* of the list into alphabetical order. Hence, on one procedure to sort a list, the task of sorting the whole list can be broken down into the tasks of sorting its parts. Each such a task can be accomplished by executing certain further tasks, such as

⁴¹See Downey(43) for this intuitive characterization of programs.

⁴²See http://en.wikipedia.org/wiki/Pasta_e_fagioli for information about *pasta and fagioli*.

checking that the relevant part of the list is sorted and taking certain actions in case it is not, such as ordering it. Those tasks—checking whether a certain part of the list is sorted and ordering it in case it is not—are sub-tasks of the task of sorting that part of the list. By performing the tasks of sorting a part of the list, for each part of the list, one can perform the complex task of sorting a whole list of names into alphabetical order. This complex task has as subtasks the tasks of sorting a part of the list, as well as the tasks that are sub-tasks of those tasks. Thus, procedures have parts in the following sense: a Procedure for executing a task ϕ has as its parts the sub-procedures for performing the sub-tasks of the whole task ϕ .

Some of these sub-tasks do not further sub-divide into sub-tasks—i.e., they are *minimal*, in so far as their internal structure does *not* matter for the execution of the complex task. Thus, for instance, consider again the task of unshelling the beans. It does not matter to the execution of the task whether I take each bean from the basket with bare hands, or gloves, or if I do it while repeatedly hitting my head with a stick. Other sub-tasks are not simple in the same sense: thus, for instance, the task of repeatedly taking the bean, peeling it, and so on, until the basket is empty must involve the execution of those simpler tasks in a certain *sequence* and *until* a certain condition obtains—i.e., until the basket is empty. Since the right sequencing and control are crucial to the overall execution of the complex task, some sub-tasks cannot be fully reduced to conjunctions of minimal tasks.⁴³ Hence, some parts of a procedure are or can be reduced to minimal parts. Others cannot. A complete

⁴³These particular kinds of sub-tasks that are not structurally simple correspond to what are known as *loops* in the computer science literature. Loops involve the execution of a collection of instructions under a *control* process. Loops have essential internal structure for the execution of the task: The control of a loop involves the three activities of *initialize*, *test* and *modify*. See Brookshear(26) pp. 232–240 for a discussion and a characterization of loops. Other non-minimal kinds of sub-tasks are *recursive structures*. Whereas a loop involves repeating a set of instructions in a manner in which the set is completed and then repeated, recursion involves repeating the set of instructions as a subtask of itself. Like loops, recursive sub-tasks require proper control and involve *initializing*, *modification*, and *tests* for termination.

determination of the procedures for preparing beans depends on the specification of both minimal tasks and of tasks that are not minimal.

A practical proposition represents a way to ϕ through a certain procedure \mathcal{P} . A practical answer in turn is an ordered pair that has a practical proposition as its first element. A practical answer is associated with an algebra of propositions which are the parts of that answer. We can think of reaching a practical answer as the process of answering the smallest sub-questions of the practical question *How to ϕ* first, then the tasks of which those are sub-questions, upper and upper in the tree of sub-tasks, until the complete answer to *How to ϕ* is reached.

On this understanding, a practical answer has as many parts as there are sub-questions to the question that it answers. But there are as many sub-questions to the question *How to ϕ* as there are sub-procedures to the procedure for performing the task of ϕ -ing. Hence, a practical answer $\lceil \alpha_{\mathcal{P}} \rceil$ to the question *How to ϕ* has as many parts as there are sub-procedures to \mathcal{P} . So the parts of a practical answer $\lceil \alpha_{\mathcal{P}} \rceil$ are in one-to-one correspondence with the parts of the procedure \mathcal{P} for ϕ -ing. A procedure \mathcal{P} for ϕ -ing is a way of breaking down the task of ϕ -ing into sub-tasks. Parts of a practical answer to the question *How to ϕ* are propositions specifying how to perform every one of those sub-tasks.

This way of thinking of parts of practical answers ensures that the decomposition of practical answers into their parts will not proceed *ad infinitum*. Programs are *effective*—i.e., *finitely specifiable*. In analogy with programs, procedures are to be thought of as *finitely specifiable* ways of breaking down a complex task into simpler tasks.⁴⁴ Because the decomposition of each practical answer into its parts tracks

⁴⁴Strictly speaking, the finiteness requirement could be dropped. The practical answers could be associated with *infinitary* algebras. A measure can be defined over an infinitary algebra, roughly in the way in which probability measures are defined over infinite probabilistic spaces. But dropping the finiteness requirement would no doubt highly complicate matters. See Girard (63) for a very sophisticated (and complicated) analysis of parts of open-ended procedures. However, finiteness can

a particular procedure's way of breaking down a complex task into its smaller and smaller tasks, by thinking of procedures as themselves finitely specifiable, we are guaranteed that the resulting set of parts will end up being finite.

1.3.2.3 Issues and replies

I provided a compositional semantics for quantitatively graded ascriptions of know how and I have explained in some detail what practical answers and their parts are. I conclude my analysis of quantitative gradability by rebutting some possible objections to my analysis.

1. If things stand as my analysis alleges, then why are not *quantitative* comparisons possible between knowing how ascriptions? Why, in other words, cannot we say the following:

(44) ??Marco knew how to solve the problem more than any of the other students in the class.

After all, on my analysis, one can know how to ϕ to a smaller or greater extent if one knows a smaller or greater extent of the practical answer to the question *How to ϕ* . Now, (44) is indeed not acceptable. However, quantitative comparisons *are* possible with modifiers other than “more than”, as in (45):

also be guaranteed in another way. We can think of the grinding of answers into their sub-answers as being done against a set of background assumptions, a *context set*. A context set can be seen as a partition of a set of possible worlds into cells, roughly corresponding to the number of distinctions among possible worlds that have been “noticed” by the context, as in Yalcin(190). On this view of context, the only propositions that are assumed in the context are those that follow the boundaries of the cells of the partition. Since it is reasonable to assume that, at each time, the context has noticed only a finite number of distinctions among possible worlds, it is reasonable to assume that the number of cells will, at each time, be finite. In this case, the number of propositions that are assumed in the context at each time is also finite. Hence, relative to a similar context, the set of sub-answers into which a practical answer can be broken down cannot be but finite.

- (45) Marco knew how to solve the problem to a larger extent than any of the other students in the class.

It is indeed an interesting phenomenon why the construction “more than” is not available in English to compare knowing how ascriptions and why instead the locution “to greater/smaller extent” is available. However, observe that there is a parallel here again with color sentences:

- (46) This leaf is more green than that leaf.

(46) cannot be used to say that the leaf is green to a greater extent than the other—i.e., that the green area of one leaf is greater than the green area of the other leaf. And yet quantitative comparisons are possible for color sentences as in (47) with the other locution—i.e., “to a greater/larger extent:”

- (47) This leaf is green to a greater extent than that leaf.

2. Suppose I learn how to turn on the oven. I may not thereby know in part how to make lasagna, even though, plausibly, knowing how to make lasagna requires that I know how to turn on the oven. It seems that not every minimal answer to the question *How to ϕ* needs to count as part of an answer to that question. Hence, against my account, not every sub-answer is a part of the practical answer. This conclusion can be conceded on my analysis. On my formal analysis, what is responsible for counting the *parts* of a practical answer is a *measure* defined over the corresponding algebra. This measure is a function from elements of the algebra associated with that practical answer to the number of those elements’ parts. Such a measure is associated with the meaning of the mass quantifiers “in part” and “for the most part.” But different measures can be

defined over an algebra, each possibly defining different parthood relations over it. An adequate analysis of the linguistic phenomenon of quantitative gradability does not need to specify *which* measure happens to be picked out by those quantifiers. It may even be a context-sensitivity issue. Whatever part-hood relation(s) happens to be picked out by the quantifiers “in part” and “for the most part” in English (perhaps at a context) is the one relevant for the truth conditions of ascriptions of know how.

3. By analogy with *deterministic* programs, I have been assuming that procedures can be thought of as complete specifications of each of their parts. Given this conception of procedures, the following will be true: for any practical question \mathcal{Q} , for a practical answer $\alpha_{\mathcal{P}}$ in \mathcal{Q} , and for any sub-question sub- \mathcal{Q} of \mathcal{Q} , only one practical answer in sub- \mathcal{Q} is a sub-practical answer for $\alpha_{\mathcal{P}}$. This fact ensures that there be just one set of sub-practical answers for each practical answer.

However, this raises an issue. Most procedures we are interested in when describing human actions ordinarily leave margins for variation in how every step of the procedure can be executed—i.e., they are, in other words, *nondeterministic*.⁴⁵ A non-deterministic procedure divides into smaller sub-tasks, each of which can be performed in different ways. If procedures are nondeterministic, there may indeed be more than one set of sub-practical answers for each practical answer. So with non-deterministic procedures, we cannot expect there to be a unique decomposition of answers into parts.

There is an easy fix. We can map each non-deterministic procedure \mathcal{P} into a

⁴⁵There are two senses of *nondeterministic* algorithms/procedures that are often conflated but that are worth distinguishing. The first sense is one on which an algorithm is *nondeterministic* if the outcome of executing it cannot be predicted in advance. On a second sense, an algorithm/procedure is *nondeterministic* if its execution may involve taking different steps from time to time. It is the second sense of *nondeterministic* that I have in mind in the main text. See Brookshear (26) for a characterization of nondeterministic procedures.

set of deterministic procedures, each of which “settles” the choices left open by \mathcal{P} . Each deterministic procedure in the set will correspond to *one* possible way of disambiguating \mathcal{P} . Accordingly, a practical answer anchored to a nondeterministic procedure will correspond to a set of answers anchored to *deterministic* procedures. Being anchored to a deterministic procedure, each answer in that set will be susceptible of a unique decomposition into its parts. Hence, each of those answers will correspond to a different way of grinding a practical answer into parts. On this way of proceeding, for it to be true that one knows in part how to ϕ we require that, for some practical answer to the question *How to* ϕ and for *some* possible way of grinding that practical answer into parts, one know part of that practical answer.

1.4 Qualitative Gradability

So far, I have only focused on the *quantitative* dimension of the phenomenon of gradability. However, there seems to be a further, more *qualitative* dimension to the gradability of know how, which simply cannot be captured on the quantitative analysis I just proposed. It appears as if one’s know how with respect to a subject matter can be compared with another’s know how with respect to the same subject matter, as in (48), or with one’s or another’s know how with respect to a *different* subject matter, as in (49):

- (48) Louis Armstrong knew how to play the trumpet better than any of his contemporaries.
- (49) Woody Allen knows how to play the clarinet almost as well as he knows how to direct his movies.

Sentences such as (48) and (49) are ambiguous between *two* interpretations, that depend on the scope in them of the comparative clauses—i.e., “-er than any of his contemporaries” and “as well as he knows how to direct movies.” On a narrow scope reading of the comparative clause, a sentence such as (48) is truth-conditionally equivalent to (50):

- (50) Louis Armstrong knew an answer to the question “How to play the trumpet better than any of his contemporaries.”

In (50) “-er than any of his contemporaries” takes narrow scope with respect to the embedded interrogative. This is however not the only reading of (50). There is a wide-scope reading that is distinct from (50). Suppose Louis Armstrong did not know how any of his contemporaries played the trumpet. On the wide-scope reading of the comparison construction in (48), it could still be true that Louis Armstrong knew how to play the trumpet better than any of his contemporaries. It is an understanding of this reading that is at issue here. Qualitative comparisons of know how are readings of sentences such as (48) and (49) on which their comparative clauses are interpreted as taking wide-scope with respect to the embedded interrogatives.

As mentioned at the outset, QUANTITATIVE GRADABILITY cannot account for every such comparison of know how. For suppose comparative constructions also could be analyzed in terms of the degree of completeness of one’s practical answers to the relevant question. It would follow that, if two subjects each knew a practical answer to a question *How to ϕ* (not necessarily the same), then they would know how to ϕ equally well. But this does not seem to be true. If each student in a math class found exactly one method to solve a certain problem, it still could be John’s technique is more elegant and straightforward. It would seem correct to say, in such a case, that John knew how to solve the problem better than anybody else in the class, even though everybody else also knew how to solve the problem. Moreover,

comparative constructions into ascriptions of know how cannot be entirely explained in terms of the *number* of practical answers known by one. For instance, Mariano Rivera is arguably the best closer of all times. But, as compared to many other great relief pitchers, Rivera’s pitch repertoire was ridiculously small: he famously only knew how to close in one way, by a cut fastball. And yet, despite the fact that he knew one practical answer to the question *How to close*, it still seems to be true of him that he knew how to close better than anybody else.⁴⁶ All in all, the gradability of ascriptions of know how is not always a merely quantitative matter. There is a *qualitative* dimension to the phenomenon of gradability that requires a separate treatment.⁴⁷

It is tempting to construe this *qualitative* dimension of gradability as affecting the *state* expressed by main verb of the report. But as with quantitative gradability, this is *not* the only possible analysis of comparative constructions, let alone the most plausible. Several considerations indicate that it is instead highly plausible that in

⁴⁶One may object that this is not the correct description of Marian Rivera’s *actual* situation. Perhaps, Rivera happened to know different practical answers to the question *How to close*, but he only closed in one way because that was the only way he *could* close. Or perhaps, Rivera chose to employ only that particular way of closing when playing. Note however that whether or not my description accurately describes Rivera’s actual knowledge does not really matter for the point being made. Imagine a situation, possibly counterfactual, in which Rivera knows only one way of closing, the one that he is actually famous for using. Moreover, suppose that closing in Rivera’s way is the best method for closing. In such a situation, we would still want to say that Rivera knows how to close better than anybody else.

⁴⁷Here is another way of seeing the same point. Compare ascriptions of the form ‘S knows how to ϕ ’ to ascriptions of the form ‘S knows about X’—i.e., topical knowledge ascriptions. A predicate of the form ‘know about X’ can directly enter in comparative constructions as in “John knows more about math than he does about chemistry” or in “John knows more about where to find girls than how to talk to them.” Yet, whereas in comparative constructions, ascriptions of topical knowledge privilege *quantitative* adverbs, such as “less” and “much,” ascriptions of the form ‘S knows how to ϕ ’ and other knowledge *wh* reports in general privilege qualitative comparative clauses, such as “better” and “as well as”. Thus, for example, “John knows how to cook more than James does” and “John does not know much how to cook” are infelicitous, as compared to “John knows how to cook better than James does.” On the other hand, “John knows about how to cook more than James does” is better than “John knows about how to cook better than James does.” See Kennedy(94), pp. 345-7 on the different distributions of different kinds of degree modifiers. The difference between the kinds of modifiers allowed by “know-*wh*” and by those allowed by “know about” strongly suggests that the explanation for their gradability must be different.

such comparative constructions, what is compared are the sets of answers known by the two subjects of the ascription—i.e., the “Answer Sets,” as I will call them.

Far from being exclusive to ascriptions of know how, comparisons are systematic across knowledge-*wh* ascriptions:

- (51) a. Gianni knows better than John where to find a good Italian restaurant.
 b. Mark knows better than anyone else what gift will make Gianna happy.
 c. Ann knows better than Mary when to act tough.

By contrast, recall that comparisons are generally infelicitous into know-that ascriptions:

- (52) a. ??Kim knows better than Lee does that *bucatini all’ amatriciana* is a classic element of Cucina Romana.
 b. ??Mark knows better than John that an Ipod for Christmas will make Gianna happy.

Their distribution corroborates the hypothesis that qualitative comparisons between knowledge-*wh* ascriptions are licensed by the presence of interrogative constructions.

Other verbs besides “know” show the exact same pattern: they allow for qualitative constructions *when* embedding interrogatives, while being fully non-gradable *when* embedding *that*-clauses. Consider for instance “decide” and “advise:”

- (53) a. ??Marco decided that he will go to Harvard for his PhD better than anybody.
 b. ??Marco advised me that I should get married better than my best friend.

- (54) a. How can I best decide whether I should study for a Masters or a PhD

degree?⁴⁸

- b. How does one best decide which location is appropriate for one's store front?⁴⁹
- c. To better decide where to go for graduate school, prospective students should get plenty of advice from other graduate students.
- d. Nobody decides whom to marry better than those who marry the person they love.
- e. My enemies ended up advising me what to do better than any of my friends.

Because reports with “decide” and “advise” only license qualitative modification and comparison constructions when embedding an interrogative, in those reports the modification must be licensed by the presence of the interrogative construction. Qualitative gradability distributes across most knowledge *wh* ascriptions as well as across other *extensional* attitude reports embedding interrogatives but is not allowed when those very same attitude verbs take *that*-clauses as complements. This corroborates the hypothesis that, in ascriptions of know how, it is the practical answers known by the subjects that are being compared. If so, it is legitimate to infer that the gradability is licensed by the distinctive semantic contribution of the embedded interrogatives.⁵⁰ Because the distinctive semantic contribution of interrogatives are sets

⁴⁸<http://www.quora.com/How-can-I-best-decide-whether-I-should-study-a-Masters-or-a-PhD-degree>

⁴⁹<http://www.quora.com/Small-Businesses/How-does-one-best-decide-which-location-is-appropriate-for-their-store-front>

⁵⁰Of course, the claim is *not* that qualitative modification and comparison constructions are licensed *only* with reports embedding interrogatives. Some reports embedding *that*-clauses *also* license comparisons, such as:

- (i) Marco remembers better than John that they should buy their mum a present.

Rather, the claim is that *if* an attitude verb only licenses gradability when embedding interrogatives—and does not when embedding *that*-clauses—that is evidence that what licenses the

of answers, we should infer that what is being qualitatively assessed and compared are sets of answers—i.e., the sets of answers known by the subjects of the ascriptions, or *Answer Sets*.

Further evidence for this analysis comes from the behavior of *intensional* verbs. Standardly, *intensional* verbs such as “wonder” and “ask” are thought to take as argument the *intension* of the embedded interrogatives—i.e., the question they express—as opposed to the *denotation* of the interrogative—i.e., the set of true answers. My analysis predicts that reports with *intensional* verbs should not allow for comparative constructions. And this prediction is borne out:

- (55) a. ??Kim wonders better than Lee does how to make *bucatini all'amatriciana*.
 b. ??Kim asked better than Lee did how to make *bucatini all'amatriciana*.

It is worth noting that this pattern of qualitative adverbial modification is not unique to the examples just considered, all involving the adverb “well.” Consider the following examples, featuring different adverbs:

- (56) a. Mark knows exactly what he has to do to pass the exam.
 b. Mark told me exactly where the party is.
 c. Mark suggested to me precisely what to say to my parents.

In (56-a), the *manner* adverb “exactly” must grammatically modify the predicate “know what he has to do to pass the exam:” “exactly” *cannot* be grammatically modifying the question word for the simple reason that “what” is a noun phrase, and noun phrases do not grammatically accept adverbs as modifiers. Hence, “exactly”

comparison in those cases is the presence of the interrogative clause. By contrast, the availability of comparison constructions with “remember”—also when this attitude verb embeds *that*-clauses—presumably shows that the state of propositional remembering can hold to different degrees of a subject.

must be modifying the predicate. Interestingly, moreover, sentences (56-a)-(56-c) can be paraphrased *via* the following sentences (57-a)-(57-c):

- (57) a. Mark knows the exact answer to what he has to do to pass the exam.
 b. Mark told me the exact answer to where the party is.
 c. Mark suggested to me the exact answer to what I should say to my parents.

where it is the complement that is both grammatically and semantically modified by the qualitative construction.⁵¹ It is extremely natural to take the modifier “well” to instantiate the same pattern of modification illustrated by “exactly” and “precisely.”

To sum up; A predicate of the form ‘knows how to ϕ ’ is qualitatively gradable, whereas a predicate of the form ‘knows that p ’ is not. However, the distribution of the phenomenon indicates that the source of the gradability is the interrogative construction. If the source of gradability is the interrogative construction, then the qualitative comparisons should be analyzable in terms of the quality of the answers that are known by the subjects of the ascriptions to the questions expressed by that interrogative, as in QUALITATIVE GRADABILITY:

QUALITATIVE GRADABILITY

1. “S knows well how to ϕ ” is true relative to a context c if and only if
 S’s Answer Set is good relative to c .
2. “S knows how to ϕ better than S* knows how to ϕ ” is true relative

⁵¹I am assuming that we are dealing with *qualitative* gradability and not with *quantitative* gradability in these examples for two main reasons. First, “exactly” and “precisely” are manner adverbs and not *proportional* modifiers, such as “in part” and “for the most part.” Secondly, two subjects may each know a (different) complete answer to the question “where the party is”, but only one of the two know exactly where the party is. This suggests that we are dealing again with a kind of gradability that is not reducible to a merely quantitative analysis.

to a context c if and only if S 's Answer Set is better, relative to c , than S^* 's Answer Set.

According to QUALITATIVE GRADABILITY, for one to know how to ϕ better than another is for the answers known by one to be better than the answers known by the other. QUALITATIVE GRADABILITY specifies the satisfaction conditions of the gradable predicate 'knows how to ϕ ' in terms of an absolute state of knowledge. Answer sets are sets of *answers*—i.e., ordered pairs of propositions and the question they answer. Answer sets can be better or worse, just as answers can. By contrast, propositions known are not better or worse. Hence, QUALITATIVE GRADABILITY also explains why know-*that* ascriptions cannot be qualitatively compared, or at least they are not so along the same way as know how ascriptions.

1.4.1 The Compositional Semantics

We are left with the issue of how such a story could be implemented into a compositional semantics. On the approach suggested by QUALITATIVE GRADABILITY, qualitative modifiers semantically operate on the sets of complete practical answers known by the subjects of the ascriptions. However, from a syntactic point of view, the qualitative modifiers modify the whole constituent 'S knows how to ϕ '. Hence, the issue arises of how the desired truth conditions in QUALITATIVE GRADABILITY can be derived compositionally from the superficial syntax of the graded ascriptions.

Here is my suggestion. As the superficial syntax suggests, I will assume that the modifier "well" grammatically modifies the whole constituent 'S knows how to ϕ '. The comparative clauses "better than" and "as well as" compare two predicates of the form 'S' knows how to ϕ '. What is grammatically modified is a *predicate*: hence the *adverbial form* of the modifiers, rather than the adjectival form. What each predicate does is to semantically contribute an Answer Set—i.e., the set of answers known by their subjects; "well" takes an Answer Set as its input while the comparison clauses

take two. The result is as in (58-b)–(60-b) (where ‘ α ’ is as before a variable over answers—i.e., type $\langle p, \mathcal{Q} \rangle$ —i.e., $\langle \langle s, t \rangle, \langle s, \langle s, t \rangle \rangle \rangle$):

- (58) a. S knows well how to ϕ .
 b. **well** ($\lambda\alpha(\alpha = \langle p, \textit{How to } \phi \rangle$ and S know p)).
- (59) a. S knows how to ϕ better than S^* knows how to ϕ .
 b. **Better than** ($\lambda\alpha(\alpha = \langle p, \textit{How to } \phi \rangle$ and S knows p); $\lambda\alpha(\alpha = \langle q, \textit{How to } \phi \rangle$ and S^* knows q)).
- (60) a. S knows how to ϕ as well as S^* knows how to ϕ .
 b. **As well as** ($\lambda\alpha(\alpha = \langle p, \textit{How to } \phi \rangle$ and S knows p); $\lambda\alpha(\alpha = \langle q, \textit{How to } \phi \rangle$ and S^* knows q))

It is crucial to getting the right truth conditions that the answers that are qualitatively assessed (as in (58-b) or compared (as in (59-b)–(60-b)) are those *known* by the subjects. In this respect, the analysis of qualitative and comparative constructions into embedded questions raises an issue analogous to one that arises with *indefinites*. Indefinites appear to have *existential force* when occurring in *unembedded* sentences, such as:

- (61) A donkey entered the room.

However, when occurring embedded, as in donkey sentences, they can acquire universal force:

- (62) If Mary owns a donkey, she beats it.

(62) is true only if Mary beats *every* donkey she owns: it is not enough for the truth of (62) that Mary beat one donkey if she has got more than one. We are presented with

a similar problem here. The issue is that, as showed in *Quantitative Gradability*, each ascription existentially generalizes over practical answers that are (entirely) known by the subjects, as in:

(63) For some practical answer, S knows every part of that practical answer.

As described earlier and in the Appendix 2.7, the existential force derives from a QUEST operator, one which works as a lifted existential quantifier over the set of answers contributed by the interrogative. However, this is a bit problematic because we want the ascriptions to feed the adverbial modifier with *the set of all answers known* by the subject to that question. So what we want is to get (64) from (63):

(64) $\lambda\alpha (\alpha \in \text{How to } \phi \ \& \ S \text{ knows every part of } \alpha)$.

Because of the structural analogy of the problem, we can extend the usual strategies employed to deal with indefinites to the present case. According to the DRT-style analysis (Kamp(87), Heim(78)), indefinites are free variables that are bound by an external existential quantifier when they occur in unembedded sentences but acquire universal force in virtue of the embedding construction.⁵² Analogously, we can take the result of applying the QUEST operator to the interrogative to be a predicate of answers. That predicate is existentially generalized over when the ascription occurs unembedded but acquires universal force when embedded into qualitative and comparative constructions. In this way, we get that when unembedded, the know how ascription has the required existential force—i.e., its truth conditions are given as

⁵²Other approaches, such as an E-type approach (Heim(81), Reinhart(146) and Heim(80)) or a dynamic approach, are also possible, but will not be developed here. On the E-type analysis, the antecedents in donkey sentences introduce sets of individuals that satisfy the conditions by them imposed; analogously, in the above, knowledge-*wh* ascriptions introduce sets of propositions known by their subjects.

in (63); when embedded into the comparative construction, by contrast, the whole knowledge ascription has the required universal force—i.e., it contributes the set of *all* answers known by the subject to its embedded interrogative.

The next question is how to understand the semantics of modifiers such as “well” and the semantics of degree constructions such as “-er than” and “as well as.” The modifier “well” is nothing but the adverbial counterpart of the gradable predicate “good.” According to a long-standing semantic tradition—the so-called Scalar View—(Heim (79), Klein(99), Seuren(162), von Stechow(182), Kennedy(90), Cresswell(37), Bierwisch(20)), a gradable predicate expresses a relation between *degrees* and *properties*. Degrees are points or intervals ordered along some dimension (height, cost, weight, and so on) in such a way that, if something possesses the property G to a degree d_n , it possesses G to every degree d_m , with $n \geq m > 0$. On this model, the gradable adjective “good” denotes a relation between degrees of goodness d and objects x such that the cost of x equals d :

$$(65) \quad \llbracket \text{good} \rrbracket = \lambda d \lambda x (\text{GOOD}(x) = d).$$

But, as I said, “well” is an *adverb*, not an adjective. So the denotation of “well” has to be modified accordingly. I will take “well” to express a relation between degrees of goodness and a property, such that the extension of that property is good to degree d :

$$(66) \quad \llbracket \text{well} \rrbracket = \lambda d \lambda P (\text{WELL}(P) = d).$$

(66) has to be refined a little to allow for potential contextual variation. According to a rather standard analysis (von Stechow(182), Kennedy(90), Cresswell(37), Bierwisch(20)), when the gradable adjective is *not* modified by comparative expressions—i.e., when it is, in other words, in its *positive form*—the contextual variability is

licensed by a null morpheme, *POS*, which encodes the relation ‘STANDARD’. STANDARD holds of a degree *d* just in case it meets a standard of comparison for an adjective *G* with respect to a comparison class determined by *C*. *C* is a variable over properties of individuals, the value of which is determined contextually:

$$(67) \quad \llbracket POS \rrbracket = \lambda G' \lambda P \exists d [\text{STANDARD}(d)(G)(C) \ \& \ G(d)(P)].$$

The null morpheme combines with the adverb “well” to give (68-b):

$$(68) \quad \begin{array}{l} \text{a.} \quad \llbracket POS \rrbracket(\llbracket \text{well} \rrbracket) \\ \text{b.} \quad \lambda P \exists d [\text{STANDARD}(d)(\llbracket \text{well} \rrbracket)(C) \ \& \ \llbracket \text{well} \rrbracket(d, P)]. \end{array}$$

So it is rather natural to suppose that, when combining with a know how ascription, (68-b) is fed with the Answer Set contributed by that know how ascription. The result would be that an Answer Set \mathcal{A} satisfies (68-b) relative to a context *c* just in case there is a degree *d* of goodness on the scale determined by the contextual comparison class that \mathcal{A} meets. (69-b) gives the truth conditions of graded ascriptions such as ‘*S* knows how to ϕ well’:

$$(69) \quad \begin{array}{l} \text{a.} \quad \llbracket POS \rrbracket(\llbracket \text{well} \rrbracket) \\ \text{b.} \quad \lambda P \exists d [\text{STANDARD}(d)(\llbracket \text{well} \rrbracket)(C) \ \& \ \llbracket \text{well} \rrbracket(d, P)]. \end{array}$$

Things look a little different when the gradable adjective/adverb is embedded into a comparative construction. When embedded into a comparative construction, such as “-er than” and “as — as,” the standards set by the context play no role in the truth conditions of the comparative construction (Kennedy(90), (91)). To illustrate, although an object may not be big *simpliciter* relative to a context, it may still be bigger than another object (according to the same dimension) relative to the same context. Or to use an example more relevant to our purposes, a professional cook

may not know how to cook well according to the standards of the context and yet may know how to cook much better than a non-professional cook. Into comparative constructions involving the adverb “well”, the contextual variability seems to be restricted to the *dimensions* (respects) along which something can be or not be better than something else. A compositional implementation of QUALITATIVE GRADABILITY has to account for this phenomenon.

On the standard semantic analysis of degree constructions, the so-called “Scalar View,”⁵³ the comparative morpheme “-er” in a comparative sentence such as “*X* is *G*-er than *Y*” existentially quantifies on the degrees in a *G*-ness scale that are achieved by *X* and *Y*. In other words, the comparative morpheme plays the role played by *POS* in the positive form of the adjective. In contrast with *POS*, however, the comparative morpheme does not allow for contextual variability in the relevant standard of comparison. As observed, this is as it should be. However, we still want to allow for contextual variability in the *dimensions* of goodness along which Answer Sets can be assessed. For this purpose, I propose we revise the positive form of “good” and “well:”

$$(70) \quad \llbracket \text{good} \rrbracket^c = \lambda d \lambda x (\text{GOOD}(D)(x) = d).$$

$$(71) \quad \llbracket \text{well} \rrbracket^c = \lambda d \lambda P (\text{WELL}(D)(P) = d)$$

where *D* is a free variable over properties, capturing possibly different contextual dimensions of goodness. When *POS* applies to the bare adjective/adverb’s semantic value, it adds a further dimension of contextual variability, that has to do with the standards of comparison. The comparative construction, by contrast, does not allow

⁵³The Scalar View has been first sketched by Lewis(109) and successively developed by Kamp(86), Klein(98), and Schwarzschild&Wilkinson(158), and Kennedy(90), Kennedy(94), Kennedy(91). See also Schwarzschild(157) for a helpful presentation and defense of the view.

for any further contextual variability, as can be seen by the following:

$$(72) \quad \lambda G \lambda x \lambda y \exists d [G(d, x) \text{ and not } G(d, y)].$$

(72) takes a property G into a relation between individuals x and y such that for some degree d on a scale of G -ness, x is G to a degree d , and y is not G to that degree. Because G stands for some property F -along-a- D -dimension, where the dimension D may vary across contexts, we get the kind of contextual variability that we want without having it to be induced by the comparative construction. The comparative construction has to be lifted to take adverbs as arguments. The result is as in (73):

$$(73) \quad \lambda G' \lambda P \lambda Q \exists d [G(d, P) \text{ and not } G(d, Q)].$$

The composition of the comparative construction “-er than” and “well” returns (74-b):

$$(74) \quad \begin{array}{l} \text{a. } \llbracket \text{-er than} \rrbracket (\llbracket \text{well} \rrbracket^c) = \\ \text{b. } \lambda P \lambda Q \exists d (\llbracket \text{well} \rrbracket^c(P) = d) \text{ and not } (\llbracket \text{well} \rrbracket^c(Q) = d). \end{array}$$

(74-b) takes two Answer Sets as its arguments P and Q , each contributed by the two know how ascriptions.⁵⁴ The upshot is that a sentence of the form ‘ S knows how to ϕ better than S^* ’ is true relative to a context c if and only if there is a degree of goodness on the scale determined by the dimension of goodness selected by the context c that is achieved by X ’s Answer Set and that is not achieved by Y ’s Answer Set.⁵⁵

⁵⁴Analogous clauses for other comparative constructions are provided in the Appendix.

