Spatial Vector Autoregressions

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Spatial Vector Autoregressions

Michael Beenstock and Daniel Felsenstein (2007)

Spatial Economic Analysis 2 (2): 167-196

Outline

- Classical (aspatial) vector autoregression (VAR)
- Structural analysis
- Spatial vector autoregression (SpVAR)
- Future research
- References

Classical Vector Autoregression

Vector Autoregression

- VAR credited to Sims (1980)
- The technique is considered one of the most flexible and easy to use models for the analysis of multivariate time series
- Determines how each endogenous variable responds over time to a shock in its own value and in every other variable
- Allows the data to guide the researcher

Vector Autoregression

- VAR(p) Equation

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + ... + \Pi_p Y_{t-p} + \epsilon_t, \quad t = 1, 2, ..., T$$

- Assumes all current innovations are unanticipated but become part of the information set in the next period
- VAR(p) is SUR with lagged and deterministic variables as common regressors
- Since these variables are the same across all equations, the system can be estimated with OLS without loss of efficiency
- Requirements: stationary series (in levels or differences) and free of cointegration

Selecting the lag length

- The optimal lag length (p) is usually determined using one of the following popular

$$\text{Akaike IC}(p) = \ln|\tilde{\Sigma}(p)| + \frac{2}{T}pn^2$$

$$\text{Shwartz-Bayesian IC}(p) = \ln|\tilde{\Sigma}(p)| + \frac{\ln T}{T}pn^2$$

Bayesian VAR

- A common critique of VAR is that parameter estimates are not very precise so forecasts appear more precise than they really are
- Incorporates a *prior* belief of the parameter distribution – coefficients on long lags are more likely to be close to zero than coefficients on shorter lags
- The Litterman Prior (1986) suggest the prior mean of the first lagged value of the dependent variable should be set to one while all other coefficients should be set close to zero with varying degrees of uncertainty

Critiques

- There is little structural interpretation of parameter estimates – except for Structural VARs
- The estimates are of little value for hypothesis testing
- "It's just statistics"
- Many economists believe we can not acquire enough data to properly describe the behavior of a lot of economic data

Critiques

This is not a body of literature that I have much respect for, because the models are usually inadequate. The restrictions used to reduce sampling error are generally arbitrary, and thus the forecasts can be poor.

Structural Analysis

Interpreting VAR Results

Structural Analysis

- VAR coefficients capture the anticipated impact of a variable. However, there are often a lot of coefficients to interpret
- It is more common to examine the model's residuals which represent unforeseen contemporaneous events
- Common techniques:
  - Granger Causality
  - Impulse Response Functions
  - Forecast Error Variance Decomposition

Granger causality

- If y_1 is a helpful predictor of y_2 then y_1 is said to Granger cause y_2.
- Granger causality can be tested with a Wald or F test on all of the coefficients for lagged values of y_1 in the y_2 equation