⁵⁵There may be some few cases in which two know wh ascriptions are being compared and in which it cannot plausibly be the quality of the answers that is being compared, because the answers known by the two subjects are the same. For example, suppose both Pete and Mark know whether it is raining, but one is more justified than the other (Dutant (46) has an example along these lines). One

QUALITATIVE GRADABILITY gives the truth conditions for qualitative comparisons of know how in terms of propositional knowledge, as desired.⁵⁶ Such an analysis is then compatible with the view that ascriptions of the form ‘S knows how to ϕ ’ ascribe a propositional knowledge state. Moreover, because it individuates the source of gradability in the semantic contribution of the embedded interrogative, such an analysis has the virtue of being easily generalizable to account for *any* qualitative comparisons between know *wh* ascriptions as well as between other reports embedding interrogatives, as desired. I have also shown how the desired truth conditions can be derived from a compositional assignment of semantic values. Such a semantics can

possible response to this challenge is to think of those cases as instances of the same phenomenon that, in the set up of this essay, I called “marginal gradability” and that we observe with know-*that* ascriptions. Know *that* ascriptions are marginally gradable. Whatever explanation works for the marginal gradability of know-*that* ascriptions can be extended to deal with this (exceptional) instance of qualitative gradability into know *wh* ascriptions. Note that this kind of gradability is surely not very common with know how ascriptions. Rarely we would say that Mark knows how to cook lasagna better than Mary, just in virtue of Mark’s having more evidence for his answers. When we compare know how ascriptions, we are, in the majority of the cases, not very concerned about the level of justification subjects have for their answers. Moreover practical answers represent maximally specific methods for acting. And so it will happen rather rarely that two subjects both know the same practical answers. Another possible response is to appeal to something like Larsersons pragmatic halo. On this response, we *are* comparing the epistemic quality of answers to questions, even in those cases where the answers known are exactly the same for the two subjects. However, in so doing, we conceptualize answers not just as ordered pairs of propositions and a question. Rather, the event of answering a question also becomes relevant to the comparison, so that one subject’s answering a question may be epistemically better off than the other subject (See for instance, Kennedy&McNally(90) and (92) on the importance of verbs’ event structure for degree modifications). Such event is not represented semantically, but it is represented in the pragmatic halo. In the pragmatic halo specifically, such an event is represented in the Answer Sets. I am equally open to both responses.

⁵⁶Another interesting issue is how the mechanism of Answer Sets’ comparison is to be understood. *Qualitative Gradability* says that one knows how to ϕ better than another, relative to a context c if and only if one’s Answer Set is better than another’s, relative to a context c . But under what circumstances is one’s Answer Set better than another’s? We should not expect there to be a unique and general answer to such a question. Quite generally, when making qualitative comparisons, the variables to be taken into account are diverse, and so are the criteria for assigning those variables relative weight into the final counting. Expecting there to be a unique mechanism of comparison would be tantamount to expecting there to be a unique criterion for ranking, say, our favorite athletes or our preferred musicians. Outlining such mechanisms is not, moreover, the semanticist’s job. It has to do with the meta-semantics of degree constructions—i.e., the question of what fixes the meaning of degree constructions relative to a context. There are several possible accounts of the meta-semantics of gradable adjectives. For a recent (and, in fact on-going) discussion of these meta-semantic issues, see Glanzberg (64) and King (97), (96).

be cashed out in terms of quite standard semantics for gradable predicates, degree modifiers and comparative constructions, and is susceptible of being implemented through a machinery familiar from the debate on donkey anaphora.

1.4.2 Objections and Replies

QUALITATIVE GRADABILITY seems to be liable to a variety of objections. But all the objections can be rebutted once certain facts concerning the logical form of know how ascriptions are brought into better focus.⁵⁷

On QUALITATIVE GRADABILITY, one knows how to ϕ better than another if and only if one's Answer Set to the question *How to ϕ* is better than the other's. Hence, QUALITATIVE GRADABILITY assumes that the truth of qualitative comparisons of know how will depend on there being at least *one* practical answer to the question *How to ϕ* . However, qualitative comparisons are possible across ascriptions of know how even in those cases where it is not obvious that there is any general practical answer to the question expressed by the embedded interrogative. This happens in particular

⁵⁷Let me set aside a worry that arises more generally for any analysis of know how ascriptions as ascribing knowledge of *answers*. The worry is that such analysis is implausible, as we ordinarily say things such as the following:

- (i)
 - a. An infant knows how to distinguish speech sounds from non-speech sounds.
 - b. A dog knows how to find an object by smell better than a human does.

And surely—the objection goes—neither infants or dogs know *answers* to questions. Does that mean that these sentences are strictly speaking false or involve a kind of presupposition failure and we're just speaking in a kind of metaphoric/figurative way here? What response we give to this sort of objection will depend on whether we are happy to take the *same* stance about the truths of the following ordinary know-*that* ascriptions:

- (ii)
 - a. My dog knows that he will get treats if he behaves.
 - b. My baby knows how much we love him.

(ii) are as common as the previous know how ascriptions. Do dogs or infants really know propositions? In other words: This worry does not specifically apply to my analysis of know how ascriptions but applies generally to any view that takes knowledge ascriptions to ascribe knowledge of a proposition.

whenever the infinitival of the embedded interrogative expresses a very general task, for which it is not obvious that any method to perform it can be plausibly supposed to exist. Call this the “NO UNIQUE TASK PROBLEM.” Here is an example that vividly illustrates the problem:

MARY, THE BABYSITTER Mary the babysitter knows how to deal with kids better than anybody else. Every kid is different, of course, and she uses different methods to deal with them. There is no general answer to the question *How to deal with kids*. Mary can know how to deal with kids better than anybody else, although there is no unique general answer that she knows.

A related objection is the BEST CHOICE PROBLEM: it seems that two subjects may both have learned the same method(s) to ϕ , hence know the same practical answers to the question *How to ϕ* , and yet one may know how to ϕ better than the other, because that one knows better how to choose among such methods in response to a variety of different circumstances. JORGEN AND ZHANG is a good illustration of this sort of case:

JORGEN AND ZHANG In ping pong, there are four main ways of hitting a ball: the forehand drive, the backhand drive, the smash and the push. Jorgen and Zhang each know equally well how to forehead drive a ball and how to backhand drive a ball. Moreover, each knows equally well how to smash a ball or push a ball at least in the following sense: given a ball, and a choice of a stroke (whether a smash or a push), they are likely to hit it equally well by using that stroke. Jorgen, however, is better at *deciding* when to smash or push a ball.

In JORGEN AND ZHANG, Jorgen can know how to hit a ping pong ball better than Zhang, although they each know the same general practical answers to the question

How to hit a ping-pong ball. What is being compared in this case cannot plausibly be Jorgen’s and Zhang’s Answer Sets, which are identical. Again, it is not clear how *Qualitative Gradability* can account for this kind of case.

Both the NO-UNIQUE-TASK PROBLEM and the BEST-CHOICE PROBLEM can be addressed by appealing to *higher-order methods*. On this response, there *are* general practical answers to the question *How to deal with kids* that Mary the Babysitter knows: They are answers concerning higher-order methods, methods that enable her to choose among different possible methods to deal with kids from circumstance to circumstance. Similarly, this line of response holds that Jorgen and Zhang’s Answer Sets are not identical: Jorgen’s is better, for he knows of a better higher-order method to choose between two different ways of hitting a ping pong ball in different circumstances. The appeal to higher-order methods is not, however, the only option. There is another way of dealing with these two problems. I will end my discussion of QUALITATIVE GRADABILITY by arguing that ascriptions of know how can quantify over situations, and qualitative comparisons are demonstrably affected by that.

Strikingly, know how ascriptions (as many other know-*wh* ascriptions) can ordinarily be restricted by *when* and *if clauses*:

(75) When the children are well behaved, Mary knows how to deal with them.

(76) If the children are not well behaved, Mary does not know how to deal with them.

It is the prevailing position among semanticists today to take restrictors such as *when* and *if clauses* to restrict adverbs of quantifications—i.e., quantifiers over situations (Berman(17), von Stechow(181)) that unrestrictedly bind every free variable within their scope (Lewis (111)). On this widespread assumption, the availability of restrictors in (76) suggests that some adverb of quantification is being restricted by the

relevant *when* and *if clauses*. To illustrate, consider (77):

(77) Mary knows how to deal with kids.

(77) can be restricted by *when* and *if clauses* as in (78):

(78) Mary knows how to deal with kids when they are well-behaved.

The availability of restrictors suggests that some adverb of quantification can be restricted in (77) roughly equivalent to (79-a) and more formally to (79-b):

- (79) a. Mary knows how to deal with kids in most circumstances.
 b. For most circumstances *c*, Mary knows in @ how to deal with kids in *c*.

According to (79-b), in the actual circumstance @, Mary knows the answers to the questions “how to deal with kids in *c*,” for most substitutions of possible circumstances to *c*.⁵⁸

This pattern of quantification over situations helps both with the NO-UNIQUE-TASK PROBLEM and with the BEST-CHOICE PROBLEM. In MARY, THE BABYSITTER, Mary knows how to deal with kids better than most people because, in most

⁵⁸Note that (77) and (79-a) are both susceptible of another reading, one in which the adverb of quantification takes wider scope with respect to the interrogative clause:

- (i) For most circumstances *c*, Mary knows in *c* how to deal with kids in *c*.

On the reading specified in (i) and (77) is a generic statement that is true just in case in most situations, it is true of Mary in that situation that she knows how to deal with kids in that situation. On this reading, then, (77) does not ascribe knowledge of answers to different situation-specific questions. Rather, it describe a generalization true of Mary—i.e., her capacity to know in different situations how to deal with kids.

Yet another possible interpretation of statements such as (77) is one in which the source of quantification is the predicate “know” itself. Schaffer&Szabo(155) argue that restrictors are available in know-*that* ascriptions, and that is evidence that the predicate “know” itself is a quantifier on situations. I leave it open in this essay whether this reading is also available for (77). If it is, this also reading can dispense with the appeal to higher-order methods.

situations involving kids, she knows how to deal with kids in those situations better than most people—i.e, because for most questions of the form *How to deal with kids in situation s*, her Answer Sets are better than most people’s Answer Sets. In JORGEN AND ZHANG, Jorgen can know how to hit a ping pong ball better than Zhang, although they are each in possession of the same general answers to the question *How to hit a ping-pong ball* because what is being compared are their answers to several *situation-specific* questions of the form *How to hit a ping-pong ball in this and this particular situation*. And Jorgen does know better answers to *those* questions.⁵⁹

⁵⁹One may object that quantification over situations will not help in every problematic case. Suppose Roger is the world’s best tennis player on grass courts; Rafael is the best tennis player on clay courts. In this situation, Novac may be, nonetheless, the world’s best tennis player, despite not being better than anybody else at playing on clay or at playing on grass, if he is sufficiently competitive at both. Call this case DJOKOVIC. In DJOKOVIC, assuming that the situations of playing tennis divide equally into situations of playing tennis on *grass courts* and into situations of play on *clay courts*, it is not the case that in most typical situations *s*, Novac’s answers to the question *How to play tennis in s* are better than Rafael’s (in fact, they are not better in half of the total situations), nor is it the case that in most typical situations *s*, Novac’s answers to *How to play tennis in s* are better than Roger’s (in fact, they are not better in the other half of the total situations). Call this objection the NO-BEST-ANSWER-SET PROBLEM. NO-BEST-ANSWER-SET PROBLEM is not a problem for QUALITATIVE GRADABILITY once it is appreciated that quantification over situations can affect the comparison between Answer Sets in *more* than one way. MARY, THE BABYSITTER or JORGEN AND ZHANG exemplify only *one* of the ways in which quantification over situations can affect qualitative gradability. When more than one question is at issue, we may be interested in comparing, for each situation-specific question, the respective Answer Sets to that particular question. But this is not the only way in which quantification over situations can affect qualitative gradability. In some cases, we may be interested *not* in comparing Answer Sets situation by situation but rather in comparing Answer Sets *across* situations. DJOKOVIC is a good example of this: the temptation to say that Novac knows how to play tennis better than the other two reflects the fact that the *average* between the best answer known by Novac to the question *How to play on clay* and the best answer known by Novac to the question *How to play on grass* is higher than the corresponding Roger’s and Rafael’s averages. In DJOKOVIC, in other words, Answer Sets are compared across (classes of) situations. Note that both mechanisms for comparison are reflected by the scope interaction between the comparative and the generic quantifier that we have seen occurs hidden in knowledge-*wh* ascriptions. On the wide scope reading of the generics, “S (generally) knows how to ϕ better than S*” has the same truth conditions as:

- (i) For most interrogatives ‘*Q*’ of the form ‘How to ϕ in situation *s*’, *S* knows *Q* better than *S**.

The reading given by (i) is the reading on which for *S* to know how to ϕ better than *S**, it must be the case that *S*’s Answer Set is better than *S** for *each* situation specific question. This reading corresponds to the reading on which Answer Sets are compared one with another for each situation specific question, as in MARY, THE BABYSITTER. On the narrow scope reading of the generics,

In conclusion, different mechanisms of comparison come into play when making qualitative comparisons of know how. Additional complications come from the fact that ascriptions of know how can quantify on situations, and the mechanisms of comparisons are demonstrably affected by that. However diverse, complex, and context-dependent the comparison can be, what is compared in typical qualitative comparisons is the quality of the answers known by the subjects of the compared ascriptions. The complexity and variability of the mechanisms employed for such a comparison should not distract us from this result.

1.5 Conclusions

Ascriptions of know how are gradable. In this paper I have argued that there are two distinct dimensions to their gradability: a *quantitative* and a *qualitative* dimension. In both cases the surface form of the graded ascriptions is deceptive. Though their gradability seems to affect the state reported by them, that is not how we should analyze it. The best semantic analysis of the phenomenon is not only compatible with, but also invites a unified picture of knowledge and inquiry.

The first dimension of gradability is *quantitative gradability*. One may know how to ϕ to different extents. I argued—by looking at the distribution of the phenomenon of gradability across reports—that quantitative gradability into knowing how ascriptions has to be analyzed as an instance of a general pattern of adverbial modification

instead, we have the following truth conditions:

- (ii) Better than (S knows how to ϕ generally), (S* knows how to ϕ generally).

In (ii), the two arguments of the comparative each feed it with a *set* of Answer Sets, the set of each Answer Set corresponding to each situation-specific question; the comparative then compares the set of all S's Answer Sets with the set of all S*'s Answer Sets. This reading corresponds to the reading on which Answer Sets are compared *across* situations, as in DJOKOVIC.

into argument position. Accordingly, knowing how to ϕ to different extents is a matter of knowing different *extents* of a practical answer. I introduced and developed a new analysis of quantitative gradability: THE PART-WHOLE APPROACH. According to THE PART-WHOLE APPROACH, knowing how to ϕ to different extents is a matter of knowing different *extents of* a practical answer to the question *How to* ϕ . Because different extents of an answer are elements of an algebra of propositions associated with that answer, THE PART-WHOLE APPROACH succeeds at specifying the satisfaction conditions of the quantitatively gradable predicate 'knowing how to ϕ ' in terms of an absolute propositional knowledge predicate, as desired.

THE PART-WHOLE APPROACH raised two main issues. The first issue was how to understand the notion of *part of a complete answer*, whether practical or not. In §1.3.2.2, I addressed this issue by giving an intuitive illustration of parts of practical answers. Parts of a practical answer to a question *How to* ϕ are in one-to-one correspondence to the parts of a procedure to ϕ . But the notion of parts of an answer can also be defined in full generality and not just for the specific case of practical answers. As showed in Appendix 1.6.1.1, parts of a complete answer can be defined through the notion of *sub-question*. The specific case of practical answers falls out from the general account of parts of answers: Part of an answer to a practical question is a proposition that completely answers some of its sub-questions. By specifying a practical question and its sub-questions, practical answers can be ground into their parts, by first getting to the sets of sub-answers, and by building an algebra for each practical answer. This way of grinding practical answers into their parts nicely illustrates how the part-whole structure of a practical answer can be brought to light, so to say, only by specifying the question that it answers.

The second issue was to explain why ascriptions of the form '*S* knows how to ϕ ' can be graded, while ascriptions of the form '*S* knows that p ' cannot. My solution to this puzzle can be made more explicit by the *slogan*: propositions can be ground

into parts *only relative to a question*, only *qua* answers to that question. I argued that only relative to a question, does it make sense to speak of *parts* of a proposition. *That*-clauses cannot be quantitatively graded into because they do not specify a question relative to which their denotation can be ground. In the formal semantics, this difference between *that*-clauses and interrogatives is captured by taking the denotation of the former to be a single proposition and the members of the denotation of that latter to be not just bare propositions, but *answers*—where an *answer* in this technical sense is an ordered pair of a proposition and a question it answers.

That concluded my analysis of the first dimension of gradability—i.e., *quantitative gradability*. I argued that there is a second dimension of gradability—i.e., *qualitative gradability*. One may know how to ϕ better than another, or as well as another, or worse than another. I argued—by looking at the distribution of the phenomenon of qualitative gradability across reports—that qualitative gradability into knowing how ascriptions affects the quality of the practical answers known by the subjects of the ascriptions. On my analysis, knowing how to ϕ better than another is a matter of one’s practical answers to the question *How to ϕ* being better than the other’s. The qualitative comparison is between Answer Sets—i.e., the sets of answers known by the subjects (§2.5-1.4.1). The satisfaction conditions of the gradable predicate can be stated in terms of the satisfaction conditions of the absolute predicate. Moreover, such intuitive truth conditions can be derived from a compositional assignment of semantic values that respects the surface form of the ascriptions. As observed in §1.4.2, issues are complicated by the fact that know how ascriptions quantify over situations, and qualitative gradability is demonstrably affected by that. However, as I explain, even quantification over situations can be accommodated by my analysis.

Neither *quantitative gradability* or *qualitative gradability* is evidence that the state ascribed by means of know how ascriptions is gradable. The availability of knowing how ascriptions to adverbial modification by means of proportional and

degree modifiers tricked us into thinking that the state ascribed by those ascriptions was gradable. But the move from the phenomenon of gradability to the gradability of the state ascribed by means of ascriptions of know how is invalid. The gradability of the ascriptions does not entail, nor is it best explained by the gradability of the state ascribed. *The Rylean Argument* fails, and with it the more general conclusion can be blocked that know how and propositional knowledge are different kinds of states.

1.6 Appendix

1.6.1 Quantitative Gradability

1.6.1.1 Sub-Questions and Parts of Answers

Thus far, I have not said much about what parts of an answer are, except for saying that parts of an answer are elements of an algebra associated with that answer. A measure associated with the meaning of the mass quantifiers such as “in part” and “for the most part” is responsible, perhaps even relative to a context, to count the elements of the algebra that qualify, relative to that context, as parts of the answer. A part of an answer is any element of the algebra p for which $\mu(p)=1$. One might think that as semanticists, we should rest content with this deflationary attitude to parts of answers, on which what counts as part of an answer is whatever is quantified over by quantifier “for some part.” It is not clear that as semanticists we should be looking for a more substantive account of what parts of answers are. That belongs to *mereology*—more precisely—to the mereology of answers. And that is metaphysics, not semantics.

There is however an important reason to want more from a semantic analysis of the phenomenon that we are studying. It would be nice to have a more *predictive* theory of the phenomenon. As it is, the semantic theory given is not very predictive. It only

really tells us that bare propositions do not have parts and that *some* answers instead do. It does not tell which answers do not have parts and why. So for instance, it does not predict the observed contrast between reports embedding whether-interrogatives and other kinds of reports. So the theory does not predict that adverbial modification is not acceptable into whether-interrogatives. To the extent to which we want to predict certain generalizations, we need to say more about what parts of answers are. To have such predictive theory, one needs to engage into some mereology of answers.

A first natural thought is to think of parts of complete answers to a question as PARTIAL ANSWERS to that questions.⁶⁰ To assess this proposal, we need to make the notion of PARTIAL ANSWER precise. The notion was first defined by Groenendijk and Stokhof(70). Roughly, according to Groenendijk and Stokhof(70), a partial answer is any proposition that can be derived by a (possibly infinite) disjunction of (true or untrue) strongly exhaustive answers. Generally we will want to exclude the trivial disjunctions of all the possible strongly exhaustive answers (yielding tautology) and none of them (yielding contradiction). Every strongly exhaustive answer is a partial answer, as is the weakly exhaustive answer and every mention-some answer.

The problem with this definition is that it is insufficiently general. It only gives us the notion of partial answer to a (strongly exhaustive) mention-all question, a question the complete answer to which specifies both the positive and the negative instances of the predicate. But we want a more general notion of partial answer, one that on which also mention-some questions can have partial answers.

As before, let a question \mathcal{Q} be a set of propositions, each completely answering \mathcal{Q} . As before, let us distinguish three kinds of questions: Depending on the kind of propositions in the set, we can distinguish between three kinds of questions: (i) mention-all questions; (ii) whether questions and (iii) mention-some questions.

⁶⁰As suggested en passant by Roberts(147).

- Mention-all questions are sets of incompatible propositions.
- Mention-some questions are sets of possibly compatible propositions.
- Whether-questions are sets of two complementary propositions.

In all three cases, the sets do not need to cover the whole logical space, because questions may come with presuppositions. Thus, for example, the question “Where does Mark live?” can presuppose that he lives somewhere and the question “What is the smallest natural number?” presupposes that there is such a number. Mention-all questions have a unique true complete answer; mention-some questions have more than one true complete answer. So much for questions. I will indicate an *answer* α_Q to a question Q as an ordered pair $\langle p, Q \rangle$, where p is a bare proposition. The challenge is to define the necessary and sufficient conditions for a proposition to be part of an answer.

In the semantic literature, it is standard to take questions to be sets of sets of possible worlds—i.e., it is standard to identify propositions with sets of possible worlds. Here I want to be neutral as to whether propositions are coarse-grained or fine-grained. Hence, logical relations between propositions cannot be just set-theoretical relations of inclusion and exclusions. Hence, instead of saying that a proposition is incompatible with another just in case they do not overlap, we say that a proposition p is *incompatible* with another proposition q just in case they cannot both be true at the same circumstances of evaluations. By means of this notion of incompatibility, we can define a PARTIAL ANSWER to a question Q as follows:

PARTIAL ANSWER A proposition p partially answers Q if and only if p is incompatible with at least one but not all of the propositions in Q .

According to PARTIAL ANSWER, any answer to a question is a partial answer to that question. But any answer is not a proper part of itself. For approximating the notion

of *proper part*, it is convenient to adopt PROPER PARTIAL ANSWER:

PROPER PARTIAL ANSWER A proposition p properly partially answers \mathcal{Q} iff p is incompatible with at least one but not all of the propositions in \mathcal{Q} without fully answering \mathcal{Q} .

As defined, however, PROPER PARTIAL ANSWER may be incompatible with every true answer to \mathcal{Q} while still be (properly) partial! So PROPER PARTIAL ANSWER cannot give us the right notion of part of a true answer. So we need to be more precise. We need to talk of ACCURATE PARTIAL ANSWER and ACCURATE PROPER PARTIAL ANSWER:

ACCURATE PARTIAL ANSWER A proposition p (accurately) partially answers \mathcal{Q} iff p is incompatible with at least one but not all of the propositions in \mathcal{Q} and it is *not* incompatible with every true proposition in \mathcal{Q} .

ACCURATE PROPER PARTIAL ANSWER A proposition p (accurately) properly partially answers \mathcal{Q} iff p is incompatible with at least one but not all of the propositions in \mathcal{Q} without fully answering \mathcal{Q} and it is *not* incompatible with every true proposition in \mathcal{Q} .

Intuitively, a ACCURATE PARTIAL ANSWER is one that *does not lead you astray*—i.e., by coming to know such an answer one progresses towards correctly answering the relevant question. This is as it should be, if we are after the notion of part of a true answer. For intuitively, in virtue of knowing in part a true answer, one does not know something that cannot be true if that answer is. Observe that ACCURATE PROPER PARTIAL ANSWER has the advantage of generality: because it defines a partial answer in terms of which possible answers in the question it eliminates, it holds both for mention-some questions and for mention-all question.

Defining *parts of true answers* in terms of *accurate and proper partial answers* has a certain amount of plausibility. For one thing, invoking accurate partial answers works well to explain why adverbial modification into reports embedding whether-interrogatives is not possible or at least very marginal, as noted earlier in this essay. Consider the contrast in acceptability between the following pairs:

- (80) a. Mark knows in part what the northern regions of Italy are.
 b. ??Mark knows in part whether Piemonte, Val d’Aosta, Lombardia, Veneto and Friuli are the northern regions of Italy.
- (81) a. Mark knows in part how Mary makes lasagna.
 b. ??Mary knows in part whether Mary makes lasagna following his aunt’s recipe.
- (82) a. Mark knows in part how to prepare lasagna.
 b. ??Mark knows in part whether a certain recipe is how to prepare lasagna.

In each minimal pair, the two embedded interrogatives may have the same Hamblin-Karttunen denotation—i.e., they may denote the same set of HK-answers. However, the first report of each pair is much more acceptable than the second. On the new envisaged analysis, the contrast between the two kinds of reports is not to be explained by appeal to their Hamblin-Karttunen denotations, which are identical in the two cases. Rather, the contrast is to be explained by the fact that the set of proper partial answers is different for the two kinds of interrogatives. Whether-interrogatives express questions (whether-questions) that have only two possible answers: the positive answer “yes” or the negative answer “no” and only one true answer. That means that whether-questions have *no* partial answers which are not also complete answers. Hence, whether-questions have no proper partial answers. That is why answers to

whether-questions cannot be ground into proper parts: they have none!

By contrast, the question expressed by the interrogative in (80-a) “what the northern regions of Italy are” has many more possible answers. Hence, it has proper partial answers. So an account of parts of true answers in terms of proper partial answers seems to share all the good predictions of APPROACH II, without sharing its bad predictions. Recall that APPROACH II predicted that singular interrogatives could not be adverbially quantified into, for they only have one HK-answer. By contrast, the questions expressed by singular interrogatives may have more than one proper and accurate *accurate and proper partial answer*. So proper partial answers may help where APPROACH II fails.

The problem with such view of parts of answers in terms of proper partial answers is that it predicts that adverbial modification is possible when it is not. Suppose Sherlock Holmes discovered that one of the main suspects, Miss Harrison, did not commit the crime, but has no idea who did. The following seems false of Sherlock:

(83) Sherlock Holmes knows in part who committed the crime.

But the question *who committed the crime* does have accurate partial answers. Suppose only Miss Wernitzky, Baroness Van Halden and Lord Farnsworth are the only possible suspects and that the crime was committed by one person only. The proposition that Miss Wernitzky did not commit the crime is incompatible with the proposition that Miss Wernitzky committed the crime. Moreover, it leaves open that either Baroness Van Halden or Lord Farnsworth did. Hence, the proposition that Miss Harrison did not commit the crime accurately partially answers the question *who committed the crime*. Another example. Consider:

(84) ??Mark knows in part who is the (only) tallest person in the room.

(84) is off. But the question *who is the tallest person in the room* does have accurate partial answers. Suppose only John, Mary and Tom are in the room, and John is the tallest. If Mark knows that Tom is not the tallest person, Mark knows a partial answer to the question *who is the tallest person in the room*, according to our definition *Partial answer*. However, it still not true of Mark that he knows in part who is the tallest person in the room. This although the proposition that Tom is not the tallest person in the room partially answers the question *who is the tallest person in the room*.

The notion of part of a true answer and the notion of proper and accurate partial answer come apart. Although every part of true answer is a proper and accurate partial answer, not every accurate and proper partial answer to a question is part of a true complete answer to a question.

My proposal is to start with the notion of subquestion and define part of an answer in terms of the notion of sub-question. The notion of sub-question was first defined by Lewis, for the partition case as follows:

LEWIS' INCLUSION FOR PARTITION-QUESTIONS

$Q_1 \subseteq Q_2$ if and only if every cell of Q_1 is a union of cells of Q_2 .

Such definition is defective for two reasons. It only works for a coarse-grained notion of questions, answers and propositions, on which questions are set of cells, and cells are themselves sets of possible worlds that can be joint or disjoint. Moreover, it only applies to mention-all questions—questions that form a partition and for which it makes sense to speak of cells. However, we want to define the notion of sub-question for mention-some questions as well and such questions do not form a partition. That is so because it is not the case that a complete answer to a mention-some questions is incompatible with any other complete answer to it. So a complete answer to a

mention-some question may not be incompatible with another complete answer to the same question.

The following slight qualification will address both concerns:

PROPER INCLUSION FOR QUESTIONS (GENERAL)

$Q_1 \subset Q_2$ if and only if every answer to Q_1 is properly entailed by some answer to Q_2 .

where entailment is just truth-preservation. One more fix is required: Because an answer to the tautological question is entailed by every answer to any question, it follows from INCLUSION FOR QUESTIONS (GENERAL) that every question includes the tautological question. For a more *exclusive* notion of inclusion, we need to add a further condition:

PROPER INCLUSION FOR QUESTIONS (GENERAL)

$Q_1 \subset Q_2$ if and only if

1. every answer to Q_1 is properly entailed by some answer to Q_2 ;
2. for any answer to Q_1 , p is incompatible with some but not all answers to Q_2 without entailing any answer to Q_2 .

Observe that if a question Q_1 satisfies PROPER INCLUSION FOR QUESTIONS (GENERAL), Q_1 is not tautological—i.e., is not answered by every answer: For not any answer to a tautological issue would partially answer Q_2 ; for the same reason, if an issue satisfies PROPER INCLUSION FOR QUESTIONS (GENERAL), then Q_1 is not completely answered by Q_2 's presuppositions.

THE PART-WHOLE APPROACH

- 'S knows in part Q ' is true at a context c if and only if for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, S knows some part of α .

- 'S knows for the most part Q' is true at a context c if and only if for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, one knows most part of α .
- 'S knows (fully) Q' is true at a context c if and only if for some complete true answer $\alpha \in \llbracket Q \rrbracket^c$, one knows every part of α .

We can start outlining some desirable features of this analysis. We observed that adverbial modification is not possible into whether-interrogatives. Thus, the following is not acceptable:

- (85) # Mark knows in part whether Paris is the capital of France and London the capital of UK.

It follows from my definition that whether-questions do not have other questions as parts:

Theorem 1. *Whether-questions do not have other questions as parts.*

Proof. According to INCLUSION FOR QUESTIONS, a question \mathcal{Q} is included into a whether-question \mathcal{Q}_ϵ only if some answers to \mathcal{Q} only partially answers \mathcal{Q}_ϵ . But no answer only partially answer a whether question. For if an answer partially answers a whether-questions, it is incompatible with one of its possible answers. But a whether-question only has two possible answers. Hence, if an answer partially answers a whether-question, it fully answers it. No question is included into a whether-question (by INCLUSION FOR QUESTIONS). Hence, for every p , $p?$ is not included into a whether-question. (By instantiation). □

Theorem 2. *Any answer to a whether-question does not have parts.*

Proof. According to THE PART-WHOLE APPROACH, S would know in part whether A iff S knows part of the true complete answer to the question whether-A. And S

knows part of the true complete answer to the question whether- A iff for some proposition p that is part of the true complete answer to the question whether- A , S knows p . For there to be a proposition that is part of either (i) or (ii), such a proposition would have to (only) partially answer the question whether- A . For it to partially answer the question whether- A , it would have to be incompatible with some proposition in the question whether- p , while at the same time being compatible with more than one proposition in that question. But any proposition that is incompatible with one proposition (say, A) in the question whether- A will entail its complement, thereby entail the full answer to whether- A . Hence no proposition can satisfy the requirement imposed by (PARTS OF AN ANSWER), if Q_2 is a whether-question. So according to INCLUSION FOR QUESTIONS, no whether-question can have other questions as parts. Hence, according to PARTS OF AN ANSWER, no answers is part of an answer to a whether-question. \square

Hence, my account of parts of answers together with THE PART-WHOLE APPROACH correctly predicts that reports embedding whether-interrogatives cannot correctly be quantified over.