Impulse response functions

- Any VAR can be modeled as a triangular moving average process

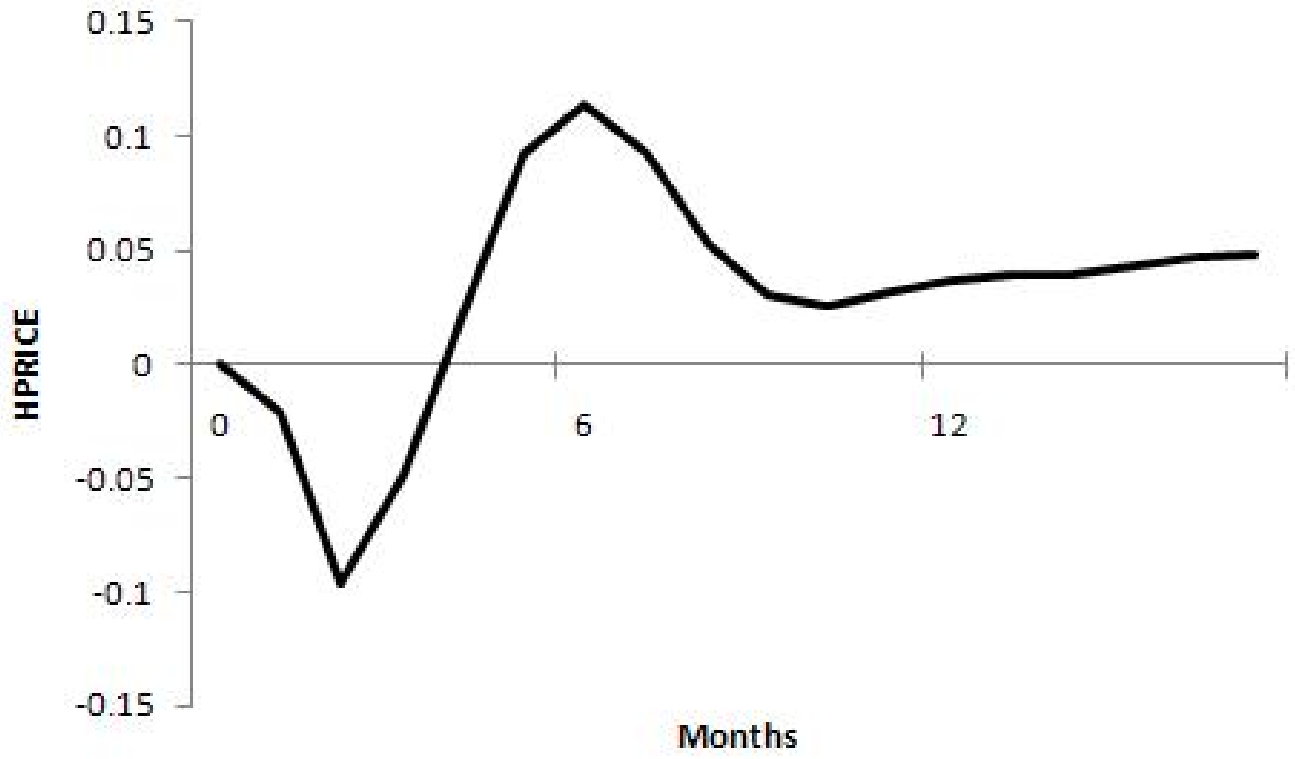
$$Y_t = \eta + \theta_0 \eta_t + \theta_1 \eta_{t-1} + \theta_2 \eta_{t-2}$$

- From this equation we can observe changes in Y_t given a change in the residual. Plotting these values maps out the "cycle" created in all variables given a "shock" in one variable

$$\frac{\partial y_{i,t+s}}{\partial \eta_{j,t}} = \frac{\partial y_{i,t}}{\partial \eta_{j,t-s}} = \theta^s_{i,j}, \quad i,j = 1,2,...,n; \quad s > 0$$

- It is common to draw bootstrapped confidence intervals around IRF

Impulse response functions



Tippecanoe County housing price cycle given a one standard deviation (positive) shock in the effective mortgage rate (Kuethe, 2007).

Variance decomposition

- The forecast error variance decomposition shows the portion of variance in the prediction for each variable in the system – its own shocks and shocks to all other variables

$$Y_{T+h} - Y_{T+h|T} = \sum_{s=0}^{h-1} \theta_s \eta_{T+h-s}$$

$$FEVD_{i,j}(h) = \frac{\sigma_{\eta_i}^2 \sum_{s=0}^{h-1} (\theta_{i,j}^s)^2}{\sigma_{\eta_i}^2 \sum_{s=0}^{h-1} (\theta_{i,j}^s)^2 + \ldots + \sigma_{\eta_n}^2 \sum_{s=0}^{h-1} (\theta_{i,n}^s)^2}, \qquad i,j = 1,\ldots,n$$

Variance decomposition

Months	Price	Mortgage Rate	Inflation	Employment	Money Supply
1	71.753	0.000	0.774	27.473	0.000
6	55.897	2.358	0.812	37.734	3.199
12	49.286	4.619	1.568	41.421	3.107
18	43.991	5.004	1.965	45.718	3.323

Housing price forecast error variance decomposition – Tippecanoe County, IN (Kuethe, 2007).