We have observed that not every partial answer to a question is part of an answer to that question. This follows from my account. Call a mention-all question a *partition*, and each member of a mention-all question a *cell* of the partition. We can prove in full generality that the complement of any member of a mention-all question is not part of a true complete answer to that mention-all question:

Theorem 3. *The complement of any cell of a partition is not part of the true answer associated with that partition.*

Proof. Suppose Q is a mention-all question, a set of three incompatible propositions p_1, p_2, p_3 . And consider the answer $\langle p_1, Q \rangle$. Consider the proposition $\neg p_2$. $\neg p_2$ partially answers Q and is entailed by p_1 . However, crucially, it is not a part of $\langle p_1,$

\mathcal{Q}). For it to be part of $\langle p_1, \mathcal{Q}_2 \rangle$, it must be the case that any proposition in the question $\neg p_2?$ only partially answers \mathcal{Q} . The question $\neg p_2?$ is a whether-question with two propositions: $\neg p_2$ and p_2 . And p_2 entails a complete answer to \mathcal{Q} , i.e. p_2 itself. Hence not every proposition in the question $\neg p_2?$ only partially answer \mathcal{Q} . This result is utterly general: the complement of any cell of a partition does not constitute parts of any answer to the question corresponding to that partition. \square

1.6.1.2 Mass Quantifiers

We want to associate each ordered pair $\alpha (= \langle p, \text{How to } \phi \rangle)$ in the denotation of the interrogative $\llbracket \text{How to } \phi \rrbracket$ with an algebra that has α 's first element p as its top. Take the set of all p 's sub-answers relative to *How to* ϕ as defined in PROPER PARTS OF AN ANSWER. The set of all p 's sub-answers is $\mathcal{A}(\alpha)$. Add the practical answer p to it. The result is $\mathcal{A}^*(\alpha)$. $\mathcal{A}^*(\alpha)$ is partially ordered by the relation *less than* $<$. $\mathcal{A}^*(\alpha)$ has a top—i.e., a practical proposition answering the question *How to* ϕ . For any element in $\mathcal{A}^*(\alpha)$, SUB-ANSWERS (a) = $\{q \in \mathcal{A}^*(\alpha) : q < a\}$

The general idea, due to Higginbotham(85), is to define the meaning of a mass quantifier in terms of a *measure*. In our particular case, we want to define a measure that *counts* the parts of the elements of $\mathcal{A}^*(\alpha)$. We define it as a function μ from subsets of $\mathcal{A}^*(\alpha)$ to the non-negative real numbers:

$$(86) \quad \mu: \mathcal{P}(\mathcal{A}^*(\alpha)) \rightarrow \mathbb{R}_+$$

Applied to the empty set, the value of μ is 0:

$$(87) \quad \text{If } X \subset \mathcal{A}^*(\alpha) = \emptyset,$$

$$\mu(X) = 0.$$

Applied to any singleton of an element a in $\mathcal{A}^*(\alpha)$, the value of μ is the number

of elements in $\mathcal{A}^*(\alpha)$ that are in SUB-ANSWERS (a):

$$(88) \quad \text{If } X \subset \mathcal{A}^*(\alpha) = \{a\} \text{ for some } a \in \mathcal{A}^*(\alpha),$$

$$\mu(\{a\}) = |\{b \in \mathcal{A}^*(\alpha) : b \in \text{SUB-ANSWERS}(a)\}|.$$

For every two disjoint elements of $\mathcal{A}^*(\alpha)$ —i.e., for any two or more elements of $\mathcal{A}^*(\alpha)$ that do not have sub-answers in common—the value of the set containing them is the same as the sum of the values of each:

$$(89) \quad [\text{COUNTABLE ADDITIVITY}] \text{ For every } a_1, \dots, a_n \in \mathcal{A}^*(\alpha), \text{ such that } \text{SUB-ANSWERS}(a_1) \cap \dots \cap \text{SUB-ANSWERS}(a_n) = \emptyset,$$

$$\mu(a_1, \dots, a_n) = \mu(a_1) + \dots + \mu(a_n)$$

(As a measure, μ has to satisfy *countable additivity*.)

For any set X of elements of $\mathcal{A}^*(\alpha)$ that are not disjoint—i.e., that have sub-answers in common—the value of μ for X is the same as the value of μ for the biggest sub-answer of that set, *if* there is one. If no element in the set is the biggest, the value of μ for X is the number of sub-answers that the elements in X have in common:

$$(90) \quad \text{If } a_1, \dots, a_n \in \mathcal{A}^*(\alpha), \text{ such that } \text{SUB-ANSWERS}(a_1) \cap \dots \cap \text{SUB-ANSWERS}(a_n) \neq \emptyset,$$

- if for some $a_i \in (a_1, \dots, a_n)$, every $b \in (a_1, \dots, a_n)$ such that $b \neq a_i$, $b \in \text{SUB-ANSWERS}(a_i)$, then $\mu(a_1, \dots, a_n) = \mu(a_i)$.
- else, $\mu(a_1, \dots, a_n) = |\text{SUB-ANSWERS}(a_1) \cap \dots \cap \text{SUB-ANSWERS}(a_n)|$.

Now that we have defined our measure, we can define the meaning of a mass quantifier in terms of our measure. In our particular case, the mass quantifiers “fully,” “in part,”

and “for the most part” take two arguments: the first argument is an ordered pair α that has as its first element a practical answer and as a second element the practical question it answers; the second argument is the set of propositions known by S^* .

Recall that according to *Quantitative Gradability*, “ S knows in part how to ϕ ” is true iff S knows part of a practical answer to the question “How to ϕ .” On this analysis, S knows part of a practical answer if and only if the value of μ for the intersection between $\mathcal{A}^*(\alpha)$ and the propositions known by S is different from zero:

$$(91) \quad \text{In part } [\alpha; \lambda p_{\langle s,t \rangle} (S \text{ knows } p)] = 1 \text{ iff } \mu(\mathcal{A}^*(\alpha) \cap \lambda p_{\langle s,t \rangle} (S \text{ knows } p)) \neq 0.$$

Less formally, according to (91), S knows part of a practical answer if and only if the number of sub-answers of that practical answer that are known by S is different from zero.

Recall that according to *Quantitative Gradability*, “ S knows for the most part how to ϕ ” is true iff S knows part of a practical answer to the question “How to ϕ .” On this analysis, S knows most part of a practical answer if and only if the value of μ for the intersection between $\mathcal{A}^*(\alpha)$ and the propositions known by S is higher than the value of μ for the intersection between $\mathcal{A}^*(\alpha)$ and the propositions not known by S :

$$(92) \quad \text{For the most part } [\alpha; \lambda p_{\langle s,t \rangle} (S \text{ knows } p)] = 1 \text{ iff } \mu(\mathcal{A}^*(\alpha) \cap \lambda p_{\langle s,t \rangle} (S \text{ knows } p)) > \mu(\mathcal{A}^*(\alpha) \cap \lambda p (S \text{ does not know } p))$$

Less formally, according to (92), S knows every part of a practical answer if and only if the number of sub-answers of that answer that are known by S is bigger than the number of sub-answers of that answer that are not known by S .

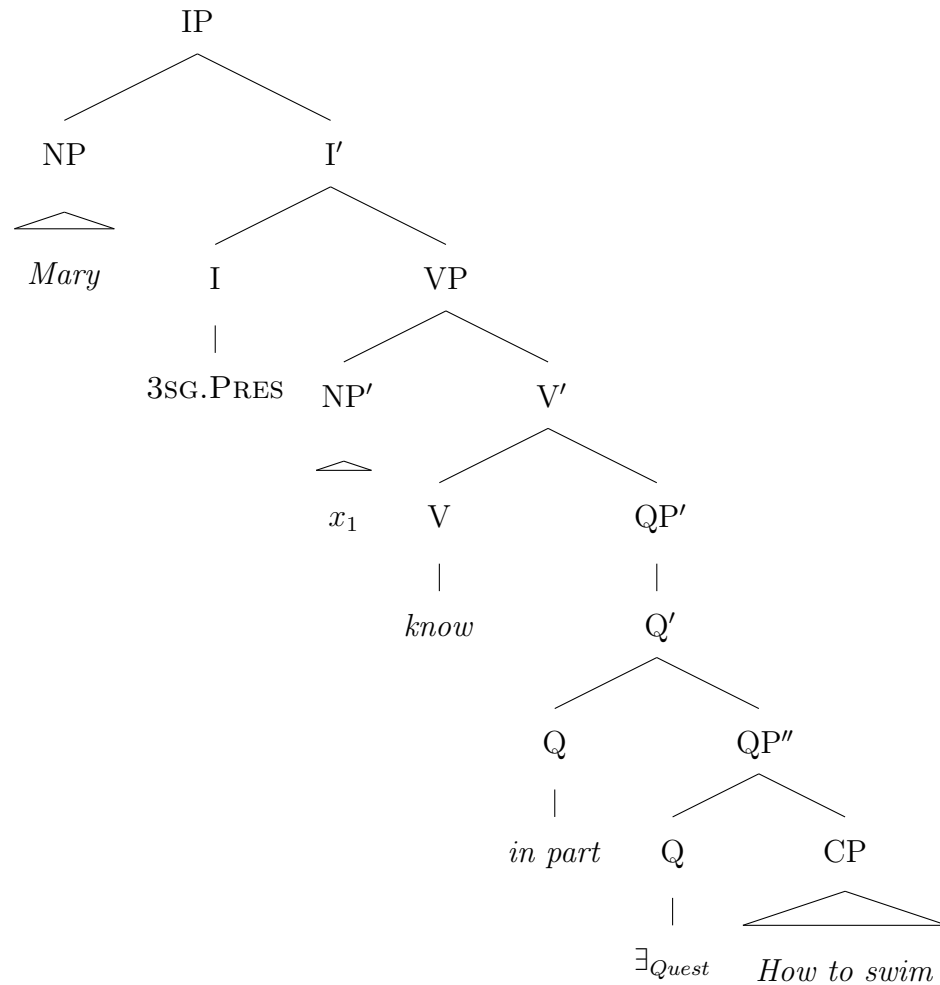
Finally, recall that according to *Quantitative Gradability*, “ S knows fully part how to ϕ ” is true iff S knows every part of a practical answer to the question “How to ϕ .”

According to (93), S knows every part of a practical answer to a question if and only if the number of sub-answers of that answer that are not known by S is equal to 0:

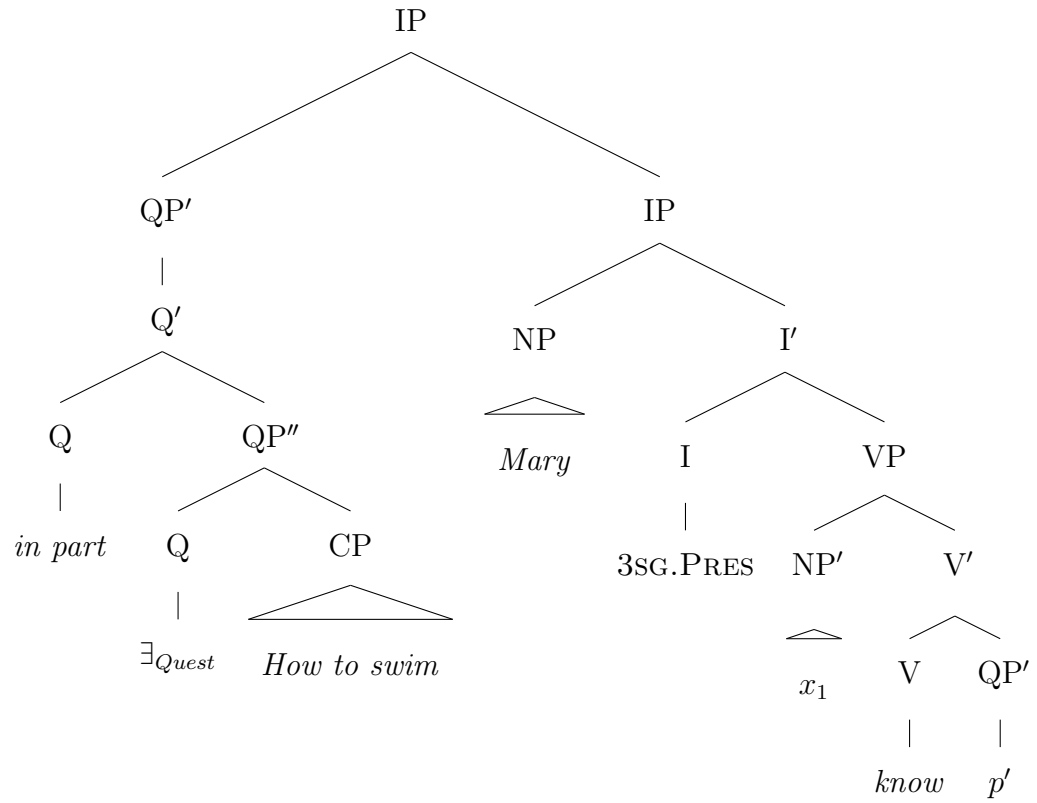
$$(93) \quad \text{Fully } [\alpha; \lambda p_{\langle s,t \rangle} (S \text{ knows } p)] = 1 \text{ iff } \mu(\mathcal{A}^*(\alpha) \cap \lambda p_{\langle s,t \rangle} (S \text{ does not know } p)) = 0.$$

1.6.1.3 Logical Form

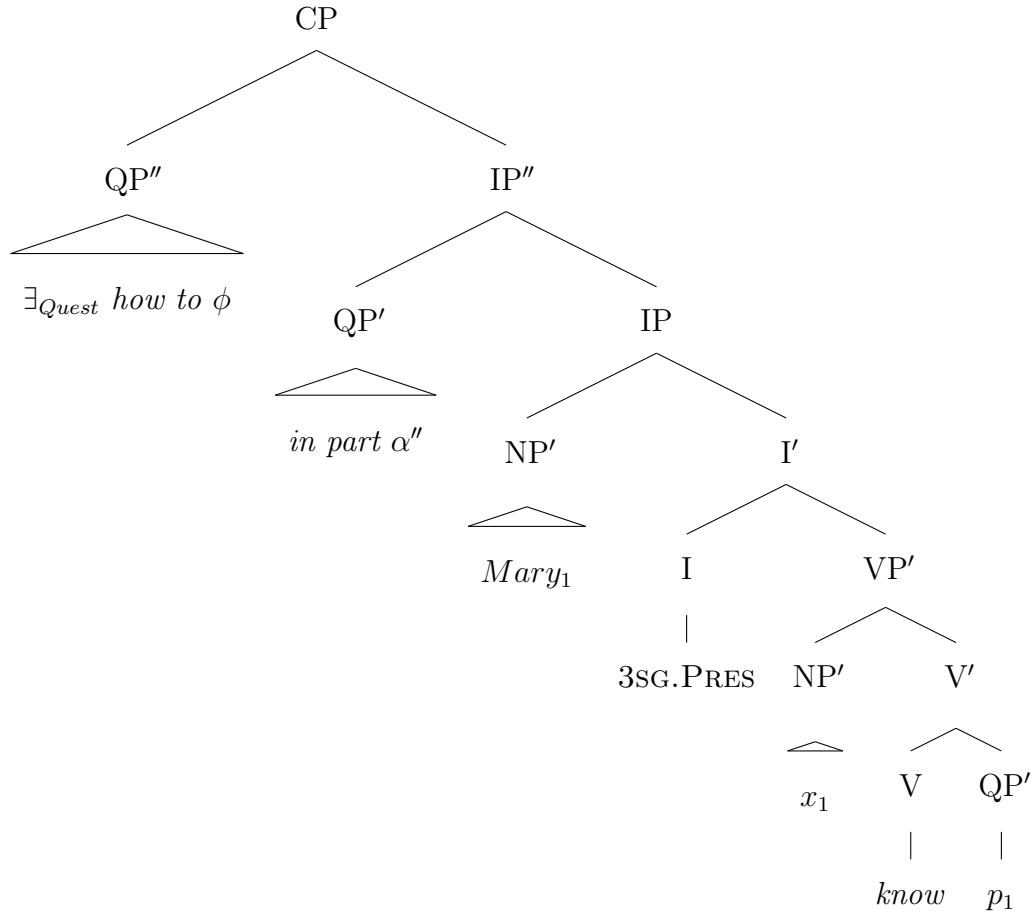
Start with the following *in situ* logical form:



Following Heim and Kratzer (1998)'s analysis of lifted quantifiers, QP' moves because of a type mismatch leaving behind a trace of type p' :



For the same reason, the second quantifier QP'' moves up to the CP 's adjunct position, leaving behind a trace of type α'' . The resulting logical form to be interpreted is:



Fragment

- $\llbracket know \rrbracket = \lambda w \lambda p_{\langle s,t \rangle} \lambda x_e (know(w)(p)(x))$.
- $\llbracket in\ part \rrbracket = \lambda \alpha \lambda P_{\langle p,t \rangle} (\mu (\mathcal{A}^*(\alpha) \cap P) \neq 0)$.
- $\llbracket \exists_{Quest} \rrbracket = \lambda P_{\langle \alpha,t \rangle} \lambda P'_{\langle \alpha,t \rangle} \exists \alpha (P(\alpha) \& P'(\alpha))$

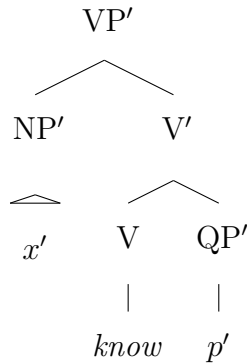
Rules of Composition The Rules of Composition are as in Heim & Kratzer(82):

1. NON-BRANCHING NODES (NN)= If α is a non-branching node, and β is its daughter node, then α is in the domain of $\llbracket \]$ if β is. In this case, $\llbracket \alpha \rrbracket = \llbracket \beta \rrbracket$.

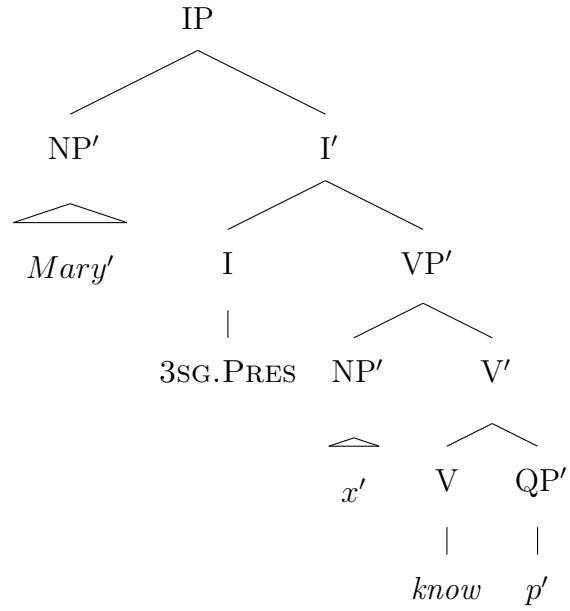
(5)

2. FUNCTIONAL APPLICATION (FA)= If α is a branching node, $\{\beta, \gamma\}$ is a set of α 's daughters, and $\llbracket\beta\rrbracket$ is a function whose domain contains $\llbracket\gamma\rrbracket$, then $\llbracket\alpha\rrbracket = \llbracket\beta\rrbracket(\llbracket\gamma\rrbracket)$.
3. PREDICATE MODIFICATION (PM)= If α is a branching node, $\{\beta, \gamma\}$ is the set of α 's daughters, and $\llbracket\beta\rrbracket$ and $\llbracket\gamma\rrbracket$ are both in D_t , for some type t , then $\llbracket\alpha\rrbracket = \lambda x_t(x_t \in \llbracket\beta\rrbracket \text{ and } x_t \in \llbracket\gamma\rrbracket)$.

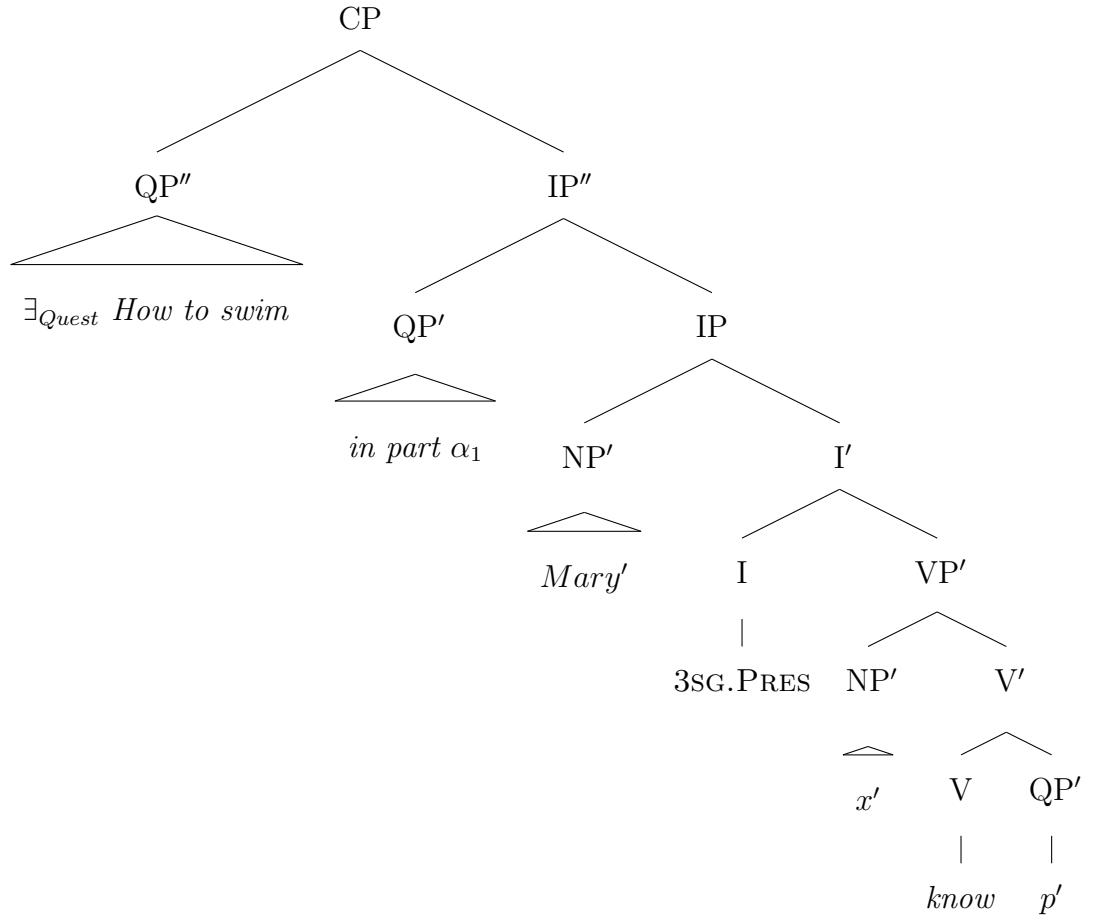
1.6.1.4 Composition



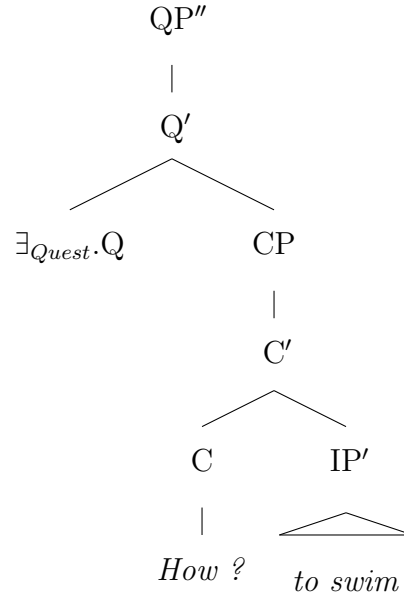
- (94) a. $\llbracket V' \rrbracket = \llbracket V \rrbracket(\llbracket QP' \rrbracket) =$
 b. $\llbracket \text{know} \rrbracket(\llbracket p' \rrbracket)$
 c. $\lambda p \lambda x(\text{know}(w^*)(p)(x))(p') =$
 d. $\lambda x(\text{know}(w^*)(p')(x))$ (by FA).
- (95) a. $\llbracket VP' \rrbracket = (\llbracket NP' \rrbracket)\llbracket V' \rrbracket =$
 b. $(\llbracket x' \rrbracket)\llbracket \text{know} \rrbracket$
 c. $(x^*)\lambda x \text{knows}(w^*)(p')(x)$
 d. $\text{know}(w^*)(p')(x')$ (by FA).



- (96) a. $\llbracket IP \rrbracket = (\llbracket NP \rrbracket) \llbracket I' \rrbracket =$
 b. $(\llbracket Mary' \rrbracket) \llbracket knows p' \rrbracket$
 c. $(Mary)(knows(w^*)(p')(x'))$
 d. $knows(w^*)(p')(Mary)$ (by FA).



- (97) a. $\llbracket IP'' \rrbracket = \llbracket QP' \rrbracket (\llbracket IP \rrbracket) =$
 b. $\llbracket in\ part\ \alpha_1 \rrbracket (\llbracket Mary\ knows\ p^*\ at\ w^* \rrbracket)$
 c. $\lambda P_{\langle p,t \rangle} (PART(\alpha) \in P)(knows(w^*)(p)(Mary))$
 d. $PART(\alpha) \in (knows(w^*)(p)(Mary))$ (by FA).



(98) $\llbracket QP'' \rrbracket = \llbracket Q' \rrbracket$ (by NN).

(99) a. $\llbracket Q' \rrbracket = \llbracket Q \rrbracket(\llbracket CP \rrbracket) =$

b. $\llbracket \exists_{Quest} \rrbracket(\llbracket How\ to\ swim \rrbracket) =$

c. $\lambda P_{\langle \alpha, t \rangle} \lambda P'_{\langle \alpha, t \rangle} \exists \alpha (P(\alpha) \ \& \ P'(\alpha))$ ($\lambda \alpha$ ($\alpha^T \in How\ to\ \phi$))

d. $\lambda P'_{\langle \alpha, t \rangle} \exists \alpha (\lambda \alpha (\alpha^T \in How\ to\ \phi)(\alpha) \ \& \ P'(\alpha))$

e. $\lambda P'_{\langle \alpha, t \rangle} \exists \alpha (\alpha^T \in How\ to\ \phi \ \& \ P(\alpha))$ (by FA).

(100) a. $\llbracket CP \rrbracket = \llbracket QP'' \rrbracket(\llbracket IP'' \rrbracket) =$

b. $\llbracket \exists_{Quest} \ How\ to\ swim \rrbracket(\llbracket in\ part\ Mary\ knows \rrbracket) =$

c. $\lambda P'_{\langle \alpha, t \rangle} \exists \alpha (\alpha^T \in How\ to\ \phi \ \& \ P(\alpha)) (\text{PART}(\alpha) \in (\text{knows}(w^*)(p)(Mary)))$

d. $\exists \alpha (\alpha^T \in How\ to\ \phi \ \& \ (\text{PART}(\alpha) \in (\text{knows}(w^*)(p)(Mary))))(\alpha)$

e. $\exists \alpha (\alpha^T \in How\ to\ \phi \ \& \ \text{PART}(\alpha) \in (\text{knows}(w^*)(p)(Mary)))$ (by FA).

f. For some practical answer to the question *How to ϕ* , *S* knows part of that practical answer.

1.6.2 Qualitative Gradability

1.6.2.1 Logical Form

As for *Quantitative Gradability*, I will now apply the general idea defended in the main text and provide a compositional semantics for qualitatively graded ascriptions of know how. The general idea is that the comparative clause 'better than' in 'S knows how to ϕ ' compares the sets of all answers (entirely) known by the two subjects. Now, as explained in the text, I am assuming that the comparative clause takes the two knowledge ascriptions 'S knows how to ϕ ' as its arguments. Now, the issue is that as showed in *Quantitative Gradability*, each ascription existentially generalizes over practical answers that are entirely known by the subjects, as in:

(101) For some practical answer, S knows every part of that practical answer.

However, this is a bit problematic because we want the ascriptions to feed the adverbial modifier with *the set of all answers known* by the subject to that question. So what we want is to get (102) as input to the adverbial modifier:

(102) $\lambda\alpha(\alpha^T \in \text{How to } \phi \ \& \ S \text{ knows every part of } \alpha)$.

Because of the structural analogy of the problem with the case of indefinites observed in the main text, I propose that we extend the usual strategies employed to deal with indefinites to the present case. In particular, here I illustrate how to extend the DRT-style analysis (Kamp(87), Heim(78) to this issue. Other approaches, such as a E-type approach (Heim(81), Reinhart(146) and Heim(80)), or a dynamic approach, are also possible, but will not be developed here.

DRT treats indefinites as variables over individuals, that are bound by an existential quantifier when unembedded, and that acquire universal force by means of

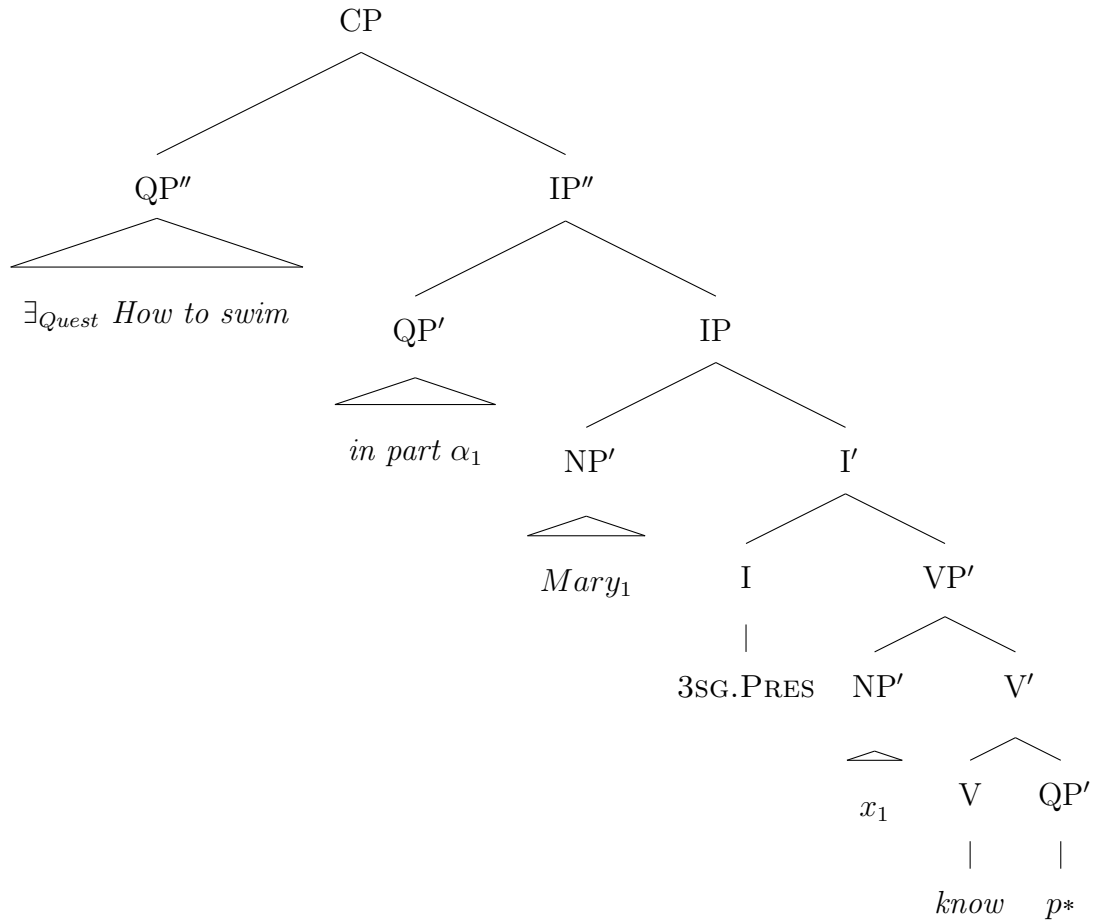
the embedding construction. Applying this strategy to this case amounts to treating, when the construction embedding an interrogative occurs embedded, QUEST *not* as a existential quantifier, as in our original fragment:

$$(103) \quad \llbracket \exists_{Quest} \rrbracket = \lambda P \lambda P' \exists \alpha (P(\alpha) \ \& \ P'(\alpha))$$