Spatial Vector Autoregression

Adding Regional Spillovers

Spatial vector autoregression (SpVAR)

- The basic model without contemporaneous spatial lags

$$
\begin{aligned}
Y_{A,t} &= \alpha_A + \beta_A Y_{A,t-1} + \lambda_A W Y_{B,t-1} + \epsilon_{A,t} \\
Y_{B,t} &= \alpha_B + \beta_B Y_{B,t-1} + \lambda_B W Y_{A,t-1} + \epsilon_{B,t}
\end{aligned}
$$

- The model introduces a *spatially lagged lag* regressors
- If $\lambda=0$, the model reduces to the classical form
- If $\lambda=1$, the data is *spatial* nonstationary
- If the spatial lags are contemporaneous Y_{At} and Y_{Bt} are no longer determined sequentially [$Cov(Y_{Bt}, \varepsilon_{At})\neq 0$] and [$Cov(Y_{Bt}, \varepsilon_{At})\neq 0$], so λ_A and λ_B not identified

Spatial vector autoregression

- Beenstock and Felsenstein also present a model that incorporates spatial autocorrelation in the error term
- This specification impacts structural analysis with IRFs

$$\epsilon_{A,t} = \frac{\rho_A e_{B,t} + e_{A,t}}{1 - \rho_A \rho_B}$$

$$\epsilon_{B,t} = \frac{\rho_B e_{A,t} + e_{B,t}}{1 - \rho_A \rho_B}$$

Estimation

- Assume shocks are correlated across regions by variable, but shocks between variables are uncorrelated

$$y_{j,k,t} = \alpha_{j,k} + \sum_{i=0}^{q}\sum_{j=1}^{n} \beta_{i,j} y_{j,k,t-1-i} + \sum_{i=0}^{q}\sum_{j=1}^{n} \beta_{i,j} \lambda y^{*}_{j,k,t-1-i} + \epsilon_{j,k,t}$$

Bias correction

- There is evidence that the estimated coefficients will be biased downwards when *T* is finite
- The bias can be overcome by GMM estimation or corrected for using the following adjustment (which is what Beenstock and Felsenstein do)

$$b = \frac{\frac{1+\beta}{T-1}\left(1 - \frac{1-\beta T}{T(1-\beta)}\right)}{\left(1 - \frac{2\beta}{(1-\beta)(T-1)}\left(1 - \frac{1-\beta T}{T(1-\beta)}\right)\right)}$$

Future Research

Where YOU can play a part

Spatial vector error correction (SVECM)

- Beenstock and Felsenstein's working paper
- "Since economic panel data are typically nonstationary either because their means and/or their variances vary over time, the need to develop spatial panel cointegration methods requires no justification."

$$\Delta y_t = a + \sum_{j=1}^{p} B_j \Delta y_{t-j} + \sum_{j=1}^{p^*} C_j \Delta y^*_{t-j} + b u_{t-1} + b^* u^*_{t-1} + d \Delta Z_t + e_t$$

Spatial vector error correction

- The spatially lagged lag variables create additional opportunities for cointegration
- *Local cointegration* – cointegration within spatial units but not between them
- *Spatial cointegration* – cointegration between spatial units but not within them
- *Global cointegration* – cointegration both locally and spatially

Other areas for future research

- These tools are ripe for application to a number of economic questions concerning forecasts of variables that may be effected by locational spillovers
  - Housing/land values
  - Macroeconomic measures
  - Employment
  - Commodity prices
- There may be a need for a more appropriate model using the Bayesian approach – setting prior beliefs on the spatial interaction parameters

Thank You

Questions, comments, concerns?

References

- Beenstock, M. and D. Felsenstein. 2007. Spatial Vector Autoregressions. *Spatial Economic Analysis* 2 (2): 167-196.
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