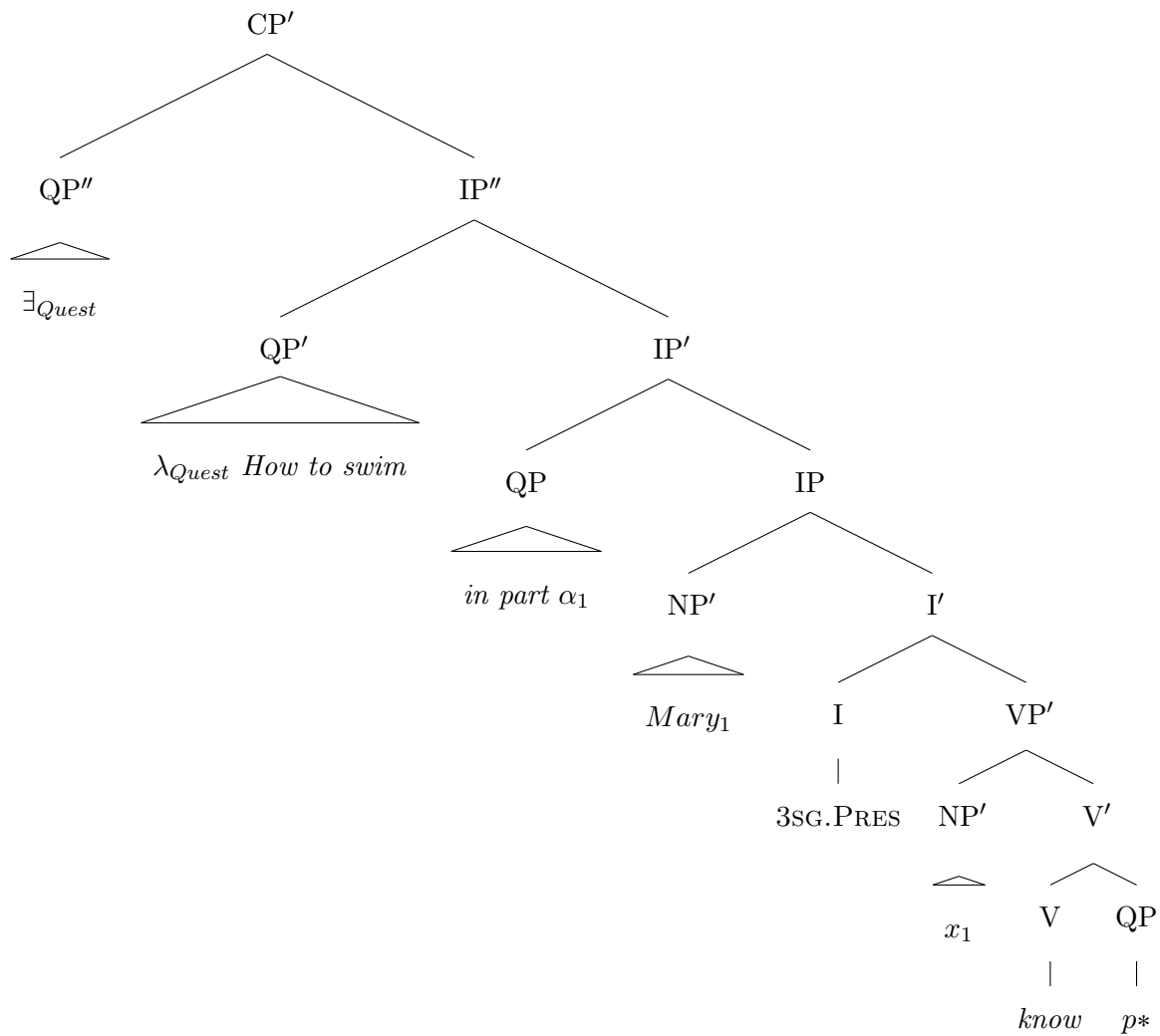
but rather as an operator that when fed two properties of answers returns the sets of answers that are in both:

$$(104) \quad \llbracket \lambda_{Quest} \rrbracket = \lambda P \lambda P' \lambda \alpha (P(\alpha) \ \& \ P'(\alpha))$$

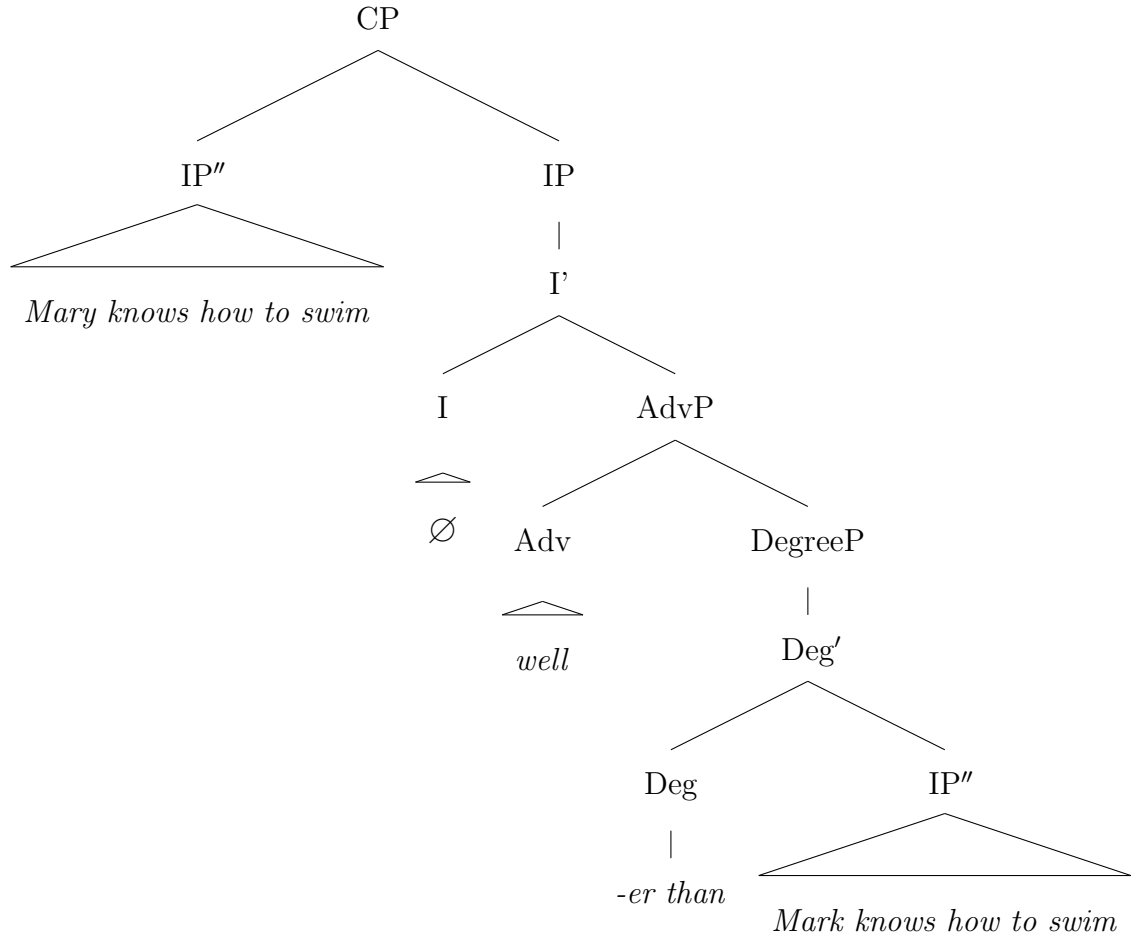
The universal force is induced by the whole construction—i.e., by embedding the ascription into the adverbial construction and into the comparative construction. By contrast, we can assume that when unembedded, the QUEST has existential force or that an external quantifier to the higher *CP* is responsible for the existential force. In this case, the logical form is not:



but rather:



Only the tree under IP'' embeds into the adverbial modifier and comparative clause:



Fragment*

- $\llbracket know \rrbracket = \lambda w_s \lambda p_{\langle s,t \rangle} \lambda x_e (know(w)(p)(x))$.
- $\llbracket fully \rrbracket = \lambda \alpha \lambda P_{\langle p,t \rangle} (\mu(\mathcal{A}^*(\alpha) \cap \lambda p_{\langle s,t \rangle} (S \text{ does not know } p)) = 0) =_{abbr} \lambda \alpha \lambda P_{\langle p,t \rangle} (FULL(\alpha) \in P)$.
- $\llbracket \exists_{Quest} \rrbracket = \lambda P_{\langle \alpha,t \rangle} \lambda P'_{\langle \alpha,t \rangle} \exists \alpha (P(\alpha) \& P'(\alpha))$
- $\llbracket well \rrbracket = \lambda d \lambda P' (\text{WELL}(D)(P') = d)$.
- $\llbracket POS \rrbracket = \lambda G'_{\langle d,t \rangle} \lambda P_{\langle \alpha,t \rangle} \exists d [\text{STANDARD}(d)(G)(C) \& G(d)(P)]$.
- $\llbracket -er \text{ than} \rrbracket = \lambda P \lambda G \lambda Q \exists d (G(d, Q) \& \text{not } G(d, P))$.
- $\llbracket as \dots as \rrbracket = \lambda P \lambda G \lambda Q \exists d (G(d, Q) \& G(d, Q) \& \text{MAX}(G)(d): G(d, P))$.
- $\llbracket \lambda_{Quest} \rrbracket = \lambda P_{\langle \alpha,t \rangle} \lambda P'_{\langle \alpha,t \rangle} \lambda \alpha (P(\alpha) \& P'(\alpha))$

- $\llbracket \text{How to } \phi \rrbracket = \lambda \alpha (\alpha^T \in \text{How to } \phi)$.

Comments on Fragment* Most developments of the scalar view of comparatives appeal to *maxima* to deal with so-called *equatives*, constructions of the form ‘ X is as tall as Y ’. Accordingly for X to be as tall as Y , there must be a degree of tallness met by Y that is also met by X , which is the maximum degree of tallness met by Y . Accordingly, a sentence such as ‘ X is as tall as Y ’ is true if and only if there is a degree of tallness achieved by X that is the maximum degree of tallness achieved by Y . The appeal to maxima is also needed for dealing with comparative constructions that involve negative polarity items. See von Stechow (182) for discussion and the Appendix for further discussion.

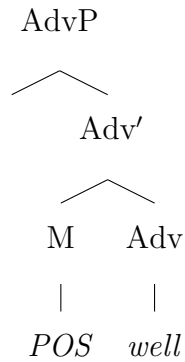
Accordingly, in the denotation of equatives “as . . . as”, I appealed to the *MAX* operator, which following von Stechow(182), p. 37, I define thus:

$$(105) \quad \text{MAX}(P)(d) =_{def} P(d) \ \& \ \forall d' (d' \neq d \ \& \ P(d') : d' < d).$$

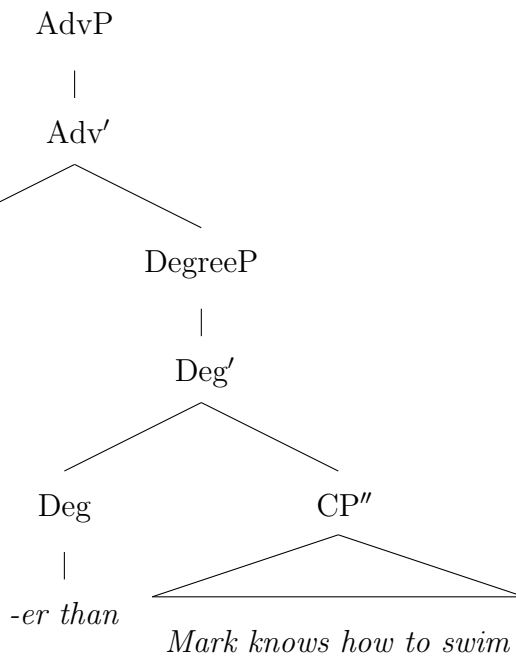
MAX takes a property of degrees P (such as *being good*) and a degree d into the truth just in case d satisfies P and d is the maximum degree that satisfies P .⁶¹

⁶¹The use of *maxima* will not work if there are infinitely many degrees. For example, suppose that degrees are rational numbers between 0 and 1, and P and Q are good to every degree less than $2/3$ (and no others). The set of degrees less than $2/3$ has no maximum. Since it does not have a maximum, the semantics predicts that P cannot be as good as Q . (Thanks to Tim Williamson for pressing this objection on me.) Note however that the problem is remarkably similar to the one arising for a Lewisian theory of counterfactuals that gives up the so-called “limit assumption” (Lewis (110)). Whatever tools used for limit assumption conditionals (whether Lewis’s own solution or a supervaluationist solution) could be added on to the account with *maxima*.

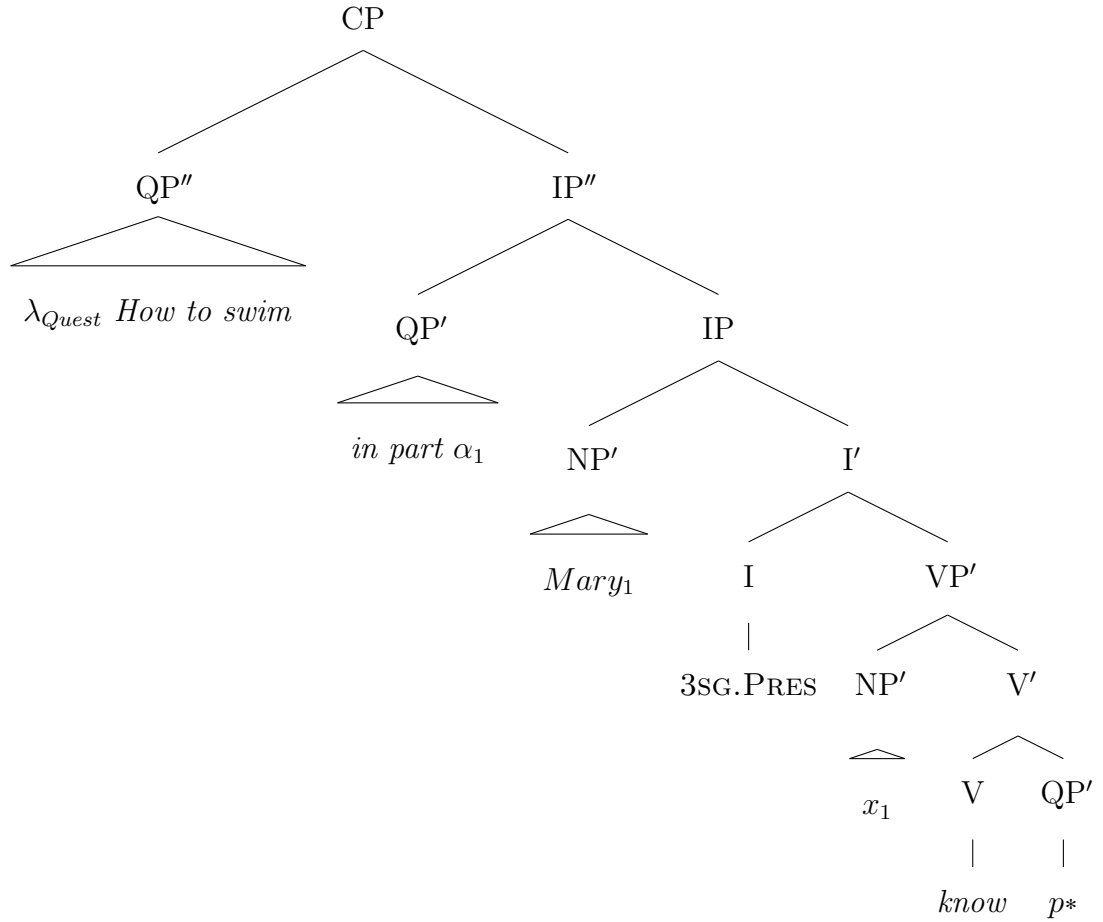
1.6.2.2 Composition



- (106)
- $\llbracket Adv' \rrbracket = \llbracket M \rrbracket (\llbracket Adv \rrbracket) =$
 - $\llbracket POS \rrbracket (\llbracket well \rrbracket) =$
 - $\lambda G' \lambda P \exists d [\text{STANDARD}(d)(G)(C) \ \& \ G'(dP)] \ (\lambda d \lambda P' (\text{WELL}(D)(P') = d))$
 - $\lambda P \exists d [\text{STANDARD}(d)(\lambda d \lambda P' (\text{WELL}(D)(P') = d))(C) \ \& \ \lambda d \lambda P' (\text{WELL}(D)(P') = d)(d, P)]$ (by FA)
 - $\lambda P \exists d [\text{STANDARD}(d)(\lambda d \lambda P' (\text{WELL}(D)(P') = d))(C) \ \& \ \text{WELL}(D)(P) = d]$ (Simplification).



- (107) a. $\llbracket DegreeP \rrbracket = \llbracket Deg' \rrbracket$ (by NN).
- (108) a. $\llbracket Deg' \rrbracket = \llbracket Deg \rrbracket(\llbracket CP'' \rrbracket) =$
 b. $\llbracket -er\ than \rrbracket(\llbracket Mark\ knows\ how\ to\ swim \rrbracket) =$
 c. $\lambda P \lambda G \lambda Q \exists d (G(d, P) \ \& \ \text{not } G(d, Q)) (\lambda \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s, t \rangle} (\text{knows}(w^*)(p)(Mark))))$
 d. $\lambda G \lambda Q \exists d (G(d, Q) \ \& \ \text{not } G(d, (\lambda \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s, t \rangle} (\text{knows}(w^*)(p)(Mark))))))$ (by FA).
- (109) a. $\llbracket AdvP \rrbracket = \llbracket Adv' \rrbracket$ (by NN).
- (110) a. $\llbracket Adv' \rrbracket = (\llbracket Adv \rrbracket) \llbracket DegreeP \rrbracket =$
 b. $(\llbracket Adv \rrbracket) \llbracket Deg' \rrbracket$ (by NN).
- (111) a. $(\llbracket Adv \rrbracket) \llbracket Deg' \rrbracket =$
 b. $(\llbracket well \rrbracket) \llbracket -er\ than\ CP'' \rrbracket =$
 c. $(\lambda d \lambda P' (\text{WELL}(D)(P') = d)) \lambda G \lambda Q \exists d [G(d, P) \ \text{and} \ \text{not } G(d, (\lambda \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s, t \rangle} (\text{knows}(w^*)(p)(Mark)))))] =$
 d. $\lambda Q \exists d (\lambda d \lambda P' (\text{WELL}(D)(P') = d)(d, P) \ \text{and} \ \text{not } (\lambda d \lambda P' (\text{WELL}(D)(P') = d))(d, (\lambda \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s, t \rangle} (\text{knows}(w^*)(p)(Mark))))))$
 e. $\lambda Q \exists d (\text{WELL}(D)(P) = d \ \text{and} \ \text{not } (\text{WELL}(D)(\lambda \alpha (\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s, t \rangle} (\text{knows}(w^*)(p)(Mark)))))) = d$ (by FA).

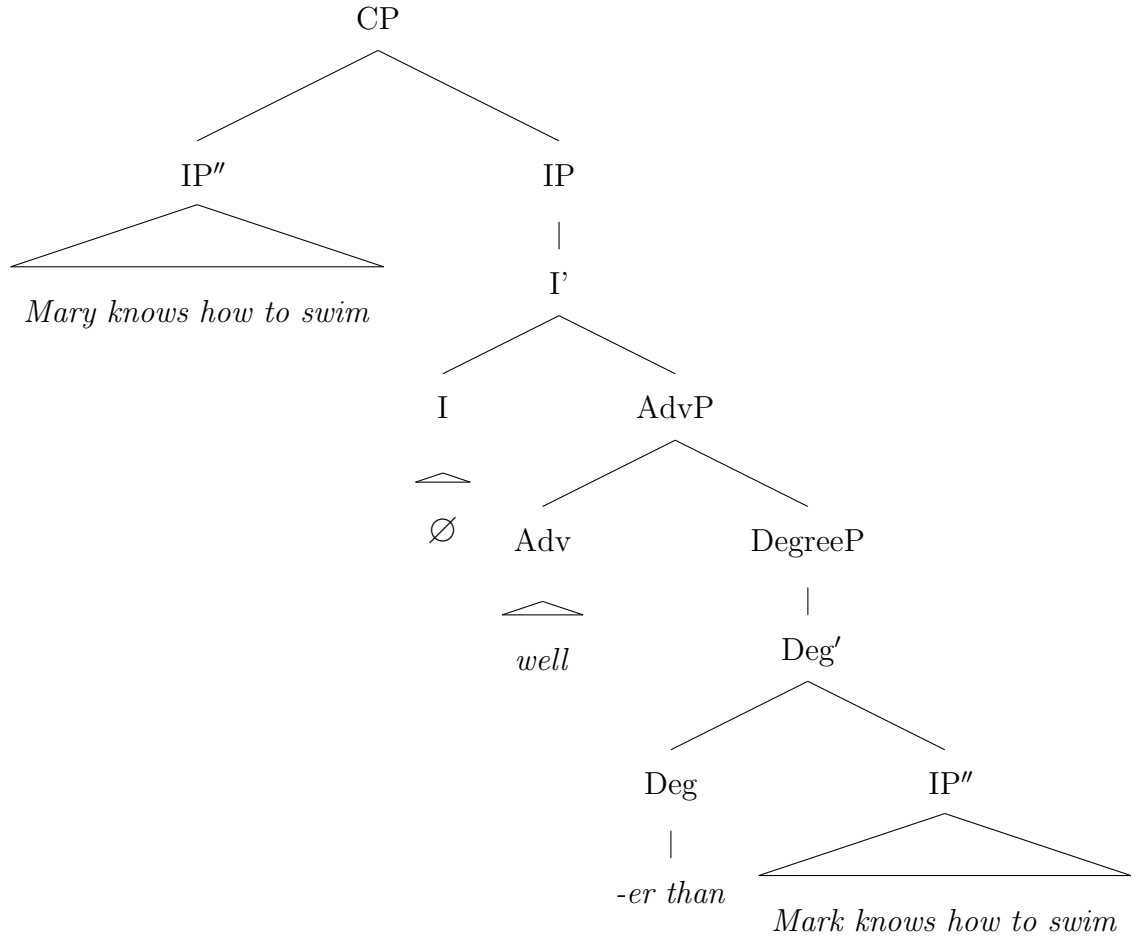


- (112) a. $[[V'] = [V]([QP']) =$
 b. $[[know]([p^*])$
 c. $\lambda p_{\langle s,t \rangle} \lambda x_e (know (w^*)(p)(x))(p^*)$.
 d. $\lambda x_e (know (w^*)(p^*)(x))$ (by FA)

- (113) a. $[[VP'] = ([NP'])[V'] =$
 b. $([x_1])[V']$
 c. $(x_1) \lambda x_e (know (w^*)(p^*)(x))$
 d. $know (w^*)(p^*)(x_1)$ (by FA)

- (114) a. $[[I'] = [I]([VP']) =$
 b. $[[3sg.Pres](know (w^*)(p^*)(x_1)) =$
 c. $knows (w^*)(p^*)(x_1)$ (by FA)

- (115) a. $\llbracket IP \rrbracket = (\llbracket NP \rrbracket) \llbracket VP \rrbracket =$
 b. $(\llbracket Mary \rrbracket) \llbracket VP \rrbracket$
 c. $(Mary) \text{ knows}(w^*)(p^*)(x_1)$
 d. $\text{knows}(w^*)(p^*)(Mary)$ (by FA)
- (116) a. $\llbracket IP'' \rrbracket = \llbracket QP' \rrbracket (\llbracket IP \rrbracket) =$
 b. $\llbracket \text{in part } \alpha_1 \rrbracket (\llbracket IP \rrbracket)$
 c. $\lambda P_{\langle p, t \rangle} (\text{FULL}(\alpha) \in P) (\text{knows}(w^*)(p^*)(Mary))$
 d. $(\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary)))$ (by FA).
- (117) a. $\llbracket CP \rrbracket = \llbracket QP'' \rrbracket (\llbracket IP'' \rrbracket) =$
 b. $\llbracket \lambda_{\text{Quest}} \text{ How to swim} \rrbracket (\llbracket IP'' \rrbracket)$
 c. $\lambda P'_{\langle \alpha, t \rangle} \lambda \alpha (P(\alpha) \ \& \ \alpha^T \in \text{How to } \phi) (\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary)))$
 d. $\lambda \alpha (\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary))) (\alpha) \ \& \ \alpha^T \in \text{How to } \phi$
 e. $\lambda \alpha (\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary)) \ \& \ \alpha^T \in \text{How to } \phi)$ (by FA).



- (118)
- a. $\llbracket CP''' \rrbracket = (\llbracket CP' \rrbracket) \llbracket AdvP \rrbracket =$
 - b. $(\llbracket Mary \text{ knows how to swim} \rrbracket) \llbracket better \text{ than Mark knows how to swim} \rrbracket =$
 - c. $(\lambda\alpha(\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary)) \ \& \ \alpha^T \in \text{How to } \phi)) \ \lambda P \exists d(\text{WELL}(D)(P) = d \text{ and not } (\text{WELL}(D)(\lambda\alpha(\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s,t \rangle} (\text{Mark knows } p)))) = d)$
 - d. $\exists d(\text{WELL}(D)(\lambda\alpha(\text{FULL}(\alpha) \in (\text{knows}(w^*)(p^*)(Mary)) \ \& \ \alpha^T \in \text{How to } \phi)) = d \text{ and not } (\text{WELL}(D)(\lambda\alpha(\alpha^T \in \text{How to } \phi \ \& \ \text{FULL}(\alpha) \in \lambda p_{\langle s,t \rangle} (\text{Mark knows } p)))) = d) \text{ (by FA)}$
 - e. There is a degree of goodness (along a dimension of goodness set by the context c) achieved by S 's Answer Set that is not achieved by S^* 's Answer Set.

Chapter 2

PRACTICAL SENSES

2.1 Introduction

According to *Intellectualism about know how*, knowing how to do something is a matter of possessing some piece of propositional knowledge. Intellectualists about know how routinely appeal to *practical modes of presentation* in characterizing the relevant propositional knowledge. According to this view, for one to know how to ϕ , one must know that a certain way is a way of ϕ -ing *under a practical mode of presentation* (Stanley and Williamson(178), Stanley(176)). But we are never told much about the nature of these modes of presentation, except about their being distinctively *practical*—hence their name. This lacuna has exposed *Intellectualism* to several charges. Among the most common charges, there is the complaint that by appealing to practical modes of presentation, *Intellectualism* is invoking exceedingly mysterious entities.

In this essay I propose and defend a new view of practical modes of presentation. I suggest that they should be thought of on the model of *programs*. I argue that this way of thinking of practical modes of presentation defuses the common charges leveled against *Intellectualism*'s appeal to them.

In section 2.2 I start by comparing two different construals of *Intellectualism about know how*. I argue that one construal is more attractive than the other. According to the most attractive construal, practical modes of presentations are genuine *Fregean senses*. I call them “practical senses”. Just as Fregean senses are constituents of propositions, a practical sense is a constituent of the proposition that is the content of the know how.

In sections 2.3-2.4 I introduce and motivate my positive account of practical senses. I argue that practical senses are naturally conceived of along the model of *computer programs*, if, as I suggest, tasks can be modeled as *functions* and if ways of solving tasks can be modeled as *algorithms*.

In sections 2.5-2.6 I then argue that my positive view overcomes the main charges against the *Intellectualism*'s appeal to practical modes of presentation: the charge of being unfit as constituents of propositional contents (Stalnaker(174)), the charge of mysteriousness (for example, Schiffer(156)), and the charge of *ad hocness* (for example, Noe(125), Glick(66)).

In embracing a particular view on the nature of practical modes of presentation, I inevitably incur some commitments. In the final part of this essay, section 2.7, I discuss the nature of such commitments. My positive proposal commits me to thinking of tasks that one knows how to solve as *algorithmic*—i.e., as solvable by an algorithm. I discuss whether the possibility of a non-algorithmic know how raises a powerful objection to thinking of practical modes of presentation on the model of programs and I argue that it does not.

2.2 Practical Modes of Presentation versus Practical Senses

According to *Intellectualism about the know how*, knowing how to ϕ , for some task ϕ , is a matter of knowing a proposition—i.e., a proposition that answers the question *How to ϕ* . What kind of answer?

It is clear that not *every* kind of answer will do. To use a variant of Stanley and Williamson(178)'s original example, suppose I look at a swimmer's swimming and my swimming instructor, pointing to the swimmer, says to me "That is a way for you to swim". I believe my instructor, and what she said is in fact true. I may thereby come to know an answer to the question *How to swim*. However, *in the relevant sense*, I may not have come to know how to swim. If I took a swimming test, I might still fail it. If thrown in the swimming pool, I might still drown. I do not know how to swim

in the relevant sense,¹ and yet I do know an answer to the question *How to swim*.

In response, Stanley and Williamson(178) have argued that not every answer to the question *How to ϕ* is such that by knowing such an answer one would thereby come to know how to ϕ , in the relevant sense. For one to come to know how to ϕ , in the relevant sense, one must come to know an answer to the question *How to ϕ* —i.e., a proposition of the form *w is a way of ϕ -ing*, for some way *w* of ϕ -ing—under a particular mode of presentation, a *practical mode of presentation*. But what is a practical mode of presentation?

It is helpful to distinguish between two main construals of *Intellectualism*, each of which corresponds to a different possible construal of practical modes of presentation. The former is a *Fregean* construal of the view. On this construal, modes of presentation are constituents of the content of the attitude, just as Fregean senses are constituents of Fregean propositions (Frege(53)). On this construal, a practical mode of presentation is part of the content of the know how. The latter construal is a *Russellian* construal of the view. On the Russellian construal, a practical mode of presentation is not part of the proposition itself. Rather it is a way whereby one stands in a certain propositional relation (such as knowledge or belief) towards that proposition.² Stanley and Williamson(178) explicitly embrace a Russellian construal

¹The qualification “in the relevant sense” is important. There is a sense in which I do come to know how to swim, by coming to know what my swimming instructor told me. As observed by Schaffer(153) (p. 396), Monty Python’s explanation of how to play the flute is as follows: “Well, you blow in one end and move your fingers up and down the outside.” One comes to know how to play the flute, in some sense, by coming to know such an explanation of how to do it, although such an explanation is not helpful. From this uncontroversial remark, however, one should not conclude that know how is just a matter of knowing *any* answer to the question *How to swim*. Know how is a mental state that is, as such, associated with a distinctive causal/functional characterization. If I am in that mental state, I possess a certain set of dispositions to behavior: for example, I am disposed to pass a swimming test and to survive if drown in the swimming pool. Arguably, those dispositions to behavior are not associated with knowledge of just any answer to the question *How to swim*. So, know how is not just knowledge of any answer to the question *How to ϕ* .

²The Russellian construal of modes of presentation goes back to Perry’s(135) work on essential indexicals and to Kaplan’s(88)’s “Demonstratives”. Other examples of Russellian construals of modes of presentations are Salmon(152) and Braun(22).

of the view (Stanley and Williamson(178), p. 427).

The Russellian construal of practical modes of presentation and of *Intellectualism* is not, however, the most attractive. In particular, there are at least three main reasons to find it unappealing.

On the Russellian construal, know how is a kind of propositional knowledge—one that consists in knowing a particular proposition—i.e., one of the form *w is a way of ϕ -ing* for some way *w* of ϕ -ing—but under a particular kind of mode of presentation, a practical mode. But what is it to know a proposition of that kind under a practical mode of presentation? We are told that thinking of a proposition under a practical mode of presentation “undoubtedly entails the possession of certain complex dispositions. It is for this reason that there are intricate connections between knowing how and dispositional states” (Stanley and Williamson(178), p. 429). But it is difficult to see what distinguishes such a view, as couched, from a view that says that know how is *in part* a matter of knowing the proposition *w is a way of ϕ -ing* and *in part* a matter of possessing a certain complex of dispositions to behavior, the ones that are characteristically associated with knowing how to ϕ . So it is unclear whether this construal does not collapse into a verbal variant of a mixed view, one on which know how is partly propositional and partly non-propositional.³

³Stanley and Williamson(178) do try to motivate their construal of *Intellectualism*, by comparing it to the case of first-personal knowledge: In the case of first-personal knowledge (at least on certain views of first-personal attitudes, such as Perry’s(135)) we also have a kind of knowledge towards a propositional content but under a mode of presentation—a first-person mode. The analogy, they claim, is fitting, because first-person modes of presentation are, just like practical modes of presentation, dispositions-entailing: Just like thinking of oneself under a first-person mode of presentation entails being disposed to behave in certain ways with respect to oneself—in the same way, thinking of a way of ϕ -ing under a practical mode of presentation entails being disposed to behave in certain ways. However, dialectically, the analogy with first-personal attitudes does not help Stanley and Williamson’s cause too much, because first-personal attitudes are not the paradigmatic case of propositional knowledge. Even Perry(135) proposes his view as an *alternative* to the “doctrine of propositions”, the view on which a mental state of believing can be fully characterized as a relation to a proposition. The puzzles raised by first-personal attitudes were also the main reason that led Lewis(112) to deem those attitudes non-propositional. Whether or not one agrees with Perry and Lewis, the following seems fair: The case for *Intellectualism* about know how should not (nor I think

Here is a second reason to find the Russellian construal unattractive. A Russellian construal of the Intellectualist view does not truly vindicate a *unitary picture of knowledge*, which I find among the most appealing features of the Intellectualist outlook on the mind. According to a unitary picture of knowledge, there is just *one* fundamental and most *natural* cognitive state that plays a fundamental explanatory role across a variety of domains. On this Russellian construal of *Intellectualism*, however, know how comes out as a *distinctive* kind of knowledge state—the state of *knowing-under-a-practical-mode-of-presentation*. The state of *knowing-under-a-practical-mode-of-presentation* is opposed to the state of *knowing-under-a-non-practical-mode-of-presentation* or to the state of *knowing-under-no-modes-of-presentation-at-all*. Even if there is something in common between these kinds of knowledge states, it is unclear it will be non-disjunctive—let alone a natural kind.

Finally, an Intellectualist view, when couched along the Russellian line, seems to me to lack in explanatory power. Let the *explananda* be sets of behavioral dispositions associated with the know how, and let the *explanans* be what this construal takes know how to be. As theorists of the mind, we want to *explain* those sets of dispositions to behavior *in terms* of the subject's being in a mental state with a certain content. Now, suppose we want to explain a certain set of behavioral dispositions \mathcal{D} that are associated with one's knowing how to cook lasagna. It will do very little explanatory work to claim that \mathcal{D} is to be explained in terms of a subject's knowing an answer to the question *How to cook lasagna* under a practical mode of presentation. For we have no antecedent sense of what it means to know *any* answer under a practical mode of presentation. On the other hand, explaining what it is to know a particular answer to the question *How to cook lasagna* under a practical mode of presentation by appealing to that very same set of dispositions \mathcal{D} will not help ei-

need to) turn on the case for the entirely propositional nature of first-personal attitudes.

ther, because such an explanation would be circular. What would be needed is a *general* functional characterization of what it is to know any proposition whatsoever under a practical mode of presentation, just as general functional characterizations are provided for beliefs in the possible-worlds tradition to give an account what it is to believe a set of possible worlds.⁴ But I am skeptical that any such a general functional characterization can be provided for the case at issue. At any rate, no Intellectualist has given one so far.⁵

Comparatively, the Fregean construal of *Intellectualism* looks much more appealing. On the Fregean construal of the view, practical modes of presentation are just Fregean senses—“practical senses”, as I will call them. As Fregean senses, practical modes of presentation are components of the content that is putatively known when one knows how to do something. Because, as Fregean senses, they are ways of determining their referents—i.e., ways of ϕ -ing—it is a subject’s grasping them that explains the complex dispositions to behavior that are associated with knowing how to ϕ . I do believe, and will show later in this essay, that appropriately developed, the view is fully explanatory.

⁴As a recent illustrative example of this kind of general functional characterization, consider the following quote from Yalcin(191):

For an agent x to be in a total state of belief whose content is given by the set of centered worlds P is (at least) for x to be disposed to act in ways that would tend to satisfy x ’s desires in worlds w such that $\langle w, x \rangle \in P$.

Another example of a similar kind of general functional characterization of what it is to believe a set of possibilities can be found in Stalnaker(173).

⁵I am not suggesting that the functional characterizations that are provided in the possible worlds tradition are adequate. I am just giving an illustration of how I think the dialectic stands. Because we have no antecedent sense of what believing a set of possibilities amounts to, in the possible worlds tradition people have felt the need to provide a general functional characterization of such a notion before appealing to it in providing an explanation of a subject’s dispositions to behavior. I am suggesting that *mutatis mutandis*, we would need something similar from a Russellian construal of *Intellectualism about know how*: Because we have no antecedent sense of what it means to know a proposition under a practical mode of presentation, for such a mental state, so characterized, to play any explanatory role in an explanation of a subject’s dispositions to behavior, what would be needed is an antecedent general functional characterization of it.

Moreover, a Fregean construal of *Intellectualism* does not lend itself to the objection that the view is not really Intellectualist but only a verbal variant of a mixed view. For on this view, know how is knowledge of a proposition—full stop. It is just that the proposition is a practical proposition—a proposition that has as constituent a practical sense.

Finally, the Fregean construal does vindicate a unitary picture of knowledge. On the Fregean construal of *Intellectualism about know how*, there is just one kind of knowledge state—knowledge of propositions. It is just that propositions come in a larger variety than we originally thought, encompassing practical as well as non-practical propositions.

Of course, a Fregean construal of *Intellectualism* needs to be substantiated by an account of what *kinds* of entities practical senses are. Only with this account on the table, can the explanatory power of the view be assessed. Moreover, not *any* view of the nature of practical senses will do the work. In particular, practical senses had better come out to be the kind of things that can enter as constituents of a proposition.

This last point is important to assess an exchange between Stalnaker(174) and Stanley(176) on the nature of practical modes of presentation. In his recent book, Stanley explicitly commits himself to taking modes of presentation, whether practical or not, to be *part* of the content of propositional attitudes (Stanley(176), chapter 3, especially p. 85 and following).⁶ So Stanley here explicitly advocates a Fregean

⁶Here is a quote where Stanley(176) commits himself to a Fregean construal of first-person ways of thinking (chapter 3, pp. 85-86):

Take two cognitive relations R_i and R_j between persons, objects, and times—intuitively such a relation states the conditions under which one is thinking of that object in a certain way at that time. R_i and R_j determine the same way of thinking of a given object o at a time t if and only if $\forall x(R_i(x, o, t) \leftrightarrow R_j(x, o, t))$. If we replace R_i and R_j by Evans' R_1 [where R_1 is a relation that can be satisfied by a triple S, S' and t if and only if S thinks of S' at t and $S=S'$], we obtain a contextual definition of certain entities, first-person ways of thinking. It

construal of the Intellectualist view. His aim is to dispel the worry that modes of presentation, construed along a Fregean line, are mysterious and obscure creatures. Stanley takes as a case-study the case of first-person modes of presentation and proposes a view of their nature (Stanley(176), chapter 3, pp. 85-86). On his proposal, which echoes Evans'(48), a first-person mode of presentation is a relation between a subject and himself: a relation that holds between a subject and himself just in case *one thinks of himself as oneself*. Stanley(176) seems happy to extend his proposal to the case of practical modes of presentation.⁷ On this construal, a practical mode of presentation is a relation that holds between a subject and a way of ϕ -ing just in case *the subject thinks of that way practically* (Stanley(176), pp. 122-125).

Although Stanley's Fregean proposal of what modes of presentation are may succeed in dispelling the worries that I argued afflict the Russellian construal of *Intellectualism about know how*, his positive story exposes his form of *Intellectualism to*

follows from the nature of R_1 that one can only stand in that relation to that object at a time if that object is oneself—i.e., $\forall x \forall y (R_1(x, y, t) \rightarrow x=y)$. The fact that certain thoughts contain these first-person ways of thinking as constituents is what makes them first-person thoughts.

The framework I have described yields a fully satisfactory account of Lewis's two gods. On a Fregean account of propositions, Lewis's two gods do not know every proposition true at their world. Manna does not know the proposition he may express to himself by "I throw down manna", and Thunderbolt does not know the proposition he may express to himself by "I thrown down thunderbolts". The true proposition Manna does not know involves a first-person way of thinking of Manna, and the true proposition that Thunderbolt does not know involves a first-person way of thinking of Manna and Thunderbolt. But the fact that these propositions contain first-person ways of thinking of Manna and Thunderbolt does not preclude them from being propositions true at their world.

⁷At p. 124, Stanley draws an analogy between "first-person ways of thinking" and "practical ways of thinking", one that suggests he would endorse extending his account of first-person modes of presentation to the case of practical modes:

To think of an object in a first-person way is for that object to occupy a certain functional role—to be something towards which first-person dispositions are directed. Similarly, explaining what it is to think of a way of doing something in a practical way is, as Peacocke makes vivid, a matter of spelling out the distinctive practical functional role that way occupies in the mental life of the speaker.

a different charge—i.e., the charge of appealing to a kind of entity that are *unfit* to be components of propositions. It is precisely along these lines that Stalnaker(174) has recently objected to the appeal to practical modes of presentation, if they are construed as on Stanley’s view:

The problem with modes of presentation is that the equivocation persists, and I think it persists in Stanley’s characterization of them. The equivocation is between a constituent of propositional content on the one hand, and a relation between a thinker and the content of his or her thought on the other. Stanley is explicit in saying that modes of presentation, as he understands them, are “ways of hypostasizing our relations to objects in virtue of which we can think about them.” But these “ways” are also constituents of the propositions themselves (and since facts are true propositions, according to Stanley, constituents of facts). I do not think anything can play both of these roles, since it is part of the idea of a proposition (an idea that was important to Frege, and that is important to Stanley’s general thesis) that propositions, or Thoughts, be characterizations of the way the world is, characterizations that are intelligible independently of thinkers that think the Thoughts, or speakers who assert them. It seems particularly implausible to hold that a fact has as constituents, relations to thinkers and speakers who think and talk about those facts.

Stalnaker’s remark is important. Stanley(176)’s practical modes of presentation are relations that hold between a way of ϕ -ing and a subject just in case the subject thinks of that way practically. So practical modes of presentation are defined in terms of the thinkers that think of a way of ϕ -ing in a certain mode. Hence, on Stanley’s view, a practical mode of presentation of a way of ϕ -ing does not exist nor is it intelligible *independently* of the particular thinkers who think of that way in that particular mode. As such, practical modes of presentation are not legitimate constituents of propositions, if propositions are characterizations of the way the world is that are to exist and to be intelligible independently of the particular thinkers who think them.

So Stalnaker’s challenge to a Fregean construal of *Intellectualism about know how* is a sound one. An advocate of *Intellectualism about know how* ought to respond to it. Responding to Stalnaker does not require abandoning a Fregean construal of *Intellectualism*. Stalnaker is not objecting to *every* Fregean construal of modes of

presentation. As Stalnaker(174) himself observes, the notion of sense that is most faithful to Frege’s understanding of senses is not similarly problematic:

Now while the equivocation I am worried about may have been implicit in Frege’s, most of what he said about sense is clear and unproblematic if the sense of a name is taken to be a property that identifies the referent. The sense of “Aristotle”, as used by one speaker, might be the property of being the last great philosopher of antiquity, in which case “Aristotle was fond of dogs” expresses the Thought that the last great philosopher of antiquity was fond of dogs—a proposition that might have been true if someone other than Aristotle had been the last great philosopher of antiquity, and had been fond of dogs. “Hesperus is Phosphorus” expresses a contingently true proposition, on this account, in virtue of the fact that it is contingent that the thing meeting the condition imposed by the sense of “Hesperus” is identical to the thing meeting the condition imposed by the sense of “Phosphorus”. On this understanding of sense, which I think is faithful to most of what Frege says, it is clear what a mode of presentation is, and this account of modes of presentation is compatible with the idea that a proposition is something independent of its relation to the thinkers who grasp or entertain them.

Fregean senses are ways of identifying their referent existing and intelligible independently of a particular subject and of a particular subject’s psychological life. On this understanding, it is clear what a mode of presentation is and how it can be part of the proposition that is graspable by a subject. The challenge for the Intellectualist is to provide an account of the nature of practical senses on which they are plausible *qua* Fregean senses.

In the following, I develop a view of practical senses that meets the challenge raised by Stalnaker. On this view, practical senses are genuine Fregean senses, existing and intelligible independently of the subjects that grasp them. As such, they are legitimate constituents of propositions. I argue that my view of practical senses also overcomes other complaints that have been raised in the literature against *Intellectualism*’s appeal to practical modes of presentation, such as their *ad hocness*, the mysteriousness of their nature, and their dubious ontological status. In embracing a particular view of what practical senses and practical modes of presentation are, I

inevitably incur some commitments that I explore in the last part of this essay. But I argue that those commitments are not as burdensome as one may think at first.

2.3 From Functions to Algorithms and from Algorithms to Programs

Every know how can be thought of as a know how to solve some *task* or other. If one knows how to ϕ , one knows how to solve the task of ϕ -ing. What is a task?

In the simplest cases, a task can be thought of as a function from *inputs* to *outputs*. What kinds of inputs (and what kinds of outputs) will vary with the kind of task at issue. For instance, the task of *adding* ranges from ordered pairs of integers to integers, while the task of *counting* will have as input a set of objects and as output a natural number. In many of the other kinds of tasks which interest us, the inputs and outputs are more complex kinds of entities. The inputs and the outputs may be partial specifications of states of affairs—or situations. So, for instance, the task of *babysitting* will have as inputs situations that encompass some children.

Can tasks be plausibly modeled as functions, mapping inputs to *single* outputs? One may think not, as there seem to be tasks that may have more than one possible output, given the same input. Consider, for example, the task of *totally ordering a list of people alphabetically according to their last names*. Take as input a list that encompasses more than one person with the same last name. In this case, there will not be a single output to the task but at least two outputs.

Or consider yet another, perhaps better example. Consider the task of “the eight queens”. The task consists in positioning eight queens on the chessboard so that no two are on the same row, or on the same column or on the same diagonal. Given the same input—an empty chessboard and eight queens—it turns out there are as many as 92 solutions to the task (Floyd(50)). So some tasks may have different possible

outputs, even given the same input.

We can still cling to the idea of modeling tasks as functions. Generalizing to encompass these kinds of cases, we can think of a task as a function from inputs to *sets of outputs*—call such a set the “*outcome*”. Each output will correspond to a possible solution to the tasks, for a certain input. In the simplest cases, the outcomes are singleton with a solution. In other less simple cases, the outcomes will be sets of higher cardinality than one. Tasks are functions *from inputs to outcomes*.

If we think of tasks as functions in this fashion, it is very natural to think of ways of solving tasks as *algorithms*. If one knows a way of solving a task, one knows an algorithm to solve it. The contrast between functions and algorithms is familiar from Marr(118)’s Thi-level hypothesis. Although many other examples would make the basic point, for presentational purposes the following example is particularly helpful. Consider calculating the sequence of *Fibonacci* numbers, where each number in the sequence is the sum of the previous two:

0, 1, 1, 2, 3, 5, 8, 13, 21

Fibonacci’s function can be defined as follows:

$$fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n - 1) + fib(n - 2) & \text{otherwise} \end{cases}$$

This function translates very naturally into the following algorithm for calculating it—call it the **RECURSIVE METHOD**:

- if $n = 0$, then $fib(n) = 0$;
- if $n = 1$, then $fib(n) = 1$;
- else $fib(n) = fib(n-1) + fib(n-2)$.

Although the foregoing algorithm is very natural, given our definition of *Fibonacci's* function, it is not the most efficient method to calculate a *Fibonacci* number. It involves a lot of unnecessary calculating. To calculate $fib(n)$, for any n , it requires that we calculate $fib(m)$, for every $m < n$. So instead of the RECURSIVE METHOD given above, it is more efficient to use a *closed solution*. BINET'S FORMULA provides one:

$$fib(n) = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}}. \quad (2.1)$$

This example well illustrates the algorithms/function distinction. BINET'S FORMULA and RECURSIVE METHOD are uncontroversially different methods—for one method is widely more efficient than the other. But they are different methods to calculate the *same* function—i.e., the same mapping from natural numbers to *Fibonacci* numbers. So algorithms are ways of calculating a function—ways to calculate an output of the function given any input. Although strict identity conditions for them are hard to come by (Blass, Dershowitz, Gurevich(21), and Buss, Kechris, Pillay and Shore(27), and Moschovakis(123), (124)), algorithms can be individuated at least as finely as their efficiency in calculating a function.

If tasks can be thought of as functions, then ways of solving them can be thought of as algorithms. Algorithms are ways of finding an output of a function given any input. On this construal, a way of ϕ -ing for some task ϕ is a way of finding an output of the task given any input. Some such ways are *nondeterministic*: They do not always return the same output, given the same input. Some such ways are *probabilistic*: they return a certain output only with a certain likelihood. Finally, just as there may be more than one way of calculating a function, so there may be more than one way of solving a task.

If we think of tasks as functions and we think of ways of solving a task as algorithms, then it is very natural to think of *practical senses*—modes of presentation of

algorithms—on the model of *computer programs*. A program is standardly understood as a *representation* of an algorithm (Brookshear(26), Knuth(101), Knuth(100)). This distinction between algorithms and programs in terms of what it is represented and what represents is rather standard. Next, I argue that programs stand to algorithms as *Fregean senses* stand to their referents.⁸

2.4 Programs as senses

It is uncontroversial that the relation between programs and algorithms is a *representational* relation. I contend that a helpful way to think of this representational relation is to think of it on the model of the philosophically more familiar relation between *Fregean senses* and their *referents*. Not even Frege has ever provided a necessary and sufficient set of conditions for something to count as a sense (Frege(52), (53)). However, the following features are generally accepted to be *necessary* of Fregean senses, properly said:

1. Senses are ways of representing their referents;
2. Senses uniquely determine their referents;

⁸There may well be *two* different layers of a sense/referent distinction here. It is not uncommon to find people who think of the *algorithm/function* relation in terms of a sense/referent relation. See, for example, Girard(63). However, this way of thinking of the *algorithm/function* relation is not entirely beyond question. As traditionally thought of, senses are supposed to uniquely determine their referents. However, certain kinds of algorithms—such as *probabilistic algorithms*—do not seem to uniquely determine a function. I will not try to solve this difficult question here. The level I am interested in is the *program/algorithm* level—not the *algorithm/function* level. Here is my rationale for thinking that the sense/referent relation that is relevant for describing the propositional content of the know how is located at the *program/algorithm* level and not at the *algorithm/function* level: It is important to allow that one may be able think of an algorithm *under different senses* to account for cases such as the ski instructor (Stanley and Williamson(178), Bengson and Moffett(15), Bengson and Moffett(16), Bengson and Moffett(14)) who knows how to teach his athletes to perform complex ski stunts but does not know how to do them himself. He *must* know an algorithm for performing those ski stunts—at least he must know it under a mode of presentation that enables him to teach it to his athletes. So in order to describe the content of the ski instructor’s competence, it seems advantageous to think of algorithms as ways to perform actions that one can represent under different senses. I will discuss the ski instructor later in the text.

3. Senses stand to their referents in a many-one relation;
4. Senses are intermediaries between the mind and the world:
 - a) mind-independent
 - b) abstract—i.e., non-physical.

My goal in this section is to argue that programs rightly construed satisfy all the necessary conditions for standing in a sense/referent relation to algorithms.

That programs satisfy (1)-(3) is relatively uncontroversial. By definition, programs are representations of algorithms, so they satisfy (1). Frege thought of senses as ways of individuating (or determining) their referents. Just as senses uniquely determine their referents, similarly, programs uniquely determine the algorithms they compute, for there cannot be any difference in algorithms that is not reflected in a difference in the programs that compute them. So programs satisfy (2). Finally, just as senses stand to referents in a many to one relation, similarly, to the same algorithm, there will typically correspond several different programs. So programs satisfy (3).

The more controversial question is whether programs can be thought of intermediaries between a mind (a system) and the world. I take it that an answer to this question will depend on whether programs can be thought of as existing independently of the systems that run them (mind-independent) and on whether they can be thought of as abstract objects—non-physical objects.

The analogy with senses is implausible if programs are thought as essentially *physical* representations. Senses are *not* physical signs—they are *expressed* by physical signs. By contrast, programs, as standardly introduced, are. The most standard examples of programs are physical representations (Brookshear(26), Abelson&Sussman(1)). On this construal, a program is a physical way of representing a finite set of executable steps by defining each step in terms of the syntactically simplest expressions of the

language of the system together with the functional expressions of that language. In particular, often what computer scientists call “programs” are physical objects with a certain syntax (Abelson&Sussman(1)).

Although this way of thinking of programs is surely widespread, it is not the only to think of programs. Programs do not need to be thought of as physical objects. Semantics for programming languages have been proposed and of a variety of kinds. These semantics map a *program text*—the physical instantiation of a program—into mathematical objects of various complexity. My proposal is that we identify programs with whatever mathematical objects that are mapped to a program text by a suitable such semantics. Because, however, some such semantics are less fine-grained than others, not every such semantics will serve our purposes equally well.

An example of very coarse-grained semantics for programming languages are Scott-style semantics—also called *Denotational semantics* (Stoy(179), Scott(159), Gunter(72)). *Denotational semantics* map program texts into functions from inputs to outputs. To give the reader a sense of how coarse-grained this kind of semantics is, a program computing Fibonacci’s function would be mapped by a *Denotational semantics* into that very same Fibonacci’s function. So semantics of this kind are too coarse-grained for our purposes: By mapping programs into functions, they collapse the distinction between programs and algorithms and even the distinction between algorithms and functions.

However, not every semantics for programming languages shares this feature. Some semantics—also called *Operational semantics*—allow preserving not only the distinction between algorithms and functions but also the desired distinction between algorithms and programs (Plotkin(138), Plotkin(139), Hennessy(83)). *Operational semantics* map program texts into set of *transitions*—where a transition is a move from a configuration to a configuration, and a configuration is an ordered pair of abstract syntax and *data*. The set of transitions abstractly describes step by step the unfold-

ing of the computation resulting from some system or other running—i.e., evaluating *and* executing—the program text. Instead of thinking of a program as the program text that is being interpreted by those semantics, programs can be thought of as whatever those semantics map the program text into—the abstract representations that correspond to that set of transitions. Although it is a helpful convention to talk of programs as physical texts, programs do not need to be thought as such.

This completes my argument for the claim that programs can be profitably thought of as Fregean senses. If programs are identified with the semantic values assigned by operational-style semantics to program texts, then they clearly are abstract objects: They are just a set of transitions. And sets are the paradigmatic example of abstract objects. Those semantic values are *abstract* representations, as they are not physical texts. Finally, they are system-independent, as programs so conceived exist and are intelligible independently of the particular system that implements them. The interested reader is referred to the final appendix for the sketch of an operational semantics for a program text in a very simple programming language—one that follows closely Plotkin(138), Plotkin(139)-style operational semantics.

2.5 Practical Senses as Programs

The appeal to practical modes of presentation has exposed *Intellectualism about the know how* to several charges. But if we think of practical modes of presentation as practical senses, and if we think of practical senses on the model of programs, those charges simply do not apply.

The first objection is that practical modes of presentation are *sui generis* Fregean senses. At the outset, I considered one way of pressing such an objection: Stalnaker objected to Stanley’s way of characterizing practical modes of presentation that, if practical senses are to be components of practical propositions, they had better be fit

for such a role. This objection afflicted Stanley's(176) positive view of what practical senses are, as Stanley thought of practical senses as relations between ways of ϕ -ing and subjects—i.e., relations that exist by virtue of the particular subjects that think them and are intelligible only in relation to those subjects. In the foregoing section, I have already argued that programs are very naturally thought of as Fregean senses. Programs are abstract objects that both exist independently of the systems that implement them, and they are intelligible independently of any particular such system. So Stalnaker's objection does not apply to my construal of practical modes of presentation.

Critics of practical modes of presentation have leveled several other criticisms to *Intellectualism's* appeal to them. Some accused them of being obscure and mysterious. Thus, for instance, Schiffer(156) complains that “We are not given the slightest clue as to what ‘practical modes of presentation’ are like” (p. 201). In the last two sections I argued that practical modes of presentation should be thought of as practical senses and that practical senses should be conceived on the model of programs. Because programs are entities that are susceptible of a transparent characterization, my particular view of practical senses is immune to the objection that practical modes of presentation are mysterious and obscure entities.

Another common objection to practical modes of presentation is that they are ontologically problematic. As I understand the objection, the concern is that, by appealing to practical modes of presentation, we may ontologically commit ourselves to entities of an *unprecedented* kind. But, if my positive proposal is correct, practical modes of presentation are not at all ontologically mysterious or problematic—they are program-like objects. And programs are entities to which we are ontologically committed any way. So thinking of practical modes of presentation as practical senses and of practical senses as programs does not impose any new ontological commitment upon us.

Others have claimed that the appeal to practical modes of presentation is *ad hoc* (Glick(66), Noe(125)). This objection will occupy me next. I take it that the objection is that, unless the appeal to practical modes of presentation helps cast light on some feature of the know how, the appeal to them is not fully motivated by the Intellectualist. But, as I will argue, neither does this charge apply to practical senses, as I am thinking of them. In particular, by appealing to practical senses in our characterization of the content of the know how, we can explain two striking features of the know how. If I am right, far from its being *ad hoc*, we should welcome the appeal to practical senses in our characterization of the know how.

2.6 The Explanatory Power of Practical Senses

Practical senses should be thought of along the model of programs. A program is a representation of an algorithm. Of course, not every representation of an algorithm is a program. An algorithm can be represented by a variety of means. Consider Binet's formula for calculating Fibonacci's function. Such a formula can be referred to through a variety of means, such as *The Binet's formula*, or *The linear formula for calculating Fibonacci number*, or *The formula that was given at page n of the paper "Practical Senses"*, or other related kinds of representations. None of these ways of representing *The Binet's formula* is a program. Programs are ways of representing algorithms with *distinctive features*.

Two important features stand out. Firstly, programs are representations with a distinctive *structure*. Secondly, programs are representations of algorithms only for systems that *have a certain set of abilities*. In the next two sections, I explain these two distinctive features of programs *qua* representations of algorithms. I claim that, if we think of practical senses as inheriting these two distinctive features of programs, we are in position to account for two important aspects of know how: *its*

open-endedness (Section 2.6.1) and *its connection with abilities* (Section 2.6.2).

2.6.1 Know how and open-endedness

The most precious know how we can possess is *open-ended*—or productive. If one knows how to order lists, one knows how to order lists of arbitrary finite length and according to a variety of ordering relations. If one knows how to parse English sentences, one knows how to parse English sentences of arbitrary finite length and made out of a great variety of English words. If one knows how to babysit, one knows how to deal with a variety of different children and a variety of different annoying parents. And so on. The more open-ended one's know how is, the more precious it is—for the less we have got left to learn when dealing with new instances of the task.

Every theory of the know how has to explain its open-endedness. An explanation of the open-endedness of the know how has to give an account of how it is possible and not simply predict it. For example, suppose one identifies knowing how to order lists with *one's disposition to order lists correctly*. Unless a lot is said about what grounds such a disposition and how it is acquired, such a view does not really explain the productivity of one's know how, although it does predict it.

Practical senses, as I am thinking of them, provide a very natural and elegant explanation of the open-endedness of the know how. To appreciate this point, however, we need to look more closely into the structure of programs.

Programs are representations with a distinctive structure. The notion of a program that appears in computer science textbooks (for example, see Downey(43)) does not always stress this distinctive feature. A program is often defined as *a way of breaking down, in a language, a task ϕ into an ordered set of executable instructions*, each of which defined in terms of the smallest instructions of that program's language. Assuming that to every such way of breaking down a task there corresponds one and only one list of instructions, a program for a system is often identified with *that* list of

instructions. Fodor(51) himself thinks of a program as a list of most basic linguistic instructions that a system can carry out:

So, given a list of elementary instructions for producing a type of behavior, we need no further instructions for carrying out the instructions on the list and no little men to supervise their execution. The nervous system carries out its complex operations in some way or other (i.e., by performing one or another sequence of elementary operations). [...] If every operation of the nervous system is identical with some sequence of elementary operations, we get around proliferating little men by constraining a completed psychological theory to be written in sequences of elementary instructions (or, of course, in abbreviations of such sequences).

Pace Fodor(51), as defined, this notion of a program is problematic, for it gives rise to a *distinctive* kind of regress. It is at least conceivable that a system may be able to carry out each basic instruction in the list—hence the whole program, in this sense of program—without being able to carry out any of the combinations of the single instructions, hence without being able to carry out the whole task. In this circumstance, for a system to be able to carry out the whole task, there must be a further program that specifies how to carry out every combination of the single basic instructions (each of which the system can perform by assumption) in terms of instructions that the system can also perform. But there is no guarantee that the new program will be one that the system can carry out: For although every one of its single basic instructions is defined in terms of instructions that the system can perform, that is not necessarily so for any combinations of those instructions. Hence, there must be a further program specifying how to perform any combination of the instructions of that program in terms of instructions that the system can perform. And so on. A possibly infinite proliferation of programs has started.⁹

⁹It is worth noting that this kind of regress is avoided by adding to programming languages a bit of extra functional expressions—i.e., a couple of general commands, such as *sequence*—i.e., ‘;’:

- *sequence*: evaluate “A; B;” by evaluating “A;” then evaluating “B;”

The regress is avoided if we think of programs as representations with *parts*. Start with a way of breaking down a complex task into a list of linguistic instructions. Let the list be \mathcal{L} . A program to ϕ is the collection of all parts that one can obtain from \mathcal{L} , where a “part” is, in first approximation, any combination of successive steps in \mathcal{L} . This way of thinking of programs overcomes the regress problem above. Suppose (simplifying a bit) we start with a list of three steps: step 1, step 2 and step 3. To get a program for a system out of that list, any combinations of successive steps in that list has to be defined in terms of operations the system can perform. That gives us the following collection of parts: [step 1], [step 2], [step 3], [step 1; step 2], [step 2; step 3], [step 1; step 2, step 3]. If each such part is defined in terms of operations that the system can perform, such collection is a program for that system to ϕ . This notion of a program is such that if a representation of a way of breaking down a complex task into parts is a program for a system in this sense, then the system *must* be able to carry out the whole task. No additional other program is needed to specify how any combination of the steps is to be performed by the system.¹⁰

We should think of a program as a representation of a way of breaking down a task into executable parts. A program is a program *for a system* only if it represents a way of breaking down a task into parts that that system can carry out. Such representation has itself a structure that mirrors the structure of a way of breaking down a task into parts.

What can be said about the parts of a program? Some parts cannot be further

As I show in the Final Appendix, one can then define a part of a program text by recursively defining commands in that programming language and by saying that, if a command occurs in the program text, then it is part of the program and that nothing else is part of the program. The interested reader is referred to the Appendix 2.8, where I provide a definition of commands for a very simple programming language (Appendix Section 2.8.1) and then define the notion of a part of a program (Appendix Section 2.8.2).

¹⁰This definition of parts of a program is a simplification because, in the case of *parallel programs*, parts of a program may not be successive. For a more rigorous definition of parts of a program text, see Appendix Section 2.8.2.

broken down into smaller parts: They are basic—i.e., structurally simple. Others are not similarly basic, nor are they reducible to a list of basic parts. The distinction between basic parts and parts that are not reducible to basic parts is important and will play an important role in my explanation of open-endedness of the know how. It is helpful to consider two illustrative examples.

Example 1 Suppose that we want to write a program to prepare pasta and fagioli. This task can be broken down into two simpler tasks, that of preparing the fagioli—i.e., the beans—and that of preparing the pasta. But the sub-task of preparing the beans is itself complex. Among other things, it requires to unshelling the beans. And unshelling the beans itself requires one to obtain a basket of unshelled beans and an empty bowl in which to put them; it requires one to taking a bean from the basket, breaking open the bean pod, dumping the bean from the pod into the bowl, and discarding the pod. Finally, it requires performing these operations in this order as long as there are beans in the basket. Only by addressing each sub-task, will one be in position to prepare the beans for pasta and fagioli.

Note that some of these sub-tasks do not further sub-divide into sub-tasks—i.e., they are basic in so far as their internal structure does not matter for the execution of the complex task. Thus, for instance, it does not matter whether I take each bean from the basket with bare hands or gloves, or if I do it while repeatedly hitting my head with a stick. Other sub-tasks are not simple in the same sense: Thus, for instance, the task of repeatedly taking the bean, peeling it, and so on, until the basket is empty must involve the execution of those simpler tasks in a certain sequence and until certain condition obtains—i.e., until the basket is empty. Because we cannot specify in advance how many basic operations will be needed to perform the task of unshelling all the beans, we need operations that are not reducible to basic operations or even to lists of basic operations.

Because the tasks we are interested in describing are *open-ended* in this sense, the distinction between parts of a procedure and basic instructions (or lists of basic instructions) is important. Thus, for instance, what cognitive scientists call “loops” are *not* basic operations, in that they have a *distinctive* internal structure. Moreover, they are not equivalent to any list of basic operations. Thus for instance, consider the loop that tells us to perform a certain series of basic operations as long as there are beans in the bowl:

- As long as there are unshelled beans in the basket continue to execute the following steps:
 1. Take a bean from the basket.
 2. Break open the bean pod.
 3. Dump the bean from the pod into the bowl.
 4. Discard the pod.

This loop is not equivalent to any given number of repetitions of the sequence, because we may not know how many unshelled beans there will be in the basket. Because we cannot specify in advance how many basic operations are required for the execution of a task, we need loops and recursive structures.¹¹

Example 2 Another—non-culinary—example can be used to illustrate the same general idea. Suppose one wants to sort a list of names into alphabetical order. This task can be easily accomplished by sorting every subset of the list into alphabetical order. Hence, on this way of sorting a list, the task of sorting the whole list can be broken down into the tasks of sorting its parts. Each such a task can be accomplished

¹¹On the structure of loops and recursive steps, Brookshear(26) provides an excellent introduction. See also Downey(43) for an helpful overview.

by executing certain further tasks, like checking that the relevant part of the list is sorted, and taking certain actions in case it is not, such as ordering it. Those tasks—checking whether a certain part of the list is sorted and ordering it in case it is not—are sub-tasks of the task of sorting that part of the list. By performing the tasks of sorting a part of the list, for each part of the list, one can perform the complex task of sorting a whole list of names into alphabetical order. This complex task has as subtasks the tasks of sorting a part of the list as well as the tasks that are sub-tasks of those tasks.

If we are looking for a program that allows a system to sort any list of names into alphabetical order, then the program cannot just be made of parts that are reducible to a list of smallest parts. For each part of the list, the program will tell us how to order it alphabetically. So it will tell us to keep ordering it *until* that part of the list is alphabetically ordered. This command is again not equivalent to any number of repetitions of smallest operations, if it has to be sufficiently general to apply to parts of lists of arbitrary finite length. If a program is a program for a system to execute a task that is open ended in this sense, then that program will of necessity encompass parts that are not reducible to any list of basic parts.

A program is a way of representing a breaking down of a task into executable parts. A program is a program for a system to ϕ just in case it represents a breaking down of the task of ϕ -ing into parts that the system can carry out. Some of these parts are reducible to smallest parts—i.e., the basic operations for the system. Other parts—such as loops, recursive structures and conditional statements—are not so reducible. The presence of these kinds of parts is required to perform tasks that are open-ended—tasks for which we cannot tell in advance how many basic steps will be required to perform them. Because tasks of this variety are among those we can say a system knows how to perform, the appeal to loops and recursive structures is essential in defining programs for executing those tasks.

We are now in position to appreciate how, by thinking of practical senses on the model of programs, we become able explain the open-endedness of the know how. A practical answer to the question *How to ϕ* has as constituent a practical sense. So, by coming to know a practical answer to the question *How to ϕ* , we come to represent an open-ended task through a practical sense. Like programs, practical senses are ways of representing a breaking down of a complex task into executable parts. Among those parts there are loops, recursive structures, and conditional statements. So by coming to know a practical answer, a subject represents an open-ended task through representing parts of that task such as loops, conditionals, and recursive structures—precisely those parts that account for its open-endedness. It is hard to envision a better explanation for the open-endedness of the know how.

2.6.2 Know How and Abilities

The open-endedness of the know how is not the only aspect of the know how that the appeal to practical senses can help explain. Know how has a close connection to *abilities*. If we think of practical senses as programs, we can explain this important feature of the know how.

Intellectualists tend to deny know how's connection with abilities (Ginet(62), Stanley and Williamson(178), Snowdon(163)). But their grounds for denying it are rather slim.

It is true that one may know how to ϕ without being able to ϕ , for one may be physically unable to ϕ , like Ginet(62)'s son, who knows how to lift weights without having the necessary strength; or like Hannah the pianist who has lost both arms in a car accident (Ginet(62), Bengson and Moffett(16)). However, these examples do not show that know how does not have the relevant connection with abilities. Ability talk is notoriously context-dependent, as Lewis(113) has taught us. As fleshed out by Lewis ((113), p. 77), the context sensitivity of the locution '*S can ϕ* ' has to do

with the range of facts S 's ϕ -ing is taken to be *compossible* with. When we think of the pianist's mental facts, and them only, the pianist Hannah counts as being able to play the piano, for her playing the piano is compossible with her cognitive and mental capacities. But when we think of facts about the pianist that are not necessarily mental, such as her just having lost her arms, she does not count as having the ability to ϕ , for her ϕ -ing is not compossible with that larger set of facts. S 's knowing how to ϕ does not entail S 's ability to ϕ *tout court*—i.e., ability that requires the compossibility of S 's ϕ -ing with *all* the facts. Know how entails *mental* ability, for which only *compossibility with the mental facts* is required.¹²

It is also true that one may know how to ϕ , in the sense of knowing *some* answer to the question *How to ϕ* , without being mentally or physically able to ϕ , like the ski instructor who can teach how to perform ski stunts but cannot do them himself.¹³ But again, this example does not provide evidence against the entailment.¹⁴ There is an important sense in which the ski instructor does not know how to perform ski stunts himself.¹⁵ The ski instructor may know an answer to the question *how to perform the ski stunt* in the sense that he knows an algorithm to ϕ under a description that enables him to teach it. But not every description is a *practical sense*. The ski instructor does not have the *relevant* ability to ϕ , because he does not know how to perform the ski stunts *in the relevant sense*.¹⁶

¹²This argument is developed in much greater detail in Pavese(130).

¹³The ski instructor case is discussed, for example, by Stanley and Williamson(178) and Bengson and Moffett(16).

¹⁴Glick(65) also argues that know how entails abilities, and he has a few more arguments for this conclusion. Glick(65) uses this entailment to argue against *Intellectualism about know how*. As it will be clear in the following, however, the entailment can be perfectly explained by my form of Intellectualism.

¹⁵As Stanley(176) also observes but explains in the different way from mine.

¹⁶On my view, the ski instructor does not represent that algorithm through a program; hence he does not know how to ϕ in the relevant sense. After all, it is not at all clear that a description that enables one to teach people to ϕ should also enable one to ϕ oneself.

Instead, it is extremely intuitive that know how to ϕ entails the ability to ϕ , in the sense of “ability” that has been clarified. If so, every view of know how has to explain such an entailment. Part of what it means to explain the entailment is to say what it is about know how possession that endows one with a certain set of mental abilities. If those abilities only go together with know how, we would want such an explanation to cast light on the nature of those abilities, too. Many views of know how may predict the entailment without *explaining* it in this stronger sense. Suppose one simply identifies knowing how to ϕ with the ability to ϕ .¹⁷ Unless lots is said about what an ability to ϕ is, how it is acquired, how basic and less basic abilities are related to each other, and so on, this view obviously predicts the connection of know how with abilities but does not really explain such connection. It just stipulates that it holds.

By appealing to practical senses and by thinking of practical senses on the model of programs, we can instead very elegantly explain the connection of know how with abilities.

Programs are an example of practical representations: they are ways of practical representing an algorithm for systems that *have a certain set of abilities*. For simplicity, let us assume that programs are linguistic representations. On this linguistic understanding of programs, a program for a system cashed out in a certain programming language represents each part of a task by defining it in terms of the most syntactically simple expressions of that language together with its functional expressions. Part of what it means for a system to “understand” the instructions as specified by the program is that the system can execute them *as* they are specified. If a representation of an algorithm is a program for a system, the system can execute every instruction as specified by the program. So if a system represents an algorithm

¹⁷Some people seem to endorse such as view; see for instance Glick(65).

through a program to ϕ , the system must possess a certain set of abilities. Hence, a program represents an algorithm for the system just in case the system has a certain set of abilities.

If we understand practical senses on the model of programs, we can say that a practical sense represents a way of ϕ -ing for a subject just in case the subject possesses a certain set of abilities. Putting this in slightly different but perhaps more intuitive terms, a practical sense can be “grasped” by a subject just in case the subject possesses a certain set of abilities. What abilities?

A couple of paragraphs above, I suggested that we should think of a program as *a representation with parts*. In particular, a program represents a way of breaking down a task into its executable parts and has as many parts as the parts of that task (according to that particular breaking down). The abilities that are relevant are the ones for executing every part of that task. On this model, a practical sense representing a way of ϕ -ing can be grasped by a subject just in case the subject possesses a certain set of abilities—i.e., the ability to execute every part of a task of ϕ -ing.

However, the ability to execute every part of a task of ϕ -ing (including the improper part) is not quite yet the ability to ϕ .¹⁸ To illustrate where they come apart, consider the following example. Given a recipe for cooking Indian curry, a well-trained cook may be able to execute every part of the recipe. The same cook may well be unable to cook Indian curry in the relevant sense, for example, if he does not even know of the existence of that recipe or of any other recipe for cooking Indian curry. After all, the following counterfactuals differ in their intuitive truth values:

¹⁸This is predicted also by views of actions such as in Goldman(67), according to which two descriptions such as ‘executing every part of a task of ϕ -ing’ and ‘ ϕ -ing’ denote different actions, which may, however, stand with each other in a *generation relation*. I discuss this view and its implications in Pavese(130).

- (1) If the cook were to try to cook Indian curry, he would succeed.
- (2) If the cook were to try to cook Indian curry by following a particular recipe for doing so, he would succeed.

(1) is false: because the cook does not know any recipe for cooking Indian curry, if he tried to cook Indian curry, he would fail. (2), on the other hand, is true: if the cook were to try to cook curry by following a particular recipe for doing so, he would succeed.¹⁹ In addition to the ability to perform every part of a task of ϕ -ing the cook must know an algorithm to cook the curry, in order for him to possess the ability to cook it, in the relevant sense.

If one comes to know a practical answer, one must both have the ability to perform every part of ϕ -ing—i.e., on my view, the precondition for grasping the relevant practical sense—and come to know an algorithm to ϕ through that practical sense. But, if one both has the ability to perform every part of ϕ -ing and also knows an algorithm for ϕ -ing, one must have the relevant ability to ϕ . But then, if one knows a practical answer (as specified) to the question *How to ϕ* , one will have the ability to ϕ , in the relevant sense. So, by appealing to practical senses and by thinking of them as programs, the Intellectualist can both predict and explain that know how entails ability possession, in the relevant sense.

2.7 The Possibility of non-Algorithmic Know how

According to standard presentations of *Intellectualism about know how*, knowing how to ϕ for some task ϕ is a matter of knowing a proposition that answers the question *How to ϕ* under a practical mode of presentation. Against standard presentations

¹⁹I develop in much more detail my views of abilities in Pavese(130).

of *Intellectualism about know how*, I suggested we should think of practical modes of presentation as constituents of the relevant proposition—as genuine Fregean practical senses. I recommended a positive picture about what practical senses are. On my picture, practical senses should be thought of on the model of programs. Like a program, a practical sense represents a way of ϕ -ing. A proposition answering the question *How to ϕ* that has a constituent a practical sense is a *practical proposition*. Know how is knowledge of a practical proposition.

My form of *Intellectualism about know how* appeals to practical modes of presentation, exactly like standard presentations of the view. But the way I conceive of those practical modes of presentation is not similarly problematic. My view has plenty to say about the nature of those modes of presentation. Moreover, the appeal to them is not *ad hoc*, because by thinking of them on the model of programs, it is possible to cast light on features of know how that would otherwise be hard to explain, such as its open-endedness as well as its connections with abilities. The present view is a big improvement over standard presentations of *Intellectualism about know how*.

Or so it would seem. In response, one may object that I dispelled the charge of mysteriousness only at the cost of incurring substantial and controversial commitments about the nature of know how. It is time to assess the nature of such commitments.

First of all, one may worry that my view is committed to *Computationalism about the mind*—to the view that the mind is like a computer. Although I do myself find *Computationalism* very attractive, I am not committed to such a strong view. For one thing, the scope of my claim is much narrower than that of *Computationalism*. *Computationalism* is usually conceived of as a general view about mental processes—i.e., the view according which every mental process is a kind of computation. The analogy with programs I am recommending only holds for *tasks* that one can know how to perform. Arguably, not every mental process is a task. For one thing, tasks

are intentional, while mental processes do not need to be.²⁰ To the extent to which not every mental process is a task one can know how to perform, the scope of my claim is narrower than that of *Computationalism*.

Moreover, even within this scope, the extent of my commitment to *Computationalism* should not be overstated. Nothing in my view commits me to thinking of the mind as having the same *architecture* as computers. And nothing in my view commits me to subscribing to a certain view of which level of operations is represented by practical senses, or to a view of what the basic operations are. So many features by which we tend to identify programs are abstracted away on my account.

Second of all, the notion of a program with which I am helping myself is a very *general* notion. A program is essentially a way of representing an algorithm—i.e., a way of representing a breaking down of a complex task into executable parts. Some parts are not reducible to simpler parts. Others, instead, are. That is literally all there is to the notion of a program with which I am helping myself. Trivially, such a way of representing an algorithm must be a representation. But we can be fairly neutral about the nature of such a representation. Though computer programs are essentially linguistic and although, for convenience (in this essay and in the final Appendix= I myself spoke of programs as linguistic entities, there is nothing for which I have used the notion of programs that requires programs to be linguistic.

The real commitment is to the following claim; According to my view, there cannot be tasks that we know how to perform that are not algorithmic—i.e., that are not solvable by an algorithm. That follows from how I characterized *tasks* and *ways of performing them*. A task is a function from inputs to outcomes. And a way

²⁰The intentionality of tasks that one can know how to perform is a very widespread assumption of the current debate on the know how, shared by Intellectualists and Anti-Intellectualists alike. For example, Ryle(151) says that the tasks that we know how to perform are those that are done “on purpose” (pp. 42-50). Along the exact same lines, see Stanley and Williamson(178), Setiya(160), Glick(65), Bengson and Moffett(15), Bengson and Moffett(16) and Bengson and Moffett(14).

of performing a task is a way of calculating the function for any input. So, if there is a way of performing the task, the task has to be *decidable*—i.e., there has to be an algorithm for solving it. If one knows how to perform a task, there must be a way of performing a task; hence, on my view, there must be an algorithm for solving that task. Hence, we can know how to perform only decidable—i.e., algorithmic—tasks. But the claim that we can only know how to perform algorithmic tasks may be controversial (Copeland(36)).

In some cases, however, the suspicion towards that claim derives from an arbitrarily *narrow* understanding of what algorithms are. So for example, many have objected to me that Paul Churchland(34) has conclusively shown that some tasks, such as *facial recognition*, are not algorithmic. But this interpretation of Churchland relies on an unjustifiably narrow understanding of “algorithmic”. Churchland(34) has indeed brought to the attention of philosophers the existence of mental processes that appear to require a *parallel* kind of computation—i.e., a *non-serial* form of computation. But, on a broader understanding of algorithms, *parallel algorithms* are no less algorithms than *serial algorithms*. On this broader understanding of algorithm, for a way of solving a task to be an algorithm, it suffices that there be a program that computes it. And there are parallel programs, just as there are serial programs.²¹ Nor is the distinction between those kinds of algorithms at all mysterious. The difference between *parallel algorithms* and *serial algorithms* can be cashed out in the same terms by which I described programs in this essay. Recall that programs should be thought of as having *parts*. Parallel programs are ones *distinct parts* of which can be run simultaneously, as opposed to successively. So *per se* the existence of “parallelly-algorithmic” tasks is no evidence for the existence of non-algorithmic

²¹For just some examples of parallel programs, see Akl(2), Akl and Lyons(3) and Almasi and Gottlieb(4).

know how—know how to perform non-algorithmic tasks.

The question is whether there could be non-algorithmic tasks—tasks for which there cannot be, on pain of contradiction, any algorithm to solve them—such that one could nonetheless know how to solve. Such a possibility cannot be ruled out *a priori*. However, there is no evidence that there is such non-algorithmic know how. Nor is it clear that there could be such evidence were the possibility of non-algorithmic know how to be actual. Finally, my view delivers all those kinds of *related* know how for which there is indeed evidence and which it is legitimate to insist that people actually have. So it is not at all obvious that the possibility of non-algorithmic know how is a possibility we should care about when theorizing about the know how. Or so I am going to argue.

The paradigm of a non-algorithmic task is the *Halting Problem*. In my discussion, I will take the *Halting Problem* as illustrative example, but what I will say extends to other tasks that have been proved to be undecidable on the basis of the undecidability of the *Halting Problem*, for they share the relevant features.

The *Halting Problem* is a problem of deciding whether, given *any* arbitrary program and *any* input, the program will halt or will continue running forever. That the *Halting Problem* is non-algorithmic has been proved to everybody's satisfaction by Alan Turing(180) in the 1930s.²² Suppose one nonetheless could know how to solve the *Halting Problem*. How could we ever know whether such a possibility is actual?

One could not prove that one does know how to solve it, as proving it would amount to finding an algorithm to solve the *Halting Problem*, and by assumption there is no such algorithm to be found. So there would be no way of proving that one has solved the *Halting Problem*, hence no way of proving that one does know

²²For philosophical discussions of the *Halting Problem*, see Copeland(36), Davis(39), (40), (41) and Penrose(133).

how to solve it. Nor could we ascertain that one has solved it by *induction*. By induction, one could presumably only establish that one can decide whether programs of a certain kind will halt for inputs of a certain kind, whereas solving the *Halting Problem* requires that one can so decide in full generality. Nor could we know that one has solved it by *inference to the best explanation*. If one could reliably predict whether a program halts or does not for some inputs, the best explanation for that would *not* be that one has solved the *Halting Problem* in all of its generality. There is a much more conservative and simpler explanation for that: that one has found heuristics to decide whether a particular program—or a program of a certain kind and for certain kinds of inputs—halts or does not. One may be good at finding *heuristics* to decide whether a particular program halts or does not without having solved the *Halting Problem*—which requires to find a general algorithm that will work in every possible case. So even by inference to the best explanation, we would not come to know whether one has solved the *Halting Problem*. So, not only is there no evidence for the possibility of a non-algorithmic know how; it is not even clear there could be such evidence, were that possibility to be actual.

One may reasonably ask: Could not ascriptions such as (3) and (4) ever be true?

- (3) Mark knows how to solve the *Halting Problem*.
- (4) Mark knows how to decide whether a certain program halts for a certain input.

In response, it should be observed that one can predict that (3) and (4) could be true, without having to allow for a non-algorithmic know how. In general, when we say that somebody knows how to ϕ an x , we do not require that they be able to ϕ absolutely any x under absolutely any circumstances. As I argue in Pavese(129), the interrogatives embedded in know how ascriptions quantify over situations, but

the relevant quantifier is *not* universal. Rather, it is more similar to a *generic* kind of quantification. So, to make ascriptions such as (4) true, we do not require that the person implement a program that determines an algorithm that takes any old x under any circumstances whatsoever to a successful ϕ -ing. We just require that such a person implement a program that takes enough—or the relevant (kinds of) x s in enough or the relevant (kinds of) circumstances—to successful ϕ -ings. There are indeed algorithms for determining whether programs *of a certain kind* will halt for inputs *of a certain kind* (Copeland(36) and Davis(39), (40), (41)). So, if one can be correctly said to know how to decide whether a certain program halts for a certain kind of inputs, my view can predict the truth of such an ascription. There is no need to allow for the kind of super-ambitious, non-algorithmic know how that would be involved in knowing how to solve a highly general task such as the *Halting Problem*.

To be sure, the possibility of non-algorithmic know how cannot be ruled out a priori. But it is a serious methodological question whether we should care about such a possibility when theorizing about know how. Even if such a possibility were actual, we would not be in position to know that it is. Moreover, we do not need to allow for such a super-ambitious kind of know how in order to make true (or possibly true) an ascription such as (4). My view delivers all those kinds of know how which are related to knowing how to solve *Halting Problem* for which there is indeed evidence and that it is legitimate to insist that people actually have and that suffice for making ordinary ascriptions of know how true.

The possibility of non-algorithmic know how cannot be ruled out a priori. That is, however, compatible with it being entirely negligible when theorizing about know how. I have argued there are indeed good reasons to neglect it.

2.8 Appendix

2.8.1 A very simple programming language \mathcal{L}

Basic Syntactic Sets

- **Variables**= $\{x_1, \dots, x_n\}$
- **Constant Numbers**= $\{m_0, \dots, m_n\}$
- **Constant Truth Values**= $\{tt, ff\}$

In the following, v is a metavariable over **Variables**; m, n are metavariables over **Numbers**; t is a metavariable over **Truth Values**. a is a generic metavariable over strings of \mathcal{L} .

Derived Syntactic Sets e, e' are metavariable over Expressions—i.e., **Exp**.

Expressions

- v is an expression.
- m, n are expressions.
- tt, ff are expressions.
- if $a: e+e'$, then a is an expression.
- if $a: e \times e'$, then a is an expression.
- if $a: e=n$, then a is an expression.
- Nothing else is an expression.

b, b' are a metavariables over Boolean Expressions—i.e., **Bexp**.

Boolean Expressions

- if $a: b=t$, then a is a Boolean expression.
- if $a: e=e'$, then a is a Boolean expression.
- if $a: b \text{ or } b'$, then a is a Boolean expression.
- if $a: \neg b$, then a is a Boolean expression.
- Nothing else is a Boolean Expression.

c is a metavariable over commands—i.e., **Com**.

Commands

- **IF THEN**= If $a: \text{If } b \text{ then } c, \text{ else } c', ,$ then a is a command.
- **WHILE DO**= if $a: \text{While } b \text{ do } c,$ then a is a command.
- **NIL**= If $a: c=\text{NIL},$ then a is a command.
- **ASSIGNMENT**= If $a: v=e,$ then a is a command.
- **SEQUENCE**= If $a: c; c',$ then a is a command.
- Nothing else is a command.

2.8.2 Parts of a Program Text $\mathcal{P}_{\mathcal{L}}$

- If a command c of \mathcal{L} occurs in $\mathcal{P}_{\mathcal{L}}$, then c is part of $\mathcal{P}_{\mathcal{L}}$.
- Nothing else is part of $\mathcal{P}_{\mathcal{L}}$.

2.8.3 An operational semantics for \mathcal{L}

Operational semantics thinks of programs dynamically, as a set of transitions from configurations to other configurations. Intuitively, such a set of transitions describes the behavior of a system that is running a program text. A configuration is a set of syntactic objects—the program text and the data of a system.

The set of programs that can be specified for a system is a Transition system: a structure $\langle \Gamma, \rightarrow \rangle$, where Γ is a set of configurations γ , and \rightarrow expresses the transition relation. $\rightarrow \subseteq \Gamma \times \Gamma$. ' $\gamma \rightarrow \gamma'$ ' is read as “there is a transition from configuration γ to configuration γ' ”. A configuration will be indicated by $\gamma = \langle \alpha, \sigma \rangle$. We can think of σ as an assignment function to variables. The domain of the assignment function is the set of natural numbers. m, n are metavariable over such a domain.

To specify such a set of transitions corresponding to a program, we first need to specify rules of interpretations for the basic syntactic expressions of the programming language, for the derived expressions, as well as for the boolean expressions and the commands.

The Semantics of Expressions Any constant (whether a number constant, m, n , or a truth value constant, tt, ff , is evaluated to itself:

CONSTANT $\langle m, \sigma \rangle \rightarrow \langle m, \sigma \rangle$

A variable gets as a semantic value the value assigned to it by the store σ :

VARIABLE $\langle v, \sigma \rangle \rightarrow \langle \sigma(v), \sigma \rangle$

Consider sums and multiplications. To evaluate $e_0 + e_1$, evaluate e_0 obtaining m_0 . Evaluate e_1 obtaining m_1 . Add m_0 and m_1 to obtain m_2 . This finishes the evaluation:

$$\begin{aligned}
\text{SUM (1)} & \frac{\langle e_0, \sigma \rangle \rightarrow \langle e'_0, \sigma \rangle}{\langle e_0 + e_1, \sigma \rangle \rightarrow \langle e'_0 + e_1, \sigma \rangle} \\
\text{SUM (2)} & \frac{\langle e_1, \sigma \rangle \rightarrow \langle e'_1, \sigma \rangle}{\langle m + e_1, \sigma \rangle \rightarrow \langle m + e'_1, \sigma \rangle} \\
\text{SUM (3)} & \langle m + m', \sigma \rangle \rightarrow \langle n, \sigma \rangle \text{ if } n = m + m'.
\end{aligned}$$

To evaluate $e_0 \times e_1$, evaluate e_0 obtaining m_0 . Evaluate m_1 obtaining m_1 . Multiply m_0 by m_1 to obtain m_2 . This finishes the evaluation:

$$\begin{aligned}
\text{TIMES (1)} & \frac{\langle e_0, \sigma \rangle \rightarrow \langle e'_0, \sigma \rangle}{\langle e_0 \times e_1, \sigma \rangle \rightarrow \langle e'_0 \times e_1, \sigma \rangle} \\
\text{TIMES (2)} & \frac{\langle e_1, \sigma \rangle \rightarrow \langle e'_1, \sigma \rangle}{\langle m \times e_1, \sigma \rangle \rightarrow \langle m \times e'_1, \sigma \rangle} \\
\text{TIMES (3)} & \langle m \times m', \sigma \rangle \rightarrow \langle n, \sigma \rangle \text{ if } n = m \times m'.
\end{aligned}$$

The Semantics of Boolean Expressions Let us start with **or** .

$$\begin{aligned}
\text{or (1)} & \frac{\langle b_0, \sigma \rangle \rightarrow \langle b'_0, \sigma \rangle}{\langle b_0 \text{or} b_1, \sigma \rangle \rightarrow \langle b'_0 \text{or} b_1, \sigma \rangle} \\
\text{or (2)} & \frac{\langle b_1, \sigma \rangle \rightarrow \langle b'_1, \sigma \rangle}{\langle t \text{or} b_1, \sigma \rangle \rightarrow \langle t \text{or} b'_1, \sigma \rangle} \\
\text{or (3)} & t \text{ or } t' \rightarrow t'', \text{ where } t'' = t \text{ or } t'.
\end{aligned}$$

Next to equalities:

$$\begin{aligned}
& = (1) \frac{\langle e_0, \sigma \rangle \rightarrow \langle e'_0, \sigma \rangle}{\langle e_0 = e_1, \sigma \rangle \rightarrow \langle e'_0 = e_1, \sigma \rangle} \\
& = (2) \frac{\langle e_1, \sigma \rangle \rightarrow \langle e'_1, \sigma \rangle}{\langle m = e_1, \sigma \rangle \rightarrow \langle m = e'_1, \sigma \rangle} \\
& = (3) m = n \rightarrow t, \text{ where } t = tt \text{ if } m = n \text{ or } ff \text{ if it is not.}
\end{aligned}$$

Finally, the following two rules deal with negation:

$$\begin{aligned}
\neg (1) & \frac{\langle b, \sigma \rangle \rightarrow \langle b', \sigma \rangle}{\langle \neg b, \sigma \rangle \rightarrow \langle \neg b', \sigma \rangle} \\
\neg (2) & \neg t \rightarrow t', \text{ where } t' = \neg t.
\end{aligned}$$

The Semantics of Commands To execute **NIL** from store σ take no action and terminate with σ as the final store of the execution:

$$\mathbf{NIL} \langle \mathbf{NIL}, \sigma \rangle \rightarrow \sigma$$

To execute $v=e$ from store σ evaluate e , and if the result is m , change σ to $\sigma[m=v]$ (the final store of the execution):

$$\mathbf{ASSIGNMENT} \frac{\langle e, \sigma \rangle \rightarrow \langle m, \sigma \rangle}{\langle v = e, \sigma \rangle \rightarrow \sigma[m/v]}$$

To execute $c; c'$ from store σ , execute c from store σ obtaining a final store σ' , say, if say, if this execution terminates. Execute c' from the store σ' . The final store of this execution is also the final store of the execution of $c; c'$.

$$\mathbf{SEQUENCE} (1) \frac{\langle c_0, \sigma \rangle \rightarrow \langle c'_0, \sigma' \rangle}{\langle c_0; c_1, \sigma \rangle \rightarrow \langle c'_0; c_1, \sigma' \rangle}$$

$$\mathbf{SEQUENCE} (2) \frac{\langle c_0, \sigma \rangle \rightarrow \sigma'}{\langle c_0; c_1, \sigma \rangle \rightarrow \langle c_1, \sigma' \rangle}$$

To execute **if b then c else** from σ , evaluate b in σ . If the result is the true, then execute c from σ . If the result is the false, then execute c' from σ :

$$\mathbf{IF THEN} (1) \frac{\langle b, \sigma \rangle \rightarrow \langle tt, \sigma \rangle}{\langle \mathbf{If } b \mathbf{ Then } c \mathbf{ Else } c', \sigma \rangle \rightarrow \langle c, \sigma \rangle}$$

$$\mathbf{IF THEN} (2) \frac{\langle b, \sigma \rangle \rightarrow \langle ff, \sigma \rangle}{\langle \mathbf{If } b \mathbf{ Then } c \mathbf{ Else } c', \sigma \rangle \rightarrow \langle c', \sigma \rangle}$$

To execute **While b do c** from σ , first evaluate b . If the result is true, then execute c from σ . If that terminates with final state σ' , execute **While b do c** from σ' . If the result is the false, the execution is finished and the final state is σ :

$$\mathbf{WHILE DO} (1) \frac{\langle b, \sigma \rangle \rightarrow \langle tt, \sigma \rangle}{\langle \mathbf{While } b \mathbf{ do } c, \sigma \rangle \rightarrow \langle c; \mathbf{While } b \mathbf{ do } c, \sigma \rangle}$$

$$\mathbf{WHILE DO} (2) \frac{\langle b, \sigma \rangle \rightarrow \langle ff, \sigma \rangle}{\langle \mathbf{While } b \mathbf{ do } c, \sigma \rangle \rightarrow \sigma}$$

Chapter 3

SKILLS AS KNOWLEDGE

3.1 Introduction

In a variety of domains, humans differ in their capacity to act efficiently and intelligently. People differ in their *skills*. What explains such differences in skills?

According to the *Knowledge Hypothesis*, differences in skills are differences in what the subjects *know*. In the analytic tradition, philosophers have discarded the *Knowledge Hypothesis* as obviously wrong. Thus, Gilbert Ryle(150) (p. 5) took the falsity of the *Knowledge Hypothesis* to be platitudinous:

What facts or what sorts of facts are known to the sensible which are not known to the silly? For example, what truths does the clever chess-player know which would be news to his stupid opponent? Obviously there is no truth or set of truths of which we could say “If only the stupid player had been informed of them, he would be a clever player”, or “When once he had been apprised of these truths he would play well”.

Although virtually every other aspect of Gilbert Ryle’s outlook on the mind has fallen out of favor, it is still a widespread assumption among philosophers today that Ryle was right at least in thinking of skills as irreducible to knowledge.

The recent debate on the know how has only reinforced the general feeling that such a conclusion must be right. Intellectualists about know how have argued that knowing how to do something is a matter of knowing some truths—of knowing a true answer to the question *How to ϕ* . But Intellectualists have defended this view only by *divorcing* skills from know how—i.e., by questioning the identification between skills and know how that was arguably at the basis of Ryle(151)’s criticism of the *Intellectualist Agenda*.¹ Skills, Intellectualists have claimed, are not the same as know how, for one may know how to ϕ without having the skill to ϕ (Ginet(62),

¹Throughout all chapter II of *The Concept of Mind*, Ryle seems to endorse the claim that one’s behavior is skillful at a task only if one knows how to perform it. Moreover generally, the words “skill” and “know how” are used there as synonymous.

Stanley&Williamson(178), Snowdon(163), and Stanley(176)). So Intellectualism can be true about know how without being true about skills.

At the root of the widespread conviction that skills are irreducible to knowledge is the platitude that possession of skills requires possession of *abilities* that are not, and cannot plausibly be, reducible to knowledge possession. As Ryle(151) (pp. 27-28) put it:

When a person is described by one or other of the intelligence epithets such as “shrewd”, or “silly”, “prudent” or “imprudent”, the description imputes to him not the knowledge or ignorance of this or that truth, but *the ability or inability to do certain things* (my italics).

Philosophers have all convened with Ryle that the possession of certain abilities—i.e., the ability to do certain things, or, better, *the ability to perform certain kind of operations*—is at least necessary for skill possession, and concluded that, because of that, skills cannot possibly be entirely reducible to knowledge. But this thought, platitudinous as it is, is *not* inconsistent with the *Knowledge Hypothesis*. Grant that having a skill to ϕ requires possessing a certain set of abilities. It could be that those abilities that are required for possessing a certain skill are among the *prerequisites* for acquiring and retaining the knowledge that that skill consists in. If so, the platitude that skills require the possession of abilities can be reconciled with *Knowledge Hypothesis*.

This chapter defends the *Knowledge Hypothesis*. In the first part of the chapter, drawing on the view of content defended in Chapter II, I propose a particular view of the propositional content of skills—i.e., a view that not only is compatible with but that even entails the platitude that having a skill requires the possession of certain abilities. In the second part of the chapter, I propose a positive argument for thinking that having a skill is a matter of being in a standing knowledge state with *that* kind of propositional content. It follows that every difference in skill must be a difference

in what a subject knows. The last part of the chapter defends the resulting view against the most straightforward challenges to the *Knowledge Hypothesis*.

3.2 The Scope of the Claim

The scope of this chapter is limited. It concerns a *restricted* notion of skill, but one that has a long philosophical pedigree. It is a notion that goes back to the Greek notion of *techne*—i.e., *τεχνη*. It is also demonstrably the notion of skill that is central to Ryle's(150), (151) discussion.

This tradition takes skills to be *distinctively mental*. Aristotle(7), (1140a10) defines skills (*τεχνη*) as a *reasoned* (*μετα λογου*) disposition to production (*εξισ μετα λογου αληθουσ ποιητικη*). Ryle(151) (p. 33) describes the manifestations of a skill *as* the workings of one's mind:

The clown's tripping and tumblings are the workings of his mind, for they are his jokes; but the visibly similar trippings and tumblings of a clumsy man are not the workings of that man's mind.

On this conception of skill, completely non-mental abilities and powers do not count as skills. For example, merely bodily functions, such as digesting one's food and uncontrolled breathing, are not skills. On this mental notion of skill, moreover, a *brain in a vat* could be *as* skillful at a certain task as the person in the best physical shape. Though better physical conditions—such as strength, fitness, motor acuity, and so on—may enable or facilitate the manifestations of one's skill, they are not themselves part of it.

Skills are distinguished from *mental traits*, such as intelligence, stupidity, sharpness, perspicacity, sensibility, inventiveness, etc. What tells skills apart from mental traits is their *domain-specificity*. Mental traits do not need be domain-specific: they are not about any specific subject matter, nor do they concern a specific range of ac-

tivities (von Wright(183) (p. 139), Fodor(51)). Though some may not be about any specific subject matter either, such as for instance the philosophical skill of coming up with counterexamples (Perkins&Salomon(134)), skills are domain specific in the sense that they are always tied to specific types of activities and are connected with actions of a certain specifiable sort (Zagzebski(192), p. 115).

In accordance with this tradition, I will assume that skills have a canonical description—i.e., that there is a some way of describing an activity—such as ‘to ϕ ’—such that a skill can be canonically described as ‘the skill to ϕ ’. A skill to ϕ *characteristically manifests* itself through acts of ϕ -ing: Acts of ϕ -ing, and them only, are the characteristic manifestations of a skill to ϕ . An act of ϕ -ing is *skillful* only if that act manifests a skill to ϕ .²

Another important feature of skills is suggested from assuming that they have a canonical description. Any skill is a skill to ϕ , for some action type ϕ . ‘ ϕ ’ stands for an *action type*. An action type is repeatable. A *highly specific ability*—i.e., an ability to perform a highly specific not repeatable action—is not a skill. Something even stronger of skills seems true: if one has a skill to ϕ , one must have the ability to ϕ in a variety of different circumstances.³ I will characterize this aspect of skills by saying that skills are *productive*.

The characteristic manifestations of skills are *intentional* actions. Mere reflexes or involuntary actions do not manifest any skill. An archer may have the power to hit the bull’s eye with his shot. But his hitting the bull’s eye is not skillful if that is not something that he achieved intentionally. Aristotle(7), (8) (1140b20) characterizes

²That an act manifests the skill to ϕ is only a necessary condition for that act to count as skillful. “Skillful” is a context-sensitive gradable adjective—like “tall” and “good”. Whether something can be said to be “skillful” at a context depends on the standards fixed by the context. So “is skillful” is true of an act of ϕ -ing at a context c just in case it manifests a skill to ϕ and satisfies the predicate “is skillful” to the contextually fixed degree d_c .

³This connection of skills and abilities should be qualified. I will return to it later in the main parts of the chapter.

somebody as skillful at ϕ -ing, if one can voluntarily make errors in ϕ -ing:

αλλὰ μὴν τεχνῆσ μὲν ἀρητῆ, φρονησῶσ. δ' οὐκ ἐστίν; καὶ ἐν μὲν τέχῃ ο
ἐκῶν ἀμαρτανῶν ἀρετῶτεροσ, περὶ δὲ φρωνῆσι ν ἡττν, ὡσπερ καὶ περὶ τὰσ
ἀρετὰσ.

... of a skill, there is a virtue; but there is no virtue of wisdom: in other words, in a skill, somebody who makes errors voluntarily is preferable to one that does not; by contrast, in wisdom, as in other virtues, who makes errors voluntarily is worse (my translation).

Along the same lines, Ryle(151), (p. 33) observes that what distinguishes a skillful clown from a merely clumsy person is that the former acts clumsily on *purpose*:

The cleverness of the clown may be exhibited in his tripping and tumbling. He trips and tumbles just as the clumsy people do, except that he trips and tumbles on purpose and after much rehearsal and at the golden moment and where the children can see him and so as not to hurt himself. The spectators applaud his skill at seeming clumsy [...]. The clown's tripping and tumblings are the workings of his mind, for they are his jokes; but the visibly similar trippings and tumblings of a clumsy man are not the workings of that man's mind. For he does not trip on purpose.

The intentionality of their manifestations is crucial to the notion of skills that philosophers such as Aristotle and Gilbert Ryle have divorced from knowledge. This feature sensibly restricts the scope of my discussion. If the manifestations of a skill have to be intentional, then abilities such as *bare perceptual* or *recognitional discrimination* do not count as skills, if their characteristic manifestations are not intentional actions, but mere perceptual beliefs. This upshot is, I believe, in full accordance with what this philosophical tradition takes skills to be: It cannot be an accident that perceptual or recognitional abilities are never adduced as examples of skills by either Aristotle(7), (8) or Ryle(150), (151).

There are, however, cases of abilities that involve a lot of perceptual and recognitional abilities, without being entirely reducible to those. These cases of abilities may seem to provide a counterexample to my demarcation: it is the case of the

so-called *expert perception*, such as bird watching or chicken-sexing. Chicken-sexing is the paradigm of a skill. If the characteristic manifestations of chicken-sexing are perceptual belief states, then my demarcation yields that chicken-sexing is not a skill. This does not seem correct.

But this argument is too quick. It is simply a mistake to assume that the characteristic manifestations of chicken sexing must be the *output perceptual beliefs*. A skillful chicken sexer is skillful at *finding out the sex of the chicken*. Finding out the sex of the chicken is the characteristic manifestation of chicken sexing. Such a characteristic manifestation is an *activity*: one that in turn involves a variety of other activities, such as touching the chicken in the right places, directing attention to the smell of the chicken and so on. As argued by Biederman and Shiffrar(19) and Pylyshyn(144), what distinguishes a good chicken-sexer from a bad chicken-sexer is exactly how one performs those activities: where one directs one's attention to—for example, where one looks at—when checking the sex of a chicken, as well as what one *does* to the chicken to find out its sex. If that is correct, then the characteristic manifestations of the skill of chicken-sexing are the *activities* involved in finding out the sex of chickens. And those are clearly intentional. If so, the case of expert perceivers can be encompassed within the scope of my discussion.

So, skills are mental abilities, the characteristic manifestations of which are intentional. There is a final aspect that is important to this conception of skills. Skills are distinct from *aptitudes* (or *talents*). Of course, aptitudes (such as an aptitude to music) may facilitate the acquisition of skills (such as the acquisition of a skill to play the piano) and even affect the development of skills. However, aptitudes are not skills. One important dimension of difference is that skills are *learnable*, while aptitudes and talents are not.

To say that skills are “learnable” means that they are acquirable and so from *experience*. By “experience” I mean: by possibly repeated exposition to the task

and through learning from mistakes and upon successes. The claim that skills are learnable is not the claim that skills are *necessarily* learned that way. That would be implausible: *The Matrix*'s characters are able to acquire all sort of skills just by uploading the right software into their brains. Nor is it the claim that it is necessary that skills be *learned*—or even just *acquired*: After all, Davidson(38)'s swampman may well possess all sorts of unacquired skills. Finally, it does not mean that every skill is such that anybody could learn it that way, or could learn it at all. There certainly are many skills we could never acquire, no matter how hard we tried. The *learnability* of skills is a much weaker requirement: it means that for beings like us, with our mental, psychological and sensorial constitution, if something is a skill one can acquire, it must be possible for one to acquire it from experience, in the sense of “experience” just outlined, although it is not necessary that one acquire it that way and although one may even well possess it innately.

Ordinary parlance also underwrites the learnability of skills: If a skill to ϕ is a skill that one can acquire, then one *can learn to ϕ* . The learnability of skills in roughly this sense seems also crucial to Ryle's(151) understanding of what skills are, as evidenced by the following passage:

A person's performance is described as careful or skillful if in his operations he is ready to detect or correct lapses to repeat and improve upon successes, to profit from the examples of others and so forth. He applies criteria in performing critically, that is, in trying to get things right (Ryle(151), (p. 29)).

Along the same lines, Adams(151), pp. 110-111 thus distinguishes between skills and mere abilities/capabilities:

In either case, it seems that there is a difference between intentionally practicing and improving a skill and setting up circumstances or conditions so that a capability such as digestion can function more efficiently. Here is a key difference between the two: when developing a skill such as riding a bike, one improves this skill by repeated, intentional engagement in the activity, but one does not, however, improve one's digestion by digesting repeatedly. The skills involved

in the mirror-reading study mentioned above, however, *were developed and improved only because the subjects repeatedly directed their attention toward the task at hand*. The example of digestion helps make clear two components that are involved in skill acquisition. First, one intentionally engages in the activity so as to improve one's skill. Second, one may manipulate circumstances or conditions so that one can engage in the activity more efficiently. For example, one may lose weight or take steroids so that one is able to kayak more skillfully. Capabilities and skills share the second component in common, but the first component—i.e., *the need to intentionally engage in the activity to improve one's skill, is unique to skills and skill acquisition* (my emphasis).

The scope of this essay encompasses *domain-specific mental competences that are learnable in the sense specified, that are productive as specified and the characteristic manifestations of which are intentional actions*. No doubt there are mental competences that satisfy most but not all of the features just listed. Others may simply resist this classification. My essay will not have much to say about skills that do not neatly fall under this description.

3.3 The Structure of The Argument

Although its scope is limited, the view that skills as characterized are propositional knowledge states is far from trivial. There is a lot of arguing to do. My argument is in three parts.

What kind of propositional content could skills possibly have? Much resistance to the *Knowledge Hypothesis* derives from the difficulty in finding an adequate answer to this question. The first part of this chapter, sections 3.4-3.5, is devoted to defusing a widespread skepticism about the possibility of articulating the propositional content of skills.

Why should we think of skills as knowledge? In the second part of this chapter, I sketch a general argument for why one should think that skills are be propositional knowledge states. The argument takes the form of an *inference to the best explanation*. Here is its general form:

1. Skills are whatever explain their characteristic manifestations.
2. Only a certain kind of standing propositional knowledge state can explain a skill's characteristic manifestations.
3. Skills are knowledge with propositional content.

The final part of this chapter, section 3.8, rebuts some of the most common objections to the *Knowledge Hypothesis*.

3.4 Towards an Account of Skills as Knowledge

What kind of propositional knowledge could skills possibly be? Most propositions about how to ϕ are such that by knowing them, one may nonetheless lack the skill to ϕ . So what kind of proposition is such that knowledge of it could plausibly endow one with the skill to ϕ ?

The detractors of the *Knowledge Hypothesis* have simply failed to identify the best candidate knowledge. The fiercest detractor was Ryle. According to Ryle(150), (151), having a skill to ϕ is a matter of *knowing how to ϕ* . According to him, however, knowing how to ϕ could not possibly be a matter of knowing some proposition. The main target of Ryle's criticisms was a view on which knowing how to ϕ is a matter of knowing *general principles* that regulate the task of ϕ -ing, as is evidenced by many passages such as the following, where the expressions "theory", "rules", "canons" are used to define Intellectualism:

Intelligence practice is not a step-child of theory. [...]

The practice of humor is not a client of its theory. The canons of aesthetic taste, of tactful manners, and of inventive technique similarly remain unpounded without impediments to the exercise of those gifts. [...]

Rules of correct reasoning were first extracted by Aristotle, yet men knew how to avoid and detect their fallacies before they learn his lessons [...] (p. 30).

Efficient practice precedes the theory of it. [...] (p. 30)

... supposing still that to act reasonably I must first perpend the reason for so acting, how am I led to make a suitable application of the reason to the particular situation which my action is to meet? For the reason, or maxim, is inevitably a proposition of some generality. It cannot embody specifications to fit every detail of the particular state of affairs. Clearly, once more, I must be sensible and not stupid, and this good sense cannot itself be the product of the intellectual acknowledgment of any general principle. A soldier does not become a shrewd general merely by endorsing the strategic principles of Clausewitz; he must also be competent to apply them. Knowing how to apply maxims cannot be reduced to, or derived from, the acceptance of those or any other maxims. (Ryle(151)(pp. 30-31)).

As far as I know, nobody has ever dreamt of defending a view that identifies skills with knowledge of a set of general principles.⁴ And yet this kind of view was the main target of Ryle's most influential argument against the *Knowledge Hypothesis*—i.e., Carroll(28)'s regress argument.⁵ In this version of Carroll's regress we are asked to imagine a student who cannot follow an argument—i.e., cannot see that the conclusion follows from its premises, despite knowing that if the premises of the argument are true, then the conclusion is true too. The thought experiment is supposed to show

⁴An example of general principles of this kind is the set of valid rules for the predicative calculus, specifying how to validly infer in that calculus. Theories of this kind are often proposed in cognitive psychology—such as the theory of counting proposed by Gelman and others (Gelman&Meck(56), Gelman&Greeno&Riley(55), Gelman&Greeno(54)).

⁵Here is the entire quote from Ryle(150):

A pupil fails to follow an argument. He understands the premises and he understands the conclusion. But he fails to see that the conclusion follows from the premises. The teacher thinks him rather dull but tries to help. So he tells him that there is an ulterior proposition which he has not considered, namely, that if these premises are true, the conclusion is true. The pupil understands this and dutifully recites it alongside the premises, and still fails to see that the conclusion follows from the premises even when accompanied by the assertion that these premises entail this conclusion. So a second hypothetical proposition is added to his store; namely, that the conclusion is true if the premises are true as well as the first hypothetical proposition that if the premises are true the conclusion is true. And still the pupil fails to see. And so on for ever. He accepts rules in theory but this does not force him to apply them in practice. He considers reasons, but he fails to reason. (This is Lewis Carroll's puzzle in "What the Tortoise said to Achilles". I have met no successful attempt to solve it).

that knowledge of principles governing valid reasoning may not be accompanied by the skills to build or follow an argument.

Ryle(150)'s version of Carroll's regress argument has not convinced everybody. The crucial premise that the pupil knows the relevant logical principle (*that if the premises of an argument are true then the conclusion is true*) is wanting. Many have found it unpersuasive.⁶ However, Ryle(150)'s general kind of worry can be reformulated into a more cogent objection to a view that identifies skills with knowledge of general principles. Let \mathcal{T} be the set of principles for ϕ -ing correctly. Suppose one could reliably detect violations of any principle in \mathcal{T} in others' or oneself's performances. If one were in such position, one could know what it is to correctly ϕ —i.e., as knowing a theory for ϕ -ing correctly. But clearly, one *could* be reliable in detecting performance errors, without being able to perform the task correctly, when asked to. Students in a logic class may be able to detect and correct mistakes in their mates' and their instructor's proofs without being able to produce the proofs themselves, and so perform poorly at the logic exam. They *must* have learned the logical rules, else it would be hard to explain how they can reliably detect logical mistakes. But they clearly have not yet acquired the skill to do the logical exercises themselves.

Logic is not the only domain where skills and knowledge of general principles dissociate. Several experimental results show that pre-school children can detect and correct mistakes in counting tasks much before they develop the skill to count themselves (Gelman&Meck(56), Gelman&Greeno&Riley(55), Gelman&Greeno(54)). As argued by the authors of those experiments, the children must know the principles for counting correctly much before they acquire the skill. This suggests that in several task domains, knowledge of a set of principles for ϕ -ing correctly can not be sufficient

⁶Stanley(176) (Chapter 1, section 4) for discussion.

for skill possession.⁷ One can know a theory of ϕ -ing, while lacking the skill to ϕ . Skills are not knowledge of general principles.

It does not follow that know how and skills are not propositional knowledge, for surely *theoretical* knowledge—i.e., knowledge of a theory—does not exhaust all propositional knowledge that we have or can have. Perhaps skills are knowledge *not* of general principles, but of propositions concerning particular *ways* to ϕ . Here is an issue on which the recent literature on know how provides insight. Intellectualists about know how have argued that knowing how to ϕ is a matter of knowing a proposition of the form *w is how to ϕ* under a practical mode of presentation. If having a skill to ϕ is just a matter of knowing how to ϕ , as Ryle himself thought, then perhaps we can identify skills with know how. Skills might be knowledge of a proposition of the form *w is how to ϕ* under a practical mode of presentation.

This route is one that many have judged (Intellectualists about know how included) hopeless. It has seemed to many that knowledge of a proposition of the form *w is how to ϕ* under a practical mode of presentation cannot be sufficient for skill possession (Stanley & Williamson(178), Bengson & Moffett(15), Bengson & Moffett (16)). An allegedly clear case of dissociation of skills and know how is the SKI INSTRUCTOR case. Suppose Pat has been a ski instructor for twenty years, teaching people how to do complex ski stunts. Although an accomplished skier, he has never been able to do the stunts himself. Nonetheless, over the years he has taught many people how to do them well. In fact, a number of his students have won medals in international competitions and competed in the Olympic games. Pat knows a way to perform the stunts. Moreover, he knows of that way under some mode of presenta-

⁷This point is no news to cognitive psychologists: for the possibility that knowledge of a theory and skills can come apart in this way is arguably the main rationale for the cornerstone distinction in cognitive psychology between Theories of Competence (articulating principles of a task domain) and Theories of performance (articulating the necessary conditions for successful performance at a task) (Pylyshyn(143)).

tion that has *practical relevance*, so to say, as it enables him to teach the complex ski stunts to his students. Patently, however, he does not have the skill to do it himself.⁸

A view of skills as knowledge has to explain the SKI INSTRUCTOR case. I take it that the challenge in particular is to give an account of *practical modes of presentation*, one on which it is plausible to claim that the ski instructor does not know of a way to make ski stunts under a practical mode of presentation. In Chapter II, I developed a view of practical modes of presentation that is up to this task. I argued we should think of practical modes of presentations on the model of programs. A program for a system is a *representation* of an algorithm in a representational means that the system understands (Brookshear(26), Knuth(101), Knuth(100)). Such representations have interesting features. A program is a way of representing an algorithm, much in the way senses are ways of representing their referents. But it is not *any* way of representing an algorithm. A program is a way of representing an algorithm *only for systems that have a certain set of abilities*.

If we think of practical modes of presentation on the model of programs, we can say that a practical mode of presentation represents a way of ϕ -ing for a subject just in case the subject possesses a certain set of abilities. What abilities? As explained in Chapter II, a program is *a representation with parts*. In particular, a program

⁸Another example of dissociation between skills and knowledge of an algorithm is offered by the well-studied case of **Ideomotor Apraxia** (Rapcsak, Ochipa, Anderson and Poizner(145)). Ideomotor Apraxia is a condition that is associated with a progressive degenerative process of the parietal lobes. People affected by it have serious difficulty performing transitive movements, movements involving the handling of tools, such as domestic utensils. In these series of experiments, the subjects were asked to verbally describe all the steps involved in the seven serial tasks involving the use of domestic tools that they previously attempted to perform without success. They readily described all the necessary steps in their correct order. Not only would they perform flawlessly on *verbal* tasks. When given on a table and in a random order seven sets of four to six color photographs each containing pictures depicting the component actions involved in one of the seven serial action tasks, they could arrange the action pictures in their correct order. They would consistently perform flawlessly on that task too. As in the case of **Ski Instructor**, subjects with Ideomotor Apraxia may know (or may in position to know) an algorithm for performing transitive movements, although they have lost the skill to perform them. In both cases, the knowledge of an algorithm and the possession of skills come apart.

represents a way of breaking down a task into its executable parts, and has as many parts as the parts of that task (according to that particular breaking down). The abilities that are relevant are the ones for executing every part of that task. On this model, a practical modes of presentation representing a way of ϕ -ing can be grasped by a subject just in case the subject possesses a certain set of abilities—i.e., the ability to execute every part of a task of ϕ -ing, according to a particular way of breaking down that task into parts.

We have now the elements to introduce my view. Let an answer to the question *How to ϕ* be a Fregean proposition of the form *P is how to ϕ* , where *P* is a variable over a program-like object—or as I called it in Chapter II, a *practical sense*. In this chapter, it is convenient to change the terminology slightly: a representation that is analogous to computer programs will be referred to as a *procedure*.⁹ Just as a program represents an algorithm only for systems with certain abilities, in a similar way a procedure represents an algorithm in a way that only a subject with certain abilities can grasp. Call such answer a “practical answer”. My proposal is that skills are knowledge of practical answers, as defined. PROCEDURALISM is the view that skills are knowledge of practical answers.¹⁰

⁹This is a slight simplification because, as I made clear in Chapter I, an answer is a kind of content with parts, whereas propositions properly speaking are not.

¹⁰Although it does not need to be formulated in Fregean terms, the view is best understood in those terms. An alternative presentation of the view is *Russellian*. On this construal, a procedural answer is a proposition that has as its component a worldly entity—a procedure. The Fregean presentation is superior to the Russellian presentation of the view. If *Latitudinarianism* is right about singular thoughts, it may well be very easy for one to have a thought about object. (Roderick Chisholm (31) was to my knowledge the first to use the term “latitudinarian” to apply to views of de re beliefs. See also Chisholm (30), Chisholm (32), Chisholm (33). Early latitudinarianists were Sosa (164), (165) and Pastin (127). See Baker(10). For recent versions of the view, see Hawthorne&Manley (76).) According to *Latitudinarianism*, one can have a singular thought about an object, just in virtue of possessing some uniquely identifying description of that object. So on *Latitudinarianism*, one may have a singular thought about a procedure, just in virtue of possessing some uniquely identifying description of that procedure. If so, it is not clear that knowing a practical answer in this sense will explain one’s skill to ϕ , because it will not be guaranteed that by knowing such answer, one will thereby be endowed with a certain set of abilities. If *Latitudinarianism* is correct, the picture of practical answers I am recommending is not entirely plausible. The Fregean construal avoids

This view can straightforwardly deal with the SKI INSTRUCTOR case. The ski instructor may know an answer to the question *How to perform the ski stunt* in the sense that he knows an algorithm to ϕ under a description that enables him to teach it to his athletes. But not every description that is practically relevant is a *procedure*. Because the ski instructor lacks certain abilities for performing the ski stunts, the ski instructor cannot represent that algorithm through a procedure, hence cannot know a practical answer to the question *How to perform ski stunts*. Because he does not know a practical answer to that question, he lacks the skill.

3.5 Objections to Proceduralism and replies

PROCEDURALISM raises several issues, that ought to be addressed.

It might be objected that one has not successfully reduced skills to knowledge by reducing skills to knowledge of practical answers, for the notion of procedure essentially employs the notion of *ability to perform every part of a task*. So one may worry that PROCEDURALISM is circular as a theory of skills.

But this worry is misplaced. The ability to execute every part of a task of ϕ -ing is not the same as the skill to ϕ , for one may be able to execute every part of a task of ϕ -ing without having the skill to ϕ . To illustrate, consider a highly trained cook, Julianne, who through years of training has developed the ability to possibly execute all kinds of recipes. He may not have heard of a recipe to prepare *bagna cauda*, and hence not have learned to prepare it yet, although he is in better position than anyone to learning to prepare it. Julianne has the ability to execute the recipe for preparing *bagna cauda*, although he has not developed the skill to prepare it quite yet. For

this worry, because it takes procedures to be not *objects* we can have thought about, but *senses* representing things we can have thoughts about. So in this essay, I will assume the Fregean construal of the view.

that, he may need to take a specialization course in Italian cuisine. Or take a math student who cannot solve a math problem, which in fact requires for its solution the application of a very simple procedure, one that anybody with a elementary school education is able to execute. The math student may not have the skill to solve the problem, and yet be able to execute a procedure for solving it.

Is not the ability to intentionally ϕ the same as the skill to ϕ ? If so, how can the ability to perform every part of the task of ϕ -ing not be identical with the skill to ϕ ? In response, it is important to distinguish between *the ability to perform every part of a task of ϕ -ing* and *the ability to (intentionally) ϕ* . The ability to intentionally ϕ is the same as the skill to ϕ . But it is not quite the same thing as the ability to perform every part of a task ϕ -ing. To illustrate how they come apart, consider again the Julianne's example. Given a recipe for preparing *bagna cauda*, a well-trained cook such as Julianne may be able to execute every part of the recipe. Julianne may well be unable to prepare *bagna cauda* in the relevant sense, for example, if he does not even know of the existence of that recipe, and of any other recipe for preparing *bagna cauda*. After all, the following counterfactuals differ in their intuitive truth values:

- (1) If Julianne were to try to prepare *bagna cauda*, he would succeed.
- (2) If Julianne were to try to prepare *bagna cauda* by following a particular recipe for doing so, he would succeed.

(1) is false: because Julianne does not know any recipe for preparing *bagna cauda*, if he tried to prepare *bagna cauda*, he would fail. (2), on the other hand, is true: if Julianne were to try to prepare *bagna cauda* by following a particular recipe for doing so, he would succeed. In addition to the ability to perform every part of a task of ϕ -ing, the cook must know a way to prepare the *bagna cauda*, in order for him to possess the ability to prepare it, in the relevant sense.

The difference between the ability to perform a procedure for ϕ -ing and the ability that is identical with the skill to ϕ can be brought to light also by noticing that the following two descriptions are not descriptions of the *same ability*:

1. the ability to perform a procedure for ϕ -ing;
2. the ability to intentionally ϕ .

The ability to execute a procedure for ϕ is one that we share with *computers*. Computers can compute, but even intuitively, they are not *skillful* at computation. The ability to compute a program for ϕ -ing does not need to characteristically manifest itself through acts of skillfully ϕ -ing. By contrast, the ability to intentionally performing an action of ϕ -ing does characteristically manifest itself through acts of skillfully ϕ -ing. Only the latter ability is the skill to ϕ . Whereas an ability may be an ability to perform an act under some non-intentional descriptions, a skill is an ability that is canonically described by means of an intentional action (or an intentional description of that action).¹¹ The notion of procedure only employs the first kind of ability—the ability to perform every part of the task of ϕ -ing, according to the particular breaking down represented by that procedure. So PROCEDURALISM does not surreptitiously employ the notion of skill in attempting to reduce it to knowledge.

It might be objected that one could be skillful at ϕ -ing, without having the ability to ϕ . On that basis, one may want to sever *abilities* and *skills* from knowledge. Consider Hannah the pianist who has lost both arms in a car accident (Ginet(62), Bengson and Moffett(16)). She does not need to have lost her skills at playing the

¹¹Observe that the distinction between the ability to perform a procedure for ϕ -ing and the ability to ϕ is entailed by certain philosophical views of actions. According to Goldman(67), for instance, *performing a procedure for ϕ -ing* and *ϕ -ing* are not different descriptions of the same action. Rather, they are different actions, albeit ones that stand in a generation relation, for performing a procedure for ϕ -ing will typically generate the action of ϕ -ing. So on Goldman(67)'s way of individuating actions, (1) and (2) describe different abilities—the abilities to perform different actions.

piano—skills are mental and she is all the same as far as her mental capacities go. But she does lack the ability.

In response, the point to note is that ability talk is notoriously *context-dependent*, as Lewis(113) has taught us. As cashed out by Lewis (113) (p.77), the context sensitivity of the locution ‘*S can ϕ* ’ has to do with the range of facts *S*’s ϕ -ing is taken to be *compossible* with. When we think of the pianist’s mental facts, and them only, the pianist Hannah counts as being able to play the piano, for her playing the piano is compossible with her cognitive and mental capacities. But when we think of facts about the pianist that are not necessarily mental, such as her just having lost her arms, she does not count as having the ability to ϕ , for her ϕ -ing is not compossible with that larger set of facts. One’s skill to ϕ do not entail one’s ability to ϕ *tout court*—a kind of ability that requires for its possession the compossibility of *S*’s ϕ -ing with *all* the facts. Skill entails *mental* abilities, and mental abilities only require *compossibility with the mental facts*.

What about the case of Gianni, a surgeon who can skillfully perform complex brain surgeries on corpses, but cannot do it in real life situations, because he tends to panic? It seems that Gianni knows how to perform the operation, although he lacks the ability to perform one. Does or not he have the skill to perform the operation?

Gianni surely does not have the ability to perform it. But here it is helpful to individuate skills *more finely*, by individuating more finely the actions one supposedly has the skill to perform. Gianni surely knows a practical answer to the question *How to perform the operation on corpses*, as he got the skill to perform the operation on corpses. What he does not know is a practical answer to the question *How to perform the operation without panicking*.¹² So he does not have the skill to perform *that* kind

¹²See Hawley(73) for a similar move, in arguing that counterfactual success is necessary for know how.

of action.¹³

PROCEDURALISM is the view that having a skill is a matter of knowing an answer to the question how to ϕ that has a procedure as its component. If one is in position to grasp that answer, then one must have a certain set of abilities—the abilities to carry out every part of a task of ϕ -ing. In virtue of knowing a practical answer, one has the mental ability to intentionally ϕ —for one has both the ability to perform every part of the task as well as the knowledge required for executing the task. So it follows from PROCEDURALISM that if one has the skill to ϕ , one must possess a certain set of abilities. PROCEDURALISM reconciles the *Knowledge Hypothesis* with the platitude that possessing a skill is a matter of possessing a certain set of abilities.

PROCEDURALISM is a theory of what skills are, but this is compatible with the theory not being a priori. As a theory of skills, however, PROCEDURALISM is committed to the *necessity* of the claim.¹⁴ So according to PROCEDURALISM, necessarily, if one has a skill to ϕ , one knows a practical answer to the question *How to ϕ* . But could not one be skillful at a task, without knowing a practical answer?

To envision a persuasive example, we need to look at beings that quite radically differ in their psychological make up from us—a kind of being that clearly does not need to represent the breaking down of a potentially open-ended task into its parts in order to successfully solve specific instances of it. Here is a persuasive example.¹⁵ Consider a super-intelligent being who has figured out a theory about everything: for any task ϕ , she knows every general principle concerning ϕ -ing. Suppose however that she is so smart and fast that she can reason immediately from first principles to specific problems that she encounters on her way, without ever the need to derive,

¹³Does this suggest that there are background standards for how to individuate the relevant actions? Probably: If I can only bubble sort, I don't really know how to sort: I should learn radix sort.

¹⁴For another example of a necessarily true claim, if true at all, that may not be a priori, see Williamson(188)'s K=E.

¹⁵I have to thank Timothy Williamson for this example.

as an intermediate step, and for any task ϕ , knowledge of a practical answer to the question *How to ϕ* . In this scenario, the super intelligent being would not know a practical answer to the question *How to ϕ* , because she would not need to derive that knowledge from the principles that she does know. Against our intuitions, my view predicts that she does not have any skills.

The PROCEDURALIST should respond that properly speaking the being just described does *not* have the skill to ϕ . Here is why: What the being has is *a collection of abilities*: the abilities to ϕ -in-*c*, for many highly specific circumstances *c*. But those abilities are not skills, as they are not *productive*. The being just described is indeed extremely successful in her endeavors. And since, for beings like us, being (even to a much lesser extent) extremely successful in our endeavors requires the possession of skills, we intuit that she also must be skilled. We judge that she must be skilled on the basis of what explains success in our *domestic* case. But the same phenomenon—i.e., the reliable success at a task—may require different explanations in different cases. The fact that the two behaviors are alike under some descriptions—in this case, their being successful—does not mean that they are alike in all respects—including in their manifesting a skill.¹⁶

If we are happy with this consequence, we should also be happy to embrace its moral—i.e., the moral being that for beings like us, ignorant of first principles and not as fast on our feet, skills are what we need to make up for our intellectual limitations. We should allow for the possibility that a being in better epistemic and intellectual shape may very well dispense with the need for skills. If we were much smarter and much more knowledgeable of the true theory of the world, we too could afford not having skills.

¹⁶See McDowell(119) for a similar line of argument to the conclusion that animals may well lack the knowledge required for acting intelligently.

3.6 Towards The Argument

In the last two sections, I developed and defended a view on which skills are knowledge of practical answers. I think it is a view with many attractions. But why should we think of skills as knowledge to begin with? Here is a general argument:

1. Skills are whatever explain their characteristic manifestations.
2. Only a certain kind of standing propositional knowledge state can explain a skill's characteristic manifestations.
3. Skills are knowledge with propositional content.

I take that the first premise is uncontroversial, but let me say a few words about how to understand it.

According to Premise (1), it must be *because* one has the skill to ϕ that one's act of ϕ -ing can be skillful. The "because" is the one of *causal explanation*. So all rules of causal explanation apply. The claim *that the cigarette has caused the fire in the woods* is true, if at all, only against a set of features of the background situation and enabling conditions—the fact that the cigarette was lighted and was not wet, the fact that there was oxygen, and so on. Similarly here: the claim that a skill causes its characteristic manifestations is true, if at all, only against a set of relevant background/enabling conditions. So for example, when we say that *the archer's skill caused the shot to be skillful*, we are tacitly presupposing that certain normal conditions were in place: that the archer was sober and awake, that his arms were working fine, that the light was favorable, the wind was normal, and so on.¹⁷ Features of the situations and the

¹⁷Sosa(166) distinguishes between the subject's *general conditions*—for example the archer's being sober and awake—and *the situation where the shot occurs*—for example, the fact that the light was favorable and the wind normal. I will return to this classification of background conditions in a little while.

subject's general conditions enter into the set of normal conditions under which the causal claim is true. What counts as normal is hard to spell out. And it is highly context-sensitive issue. But that does not undermine the validity of the causal claim.

With this clarification in place, this section defends Premise (2). According to (2), only propositional knowledge can play the causal role that skills play with respect to their characteristic manifestations. The argument is in two parts. I first argue that for them to play their causal role with respect to their characteristic manifestations, skills must be standing belief states. Then I argue that if skills are belief states, then they must be propositional knowledge states.

3.7 The Argument

Skills' manifestations are intentional. Recall the clown's tripping, that is skillful only if done on purpose—for the clumsy person could trip in the same way, but would not do it skillfully. Or recall the archer's hitting the bull's eye—skillful because it is something the archer intended to achieve.

It is a widespread assumption in the literature on intentional action that if an act has to be intentional, then it has to be caused by a *propositional attitude correctly representing how the act is to be performed*. For instance, Mele and Moser(120) say:

On a common notion of intentional action, a person performs a certain action of type *A* intentionally only if she has a representation of that action type.

Most seem to take the relevant propositional attitude to be a *belief-like state*. So for instance, Goldman(67) (pp. 51-55) offers the following necessary conditions on the intentionality of an action:

Action token *A* is intentional only if there is an act (type) *A'* such that the agent *S* wanted to do (exemplify) *A'* and *S* either believed that his doing *A*

would generate his doing *A'* or believed that his doing *A* would be on the same level as his doing *A'*.

Finally, Setiya(161) (pp. 395–396) also submits that intentionality of an action requires belief:

[...] that beliefs about what one is doing—about the action itself or about the means by which one is doing it—are essential to the operation of the will. As I have argued elsewhere, the necessary presence of belief in intentional action shows that intention involves belief. [...] It is because intention must be present in intentional action, and because intention involves belief, that beliefs about what one is doing must be present as well.

As Gibbons(61) (p. 586) vividly puts it, the belief required for the intentionality of one's action must be *a true one*:

Any explanation of an intentional action, rather than an attempt, depends on the truth of the relevant beliefs in this way. Consider a set of means-ends beliefs, or beliefs that by *A*-ing you will *B*. Restricting attention to beliefs of this sort that play a role in the production or guidance of behavior, it seems that the truth of just these beliefs is relevant to what types of action you will perform. A belief, no matter how externally individuated, can always be a mistake. And when you act on a false belief, you will not, except by accident, do what you intend.

The view that intentional action requires a true belief about how to perform it is the received view about intentional action, if there is one. If only its being based on a true belief about how to execute an action can account for its intentionality, then a true belief about how to execute them is required to explain a skill's manifestations.

The argument that true belief about how to perform an action is required for that action to be performed intentionally does not establish that skills are *standing* belief states. It is compatible with its conclusion that the manifestations of a skill are

explained by beliefs which are *occurrent at the moment of those manifestations* and yet the skill itself is not a standing belief—i.e., standing across its manifestations.¹⁸

My reason for thinking of skills as standing belief states is that that hypothesis explains the *learnability* of skills better than any other candidate hypothesis. In particular, it explains two dimensions of the learnability of skills: the possibility of acquiring skills by learning from experience; and the learnability of skills *qua* productive abilities.

1. Skills are *acquirable from experience*. As explained at the outset, part of what it means to learn a skill from experience is to learn it from mistakes and upon successes through exposition to the relevant task. It is very natural to model one's learning (or improving) from mistakes as one's *denying that a certain way to do things is correct*, or *denying that a certain way to do things is the best way to do things*; learning (or improving) upon successes, on the other hand, can be modeled as one's *confirming that a certain way to do things is correct*; or *confirming that a certain way to do things is the best way to do things*.

Another way of putting the point is to say that skills are abilities that may be acquirable and/or improvable through *rational revision*. The best model available of rational revision takes it to be a function from propositional states to propositional states. If rational revision is possible for acquiring skills, skills must quite generally be the kind of things that can be the output of rationally revision.

2. Skills are *productive*. As explained at the outset, the productivity of skills just

¹⁸For example Millar(121) has argued that cognitive abilities such as perceptual/recognition abilities are *disposition to know*. As far as I can see, Millar(121) is not talking about skills in the sense that is the topic of this paper. However, Stanley and Williamson (manuscript) are developing a view similar to Millar(121), but that has the same target as mine. So according to Stanley and Williamson, skills in my sense are dispositions to know. As far as I can see, a view of this kind is compatible with *The Argument from Intentionality of Skill's Manifestations*.

amounts to the following: If one has the skill to ϕ at a time t , one has at t the skill to ϕ in a variety of different circumstances, some of which one may not have encountered at t .

Let assume as before that a skill is learnable from experience. At each t , one will have experienced only a limited number of circumstances of ϕ -ing. The most natural model to understand learning from a finite amount of experience is generalization from seen cases to unseen cases.

And the most natural model for generalization of this kind is *inductive reasoning*. Inductive reasoning takes as inputs a set of propositions, and outputs a proposition—i.e., general proposition, one that covers unseen circumstances.

If inductive reasoning is possible for acquiring skills from experience, skills must be propositional states.¹⁹

The hypothesis that skills are standing propositional states provides a natural model to understand the learnability of skills. The opponent of the belief constraint on skills must argue that the critical process of revision and learning from mistakes through which skills can be developed and acquired is not at all analogous to rational revision or inductive reasoning. But then they owe us another model for understanding skill's learnability. In absence of such a model, the best explanation of skills's learnability is that skills are standing beliefs.

Skills are standing true beliefs. But true beliefs may well fall short of knowledge. Why think that skills must be knowledge?

¹⁹Recent studies on robotic skills (Bethany R. Leffler's(108) Dissertation, Rutgers 2008) model *reinforcement*—i.e., the process by which robots learn from experience—takes the form of a *Bayesian update*—i.e., conditionalization upon newly acquired evidence. To the extent to which Bayesian conditionalization requires propositional contents as inputs, Leffler(108)'s work underpins my argument.

The first reason for thinking that skills must be knowledge is that the manifestations of skills are *intentional*, and nothing less than knowledge can explain the intentionality of actions. If so, nothing less than knowledge can explain a skill's manifestations. The case for the claim that only knowledge can explain intentional actions is not novel. It is originally due to Gibbons(61).²⁰ My exposition here tries to improve on Gibbons' arguments.

Lottery cases provide evidence that only knowledge can explain the intentionality of actions. Contrast the following four scenarios:²¹

1. CINDY1 Cindy believes that if she buys a lottery ticket, she will lose. The lottery is fair and by buying a lottery ticket, she has only one in a million chances of winning. So she is justified in so believing. Acting upon an irrational hope of winning, she buys the ticket anyway and as expected, she loses.
2. CINDY2 Cindy has inside information on the lottery draw, and on that basis she knows that her husband Mark will win the lottery, for the lottery was rigged in his favor. In the intent of not drawing further suspicion on the fairness of the lottery, she buys the ticket anyway and, as expected, she loses.
3. CINDY3 Cindy believes that by buying a lottery ticket, she will win the lottery because a reliable source tells her that the lottery is rigged in her favor. So

²⁰For other arguments in the neighborhood, see Williamson(186).

²¹The appeal to lottery cases in arguing for the necessity of knowledge for warranted assertion goes back to Williamson(187). See Fantl and McGrath(49) and Hawthorne and Stanley(77) for an appeal to lottery cases in an argument on behalf of the Knowledge-Action principle. Gibbons(61) already uses lottery cases for the claim that intentionality of an action requires knowledge, and not just true belief. However, Gibbons' presentation is problematic in some respects. My exposition tries to improve on it. Mele and Moser(120) use Gettier(58)-like cases but to argue that there is an anti-luck ingredients to intentional action. Interestingly, they do not quite conclude that knowledge is required. Rather, they conclude instead that *skill* is required, but of course they do not uphold the view that skills are knowledge.

Cindy is justified in so believing. However, the lottery is not actually rigged in her favor and Cindy wins it out of an unbelievable luck.

4. CINDY4 Cindy believes that by buying a lottery ticket from her local store, she will win the lottery because a reliable source tells her that the lottery happens to be rigged in her favor. So Cindy is justified in so believing. However, the lottery is not rigged in Cindy's favor. It just so happens that the local store by accident gets all and only the 6 winning tickets. So Cindy turns out to be buying a winning ticket.

Consider the first two scenarios. In CINDY1, although she believed she was not going to win, Cindy did not intentionally lose the lottery. Instead, in CINDY2, Cindy lost the lottery intentionally: after all, it would be appropriate to ask why she did it.²² In CINDY1, she does not know that by buying a lottery ticket she will lose. In CINDY2, instead, she does know. One possible explanation for the contrast is that knowledge is required for intentional action.

It is not the only possible explanation: it might be objected that in CINDY1, Cindy is not acting on the full belief that she has and that the reason why she did not lose intentionally is just that she is being irrational. Although this sort of explanation may be available for CINDY1, it will not help with CINDY3. In CINDY3, Cindy believes instead that she will win the lottery by buying a lottery ticket and she is justified in so believing. She also acts on her beliefs by buying the lottery ticket. However, her winning is again too coincidental to count as intentional.

It might be objected that buying a lottery ticket when the lottery is fair is *not* a way of winning the lottery, because it is not a reliable way of winning the lottery. But then consider CINDY4, where that element is controlled for. In CINDY4, as in

²²Anscombe (6), (5) famously proposes the following criterion for intentional actions: an action is intentional only if it would be appropriate to ask why it was performed by the subject.

CINDY3, Cindy does not intentionally win the lottery, and yet in CINDY4, buying the ticket from her local store is a perfectly reliable way of winning the lottery. Again, Cindy believes something true, and she is justified. But she clearly does not know. It seems to me that only the fact that Cindy does not know can explain the lack of intentionality in CINDY4.

In CINDY3 and CINDY4, Cindy acts on a Gettiered belief. As observed by Gibbons(61), other Gettier-like cases appear to support the same conclusion. Compare Bobby's attempts at killing his uncle, each happening in two parallel universes:

1. BOBBY1 Bobby intends to kill his uncle by planting a bomb in his house and then, after moving a safe distance away, pressing the large red button on the remote control device. He thinks that pressing the button will cause the bomb to detonate. His belief is true and justified. But here is what happens. A satellite, launched by the National Security Agency and designed to prevent bombings of just this kind, intercepts Bobby's transmission; this causes the satellite to send a warning to the intended victim; but, because of an unfortunate choice of frequency, this causes the bomb to detonate.
2. BOBBY2 Bobby intends to kill his uncle by planting a bomb in his house and then, after moving a safe distance away, pressing the large red button on the remote control device. He thinks that pressing the button will cause the bomb to detonate. His belief is true and justified. He is also aware of the satellite, launched by the National Security Agency and designed to prevent bombings of just this kind, intercepts Bobby's transmission; this causes the satellite to send a warning to the intended victim; aware of this, Bob installs a device that upon the satellite's interception blocks the warning to the intended victim. Bobby has now intentionally killed his uncle.

In both universes, Bobby intends to kill his uncle. But only in BOBBY2, does Bobby

do it intentionally. In BOBBY1, his success at bombing his uncle's house is too coincidental to count as intentional. In both cases, Bobby intentionally tried to kill his uncle. But only in the latter, did he intentionally do it. Again, what tells apart these two cases is that only in the latter scenario, does Bobby know that by pressing the button will cause the bomb to detonate. In the former scenario, instead, the truth of his belief is too accidental for that belief to count as knowledge.

Both lottery cases and Gettier-like cases both corroborate the hypothesis that knowledge is required to explain intentional actions. But skillful manifestations are intentional actions. If knowledge is required for intentional action, then the hypothesis that skills are knowledge better explains the intentionality of skills' manifestations than the hypothesis that skills are mere true beliefs.

There is room for even a more direct argument from Gettier-like cases to the necessity of knowledge to explain a skill's manifestations. Go back to our last example. Not only did BOBBY2 intentionally kill his uncle: he did it *skillfully*. Not so BOBBY1. His success was too coincidental to be due to any skill. To corroborate this same point, consider the following different example:

1. MEDICAL STUDENT1 Medical student 1 is instructed to perform two highly different procedures, procedure 1 and procedure 2. Medical student 1 learns to perform both of them flawlessly. If a patient shows symptoms A, she has to be treated by using procedure 1. If she displays symptoms B, she has to be treated by using procedure 2. It takes a lot of experience to learn to discriminate between symptoms A and symptoms B. Medical student 1 does not have that kind of experience. Confronted with a patient, Medical student 1 has to decide which procedure to apply. At loss of ideas, he asks an older colleague for help. His older colleague secretly hates him and tries to mislead him into making a mistake. So his older colleague advises him to use procedure 1. But it turns out, his older colleague had made a mistake, procedure 1 was in fact the

correct procedure to apply. By correctly applying procedure 1, Medical student 1 successfully cures his patient.

2. **MEDICAL STUDENT2** Medical student 2 is instructed to perform two highly different procedures, procedure 1 and procedure 2. Medical student 2 learns to perform both of them flawlessly. If a patient shows symptoms A, she has to be treated by using procedure 1. If she displays symptoms B, she has to be treated by using procedure 2. It takes a lot of experience to learn to discriminate between symptoms A and symptoms B. After careful study and effort, Medical student 2 correctly discriminates between symptoms A and B. Confronted with a patient, Medical student 2 has to decide which procedure to apply. After examining the case, he comes to believe correctly that procedure 1 is the correct procedure. By correctly applying procedure 1, Medical student 2 successfully cures his patient.

Here our intuitions are clear: Whereas Medical student 1 did not skillfully cure his patient, Medical student 2, instead, did. What tells apart the two cases is that in **MEDICAL STUDENT 2**, the student acts on knowledge; not so in **MEDICAL STUDENT 1**.

Admittedly, in several other cases, true beliefs that fall short of knowledge appear to be sufficient to ground skillful actions, as in **THE LUCKY LIGHT BULB**, slightly modified from Cath(29):²³

THE LUCKY LIGHT BULB: Charlie wants to learn to change a light bulb, but he knows almost nothing about light fixtures or bulbs (as he has only ever seen light bulbs already installed and so he has never seen the end of a light bulb, nor the inside of a light fixture). To remedy this situation Charlie consults *The*

²³Cath(29) uses this example to argue that, in Gettier cases, one can nonetheless have know how. There is a small literature on whether know how can be gettiered. For other readings on this topic, see Poston(140) and Stanley(177).

Idiot's Guide to Everyday Jobs. Inside, he finds an accurate set of instructions describing the shape of a light fixture and bulb, and the way to change a bulb. Charlie grasps these instructions perfectly. And so there is a way, call it w_1 , such that Charlie now believes that w_1 is a way for him to change a light bulb, namely, the way described in the book. However, unbeknownst to Charlie, he is extremely lucky to have read these instructions, for the disgruntled author of *The Idiots Guide* filled her book with misleading instructions. Under every entry she intentionally misdescribed the objects involved in that job, and described a series of actions that would not constitute a way to do the job at all. However, at the printers, a computer error caused the text under the entry for “Changing a Light Bulb”, in just one copy of the book, to be randomly replaced by new text. By incredible coincidence, this new text provided the clear and accurate set of instructions. Charlie masters those instructions and successfully changes his first light bulb.

In THE LUCKY LIGHT BULB, Charlie may skillfully change his first light bulb, although his performance is based on a belief that is true by luck.

We should not thereby conclude that something less than knowledge suffices for skills. For there are parallel cases where true beliefs falling short of knowledge seem to suffice for knowledge. As observed by Hawthorne(74), in the following scenario, most would judge that the subject knows what is the capital of Boston:

Suppose I ask in an ordinary setting whether someone knows whether Boston is the capital of Massachusetts. Suppose it turns out that he does truly believe this though the epistemic credentials of his path to that belief are decidedly shaky: Perhaps he got it from a book that misprinted most of the state capitals though not this one. Perhaps he got the information from someone that he had good reason to distrust (who happened to be sincere on this occasion or else who tried to lie and accidentally told the truth on this occasion). Would your acceptance of the statement “Boston is the capital of Massachusetts” fail to be knowledge in such cases? Not so, or not clearly so.

[...] Consider though the following case: Joe is putting together a book on interesting facts about Massachusetts. He sets out to find out what the capital is. He looks at what is in fact a list of the largest cities in each state, mistakenly thinking it is a list of state capitals. He finds “Boston” on this list and writes down “Boston is the capital of Massachusetts” in the interesting facts book. I read Joe’s book and form the belief that Boston is the capital of Massachusetts. There is no causal channel here from the fact to my belief. Fred comes and tells me that he needs to go to the capital of Massachusetts. You have witnessed the whole etiology of my belief. Someone asks you whether I know that Fred

needs to go to Boston. When I present this case, the overwhelming intuition of philosophers and non-philosophers (leaving aside those who have spent too long in Arizona) is to say “Yes” in answer to the query. In answering “Yes” you presuppose in that context that “I know that Boston is the capital of Massachusetts” is true.²⁴

At the basis of the both judgment errors, however, there is an important analogy.²⁵ We are inclined to grant knowledge when there is not, as long as certain abilities that generally go along with knowledge are present—such as *the ability to correctly answer a question*. Similarly, we are inclined to deem skillful actions that are not skillful, as long as certain abilities that generally go along with skills are present—such as *the ability to perform every part of a task*. But, as argued in the first part of this chapter, a skill to ϕ is not the same as the ability to perform every part of ϕ -ing. Such ability goes along with the skill to ϕ but can be present when the skill is lacking, as in LUCKY LIGHT BULB. We are not always so fooled: when the ability to perform every part of a task is granted to be there all along, and so clearly at issue, as in BOBBY1 or as in MEDICAL STUDENT 1, we correctly judge that an action based on a belief true by luck is not skillful.

The final argument for thinking that mere true belief cannot explain skillful actions turns on there being a *safety* condition on skillful actions.²⁶ Whether or not one possesses a skill to ϕ at a time t , if at t one accidentally succeeds at ϕ -ing, that occasion of ϕ -ing is not a manifestation of one’s skill. If Tiger got lucky when he did his hole-in-one because an extraordinary gust of wind accompanied the golf ball into

²⁴Hawthorne(75) gives other similar examples of cases where true beliefs by luck seem to qualify as knowledge.

²⁵I should say that Hawthorne(74) draws a different conclusions from his examples—i.e., that in some contexts, true belief suffices for knowledge.

²⁶My argument assumes that there is a safety condition on knowledge. Some disagree. See Comesaña(35). It is fair to say, however, that the received view is that safety is necessary for knowledge. See Sosa(167), Sosa(168), Williamson(185), (189), DeRose (42), Sosa(170), Pritchard (141), (142).

the hole, his shot was not skillful. If Michael Jordan successfully scores a basket when he wanted to pass, but the ball was inadvertently deviated by another player into the basket, his hoop was not skillful. There is an anti-luck condition on skillful action, like there is an anti-luck condition on knowledge. Safety being the *par excellence* anti-luck condition on knowledge, it seems just as plausible that there is a safety condition on skillful actions.

It is just not entirely trivial how such condition has to be cashed out. A skillful hitting the bull's eye could not have easily failed. Or could it? Of course, there could have easily been a gust of wind. If there had been a gust of wind, the archer would have failed the shot. If the archer is a drunk and had he been drunk at the time of the shot, the shot would have failed. The shot can be highly skillful, even if it could have easily failed.

Safety *simpliciter* fails for skillful actions. But safety *simpliciter* also fails for knowledge.²⁷ Safety for skillful actions, just like safety for beliefs has to be relativized, perhaps to a *basis* and/or to a *set of relevantly similar circumstances*. What basis? And what set of relevant circumstances?

The *basis*, I suggest is the skill itself. An action based on *S*'s skill may easily fail if it were not based on *S*'s skill—for example *S* could easily performed it with the intention of making a mistake or *S* could easily have performed it unintentionally.

The *set of relevantly similar circumstances* includes the background conditions to the action being caused by the subject's skill. Let me explain. Consider again the observation at the outset concerning the causal role of skills in explaining their manifestations. Suppose Michael Jordan's skill caused his skillfully hitting the basket. A set of background conditions provides the context with respect to which the causal claim true. Some of those background conditions have to do with the subject's *general*

²⁷See Sosa (170), for several counterexamples to unrelativized safety.

condition: the fact that he is awake and focused, the fact that his sight is working properly, the fact that he has control over his arms, and so on. Some of those background conditions have to do with the *situation* (within certain limits) where the hoop occurs: the fact that the playing court was in normal condition, that the light was ok, that the floor was not wet, and so on.

Those very same background conditions—the subject’s general condition and the situation of the action—are those we try to control for when testing the presence of a skill. Audition judges, athletic scouts, and appointing committees do that all the time. As observed by Sosa(166), their methods for doing so are just slightly more official versions of the sort of assessment made constantly as we judge the presence of a skill. Such judgments tend to keep track of who would perform well in situations where good performance is of interest. If the player is drunk, we do not count his failing to score a goal against his being skilled. If the ground was extraordinarily wet, we do not count the playing’s failed passing against him. Quite generally in assessing people’s skills, we are interested in how they would perform *in relevant situations and under certain conditions*.

My suggestion is that the sufficiently relevant circumstances to which we want to relativize the safety of skillful actions include precisely the background conditions to the skill of a subject’s causing the action—the smallest set of facts concerning the subject’s condition and the general situation where the action occurs that provides the context with respect to which the causal claim is true that the skill of the subject caused that action. They are also the set of normal conditions that are to be in place when assessing the presence of a skill. There *has* to be such a set, and in most cases of skills we can individuate what the set is, if we are at all reliable in assessing the presence of those skills. This suggests the following ACTION-SAFETY, that relativizes the safety of a skillful action both to a *basis*—i.e., the belief that action is based on—and to *a set of relevantly similar circumstances*.

ACTION-SAFETY: An act of ϕ -ing by S is safe in the relevant sense just in case it could not have easily failed in sufficiently similar cases based on S 's skill—i.e., just in case the act of ϕ -ing by S successfully occurs in all the closest possible worlds to the actual where it is based on S 's skill and the actual background situations and S 's actual conditions hold.²⁸²⁹

The argument from the safety of skillful actions to the safety of the beliefs that ground them now goes as follows.

Suppose A 's skill to ϕ causes at t an act of ϕ -ing. Call such causal event C . Let the relevant background conditions for the causal event be the set \mathcal{C} . That occasion of ϕ -ing is safe in the relevant sense, by assumption. So it occurs in all closest worlds to the actual where it is based on S 's skill and the background conditions in \mathcal{C} obtain, by ACTION-SAFETY. Suppose the skill is an unsafe true belief. If so, it could be easily mistaken (relative to whatever methods was used in forming it). Moreover, it could be easily mistaken even were the conditions in \mathcal{C} to hold. After all, the fact that the background conditions to the causal claim C are in place do not need to have any bearing on whether the relevant belief is false.

But if that belief was false, and the background conditions \mathcal{C} were in place, an action based on it would fail. Hence, there is some closest world in which the background conditions in \mathcal{C} hold and where an action of ϕ -ing by S based on that belief fails. But this denies ACTION-SAFETY. So if skills can be unsafe true beliefs, skillful

²⁸Dutant(45) distinguishes between two notions of safety, that depend on how the closeness condition is formulated. The first notion of closeness, normalized closeness, has to do with what it could typically have happened, independently of whether it could have happened in the circumstances at hand. The second notion of closeness is metaphysical closeness, having to do with what it could in fact have happened, given the specificity of the circumstances at hand. The way I suggest to relativize safety seems to appeal to normalized closeness. More on this in “Safe beliefs and safe performances”, in progress.

²⁹Is *every* skillful action safe, even relatively to such a basis and to such a set of sufficiently similar circumstances? Certain outstanding performances may be highly risky. But they do not seem to be any less skillful because of that. Consider for instance a hit and run strategy in baseball (Thanks to Andy Egan for the example). A careful analysis of cases such as this would require a much longer discussion than I am able to sustain at the moment. This is one aspect of the project on which I am planning to work more in the next future.

actions cannot be safe in the relevant sense. If skillful actions are safe in the relevant sense, then skills must be safe true beliefs. If skillful actions are safe, it would seem that nothing less than safe beliefs can explain the safety of skillful actions.

3.8 A Defense of the Knowledge Hypothesis against Objections

A belief correctly representing how to produce an action is required to explain a skill's manifestation. I gave three different kinds of arguments for thinking that such belief must be knowledge: knowledge of a practical answer is necessary to explain a skill's manifestation. At the outset, I characterized having a skill to ϕ as a matter of having the mental ability to intentionally ϕ . Moreover, in the first part of the chapter, I argued that knowing a practical answer to the question *How to ϕ* entails having the mental ability to intentionally ϕ . If so, there is room for a more positive argument for thinking of skills as knowledge:

1. A skill to ϕ is the mental ability to intentionally ϕ .
2. One has the mental ability to intentionally ϕ just in case one knows a practical answer to the question *How to ϕ* .
3. Hence, one has a skill to ϕ just in case one knows a practical answer to the question *How to ϕ* .

The argument is valid. The conclusion follows from the premises. If knowledge of a practical answer is both necessary and sufficient for skills, we are on good grounds in identifying them: Skills are knowledge of practical answers. In this section, the *Knowledge Hypothesis* is defended against some of the most common challenges to it.

A common worry against a view of skills as knowledge is *The Argument from Practice*. According to *The Argument from Practice*, possessing a skill cannot simply

amount to propositional knowledge. Propositional knowledge only gets us so far in the process of acquiring a skill. In addition to learning certain propositions, practice and experience are needed to complete the acquisition process.

Any account of skills has to explain the role played by practice and experience in skills' development. But it simply does not follow from the necessity of practice and experience that no amount of knowledge is sufficient for skill possession. The missing and unjustified premise is that practice and experience cannot provide one with the necessary kind of knowledge.

On the present account, for one to become skillful at ϕ -ing is for one to learn a practical answer to the question *How to ϕ* . But then on this picture, the relevance of practice and experience in the process of acquiring skills can be appreciated. For on this picture, the very process of grasping a practical answer may involve practice on the part of the subject. That is so because in some cases, parts of a practical answer become accessible to one only as one becomes able to perform those operations corresponding to that part of the task that one was not previously able to perform. In these cases, practice may be required in the process of *acquiring* the ability to perform those parts of the task. In other cases, for one to learn a practical answer to the question *How to ϕ* is for one to engage in a process that resembles *programming*. The development of a program is a two-fold activity, consisting in discovering the underlying algorithm and representing that algorithm as a program (Brookshear (26)). On this model, learning to perform a task may require finding out an algorithm to ϕ and mapping it into a practical answer. On the present account, then, acquiring and developing a skill require going through a programming-like process of defining the relevant practical answer.

In both kinds of cases, practice is necessary for acquiring knowledge of a practical answer. Since we cannot know practical answers unless we know each of their parts, and because we cannot know each part unless we acquire the ability to perform every

part of the task, in some cases we cannot know a practical answer unless we practice. That is why sometimes in many cases, we cannot acquire a skill without practicing. On the view presented here, then, the necessity of practice can be fully appreciated, because it is clear on this view how and why practice and experience may be needed for coming to know a practical answer.

Another objection to a view of skills as knowledge is *The Argument from Equal Knowledge*. According to such an argument, two people may be in the same knowledge state with respect to the question *How to ϕ* , but differ in their skills because one but not the other can apply that knowledge in practice. Thus, suppose I want to learn how to swim, and I ask Phelps to teach me. Suppose moreover, he describes to me in every detail the algorithm he uses when swimming. It seems that by learning how to swim as Phelps does, I may still fall well short from being able to swim, let alone being skilled as Phelps is.

The Argument from Equal Knowledge assumes something that should be granted—i.e., that the same answers are accessible by individuals independently of their abilities. We have seen that for one to have the skill to ϕ , one has to know an answer to the question *How to ϕ* . Not every answer will do, however. The answer has to be a *practical* answer. For an answer to the question *How to ϕ* to be a practical answer that an agent can know, that answer must specify a procedure for that subject to ϕ . Procedures for an agent are thought of by analogy with computer programs. By analogy with programs, we have thought of a procedure as a way of representing an algorithm that only subjects with certain abilities can grasp. It follows from this definition of practical answers that not any answer to the question *How to ϕ* is a practical answer that an agent can know. For a procedure that an subject can grasp may not be a procedure that another subject can grasp, if the subjects differ in their set of prior abilities. For this reason, I may learn an answer to the question *How to ϕ* , without thereby acquiring the skill to swim. That is because the answer was not

packaged in the right way for me.

A final worry against a view of skills as knowledge is *The Argument from Gradability*. According to it, skills cannot consist in knowledge of truths because skills come in degree whereas knowledge of propositions does not (Ryle(151), p. 59).

As argued in Chapter I, the gradability of ascriptions of the form “*S* knows how to ϕ ” can be analyzed as the gradability of the quality and completeness of the answers to the question *How to ϕ* that are known by *S*. On the present view, skills are knowledge of practical answers to the question *How to ϕ* . The gradability of skills can be accounted for on the same model, in terms of the gradability of the quality and completeness of the procedural answers that are known by the skillful subject in question.

Could not one’s performance improve without *any* change in the content of one’s knowledge? Recent experimental results show that certain kinds of complex motor skills, such as repeatedly moving an object along a certain imposed trajectory, can be tuned and perfected to a very high degree through mere reiteration of the exercise (Krakauer and others(137), (102), (104), (103), (59), (60), and (132)). The authors of these experiments suggested to me that those changes *may* not require a change in the subject’s knowledge. Let us grant them that this is in fact the case—i.e., that those changes in the subject’s performance do not require for their explanation any change in the subject’s knowledge.

PROCEDURALISM does not need to deny this phenomenon. If skills are knowledge of practical answers, many instances of improvement in one’s skill can be plausibly blamed on one’s accessing a better and more efficient procedure for performing the task. However, PROCEDURALISM is not committed to the claim that *every* change in one’s performance must correspond to a change in one’s knowledge. That is so because not every change in one’s performance must correspond to a change in one’s skill: One may be as skillful at ϕ -ing as ever, and yet being *rusty* at it. One may be

as skillful at ϕ -ing as ever, without being still or yet at *one's best*. Even in ordinary parlance, we acknowledge a possible gap between skills and quality of performances. Every theory of skills—and in particular, every theory of skills that takes skills to be purely mental—will have to acknowledge some competence/performance distinction. Under this respect, PROCEDURALISM fares just as well as any other.

3.9 Conclusions

The *Knowledge Hypothesis* is the view that differences in skills are differences in what subjects know. Much resistance to the *Knowledge Hypothesis* derives from the platitude that skill possession requires the possession of certain *abilities*—i.e., the abilities to perform certain kinds of operations. In this chapter, drawing from Chapter II, I have developed a view on which skills are knowledge of a propositional content. On this view, PROCEDURALISM, skills are knowledge of practical answers. A practical answer has as its component a *practical sense*. And a practical sense is a *procedure*—i.e., a program-like object—practically representing a way to ϕ for subjects that have a certain set of abilities.

On some assumptions about know how, PROCEDURALISM reduces skills to know how. In Chapter II, I argued that know how is a matter of knowing a practical answer to the question *How to ϕ* . If that view of know how is correct, then PROCEDURALISM holds that one has the skill to ϕ just in case one knows how to ϕ —i.e., one knows a practical answer to the question *How to ϕ* . Like Ryle himself thought, according to PROCEDURALISM, having a skill is a matter of having some know how. Like *Intellectualism about know how*, PROCEDURALISM maintains that know how is a matter of knowing a propositional content. PROCEDURALISM goes beyond *Intellectualism about know how* in that it takes know how to be sufficient for skill possession. Because one cannot know a practical answer without possessing a certain set of abilities,

PROCEDURALISM reconciles the *Knowledge Hypothesis* with the hypothesis that skills require possession of abilities.

In the second part of the chapter, I developed an argument for thinking that skills must be that kind of knowledge. My argument took the form of an *argument to the best explanation*. According to my argument, the hypothesis that skills are standing belief states best explains their learnability. The hypothesis that skills are knowledge, and not just true beliefs, best explains the intentionality of their manifestations. Finally, Gettier cases and considerations from safety of skillful actions all corroborate the view that only knowledge can explain a skill's manifestations.

The final section puts forward a more direct argument for identifying skills and knowledge of practical answers. Skills are mental abilities to intentionally perform an action. If one knows a practical answer, then one has the mental ability to intentionally perform the relevant action. Moreover, I argued that knowledge of a practical answer is required for possessing that ability. If one can possess a mental ability to intentionally perform an action just in case one knows a practical answer, then we are on good grounds in identifying skills and knowledge. The chapter ends with a defense of my view against the usual challenges to the *Knowledge Hypothesis: The Argument from Practice, The Argument from Equal Knowledge, and The Argument from Gradability*.

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